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DESCHANEL'S
NATURAL PHILOSOPHY.

BY PROFESSOR EVERETT.

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ELEMENTARY TREATISE

ON

NATURAL PHILOSOPHY.

DESCHANEL.

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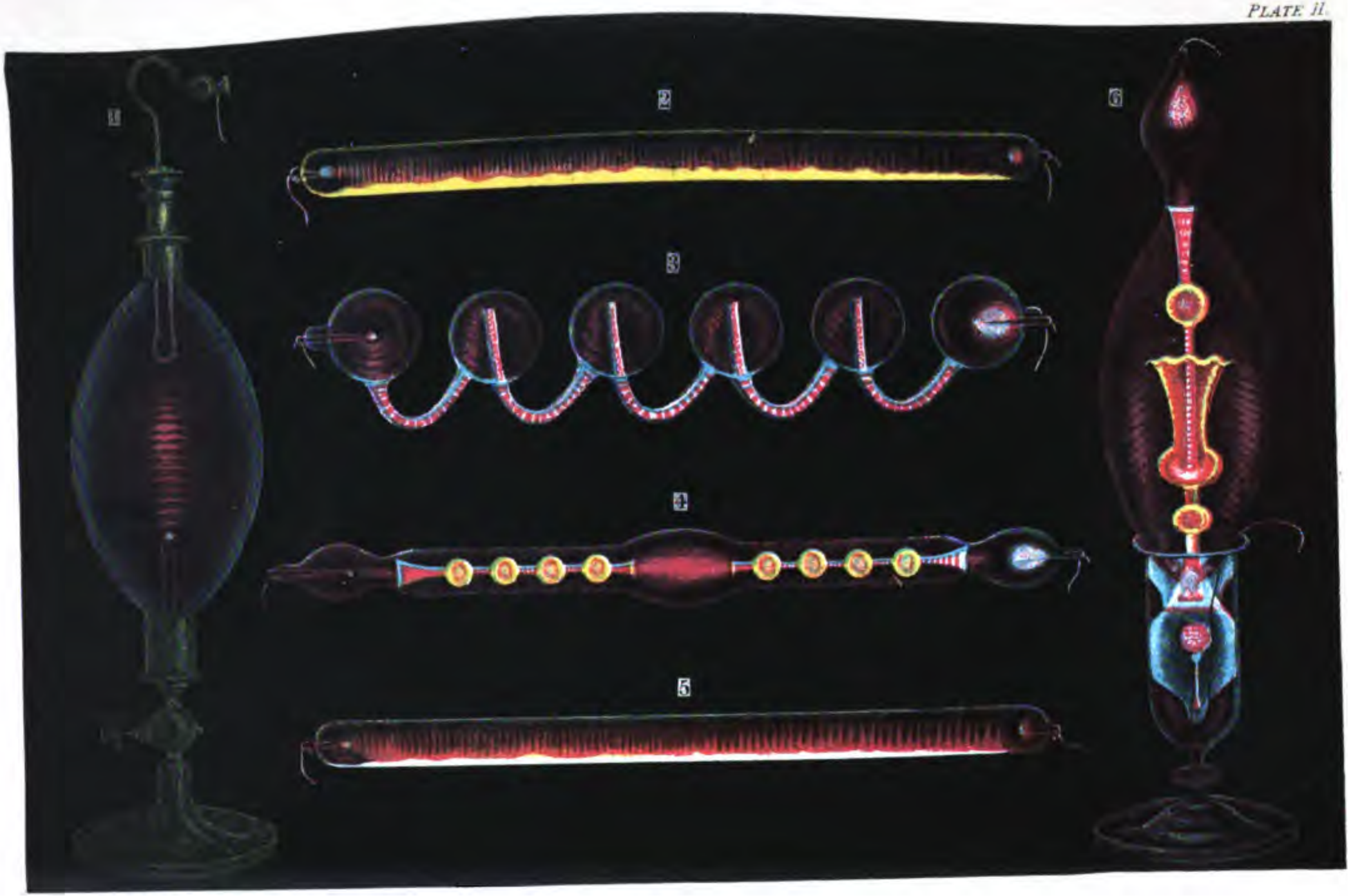

ELECTRIC DISCHARGE IN RAREFIED GASES.

1. Discharge in Vapour of Alcohol. (2. 3. 4. 5. Geissler's Tubes enclosing rarefied Gases.) 2. Shows the Fluorescence of Sulphuret of Calcium. 4. Fluorescence of Uranium glass. 5. Fluorescence of Sulphuret of Strontium. 6. Fluorescence of Uranium glass, and of Sulphate of Quinine.

ELEMENTARY TREATISE
ON
NATURAL PHILOSOPHY.

BY
A. PRIVAT DESCHANEL,
FORMERLY PROFESSOR OF PHYSICS IN THE LYCÉE LOUIS-LE-GRAND,
INSPECTOR OF THE ACADEMY OF PARIS.

TRANSLATED AND EDITED, WITH EXTENSIVE ADDITIONS,
BY J. D. EVERETT, M. A., D. C. L., F. R. S. E.,
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AUTHOR'S PREFACE.

THE importance of the study of Physics is now generally acknowledged. Besides the interest of curiosity which attaches to the observation of nature, the experimental method furnishes one of the most salutary exercises for the mind—constituting in this respect a fitting supplement to the study of the mathematical sciences. The method of deduction employed in these latter, while eminently adapted to form the habit of strict reasoning, scarcely affords any exercise for the critical faculty which plays so important a part in the physical sciences. In Physics we are called upon, not to deduce rigorous consequences from an absolute principle, but to ascend from the particular consequences which alone are known to the general principle from which they flow. In this operation there is no absolutely certain method of procedure, and even relative certainty can only be attained by a discussion which calls into profitable exercise all the faculties of the mind.

Be this as it may, physical science has now taken an important place in education, and plays a prominent part in the examinations for the different university degrees. The present treatise is intended for the assistance of young men preparing for these degrees; but I trust that it may also be read with profit by those persons who, merely for purposes of self-instruction, wish to acquire accurate knowledge of natural phenomena. Having for nearly twenty years been charged with the duty of teaching from the chair of Physics in one of the lyceums of Paris, I have been under the necessity of making continual efforts to overcome the inherent difficulties of this branch of study. I have endeavoured to turn to account the experience thus acquired in the preparation of this volume, and I shall be happy if I can thus contribute to advance the taste for a science which is at once useful and interesting.

For the convenience of candidates for the Bachelor's degree, I have appended to this treatise a number of problems, most of which have been taken from the examinations of the Faculty of Sciences of Paris or of the departments. With the same view I have made it my object to omit from the work none of the formulæ which are usually required for the solution of such questions. Beyond this point I have made very limited use of algebra. Though calculation is a precious and often indispensable auxiliary of physical science, the extent to which it can be advantageously employed varies greatly according to circumstances. There are in fact some phenomena which cannot be really understood without having recourse to measurement; but in a multitude of cases the explanation of phenomena can be rendered evident without resorting to numerical expression. In such cases calculation is of secondary importance, and may be said to be merely practical.

The physical sciences have of late years received very extensive developments. Facts have been multiplied indefinitely, and even theories have undergone great modifications. Hence arises considerable difficulty in selecting the most essential points and those which best represent the present state of science. I have done my best to cope with this difficulty, and I trust that the reader who attentively peruses my work, will be able to form a pretty accurate idea of the present position of physical science. I shall be happy in a second edition to avail myself of any observations which may be communicated to me on this or any other point.

TRANSLATOR'S PREFACE:

ORIGINALLY PREFIXED TO PART I. (CHAPTERS I. TO XVIII.)

THE "TRAITÉ ÉLÉMENTAIRE DE PHYSIQUE" of Professor Deschanel, though only published in 1868, has already obtained a high reputation in France, and has been adopted by the Minister of Instruction as the text-book for Government Schools.

I did not consent to undertake the labour of translating and editing it till a careful examination had convinced me that it was better adapted to the requirements of my own class of Experimental Physics than any other work with which I was acquainted; and in executing the translation I have steadily kept this use in view, believing that I was thus adopting the surest means of meeting the wants of teachers generally.

The treatise of Professor Deschanel is remarkable for the vigour of its style, which specially commends it as a book for private reading. But its leading excellence, as compared with the best works at present in use, is the thoroughly rational character of the information which it presents. There is great danger in the present day lest science-teaching should degenerate into the accumulation of disconnected facts and unexplained formulæ, which burden the memory without cultivating the understanding. Professor Deschanel has been eminently successful in exhibiting facts in their mutual connection; and his applications of algebra are always judicious.

The peculiarly vigorous and idiomatic style of the original would be altogether unpresentable in English; and I have not hesitated in numerous instances to sacrifice exactness of translation to effective rendering, my object being to make the book as useful as possible to English readers. For the same reason I have not scrupled to suppress or modify any statement, whether historical or philosophical, which I deemed erroneous or defective. In some instances I have endeavoured to simplify the reasonings by which propositions are established or formulæ deduced.

As regards weights and measures, rough statements of quantity have generally been expressed in British units; but in many cases the numerical values given in the original, and belonging to the metrical system, have been retained, with or without their English equivalents; as it is desirable that all students of science should familiarize themselves with a system of weights and measures which affords peculiar facilities for scientific calculation, and is extensively employed by scientific men of all countries. For convenience of reference, a complete table of metrical and British equivalents has been annexed.

The additions, which have been very extensive, relate either to subjects generally considered essential in this country to a treatise on Natural Philosophy, or to topics which have in recent years occupied an important place in physical discussions, though as yet but little known to the general public.

The sections distinguished by a letter appended to a number are all new; as also are all foot-notes, except those which are signed with the Author's initial "D."

In many instances the new matter is so interwoven with the old that it could not conveniently be indicated; and I have aimed at giving unity to the book rather than at preserving careful distinctions of authorship.

Comparison with the original will however be easy, as the numbering of the original sections has been almost invariably followed.

The chief additions in Part I. (Chap. i.–xviii.) have been under the heads of Dynamics, Capillarity, and the Barometer. The chapter on Hydrometers has also been recast.

PREFATORY NOTE TO PART II. (Chap. XIX. to XXXIV.)

In the present Part, the chapter on Thermo-dynamics is almost entirely the work of the Editor. Large portions of the chapters on Conduction and on Terrestrial Temperatures have also been re-written; and considerable additions have been made in connection with Hygrometry, the Theory of Exchanges, the Specific Heats of Gases, and the Motion of Glaciers. Minor additions and modifications have been numerous, and will easily be detected by comparison with the similarly numbered sections in the original.

The nomenclature of units of heat which has been adopted, is borrowed from Prof. G. C. Foster's article "Heat" in Watts' *Dictionary of Chemistry.*

PREFATORY NOTE TO PART III. (Chap. XXXV. to LII.)

The accurate method of treating electrical subjects which has been established in this country by Sir Wm. Thomson and his coadjutors, has not yet been adopted in France; and some of Faraday's electro-magnetic work appears to be still very imperfectly appreciated by French writers. The Editor has accordingly found it necessary to recast a considerable portion of the present volume, besides introducing two new chapters (XXXIX^A. and XLI^A.) and an Appendix. Potential and lines of force are not so much as mentioned in the original.

The elements of the theory of magnetism have been based on Sir Wm. Thomson's papers in the *Philosophical Transactions;* and the description of the apparatus used in magnetic observatories has been drawn from the recently published work of the Astronomer Royal. The account of electrical units given in the Appendix is mainly founded on the Report of the Electrical Committee of the British Association for the year 1863.

M. Deschanel's descriptions of apparatus, of which some very elaborate examples occur in the present volume, left little to be desired in point of clearness. In no instance has it been found necessary to resort to the mere verbal rendering of unintelligible details.

PREFATORY NOTE TO PART IV. (CHAP. LIII. TO LXV.)

In the present Part, the chapters relating to Consonance and Dissonance, Colour, the Undulatory Theory and Polarization, are the work of the Editor; besides numerous changes and additions in other places.

The numbering of the original sections has been preserved only to the end of Chapter LX.; the two last chapters of the original having been transposed for greater convenience of treatment. With this exception, the announcements made in the "Translator's Preface," at the beginning of Part I., are applicable to the entire work.

J. D. EVERETT.

BELFAST, *September*, 1872.

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In Fig. 356 the paper armatures are wrongly placed. Their broad parts should be exactly opposite the combs PP′, and their points *ff′*, which project through the windows, should be turned the opposite way to that represented in the figure, so that the revolving plate may pass them before it passes the combs.

TABLE OF CONSTANTS.

The pressure of one atmosphere, or 760 millimetres (29·922 inches) of mercury, is 1·033 kilogramme per square centimetre, or 14·73 lbs. per sq. inch.

The weight of a litre of dry air, at this pressure (at Paris) and 0° C., is 1·293 gramme.

Dry air at constant pressure expands by ·003665 (or $\frac{1}{273}$) of its volume at 0° C., for each degree Cent.

The specific heat of dry air at constant pressure is ·2375.

The specific heat of dry air at constant volume is ·168.

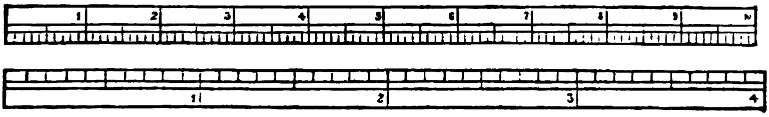
The ratio of these numbers is 1·41, and their difference ·0695.

The latent heat of fusion of ice is 79°·25 C.

The latent heat of steam at one atmosphere is 537° C.

FRENCH AND ENGLISH MEASURES.

A DECIMETRE DIVIDED INTO CENTIMETRES AND MILLIMETRES.



INCHES AND TENTHS.

TABLE FOR THE CONVERSION OF FRENCH INTO ENGLISH MEASURES.

Measures of Length.

1 Millimetre = ·03937079 inch, or about $\frac{1}{25}$ inch.

1 Centimetre = ·3937079 inch.

1 Decimetre = 3·937079 inches.

1 Metre = 39·37079 inches, or 3·2809 feet nearly.

1 Kilometre = 39370·79 inches, or 1093·6 yards nearly.

Measures of Area.

1 sq. millimetre = ·00155006 sq. inch.

1 sq. centimetre = ·155006 sq. inch.

1 sq. decimetre = 15·5006 sq. inches.

1 sq. metre = 1550·06 sq. inches, or 10·7643 sq. feet.

Measures of Volume.

1 cubic centimetre = ·0610271 cubic inch.
1 cubic decimetre = 61·0271 cubic inches.
1 cubic metre = 61027·1 cubic inches, or 35·3166 cubic feet.

The Litre (used for liquids) is the same as the cubic decimetre, and is equal to 1·76172 imperial pint, or ·220215 gallon.

Measures of Weight (or Mass).

1 milligramme = ·015432349 grain.
1 centigramme = ·15432349 grain.
1 decigramme = 1·5432349 grain.
1 gramme = 15·432349 grains.
1 kilogramme = 15432·349 grains, or 2·20462125 lbs. avoir.

Measures involving reference to two units.

1 gramme per sq. centimetre = 2·048098 lbs. per sq. foot.
1 kilogramme per sq. metre = ·2048098 " "
1 kilogramme per sq. millimetre = 204809·8 " "
1 kilogrammetre = 7·23314 foot-pounds.

1 force de cheval = 75 kilogrammetres per second, or 542½ foot-pounds per second nearly, whereas 1 horse-power (English) = 550 foot-pounds per second.

TABLE FOR THE CONVERSION OF ENGLISH INTO FRENCH MEASURES.

Measures of Length.

1 inch = 25·39954 millimetres.
1 foot = ·30479449 metre.
1 yard = ·91438347 metre.
1 mile = 1·60932 kilometre.

Measures of Area.

1 sq. inch = 645·137 sq. millimetres.
1 sq. foot = ·0928997 sq. metre.
1 sq. yard = ·8360973 sq. metre.
1 sq. mile = 2·589895 sq. kilometres.

Solid Measures.

1 cubic inch = 16386·6 cubic millimetres.
1 cubic foot = ·0283153 cubic metre.
1 cubic yard = ·7645131 cubic metre.

Measures of Capacity.

1 pint = ·5676275 litre
1 gallon = 4·54102 litres.
1 bushel = 36·32816 litres.

Measures of Weight.

1 grain = ·064799 gramme.
1 oz. avoir. = 28·3496 grammes.
1 lb. avoir. = ·453593 kilogramme.
1 ton = 1·01605 tonne = 1016·05 kilos.

Measures involving reference to two units.

1 lb. per sq. foot = 4·88261 kilos. per sq. metre.
1 lb. per sq. inch = ·0703095 kilos. per sq. centimetre.
1 foot-pound = ·138253 kilogrammetre.

ELEMENTARY TREATISE

ON

NATURAL PHILOSOPHY.

CHAPTER I.

PRELIMINARY NOTIONS.

1. Origin of Natural Philosophy.—The object of Natural Philosophy or Physics (φύσις, nature) is the study of the material world, including the phenomena which it presents to us, the laws which govern them, and the applications which can be made of them to our various wants.

In its widest sense, the study of physics must be traced back to the origin of the human race; for ever since man came into being, he must necessarily have been struck by the spectacle of the heavens and the continually changing aspect of terrestrial phenomena. But isolated and vague observations, and the barren admiration of phenomena which provoke attention or excite curiosity, do not constitute science; this can only exist where there is a mass of accurate knowledge in which the facts are related to each other and studied in connection with the causes which produce them. This process of coordination is only possible after a considerable collection of facts has been accumulated; but it then becomes inevitable, from the very constitution of the human mind. Thus, in examining the history of the nations among whom we place the cradle of our civilization, we find constant efforts of philosophers to explain the mechanism of the external world,—to bring all the facts which nature presents to us under one theory—one system. The Greek philosophers, especially, who appear to have borrowed the greater part of their physical knowledge from the Egyptian priests, have left us different systems, by the aid of which they profess to explain all natural phenomena. Thus Thales, the most celebrated of the seven wise men of Greece

(640 B.C.), made water a universal principle, which nourishes at once the sun, the earth, and the planets. Plato (398 B.C.) assumed two distinct principles, matter and form, which by their combination give birth to five elements—earth, water, fire, air, and ether. According to Anaximander, there is but one principle, the infinite, which gives birth to all bodies. According to Anaxagoras, air is the sovereign of nature. We need not stay to examine the exact meaning of these propositions, which, taken in their literal sense, appear at the present day sufficiently unintelligible. While acknowledging that these illustrious philosophers knew and taught some important facts of general physics, we are bound to remark that in the elaboration of their systems experiment played no part; that observation itself only held a secondary place; and that their theories were veritable *à priori* conceptions, to which facts had to be accommodated. Hence there is nothing in their works approaching the experimental method which serves as the foundation of modern physics. Some faint foreshadowings of this method may be traced in the works of Aristotle (383 B.C.), who was a disciple of Plato, but far superior to his master in scientific genius, besides being an eminent naturalist, and author of a history of animals, which alone would constitute an imperishable monument to his memory. Thus, to investigate the weight of air, he had recourse to a direct experiment, which consisted in weighing a skin empty and inflated. Finding no difference in the weights, he concluded wrongly that air is destitute of weight, and was thus led in his attempts at the explanation of certain phenomena to the famous principle that nature abhors a vacuum, which was universally admitted down to the time of Galileo.

It was especially in the hands of Archimedes (287 B.C.), and the philosophers of the school of Alexandria, who may be regarded as his successors, that the method of scientific observation took a distinct form, and led to important results. Every one has heard of the admirable discoveries of Archimedes respecting the theory of the lever, the determination of centres of gravity, and the measurement of specific gravities by means of the principle which bears his name; discoveries founded upon experiments which were doubtless not very accurate, but were regarded by him as necessary in order to furnish a solid basis for his investigations. After him Hipparchus (140 B.C.), by means of persevering observations, methodically directed, changed the face of astronomy, and arrived at brilliant discoveries, among which the most notable was that of the precession of the equinoxes. At a later period, Ctesibius, Hero, Posidonius, &c., following in the traces of their illustrious predecessors, advanced the boundaries of the exact knowledge already acquired, and originated several inventions displaying more or less ingenuity. To the first of these philosophers the invention of pumps appears to be due, and the fountain of Hero still finds a place in all collections of physical apparatus.

Among philosophers belonging more or less directly to the school of Alexandria, who have enriched science by important discoveries, we will only mention Plutarch, who is said to have discovered the refraction of light in its passage from air into water; and Ptolemy, the author of various works on optics and celestial physics, which constitute a better title to glory than the astronomical system which bears his name, a system which only served to retard the progress of science.

We will carry this historical review no further, but will content ourselves with remarking that, starting from the seventh century, the period of the conquest of Alexandria by the Arabs, and the burning of its celebrated library, until the time of Galileo (1564), science may be said to have been stationary. Still, some discoveries of importance belong to this period; for example, that of the mariner's compass, which was known from the thirteenth century. Shortly before Galileo, the thermometer, the microscope, and telescope were invented; but it is unquestionably to this distinguished philosopher that we owe the true scientific method—the method of experiment. His treatises upon falling bodies, the pendulum, &c., furnish admirable examples of the manner in which the physical investigator should interrogate Nature by the aid of experiment. It was by the introduction of this method that physical science became finally disentangled from the prejudices and *à priori* assumptions which had hitherto impeded its progress.

At the present day, after numberless discoveries which have introduced most material changes in our social condition, physical science has attained a very high degree of perfection. It is to the experimental method that we owe this result, and it is by remaining true to this method that we must hope to achieve fresh progress.

2. The Experimental Method.—The experimental method can easily be described in general terms: it consists in observing facts instead of trying to divine them; in carefully examining what really happens, and not in reasoning as to what ought to happen.

It is therefore entirely independent of metaphysics, which has always proved a false ally; in fact, as long as the dominion of metaphysics lasted, science continued to run in the old ruts, which it did not leave till, thanks to the teaching of Bacon and Galileo, the conviction became established that there is no way of arriving at physical truths but by the help of observation and experiment.

The experimental method is usually called by logicians the method of *observation* and *induction*. From the observation of particular facts it ascends to the general law which embraces them; being very different in this respect from the method of *deduction* employed in mathematics, in which we always descend from a certain and absolute principle to the different consequences which flow from it. Let us enter into some details upon this point.

3. Phenomena—Physical Law.—A phenomenon is any change that takes place in the condition of a body; the fall of a stone, the flowing of water, the melting of lead, the combustion of wood, for example, are phenomena. When we study the characteristics which belong to phenomena of the same class, we soon perceive that the various circumstances of their production have a mutual dependence, so that if one of them varies, the others undergo a corresponding variation. The expression of this connection constitutes a physical law.

Sometimes the law appears of itself and without difficulty, by means of observation alone. Such, for example, is the following: *All bodies left to themselves fall to the surface of the earth.* But more frequently the law is disguised by disturbing causes, whose influence should, as far as possible, be eliminated. This elimination is the object of experiment. Experiment differs from observation in this respect,—that the phenomenon is produced under conditions previously determined and regulated by the experimenter. If we wish to know, for example, what are the velocities which gravity produces in different bodies falling freely, we must not let them fall in air, because this fluid retards their movement, and that in unequal degrees for different bodies; we must operate *in vacuo,* and thus we arrive at the law, which observation alone could never have discovered, that *gravity produces the same velocity in all bodies.* It will be readily understood then that the art of experimenting, that is, of regulating the special conditions under which phenomena shall take place, and of measuring their constituent elements, is absolutely necessary to the physical investigator; and that a genius for physical science mainly consists in the possession of this aptitude in a more or less eminent degree.

We may remark that when the general law of a class of phenomena is known, the expression of this law is often called the physical cause of the particular phenomena which it includes. A phenomenon is said to be explained or accounted for, or traced to its cause, when we show that it is contained in the enunciation of a known law.

Thus, when we have once laid down the principle that the volumes of gases under different pressures vary inversely as these pressures, we are in a position to explain a crowd of facts depending on the action of a gas whose volume and pressure vary simultaneously.

When the law of observed phenomena admits of numerical statement, calculation becomes a valuable instrument for making known all its consequences, and the experimental verification of these consequences constitutes a confirmation of the physical law itself. In this way mathematical methods become powerful auxiliaries to physical science.

4. Physical Theory.—The enunciation of any one physical law, and the rational development of its consequences, constitute a partial physical theory. The assemblage of all the laws which belong to one class of phenomena, forms a more general physical theory; but it will be readily understood that these different laws may be merely corollaries of a single law.

The discovery of this single law, when it exists, marks a decided step in the progress of physical science. Thus Newton traced to the single law of gravitation all the movements of our planetary system, as well as those of bodies which fall to the surface of the earth.

In like manner the different partial theories of optics are rigorous consequences of the properties attributed to a fluid called *ether*, with which we suppose space to be filled, and whose vibrations serve for the propagation of light and heat.

This work of synthesis, however, has as yet made little progress, though these last few years have been marked by very successful efforts in this direction; but it should be considered as the true object of physical science in general, and the highest generalization will have been attained when it has been demonstrated that all the physical agents which have hitherto been regarded as distinct, are merely transformations of one and the same primordial agent.

5. Divisions of Physical Science.—Physical or natural science in general comprises the aggregate of all the phenomena of the external world; but the accumulation of discoveries in different parts of this mighty whole has necessitated the division of it into several branches, which at present constitute distinct sciences.

Natural History comprises all those facts which have reference to the different beings, organic or inorganic, which are found upon the surface of the globe; it is further subdivided into several parts. *Zoology* is occupied with the organization and habits of animals, with their regular classification, and with all the phenomena connected with their development and reproduction. *Botany* treats of the same questions with respect to vegetables. *Mineralogy* has for its object the description and methodical classification of the different inorganic bodies (minerals) which nature presents to us; the knowledge of the peculiar characteristics which serve to distinguish them from one another; and the enumeration of their principal properties as well as of the various applications that can be made of them.

Geology is the history of the earth; it recounts the different revolutions which have modified the surface of the globe and finally brought about its present configuration, the arrangement and nature of the rocks that enter into its composition, and the description of those ancient animals and vegetables whose fossil remains are still in existence, belonging in many cases to types which have since become extinct. It is the basis of the art of the mining engineer, and enables him to follow a regular method in searching for the various metals or combustible substances which are hid in the depths of the earth, and which we employ to satisfy our various requirements.

Astronomy is occupied with the laws of the movements of the heavenly bodies; thanks to the perfection to which our measuring instruments have been brought, to the progress of mathematical science, and to the discovery of the universal law of gravitation, astronomy has arrived at such a degree of perfection that it may be classed among the exact sciences.

Besides natural history and astronomy, there is room further for distinguishing physics from chemistry. This latter science, in fact, has for its object the study of phenomena in which the essential character of materials seems to be changed; phenomena in which matter seems to be destroyed, or at least metamorphosed. If we take a piece of sulphur and heat it, it will melt; if we rub it with a piece of wool, it will acquire the power of attracting light bodies, and will present the peculiar and curious properties which are characteristic of electrical excitation; but the sulphur will not have lost its proper nature, and when the different influences to which it has been submitted cease to act, it will resume all its original characteristics. The sulphur under these circumstances has displayed *physical* phenomena. If, on the other hand, we place this same body in a fire, we shall see it burn with a blue flame; at the end of some time it will have entirely disappeared, or at least will have been transformed into a gaseous substance which is dissipated with the other products of combustion. In this case the sulphur has ceased to exist as sulphur; a *chemical* phenomenon has taken place.

In a more restricted sense, then, physics or natural philosophy is understood as embracing the study of all the phenomena of the material world except those which consist in the action of vital forces or of chemical affinities. It is in this restricted sense that physics forms the subject of the present treatise. We may remark, however, that the two kinds of phenomena are often produced by the same causes, and that each is frequently the necessary consequence of the other. Thus in heating a body we render it better adapted to undergo chemical transformations; and, on the other hand, such transformations often produce a great quantity of heat. Physics and chemistry, though pursuing different ends, should yet afford each other mutual assistance. For example, our ideas of electricity would be very imperfect without a knowledge of the curious and often useful chemical phenomena which it is capable of producing.

A similar remark may also be made with regard to all the other branches of natural science. How, for example, can we separate mineralogy and chemistry, when it so often happens that the only means of recognizing a mineral is by making a chemical analysis of it? and when, on the other hand, a complete description of the substances which the chemist produces in his laboratory, must necessarily include an account of their external characteristics, such as their crystalline form, which specially belongs to the province of mineralogy?

To take another instance. Can we draw a sharp line of demarcation between zoology and botany on the one side, and physical science and chemistry on the other? Does not the tissue of organic beings undergo various chemical reactions which are a necessary accompaniment of vital phenomena? Do not physical agents in their turn produce phenomena of such a nature as completely to embarrass the physiologist and the physician, unless they are armed with a knowledge of the laws which regulate the action of these agents upon inorganic bodies?

Finally, though astronomy may seem to form a totally distinct science, consisting of the geometry and mechanics of the movements of the heavenly bodies, must it not avail itself of all the resources of physical science if it would arrive at any rational conjectures respecting their constitution? We may say, then, that all the parts of natural science are interwoven together; they form one connected whole, and the division into distinct sciences has simply arisen from the vastness of the subject, which renders it impossible for any one mind adequately to follow the development of its various branches.

CHAPTER II.

MECHANICS.

6. Principle of Inertia.—The fundamental principle of physics is the inertia of matter. Inertia does not consist in the inactivity of material particles, nor in the impossibility of changes being produced in their states of rest or motion by their mutual action; for a glance at nature is sufficient to show that repose nowhere exists, and that motion changes in an endless variety of ways. The principle of inertia is an abstract principle which must be considered as applicable to a single isolated particle. It may be enounced in the following terms:—

An isolated material point cannot change its state, whether of rest or motion. That is to say, if it be at rest it will remain at rest; if it be in motion it will continue to move in the same direction and with the same velocity.

If, then, we see a material point which was at rest begin to move, or if we observe any change in the motion of a point, we say that it has been acted on by a *force.*

Without entering upon the very obscure subject of the intimate nature of forces—without seeking to know whether they form an essential part of bodies or have a separate existence, but only regarding them in the effects which they produce, we may define them in the following manner:—

A force is any cause which tends to urge a material point in a definite direction with a definite velocity.[1]

7. Manifestations of Inertia.—The principle of inertia, as above enounced, does not admit of direct experimental verification; for we cannot observe a material point, which is a mere abstraction;

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[1] The words *with a definite velocity* only imperfectly express the idea intended to be conveyed. The correct phrase would be *with a definite acceleration.* See Chap. v.

still less an isolated material point. The principle of inertia is one of those ultimate and abstract principles which presented themselves to the minds of the founders of the science of mechanics—of Newton especially—as the key and reason of the manifold and complex characters of external phenomena. But if it is impossible to verify the principle of inertia directly, it is easy to show its influence in external phenomena, this influence reducing itself evidently to the tendency of bodies to continue in their state of rest or motion.

The tendency to continue in a state of rest is manifest to the most superficial observation. The tendency to continue in a state of uniform motion can be clearly understood from an attentive study of facts. If, for example, we make a pendulum oscillate, the amplitude of the oscillations decreases more and more; and ends, after a longer or shorter time, by becoming nothing. This is because the pendulum experiences resistance from the air, due to the successive displacement of the particles of this fluid; and because the axis of suspension rubs on its supports. These two circumstances combine to produce a diminution in the velocity of the apparatus until it is completely annihilated. If the friction at the point of suspension is diminished by suitable means, and the apparatus is made to oscillate *in vacuo*, the duration of the motion will be immensely increased.

Analogy evidently indicates that if it were possible to suppress entirely these two causes of the destruction of the pendulum's velocity, its motion would continue for an indefinite time unchanged.

This tendency to continue in motion is the cause of the effects which are produced when a carriage or railway train is suddenly stopped. The passengers are thrown in the direction of the motion, in virtue of the velocity which they possessed at the moment when the stoppage occurred. If it were possible to find a brake sufficiently powerful to stop a train suddenly at full speed, the effects of such a stoppage would be identical with those which would result from collision with another train of the same weight coming in a contrary direction with equal velocity.

Inertia is also the cause of the severe falls which are often received in alighting incautiously from a carriage in motion; all the particles of the body have, in fact, a forward motion, and the feet alone being reduced to rest, the upper portion of the body continues to move, and is thus thrown forward.

When we fix the head of a hammer on the handle by striking the end of the handle on the ground, we utilize the inertia of matter.

In fact, at the moment of the shock, and of the stoppage which results, the head continues to move, and ends after some blows by becoming firmly fixed.

8. Mechanics.—All physical phenomena fundamentally consist in motions; but these motions are in many cases too minute to admit of direct observation, and are only inferred from their effects. Thus when a solid body is heated and melted, it is certain that the liquid state results from a particular displacement of the molecules, and perhaps also from a change of their form—that is to say, from circumstances which are reducible to motions; but the liquid body thus formed has acquired peculiar properties, which form a subject of study in themselves apart from the motions to which they are due.

When motions are considered in themselves, according to their geometrical relations, and in connection with the forces which produce them, they form the subject of the science of mechanics, which must be regarded as an indispensable introduction to physics. We shall give in this chapter enunciations and illustrations of some fundamental propositions, referring the reader to special treatises on this subject for fuller information.

9. Elements required to specify a Force.—The material point submitted to the action of force is called the *point of application of the force.* It tends, in virtue of this action, to move in a certain direction, which is called the *direction of the force,* and which can be represented geometrically by a straight line drawn from the material point. It is obvious also that the force must act with some definite intensity, which is different in different cases. This intensity may manifest itself, for example, by a greater or less velocity of the point, a greater velocity corresponding to a greater force.

When two forces separately applied to the same point at rest give it the same motion, they may be called equal. The union of a number of equal forces gives a force which is a corresponding multiple of one of them, and thus the intensities of forces can be numerically compared. Forces then can be represented either by numbers or by lines; in the latter case a certain length (as an inch) being taken to represent a certain force (as the weight of a pound). It is usual to indicate the direction of a force by a line AF with which the direction of the force coincides, and to lay off on this a length AB representing (on the scale chosen) the intensity of the force.

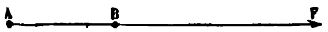

Fig. 1.

For accuracy, it is to be observed that the pound, ounce, and other units of weight are essentially units of mass, not of force. In order to render them available as accurate units of force, the *locality* must be specified, inasmuch as the force requisite to support a pound of matter is different in different localities, being for example greater at the poles of the earth than at the equator by about 1 part in 190.

10. Resultant.—When a material point or a system of points is urged by a certain number of forces, it will be readily understood that a single force of determinate magnitude, and applied at a suitable point, may be capable of producing the same effect as all the given forces acting together. This single force is called the *resultant* of the given forces, and they are called its *components.*

Thus, for example, a vessel descending a river, whether propelled by steam or wind, provided its motion be rectilinear, is really urged forward by a great number of forces applied at different points; but it is evident that a single force of proper magnitude and line of action would produce the same effect.

It is not every system of forces that has a resultant; but, in the case of those which have, it is very important to determine its magnitude and position, for the study of the body's motion will thus be evidently simplified. The following is an important case in which this determination is easily made.

11. Parallelogram of Forces.—*If a material point* A *is acted on by two forces represented in magnitude and direction by* AB *and* AC, *there is a resultant, which is exactly represented by the diagonal* AD *of the parallelogram of which* AB *and* AC *are sides.*

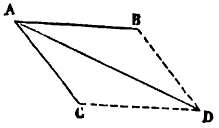

Fig. 2—Parallelogram of Forces.

This proposition can be verified experimentally by the aid of the following apparatus due to Gravesande. ABDC (Fig. 3) is a parallelogram jointed at its four corners. To the points B and C cords are fixed, which, passing over the pulleys M and N. support at their extremities weights P and P′, of 90 and 60 ounces respectively.

The lengths of the sides AB and AC are themselves proportional to the numbers 90 and 60. To the corner A is attached a weight P″ of 120 ounces. In these circumstances, the parallelogram will take a position of equilibrium, in which the cords attached to B and C will be found to form prolongations of the sides AB, AC, and the diagonal AD will be vertical. But the forces P and P′ have a resultant acting vertically at A, since their resultant must be equal and opposite to the weight P″ which balances them. The diagonal AD therefore agrees with the resultant in direction; and if this diagonal is measured, its length will be found to be 120 on the same scale on which the lengths of AB and AC are 90 and 60.

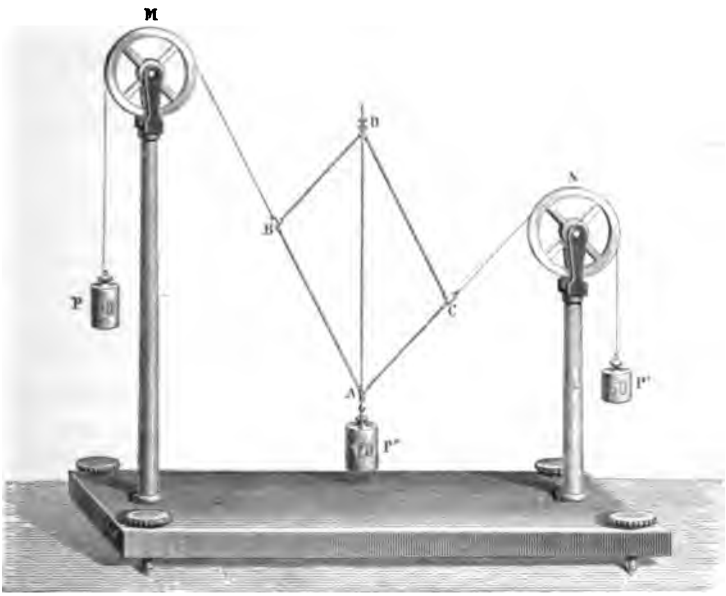

Fig. 3.—Gravesande's Apparatus.

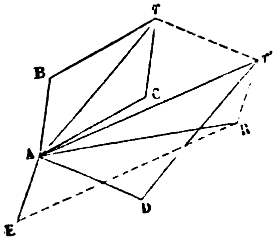

Fig. 4.—Composition of any Number of Forces.

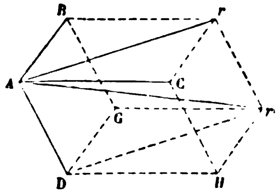

Fig. 5.—Parallelopiped of Forces.

12. Composition of Forces.—Knowing how to find the resultant of two forces, that is to say, to *compound* two forces, applied to the same point, it is easy to compound any number.

Let there be, for example (Fig. 4), four forces applied to the material point A. We may compound first the force AB with AC, which gives the resultant Ar; this, compounded with AD, gives a second partial resultant Ar', which, compounded with the fourth force, gives the complete resultant AR.

In the particular case of three forces (Fig. 5), it is easily seen that the resultant Ar' is the same thing as the diagonal of the parallelopiped constructed on the lines AB, AC, AD which represent the three forces. In the figure, the parallelopiped has been completed to render this evident, but the construction amounts, as in the preceding case, to compounding AB with AC, and their resultant Ar with AD.

13. Composition of Parallel Forces.—*When two parallel forces* F *and* F′ *are applied at the two extremities of a straight line, they have a resultant* R *equal to their sum, and acting at a point* C *which divides the straight line* AB *into parts inversely proportional to the forces.*

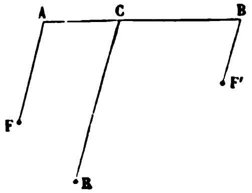

Fig. 6.—Parallel Forces.

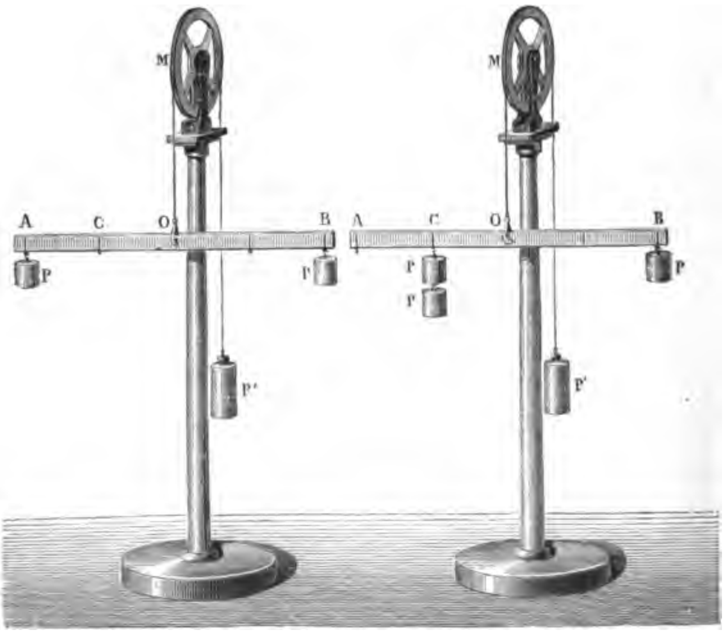

Fig. 7.—Composition of Parallel Forces.

If, for example, the two forces F and F′ are equal, the point C will be at the middle of AB; if the force F is double of F', the segment CA will be equal to half of CB.

This proposition can be verified by the aid of the following apparatus called the *arithmetical lever* (Fig. 7).

The lever AB supports two equal weights P at its extremities; it is suspended at its middle by a cord which, passing over the pulley M, sustains a weight P'. It will be found that, when the weight P' has a certain value, the lever is in equilibrium; whence it follows that the two weights P and the weights of the different parts of the lever, which we may suppose distributed two and two at equal distances from the middle point, have a resultant acting at this point, equal and opposite to the force P'. It will also be found that P' is equal to the sum of the two weights P together with the weight of the lever.

In the case of the second figure, a single weight P is placed at one of the extremities B, whilst two equal weights are suspended at the middle point C of the second half of the lever. It will be found that there is still equilibrium, provided that the weight P' is the sum of the three weights P and the weight of the lever.

To interpret this result, we may remark that the lever being balanced directly by an equal portion of P', we may neglect its weight; there remain then only two forces, of which one, that on the left, is double of the other. Now the resultant evidently passes through the point of suspension O, which is exactly twice as far from B as from C.

14. When the parallel forces F and F' act in opposite directions, there is still a resultant parallel to the components, acting in the same direction as the greater, and equal to their difference F—F'. Moreover its point of application C is so placed that the distances CA and CB are inversely proportional to the forces, a result analogous to that which holds when the forces act in the same direction.

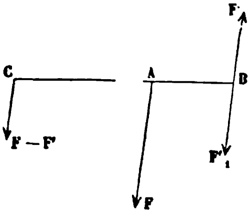

Fig. 8.—Parallel forces in Opposite Directions.

We see, as a consequence of this proposition, that if the two parallel and opposite forces differ little from one another, their resultant has a very small value, but its point of application is very remote. In the particular case in which the two forces are equal, the rule of composition is absolutely inapplicable. Such a system, consisting of two equal and parallel forces acting in opposite directions, is called a *couple.* It cannot be equilibrated or replaced by a single force, but obviously tends to produce a motion of rotation. Now in nature we frequently see bodies possessing at once a motion of translation and a motion of rotation. We may assume that the translation has been produced by a force and the rotation by a couple; this latter then presents itself as a sort of *natural element* in mechanics. The idea of couples originated with the geometrician Poinsot, and has greatly simplified many mechanical problems.

The perpendicular distance between the lines of action of the two equal forces which constitute a couple is called the *arm* of the couple. The product of one of the two equal forces by the arm is called the *moment* of the couple, and is the measure of the power of the couple to produce rotation. It is proved in treatises on mechanics that two couples acting on a body and tending to turn it in opposite directions will equilibrate each other if their moments are equal, even though they be applied at different parts of the body. Two or more couples acting on a body and tending to turn it in the same direction, may be replaced by a single couple whose moment is the sum of their moments; and any number of couples acting on a body and tending to turn it in any directions whatever, are always, except when they are in equilibrium, equivalent to a single couple.

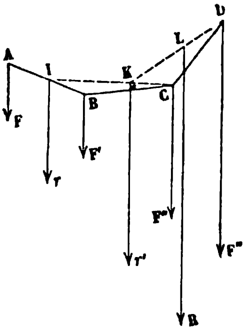

Fig. 9.—Composition of any Number of Parallel Forces.

15. Composition of any Number of Parallel Forces. To compound a given number of parallel forces, F, F', F'', F''', we may first compound the first with the second, which gives a partial resultant r; this compounded with the third force F'', gives a second resultant r', which combined with F''', gives the complete resultant R. It is clear that this procedure is applicable to any number of forces, and that the resultant is always equal to the sum of all the forces. As to its point of application, we may remark, that the point of application of the first partial resultant is obtained by dividing AB into two parts AI and BI inversely proportional to F and F'. In like manner the point of application K of r' is obtained by dividing IC into parts IK and KC, inversely proportional to r and F''; and lastly, the point of application L of the complete resultant is obtained by performing an analogous operation on the line KD. Now it will be remarked that this series of constructions is independent of the absolute direction of the forces, and only supposes that they are parallel; if then the forces were turned about their points of application in such a manner as still to remain parallel to one another, their resultant would still pass through the same point L. This point is on this account called the *centre of parallel forces.*

16. Resolution of Forces.—As any number of forces can be compounded into a single force, so a given force can be (decompounded or) resolved into two or any number of forces which would produce the same effect. Thus, for example, the force AD (Fig. 2) can be replaced by the two forces AB and AC, inasmuch as it is their resultant. It is obvious that, to resolve a force applied to a material point into two others having given directions, it is necessary to draw, through the extremity D of the line which represents the given force, lines parallel to the given directions. They determine by their intersections the points B and C, and consequently the magnitudes AB and AC of the two forces which can be substituted for the given force.

The composition of several forces into a single force constitutes an obvious simplification. As to resolution (or decomposition), its utility is not so obvious. It may appear at first sight to be a complication. Such however is not always the case, and in the course of this treatise we shall find it of continual use. It will be readily conceived, in fact, that when a force is applied to a point whose movements are constrained by guides or physical connections, the effect which the force can produce is not easily perceived. But it is sometimes possible, by resolution, to replace it by components, of which some are destroyed by the conditions of constraint, and the others can act in a manner directly appreciable.

We will confine ourselves to a single example of this, namely, the explanation of the opposite courses which can be taken by two sailing boats with the same wind.

Consider, for example, the case of the first figure. The wind, blowing in the direction Vm, tends to exert a force which can be resolved at the point m into two components, one of them tt' tangential to the sail, which has no effect, the other mn perpendicular to the sail. This latter tends to urge the boat obliquely towards the left of the figure. But as the boat can be moved much more easily in the direction of its length than in any other direction, we may resolve this latter force into two components: one perpendicular to the length, which has little effect; the other in the direction of the length, which produces the forward movement of the boat in the direction of the arrow placed below the figure.

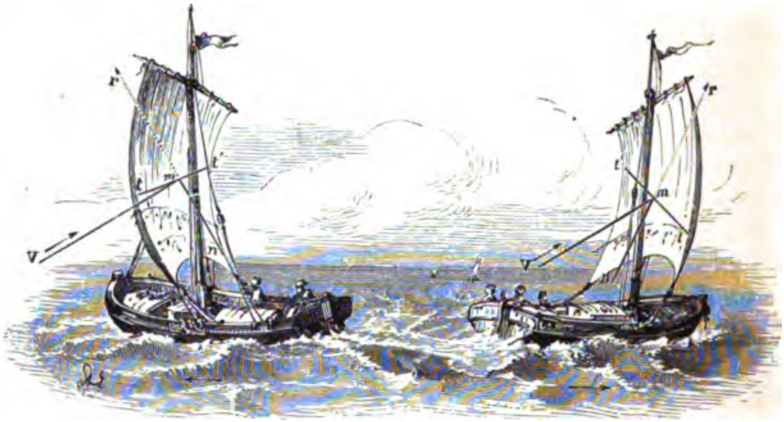

Fig. 10.—Resolution of Pressure of Wind on Sails.

In the second figure it will be seen that the same methods of resolution lead to an opposite result, and that the direction of motion is opposite, though the direction of the wind is the same. In reality, the two motions are not directly opposite, because the action of the component perpendicular to the length of the boat cannot be altogether neglected, but produces what is called leeway.

17. Work done by a Force.—In the different operations to which forces are applied, such as the raising of burdens, the compressing, piercing, and pulverizing of solid bodies, it is clear that it is always necessary to overcome a certain resistance and produce a certain displacement. Hence we are led to a special mechanical element, involving the joint consideration of force and the displacement of its point of application. This element is called *work*.

The work done by a force is the product of the force into the displacement which it produces in its point of application.

In this definition the force is supposed to be constant, and the motion of the point of application is supposed to coincide with the direction of the force.

In stating the work done by a machine it is usual to take as unit of work the foot-pound; that is to say, the work done in raising a pound through a height of a foot.

The notion of work is itself independent of time; but it is evident that in practice it is advantageous for a machine to employ little time in producing a given amount of work.

The unit employed for expressing the rate of working of a machine is the *horse-power*, and denotes 33,000 foot-pounds of work done per minute or 550 foot-pounds per second. Thus a machine which can raise 12 tons through a height of 10 feet in 2 minutes is a machine of rather more than 4 horse-power; since it does $12 \times 2240 \times 10 = 268{,}800$ foot-pounds in 2 minutes, or 134,400 in 1 minute; and 134,400 is rather more than 4 times 33,000.

17A. The above definition of *work* is only applicable to the case in which the displacement of the point of application of the force is in a direction precisely coincident with the direction of the force. It is often necessary to consider cases where (owing perhaps to circumstances of constraint, or to the action of other forces besides the one considered, or to previous motion) the point of application of a force moves in a direction oblique to that of the force. In this case the force may be resolved into two components, of which one is perpendicular and the other either coincident with or directly opposite to the direction of displacement. The former of these components is to be neglected in estimating the work done by the force; and the product of the latter component by the displacement is the work done *by* or *against* the force according as the direction of this component *coincides with* or is *opposite to* that of the displacement.[1]

The necessity of having a name to denote the idea thus defined is obvious from the following proposition, which is called the *principle of work*.

Every machine may be regarded as an instrument for transmitting work; and if we neglect friction, we may assert that *the work thus transmitted is unaltered in amount.* If, for example, the machine is driven by forces applied at points A_1, A_2, &c., and if the machine overcomes resistances at points B_1, B_2, &c., then the whole work done by the forces at A_1, A_2, &c., estimated according to the foregoing definition, will be precisely equal to the whole work done against the resistances at B_1, B_2, similarly estimated.

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[1] Or the work done by a force is the product of the force by the projection of the displacement of its point of application on the direction of the force, or is the continued product of force, displacement, and cosine of included angle. The three definitions are obviously equivalent. Work done *against* a force is to be regarded as *negative work done by* the force.

The numerous vain schemes for producing perpetual motion are founded on ignorance of this law. They are attempts to make work increase in its transmission through a machine.

Practically, work is always diminished in its transmission through a machine, owing to friction. The work thus lost leaves an equivalent in the shape of heat (see Chap. xxxii.)

CHAPTER III.

CONSTITUTION OF BODIES.

18. Different States of Matter.—As the object of physics is the study of the general properties of bodies, it is necessary for us to form some idea of the constitution of the different kinds of matter. Matter presents itself in three different states: the solid, liquid, and gaseous. Solid bodies are characterized by a kind of invariability of form; that is to say, their form cannot be changed without an effort, more or less considerable. Hence a solid body forms a firmly connected whole, so that the movement of one of its parts produces motion in the rest.

Liquids, on the contrary, appear to be formed of particles which are independent of each other and can obey individually the action of the forces which urge them, being able to slide past each other with the greatest facility. From this property the name *fluids*, by which, in common with gases, they are often designated, is derived (*fluere*, to flow). This also is the reason why a liquid moulds itself to the form of the containing vessel. Liquidity, consisting essentially in the perfect mobility of the constituent parts of a body, may evidently be met with in different degrees of perfection. Thus sulphuric ether and alcohol are more perfectly liquid than water; water itself is more liquid than oil, and so on. *Viscosity* is a name used to denote the want of independence between the particles of a liquid, which establishes a kind of intermediate state between these bodies and solids. Thus we may say that there is an insensible passage from liquids more or less perfect to viscous liquids, from these to plastic substances such as putty or moist clay, and from these last to solid bodies.

Gaseous bodies, of which the atmosphere offers us an example, are formed, like liquids, of independent particles: but these particles appear to be in a continual state of repulsion, so that a gaseous mass has a continual tendency to expand to a greater and greater volume. This property, called the expansibility of gases, is commonly illustrated by the following experiment:—

A bladder, nearly empty of air, and tied at the neck, is placed under the receiver of an air-pump. At first the air which it contains and the external air oppose each other by their mutual pressure, and are in equilibrium. But if we proceed to exhaust the receiver, and thus diminish the external pressure, the bladder gradually becomes inflated, and thus manifests the tendency of the gas which it contains to occupy a greater volume.

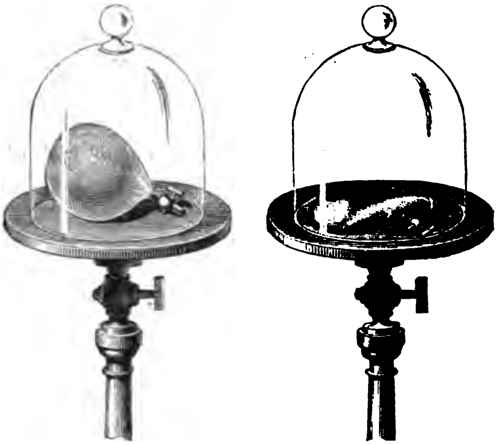

Fig. 11.—Expansibility of Gases.

It follows from this property that, however large a vessel may be, it can always be filled by *any quantity whatever* of a gas, which will always exert pressure against the sides. It is in consequence of the existence of this pressure, which is itself a result of expansibility, that the name of *elastic fluids* is often given to gases.

It is necessary to remark that the same substance may, according to its temperature, assume any one of the three states. Thus water in the cold of winter assumes the solid state and becomes ice; and, on the other hand, there is always more or less water diffused through the air in the gaseous state, called aqueous vapour. If the thermal conditions existing at the surface of the earth were to receive a notable change in either direction, some of the bodies which we habitually see in the liquid state would become either solids or vapours.

19. Molecular Constitution.—Whatever be the state under which a body presents itself, it is the general opinion of physicists that it is not composed of continuous matter, but is an aggregation of distinct parts held at a distance from each other. These constituent parts are called particles or molecules. They must be regarded as exercising two kinds of mutual actions, the one attractive, the other repulsive, which balance each other in the case of solids and liquids. In the case of gases this equilibrium does not subsist; there is a permanent repulsive force between the particles, which gives rise to expansibility or elastic force.

The molecules[1] of solids and of liquids ought not to be considered as similar. In the latter, in fact, each molecule can turn on its axis without producing any modification in the equilibrium; in other words, equilibrium depends only on the molecular distances and not at all on the form or relative disposition of the molecules. An approximate idea of this physical constitution will be obtained by assuming that the molecules of liquids are spherical, and hence that molecular equilibrium depends only on the distances between the centres of the spheres.

In solids, much depends upon the form and relative disposition of the molecules. It would seem as if these molecules (according to the ideas of some ancient philosophers) were formed with hooked projections which become locked together and so give a determinate figure to the mass. It is not, however, necessary to fall back upon such a gross image as this for the explanation of rigidity. It is sufficient to conceive that when an effort is exerted against any part of a solid body, its molecules turn on their axes, assume new directions, and take up a new position of equilibrium. Such a supposition corresponds with that invariability of form which we are accustomed to connect with the solid state. In reality this invariability is not absolute. The smallest force applied to a solid body produces some change of form, but frequently this change is only appreciable when the force is very intense.

20. Divisibility.—This hypothesis regarding the constitution of bodies amounts to assuming that matter is not infinitely divisible, but that, whatever be the means employed to produce division, there is for each body a limit below which it never descends. These parts which always remain undivided are called atoms (α, privative, τέμνω, to cut); and we are to understand by this designation not elements which it is *impossible* to divide in an absolute or metaphysical sense, but elements which are not susceptible of division by any known forces.

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[1] The hypotheses broached in the remainder of this section must be received with caution, as being merely conjectural explanations of the distinction between solids and liquids.

It is in chemistry especially that reasons are found for assuming a limit to the division of matter. In fact in chemical phenomena we see this division attain limits which, though doubtless very remote, are yet fixed. The composition of compound bodies is invariable, whatever may be the circumstances of their production;[1] their properties, which could hardly fail to be altered by a change in the size of the constituent particles, are also the same; whence it seems necessary to conclude that the elements between which chemical affinity is exerted are absolutely alike and unchangeable—have a definite existence. They are the veritable *individuals* of the mineral kingdom. These are what we mean by atoms.

The notion of an atom does not involve the idea of size; but experience teaches us that their size must be excessively minute; for we can in several different ways divide matter into extremely small parts, without finding any reason to think that we have attained or even approached the limit of division. We will cite some examples which prove the extreme divisibility of matter.

Wollaston succeeded in obtaining threads of platinum of a diameter not exceeding $\frac{1}{3000000}$ of an inch. The method which he employed for preparing them consisted in drawing a silver wire with a platinum core, and dissolving the shell of silver in nitric acid. In this way threads can be obtained so fine that they are actually invisible to direct view, and that their existence can only be detected by the aid of certain special optical phenomena. In the art of beating gold, leaves are obtained whose thickness cannot exceed $\frac{1}{200000}$ of an inch. A square inch of this leaf would weigh less than the $\frac{1}{20000}$ of an ounce, and as a square whose side is $\frac{1}{250}$ of an inch is visible to the naked eye, it follows that this square inch of leaf contains more than 60,000 visible parts.

The diffusion of colouring matters and perfumes affords a notable instance of the extreme divisibility of matter. A cubic millimetre of indigo (about $\frac{1}{16000}$ of a cubic inch) dissolved in sulphuric acid, can colour to an appreciable extent more than 10 litres (about 2 gallons) of water. Now, a litre contains a million cubic millimetres; the cubic millimetre of indigo, therefore, in this experiment is divided into ten million visible parts.

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[1] This is called the law of "definite proportions." The laws of "multiple proportions" and "equivalent proportions" furnish perhaps a still stronger argument for the atomic hypothesis.

The diffusion of odoriferous substances is still more astonishing. It is well known that a grain of musk will continue for years to supply the air, which is continually being renewed around it, with a sufficient number of particles to communicate to it its odour. The mind can hardly form an idea of the degree of tenuity which such particles must have.

21. Porosity.—Porosity is an immediate consequence of the hypothesis of molecular constitution. It consists in the existence, in the interior of all bodies, of intervals or pores between their material particles. This porosity is often so marked as to permit the passage of liquids or gases through the substance of solids. It then receives the name of *permeability*. Permeability is the property which is utilized in the employment of stone filters; the pores are large enough to allow the water to pass, and small enough to prevent the passage of the small solid bodies held in suspension in the water.

It is by means of permeability that the communication and contact of liquids takes place in organized bodies; for the vessels which contain them are nowhere open, and it is always through the substance of their walls that the final changes of elements are made which are necessary to vital action.

By using great pressure, liquids can be made to pass through metals. These latter, or at least some of them, iron and platinum for example, when raised to a high temperature, allow ready passage to different gases. Thus it is that cast-iron stoves, when red-hot, allow some of the deleterious products of combustion to pass out, and sometimes occasion serious accidents.

But even when no permeability can be detected, pores must still be assumed to exist; and the proof is found in the fact that all bodies can have their volumes increased or diminished,—that they are dilatable and compressible. The dilatation of bodies by the action of heat is a general phenomenon which will be studied further on. We will here confine ourselves to compressibility.

22. Compressibility.—Compressibility consists in the reduction of volume which bodies experience under the action of external pressure. The compressibility of solids is extremely small; that is to say, a very considerable pressure is required to produce any sensible diminution of volume. The existence of such an effect has, however, been well established, and it is found necessary to allow for it in structures.

The compressibility of liquids, though greater than that of solids, is still very small. Hence, in comparison with gases, they have often been called incompressible fluids. It is easy with the aid of Œrsted's apparatus, represented in section in Fig. 12, to show that liquids are more compressible than solids, and to measure approximately the degree of their compressibility. The liquid to be compressed is contained in a kind of large thermometer *b*, called a piezometer, whose tube has been carefully divided, and its reservoir gauged so as to determine how many divisions of the tube its volume is equal to. The tube is open at the top, and a globule of mercury, placed above the liquid column, serves for index. The apparatus is placed in a vessel of water *a*, having very thick sides.

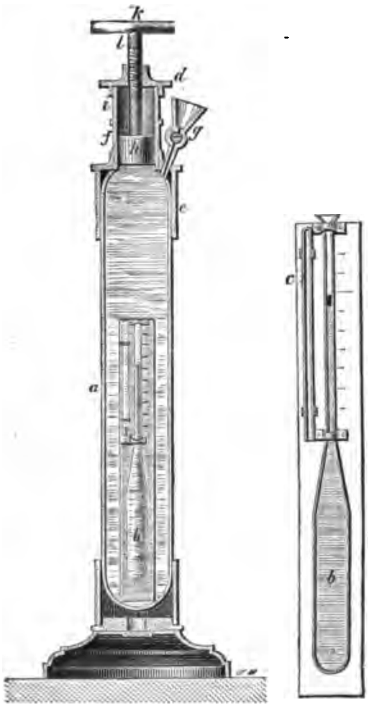

Fig. 12.—Œrsted's Piezometer.

When pressure is exerted by means of the screw-piston *klh*, the index of mercury is seen to descend, showing a diminution of the volume of the liquid. The amount of the pressure is known from the volume of air contained in the tube *c*, which serves as a manometer. In this experiment the number of divisions through which the extremity of the liquid column moves indicates the *apparent* diminution of volume; that is to say, the excess of the diminution of volume of the liquid above that of the envelope. It is easy, in fact, to understand that the piezometer itself must, under the pressure to which it is subjected, undergo a diminution of capacity, which must be taken into account. Œrsted supposed that this diminution was insensible, or nothing, since the pressure is exerted on the interior as well as the exterior of the piezometer. But this conclusion is erroneous; for if the piezometer were solid and submitted to compression, the interior shells would react with a force precisely equivalent to that which is produced when the instrument is hollow and the liquid occupies its interior. The piezometer then, in Œrsted's experiment, undergoes a diminution of volume equal to that which a solid piezometer would undergo in the same circumstances. In order, then, to find the true diminution of volume of the liquid, it is necessary to increase the apparent contraction by the contraction of the envelope. This latter element varies according to the quality of the glass of which the envelope is composed, but may be estimated at about ·0000029 per atmosphere of pressure. The true compressibility of water, according to recent experiments conducted under the direction of M. Jamin by Messrs. Amaury and Descamps, is, at the temperature of 15° centigrade, ·0000457 per atmosphere. It diminishes when the temperature increases. Alcohol and ether are rather more compressible, and their compressibility (unlike that of water) increases with the temperature. Mercury is much less compressible than water. Its variations of volume may therefore, in ordinary experiments, be neglected.

Gases are enormously more compressible than solids and liquids. This is easily shown by the pneumatic syringe, Fig. 13. It is a cylinder of very thick glass, closed at one end. A piston, which exactly fits the tube, is made to enter the other end, and can be forced in until the air is reduced to a half, a third, or a tenth of its original volume.

Hence it would seem to follow that in gases the spaces between the particles are much greater than in liquids and solids, and consequently that there is much less matter in the same volume. The same conclusion is established by the comparison of specific gravities. To give an idea of the difference, it may suffice to mention that water, when converted into steam, at the ordinary atmospheric pressure, and at the ordinary temperature of boiling water, expands 1700 times, so that a cubic inch of water gives about a cubic foot of steam.

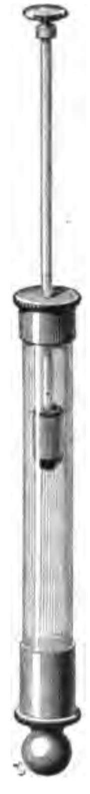

Fig. 13.—Pneumatic Syringe.

23. Elasticity.—This term, when applied to solids, is used in modern physics to denote the property in virtue of which a body tends to recover its form and dimensions when these are forcibly changed. The great majority of solid bodies possess almost perfect elasticity for small deformations; that is to say, when distorted, extended, or compressed within certain small limits, they will, on the removal of the constraint to which they have been subjected, return instantly to their original form and dimensions. These limits (which are called the limits of elasticity) are different for different substances; and when a body is distorted to an extent exceeding these limits, it takes a set, the form to which it returns being intermediate between its original form and that into which it was distorted.

When a body is distorted (using this word to include extension or compression as well as change of shape) within the limits of its elasticity, the force with which it reacts is simply proportional to the amount of distortion. For example, the force required to make the prongs of a tuning-fork approach each other by a tenth of an inch, is precisely double of that required to produce an approach of a twentieth of an inch; and if a chain is lengthened a twentieth of an inch by a weight of 1 cwt., it will be lengthened $\frac{1}{10}$th of an inch by a weight of 2 cwt., the chain being supposed to be sufficiently elastic to experience no permanent set from this greater weight. Also (within the limits of elasticity) equal and opposite distortions are resisted by equal reactions; for example, the same force that suffices to make the prongs of a tuning-fork approach by $\frac{1}{20}$th of an inch, will suffice, if applied in the opposite direction, to make them separate by the same amount.

An important consequence which can be mathematically deduced from the laws just stated, is that when a body is distorted within its limits of elasticity, the vibrations which ensue when the constraint is removed have periods which are independent of the magnitude of the distortion. For example, a common C tuning-fork makes about 528 vibrations in a second whether vibrating strongly or feebly; by whatever amount the prongs are made to approach each other, the time which elapses from their being released to the attainment of their greatest separation is $\frac{1}{1056}$ of a second, and the same time elapses from their greatest separation to their nearest approach. The sum of these two intervals, $\frac{1}{528}$ of a second, is the period of a complete vibration; and during the whole time that the vibrations are dying away until the fork finally comes to rest, this period remains unaltered, the diminution of distance moved being exactly compensated by the increasing slowness of the motion.

India-rubber has very wide limits of elasticity. Glass has sensibly perfect elasticity up to the limit at which it breaks.

Putty and wet clay are instances of bodies which are almost entirely destitute of elasticity.

The resistance of a cylindrical or prismatic bar to elongation or flexure is measured by a number called "Young's modulus of elasticity." The following are examples of its value for different substances, in kilogrammes per square millimetre:—

Flint-glass,	5,851	Iron (wrought),	19,994
Brass,	10,948	Do. (cast),	13,741
Steel,	21,793	Copper,	12,558

To illustrate the meaning of this table by the case of steel; a steel wire whose section is a square millimetre will be elongated by $\frac{1}{21793}$ of its length by a weight of one kilogramme. The elongation is inversely proportional to the section and directly proportional to the stretching weight; so that a steel wire whose section is half a millimetre will, when stretched by a weight of six kilogrammes, receive twelve times the elongation above specified.

The resistance of a cylindrical or prismatic bar or beam to bending (called its flexural rigidity) is proportional to the value of Young's modulus of elasticity for the material of the bar or beam; so that from the dimensions of the bar, the value of this modulus, and the magnitudes and directions of the externally applied forces, the amount of bending could be calculated.

The resistance of a cylindrical rod to twisting (called torsional rigidity) does not depend upon the value of Young's modulus, but upon an entirely distinct element, an element which is sometimes called simply "rigidity," and which expresses the resistance which a square of given thickness would oppose to being changed by external forces into a rhombus of the same area, having angles differing by a given small amount from right angles.

Elasticity being a molecular phenomenon, it is to be expected that all circumstances which modify the molecular constitution of a body will alter its elasticity; but in the present state of science it is impossible to predict *à priori* the nature and direction of the change, the effects being sometimes opposite for different substances. Thus tempering (that is to say heating followed by sudden cooling), which, as is well known, augments in a high degree the hardness and elasticity of steel, produces a reverse effect on the bronze of which gongs are made. This alloy, in fact, when cooled slowly, possesses the fragility of glass; whilst, when cooled suddenly, it can be wrought with the hammer.

The elasticity of springs furnishes a simple means of comparing forces. Fig. 14 represents an apparatus designed for this purpose and called a *dynamometer*. It is formed of two plates of steel, AB and A′B′, jointed at their extremities to two metallic bridles which connect them. To the middle of the upper plate is attached a ring, by means of which the apparatus can be suspended from a fixed point. To the middle of the lower plate is attached a hook, which can either receive a weight or serve as a point of application for the force which is to be tested. Under the action of the force thus applied the spring plates bend, the distance of the middle points increases, and this increase serves to measure the force itself; or the force may be measured by observing what weight must be suspended from the hook to produce the same effect.

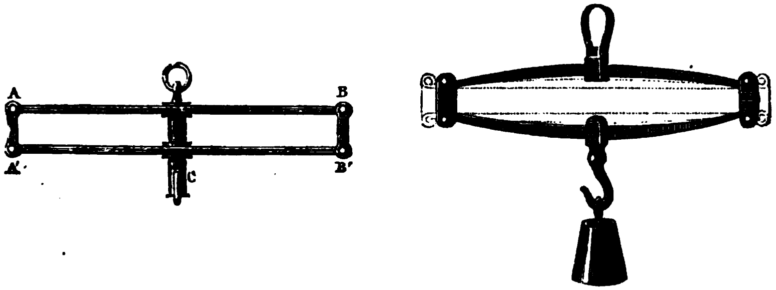

Fig. 14.—Dynamometer.

The spring-balance is another apparatus of the same kind. In its most common form it contains a spiral spring which is elongated by the application of the force which is to be measured. The equality of the graduations illustrates the law above stated of the proportionality of distortions to the forces producing them. The resistance of a spiral spring to elongation depends chiefly (as shown by Professor James Thomson) on the torsional rigidity of the wire which composes it.

It is important to remark, that whereas a pair of scales is essentially a measure of mass, a spring-balance is essentially a measure of force. Hence if a spring-balance be graduated so as to show weights correctly at a medium latitude, it will indicate too little if carried to the equator (where the force of gravity is feebler), and too much at the poles (where gravity is more intense).

CHAPTER IV.

GRAVITY.

24. Terrestrial gravity is the force in virtue of which all bodies fall to the surface of the earth. This force is general; its effects are observed in all places and for all bodies. If some of these latter, as smoke and hydrogen gas, appear to be exceptions, it is because they are sustained by the air in the same manner as cork is sustained by water. In space deprived of air, not only do all bodies fall, but, as we shall see later, they fall with equal velocities.

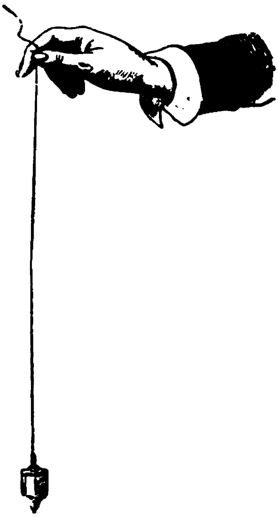

Fig. 15.—Plumb-line.

25. **Direction of Gravity.**—The direction of gravity is called the *vertical*. It is easily determined by the aid of the simple apparatus called a plumb-line, which consists of a thread fixed at one end and carrying a heavy body at the other. When the system is in equilibrium it is clear that the resultant of the actions of gravity on all the parts of the heavy body has exactly the same direction as the thread, since it is this which prevents the fall. But it can be shown that this direction does not change when the form and volume of the heavy body are altered, it must therefore be the same as the direction of the force which would act upon one of the elementary particles if suspended alone at the extremity of the thread.

It can be shown by experiment that the direction of gravity is perpendicular to the surface of a liquid in equilibrium, or to use the common expression, to the surface of still water. For this purpose a plumb-line OA is suspended over the surface of a fluid in equilibrium (which should be slightly opaque, as blackened water), and the plummet is allowed to plunge in the liquid. The image AB of the thread produced by reflection at the surface of the liquid will be seen with great distinctness, and will be observed to be exactly in a line with the thread itself. Now we shall see in a subsequent part of this treatise, that whenever reflection takes place at a plane mirror, each point of the object and the corresponding point in the image are on the same perpendicular to the mirror and at equal distances from the mirror on opposite sides. Since, then, in the experiment here described the thread and its image are in one straight line, this line must be perpendicular to the surface.

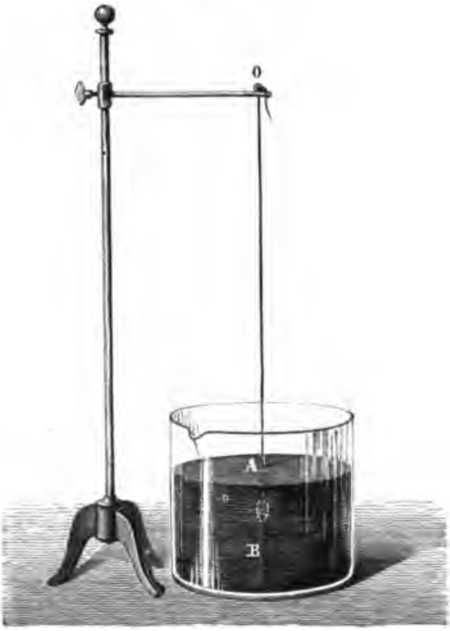

Fig. 16.—Experiment for showing that the Plumb-line is perpendicular to the surface of a fluid at rest.

The surface of still water defines in each locality what is called the surface of the earth. This expression denotes the surface of an imaginary ocean of calm water supposed to cover the whole earth. This surface is known to be sensibly spherical. It follows that the different verticals will nearly meet in the centre of the earth. The figure shows the relative position of some verticals CZ, CZ′, CZ″; it is evident that they contain angles equal to the angular distance which separates the corresponding places.

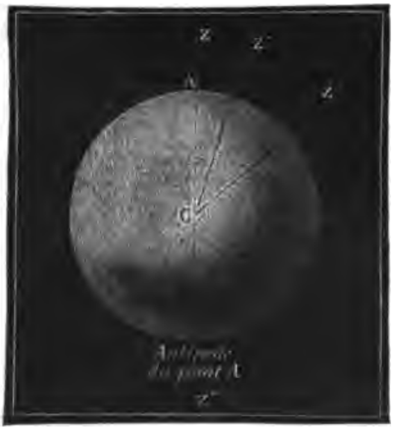

Fig. 17.—Verticals at different places.

At any one locality all verticals may be treated as parallel, on account of the immense distance of the centre of the earth. Let us calculate, for example, the angle contained between two verticals a metre (39·37 inches) apart. Ten millions of metres correspond to a quarter of the earth's circumference, that is to say, to 90°. A length of a metre, therefore, represents 90° divided by ten millions, that is to say, about $\frac{3}{100}$ of a second, a quantity quite inappreciable even with our most perfect instruments. It should be remarked, however, that the parallelism of the verticals at any one place is a physical fact, completely independent of all previous knowledge of the figure of the earth, and can be established by direct observation.

We may remark in passing, that the latitudes of places on the earth's surface are determined by the directions of the verticals. What is commonly called the latitude of a place is the angle which a vertical at the place makes with a plane perpendicular to the earth's axis of rotation. As distinguished from this, the geocentric latitude (which is required in a few astronomical problems) is the angle which a line drawn from the place to the earth's centre makes with a plane perpendicular to the axis of rotation. The difference between common and geocentric latitude generally amounts to some minutes, and attains its greatest value (11′ 29″) at latitude 45°.

26. Point of Application of Gravity—Centre of Gravity.—Gravity being a property of matter, its points of application must evidently be the different material particles which compose each body. Though a body be divided into as many parts as we please, and even reduced to the state of impalpable powder, each of the grains thus obtained will be subject to the action of gravity. The total force which urges a body to fall is the sum, or more strictly, the resultant of all the forces which are thus actually applied to its several elements. Now these forces are parallel, as has just been stated, and act in the same direction; their resultant is therefore equal to their sum, and it constitutes what is called the *weight* of the body; that is to say, the force with which it presses the obstacle which prevents it from falling. The point of application G of this resultant

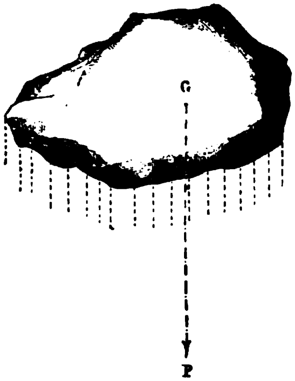

Fig. 18.—Parallelism of the Forces of Gravity on the different Points of a Body.

(Fig. 18) is called the *centre of gravity.* It follows from the property indicated in section 15, that the position of this point does not vary when the direction of the components is made to vary. The body can therefore be turned about in any manner without the centre of gravity changing its position in the body. It is a fixed point depending only on the form of the body and the distribution of the matter which composes it.

If the body has the same density throughout, the position of the centre of gravity depends only on the figure, so that in this case bodies of similar form have their centres of gravity similarly situated.

The determination of the position of the centre of gravity is a problem of mechanics which is solved by appropriate methods of general application, founded on the principle (to which we can here barely allude), that if a body be divided into a great number of equal elements, the sum of their distances from any one plane, divided by the number of elements, is equal to the distance of the centre of gravity from the same plane.

Whenever a body of uniform density contains a point which is a centre of symmetry (that is, a point which bisects all straight lines drawn through it), this point must be the centre of gravity. Hence:

1. The centre of gravity of a straight line is its middle point;

2. The centre of gravity of a circle, or of the circumference of a circle, is the centre;

3. The centre of gravity of a parallelogram is the intersection of the diagonals;

4. The centre of gravity of a sphere is its centre;

5. The centre of gravity of a cylinder is the middle point of its axis;

6. The centre of gravity of a parallelopiped is the common intersection of the diagonals, &c.

It may naturally be asked how we can speak of centres of gravity of lines and surfaces, which, being only of one or two dimensions, cannot possess weight. The answer is, that in so speaking we make an abstraction analogous to that which gives us the idea of a material point. We suppose lines or surfaces composed of elements possessing weight; and the results thus obtained can be utilized in investigating the centres of gravity of real bodies. Consider, for example, a triangular prism. It can be conceived as decomposed into elements which would be, so to speak, heavy triangles. The centre of gravity of the solid would therefore be in the line which joins the centres of gravity of all these triangles, and would be its middle point. We may say then that the centre of gravity of a triangular prism, and in general of any prism, if composed of homogeneous material, is the middle point of the line which joins the centres of gravity of the two ends.

We must guard against the error of supposing that the force of gravity on a body acts at the centre of gravity. Gravity really acts equally on all the particles of the body; but its effect is in many respects the same as that of a single imaginary force supposed to act at the centre of gravity.

The centre of gravity sometimes lies outside the body, as in the case of a ring or a hollow sphere. When this is the case, it must be regarded as rigidly connected to the body.

27. Physical Definition of the Centre of Gravity.—The centre of gravity, regarded from the mechanical point of view, is in reality merely the centre of parallel forces distributed in a determinate manner. Its position can therefore be found by a purely geometrical investigation, and apart from any physical idea of the nature of bodies. Nevertheless, it is certain that the discovery of the existence of centres of gravity had its origin in the consideration of the phenomena of equilibrium, which are exhibited by bodies under the influence of gravity. Experiment, in fact, shows that in most bodies there is a point such that, if it be supported, the body will be in equilibrium, and if it be not supported, the body will move under the action of gravity. This point is the centre of gravity, and the property in question is an obvious consequence of the mechanical definition already given. But this property may itself be used as the definition of the centre of gravity (though with some want of precision). We shall thus use it in the examination of some important cases of equilibrium.

28. Equilibrium of a Body capable of turning about an Axis or a Fixed Point.—Consider, for example, a triangular plate movable about an axis of rotation O, and let G be the position of the centre of gravity. In order that there may be equilibrium, the centre of gravity must be supported; that is to say, the vertical drawn through it must meet the axis. This condition may be fulfilled by two very different positions of the body: the centre of gravity may be either above or below the axis. In the former case (Fig. 20) it is evident that if the body be ever so little displaced from its position of equilibrium, the effect of gravity will be to make it fall still further away. In the second case, on the contrary (Fig. 19), the action of gravity tends to restore the body to the position of equilibrium. In the former case the equilibrium is *unstable;* in the latter it is *stable.*

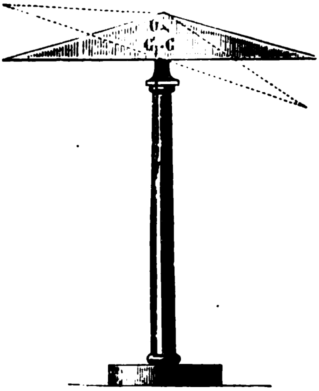

Fig. 19.—Stable Equilibrium

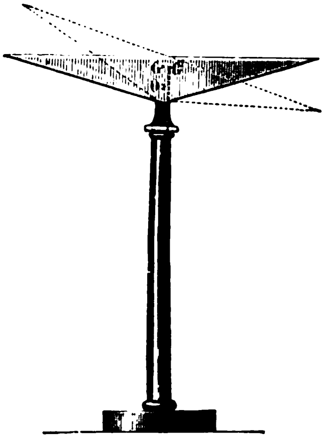

Fig. 20.—Unstable Equilibrium.

We see then that the condition of stable equilibrium is that the centre of gravity be below the axis or point of suspension.

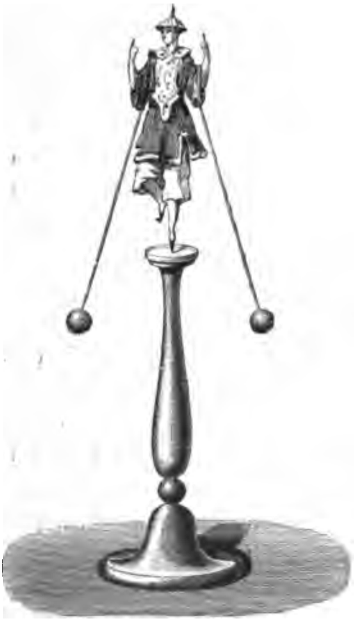

Fig. 21.—Balancer.

The toy called the balancer is an application of this principle. It consists of an ivory figure resting by one point on a small horizontal stand. Two stiff wires fixed to the figure terminate below in leaden balls. The centre of gravity of the system is thus brought below the point of support; the equilibrium is consequently stable. If we draw the figure to one side and then release it, it will perform a series of oscillations, and will end by taking a position of equilibrium such that the vertical through the centre of gravity passes through the point of support.

If a body were traversed by an axis through its centre of gravity, its equilibrium would be neutral, and the body would remain in equilibrium in all positions. This condition ought to be rigorously fulfilled by the wheels of pieces of mechanism which only serve to transmit motion.

29. Equilibrium of a Body resting on a Horizontal Plane which touches it in one Point.—Consider (Figs. 22 and 23) a body of ellipsoidal form resting on a horizontal plane. In order that there may be equilibrium it is evidently necessary and sufficient that the vertical through the centre of gravity G should meet the horizontal plane at the point of contact. We see by the figure that this condition may be realized in two ways.

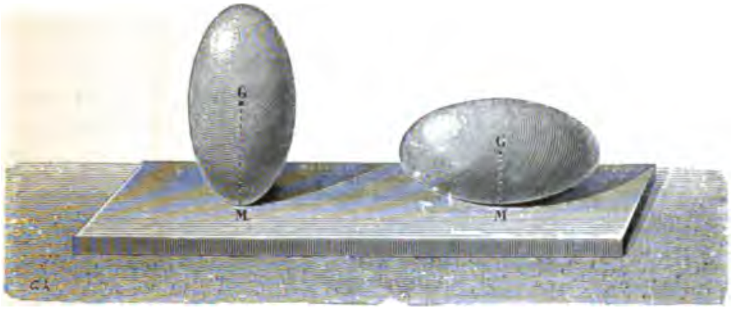

Fig. 22.—Unstable Equilibrium. Fig. 23.—Stable Equilibrium.

The first figure corresponds to *unstable*, the second to *stable* equilibrium. The figure shows that in the latter case the *centre of gravity occupies its lowest possible position.*

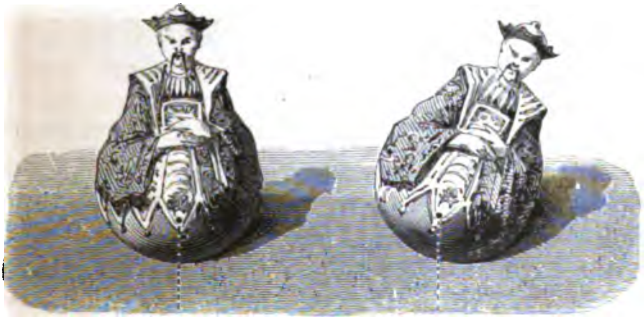

Fig. 24.—Tumblers.

The tumbler (Fig. 24) is founded on this principle. The centre of gravity being near the lower side in consequence of the accumulation of matter in this region, the apparatus is in stable equilibrium. If it be removed from its position and subjected even to very wide displacements, it always rises again and returns to its position of equilibrium after a number of oscillations.

If the centre of gravity were always at the same distance from the plane—if, for example, the body were spherical, there would be equilibrium in all positions,—the equilibrium would be neutral.

We may remark in general that a position of unstable equilibrium is only mathematically possible; it never can have a physical existence; for the smallest derangement destroys it, and in nature a multitude of causes, such as the movement of the air, the flexibility of supports, &c., introduce displacements which violate the conditions of equilibrium. If there be any actual cases of such equilibrium—if, for example, it is possible to make an egg stand on its end, it is by the help of friction, which constitutes a new force tending to prevent displacement.

30. Equilibrium of a Body resting on a Horizontal Plane at several Points.—When a body rests on a horizontal plane at several points, it is necessary for equilibrium that the vertical through the centre of gravity fall within the convex polygon which can be formed by joining the points of support. It is clear that in this case gravity will have no effect but to press the body against the plane. It is also obvious that the equilibrium will be the more stable as the centre of gravity is lower, and the distance of the vertical through it from the nearest side of the polygon greater. If this vertical is very near one side, a small force will be sufficient to overturn the body on that side, although a very great force may be required to overturn it in the opposite direction.

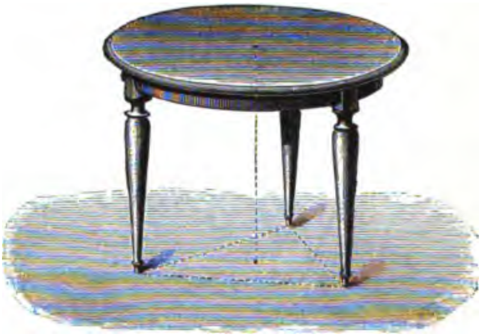

Fig. 25.—Equilibrium of a Body supported on a Horizontal Plane at three or more Points.

31. Practical Method of finding the Centre of Gravity.—The different methods which are employed in practice for the experimental determination of the centre of gravity are dependent on the principles above explained. Whatever be the particular nature of the proceeding, it always consists in placing the body in a position of equilibrium from which it can be inferred that the centre of gravity lies in a certain line or surface.

Thus, for example, if we suspend a body by one point, it is clear that the centre of gravity must lie in the prolongation of the suspending thread. If we then suspend the body by another point, a similar inference follows. Consequently, the centre of gravity must be the intersection G of the two directions thus indicated.

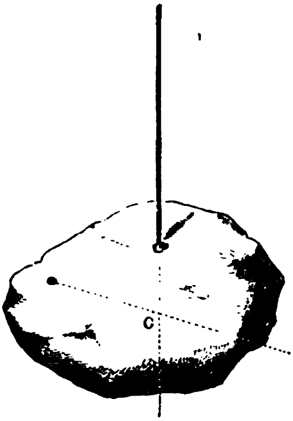

Fig. 26.—Experimental Determination of Centre of Gravity.

If we wish, for example, to pierce a plate or board by an axis which is to pass through its centre of gravity, we may begin by balancing it in a horizontal position upon two points near its circumference. The line joining them will pass vertically under the centre of gravity. By repeating the operation we may find a second line which possesses the same property, and the required axis must pass through their intersection and be perpendicular to the plate. Instead of balancing the plate upon two points, an operation which may require repeated trials, it is more expeditious, when practicable, to suspend it freely in a vertical position by a point near its circumference, and to suspend a plumb-line from the same point. The course of this line must be marked on the plate, and the operation must then be repeated, using a different point of suspension. The intersection of the two lines thus obtained will, as before, be opposite to the centre of gravity of the plate. Both the methods described in this paragraph are applicable even to plates which are not homogeneous.

CHAPTER V.

LAWS OF FALLING BODIES.

32. In air, bodies fall with unequal velocities; a sovereign or a ball of lead falls rapidly, a piece of down or thin paper slowly. It was formerly thought that this difference was inherent in the nature of the materials; but it is easy to show that this is not the case, for if we compress a mass of down or a piece of paper by rolling it into a ball, and compare it with a piece of gold-leaf, we shall find that the latter body falls more slowly than the former. The inequality of the velocities which we observe is due to the resistance of the air, which increases with the extent of surface exposed by the body.

It was Galileo who first discovered the cause of the unequal rapidity of fall of different bodies. To put the matter to the test, he prepared small balls of different substances, and let them fall at the same time from the top of the tower of Pisa; they struck the ground almost at the same instant. On changing their forms, so as to give them very different extents of surface, he observed that they fell with very unequal velocities. He was thus led to the conclusion that gravity acts on all substances with the same intensity, and that in a vacuum all bodies would fall with the same velocity.

This last proposition could not be put to the test of experiment in the time of Galileo, the air-pump not having yet been invented. The experiment was performed by Newton, and is now commonly exhibited in courses of experimental physics. For this purpose a tube from a yard and a half to two yards long is used, which can be exhausted of air, and which contains bodies of various densities, such as grains of lead, pieces of paper, and feathers. When the tube is full of air and is inverted, these different bodies are seen to fall with very unequal velocities; but if the experiment is repeated after the tube has been exhausted of air, no difference can be perceived between the times of their descent.

33. Laws of Falling Bodies.—Having found that the effect of gravity is the same on all bodies, Galileo proposed to himself the problem of determining, by experiments on one body, the law which regulates their descent; and, inasmuch as the observation of a body falling freely is very difficult, on account of the rapidity of its motion, he adopted a method of diminishing this rapidity without in other respects altering the law of motion. This method consisted in the use of the inclined plane.

Consider, in fact, a heavy body M, free to move along the inclined plane ABC. The weight of the body M being represented by MP, it can, (by § 16), be decomposed into two other forces, viz. MN perpendicular to the plane, which is destroyed by the resistance of the plane itself, and MT parallel to the plane, which alone produces the motion. Now this latter force is less than MP, but is a constant fraction of it, for at all points in the plane the parallelogram of forces will have the same form, and the ratio of MT to MP will be constant. This ratio is in fact the same as that of the height AC of the plane to its length AB, or in other words is the sine of the inclination of the plane to the horizon. The motion will therefore be less rapid, but will follow the same law as that of a body falling freely, and will be much easier to observe. The diminution of velocity has the further advantage of diminishing the relative importance of the resistance of the air, which increases very rapidly with every augmentation of velocity.

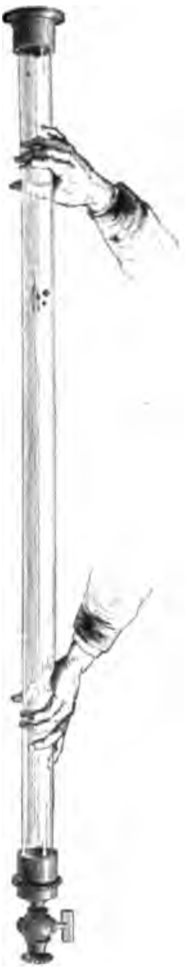

Fig. 27.—Fall of Bodies in Vacuo.

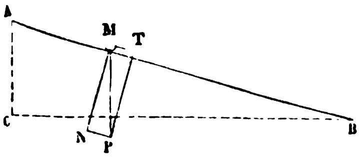

Fig. 28.—Inclined Plane.

The inclined plane employed by Galileo consisted of a long ruler, with a longitudinal groove, along which he caused a small heavy ball to roll. Having thus observed the spaces traversed in 1, 2, and 3 units of time, he found that these spaces were in the ratio of the numbers 1, 4, and 9; that is to say, when the time of descent was doubled or tripled the space traversed became 4 or 9 times greater. This law can be expressed by saying that *the spaces traversed are proportional to the squares of the times of descent.*

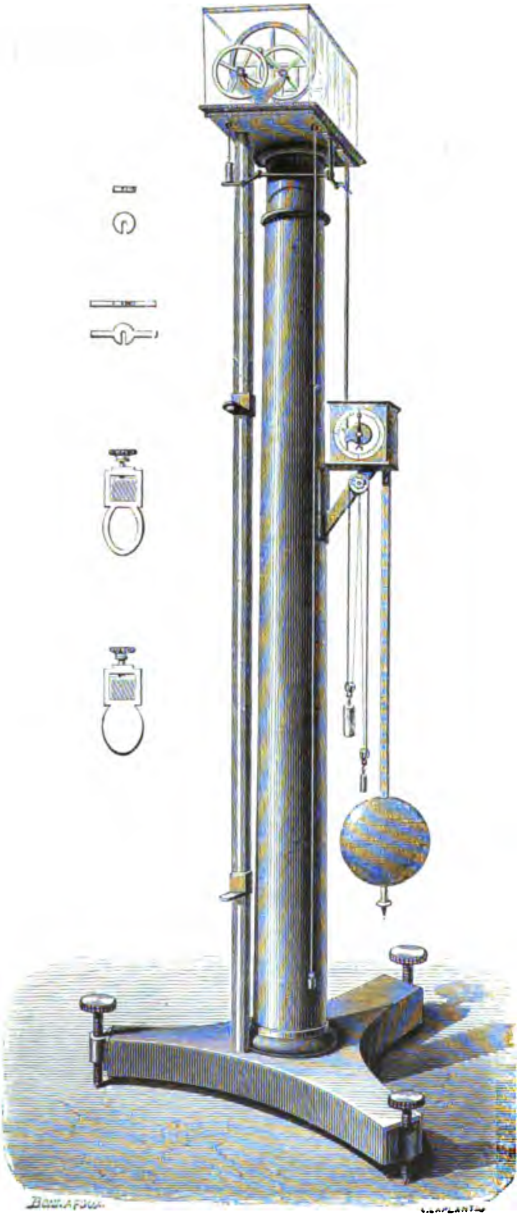

Fig. 29.—Attwood's Machine.

34. Attwood's Machine.—Attwood, a fellow and tutor of Trinity College, Cambridge, invented, towards the end of last century, a machine which affords great facilities for verifying the laws of falling bodies. It involves, like Galileo's inclined plane, a method of diminishing the velocity of descent; but this result is obtained by very different means.

The machine consists of a column, having at its top a very freely moving pulley, which forms the essential part of the apparatus. In order to obtain great freedom for the movements of the pulley, the ends of its axis are made to rest, not on fixed supports, but on the circumferences of wheels (two at each end of the axis) called friction-wheels, because this arrangement produces a great diminution of friction. Over the pulley passes a fine thread, carrying at its extremities two equal weights P. Neglecting the weight of the thread, it is obvious that these weights will be in equilibrium in every position. If however one of them be loaded with an additional weight p, the system will be put in motion, and all parts of it will move with the same velocity. We may therefore regard the moving force as distributed uniformly through it. But this force is simply the weight of p. If then, for example, the movable system $2P + p$ has 20 times the weight of p, each portion of the system is urged with a force equal to $\frac{1}{20}$ of its own weight. The force which produces motion is in general diminished, as compared with a body falling freely, in the ratio expressed by the fraction $\frac{p}{2P+p}$; and as this ratio continues constant through the whole motion, the law of the motion will be the same as that for free descent.

The following are the arrangements for observing the motion:—One of the weights moves in front of a graduated scale, and a plane stop for intercepting the descending weight can be fixed at pleasure at any part of this scale. A clock with a pendulum beating seconds serves for the measurement of time. To measure the space traversed in a second, the weight is raised to the commencement of the graduation, is then loaded with the additional weight, and is dropped precisely at one of the beats of the pendulum. The stop is placed by trial at such a point of the scale that the blow of the weight against it precisely coincides with another beat of the pendulum,—a coincidence which can be obtained with great accuracy, inasmuch as the ear easily detects the smallest interval between the two sounds. In order to insure a similar coincidence at the commencement of the fall, the weight is supported by a movable platform M (Fig. 30), which is prevented from falling by the upper end of the lever *acb*, whose lower end is guided by a cam[1] fixed to the escapement wheel

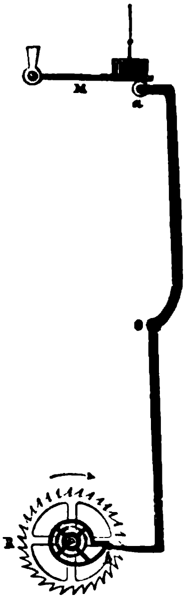

Fig. 30.—Detent in Attwood's Machine.

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[1] A cam is a rotating piece which, by means of projections or indentations in its outline, guides the movements of another piece which presses against it.

of the clock, and is kept constantly pressed against the cam by means of a spring not shown in the figure. Suppose the wheel to be turning in the direction indicated by the arrow. It is obvious that as soon as the tooth of the cam has passed the end of the lever the latter will fly to the left, and therefore the upper end will fly to the right, since the lever turns about an axis at O. The platform M is thus suddenly dropped, and the fall commences. The position of the cam must be so adjusted that this movement shall take place exactly at the instant of the escapement of one of the teeth.

It is thus easy to measure the spaces traversed by the movable system of weights in 1, 2, and 3 seconds, and the result obtained will be as follows:—Suppose that in the first second the space traversed is 11 divisions, then we shall find:

Space traversed in 2 seconds.......................... $= 44 = 11 \times 2^2$
Space traversed in 3 seconds.......................... $= 99 = 11 \times 3^2$
Space traversed in 4 seconds.......................... $= 176 = 11 \times 4^2$

We see, then, that the spaces vary as the squares of the times employed in describing them. If we use the indefinite symbol K to denote the space described in the first unit of time, the space described in the time t will be given by the formula

$$s = Kt^2. \qquad (1)$$

35. Velocities.—Attwood's machine also affords the means of studying the successive velocities which gravity imparts to the system. Before describing the means employed for attaining this end, it will be desirable to make a few remarks respecting velocity.

When a material point moves uniformly; that is to say, when it traverses equal spaces in equal times, the meaning of velocity is perfectly clear: it is the space traversed in unit time. Thus, if a point moving uniformly describes 2 feet in each second, we say that the velocity is 2 feet per second; or if it is understood that the foot and second are to be our units of space and time, we simply say that the velocity is 2. If a point moving uniformly describes 5 feet in 2 seconds, its velocity is 2½, since the space described in one second must be half that described in two; and in general, in any case of uniform motion of a point, the velocity (in feet per second) will be obtained by dividing the whole space (in feet) by the whole time occupied in its description (in seconds).

But uniform motion is in nature the exception rather than the rule. In fact, it can only occur when the moving body is acted on either by no forces at all, or by forces in equilibrium. This, in fact, is merely a statement of the principle of inertia. When a body is constantly acted on by a force, this force must evidently have the effect of continually modifying the motion, and consequently the above mode of computing velocity is not directly applicable. If, however, we suppose the action of the force suddenly to cease, the motion will become uniform, and the velocity during this uniform motion will serve as a measure of the velocity which existed at the instant when the action of the force ceased.

Attwood's machine contains an arrangement for thus suddenly arresting the action of gravity at a given instant. For this purpose, a ring large enough to allow either of the two equal weights to pass through it, is to be fixed at the point of the scale at which the moving weight arrives at the end of one second. The additional weight, which for this purpose is made long and flat, is intercepted by the ring, and the subsequent motion, being due merely to the momentum already acquired, will be uniform. The stop is to be placed at the point at which the weight arrives a second later. The distance between the ring and the stop will then represent the velocity acquired during the first second. Making this experiment under the same conditions as the foregoing experiments on spaces, we find that the velocity acquired during the first second is represented by 22 divisions. We then place the ring at the point at which the system arrives after 2, 3, &c., seconds, and the stop at the point at which it arrives a second later; we thus measure the velocities acquired in 2, 3, &c., seconds, and find them equal to 44, 66, &c. We see, then, that the *velocities acquired in different times are proportional to the times.* Further, the velocity acquired in one second, 22 (divisions per second), is double of the space, 11 (divisions), described in the first second.

In formula (1), K denotes the space described in the first unit of time; the velocity acquired is then 2K, and consequently the velocity acquired in time t is given by the formula

$$V = 2Kt. \qquad (2)$$

36. Attwood's machine, when fitted with the appliances above described, leaves little to be desired in point of accuracy, but its complication renders it expensive. We subjoin a figure representing Bourbouze's modification of Attwood's machine, which is much simpler. AB is the pulley, on the axis of which is a cylinder P,

surrounded with smoked paper. One of the iron weights, M, is held at the bottom of the apparatus by an electro-magnet, which is magnetized by means of the galvanic cell O. The weight M′, loaded with the additional weight N, is thus prevented from obeying the force of gravity. Again, the vibrating plate L, carrying a very light style for tracing a mark on the smoked cylinder, is held by the electro-magnet, E′, which is magnetized by the same cell. If at any moment the current is interrupted, the weight M′ falls, and the plate vibrates, describing an undulated curve on the surface of the cylinder. The undulations of this curve correspond to the vibrations of an elastic body, and, consequently, to equal times; while the distance of any undulation from the beginning of the curve is equal to the distance turned by the cylinder P, and is consequently proportional to the distance travelled by the weights. These distances are found to be exactly proportional to the series of numbers 1, 4, 9, &c. The ring D serves to intercept, at a given instant during the descent, the additional weight N; from this time onward the motion is uniform and the undulations of the curve are equidistant.

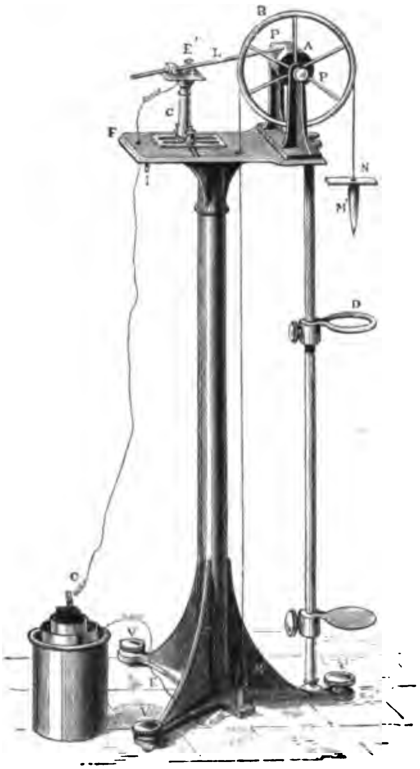

Fig. 31.—Bourbouze's Modification of Attwood's Machine.

Attwood's machine, however modified, gives only indirect evidence regarding the motion of bodies falling freely. Although this circumstance cannot affect the legitimacy or accuracy of the conclusions to which it leads, it would be interesting, if possible, to observe the phenomenon of free fall, and show that the laws just obtained are verified. This is the object of Morin's apparatus.

37. Morin's Apparatus.—Morin's apparatus consists of a wooden cylinder covered with paper, which can be set in uniform rotation about its axis by the fall of a heavy weight. The cord which supports the weight is wound upon a drum, furnished with a toothed wheel which works on one side with an endless screw on the axis of the cylinder, and on the other drives an axis carrying fans which serve to regulate the motion.

In front of the turning cylinder is a cylindro-conical weight of cast-iron carrying a pencil whose point presses against the paper, and having ears which slide on vertical threads, serving to guide it in its fall. By pressing a lever, the weight can be made to fall at a chosen moment. The proper time for this is when the motion of the cylinder has become sensibly uniform. It follows from this arrangement that during its vertical motion the pencil will meet in succession the different generating lines[1] of the revolving cylinder, and will consequently describe on its surface a certain curve, from the study of which we shall be

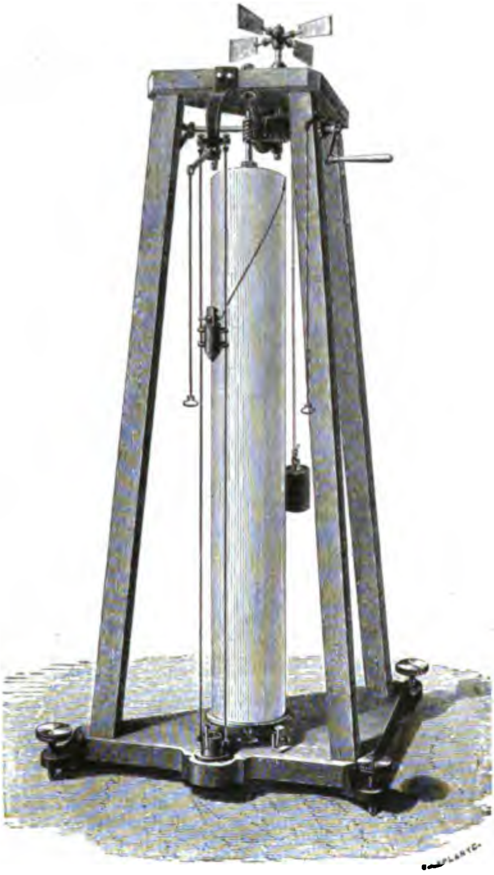

Fig. 32.—Morin's Apparatus.

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[1] That is, lines drawn on the surface of the cylinder parallel to its axis. A cylindric surface could obviously be described by the motion of a straight line in space. The line so moving is said to generate the cylindric surface, and the different positions which it successively occupies in its supposed motion are called generating lines of the cylinder. If a cylindric surface is cut open along a generating line and flattened out so as to become plane, its form will be rectangular, and its generating lines will be parallel to two sides of the rectangle.

able to gather the law of the fall of the body which has traced it. With this view, we describe (by turning the cylinder while the pencil is stationary) a circle passing through the commencement of the curve, and also draw a vertical line through this point. We cut the paper along this latter line and develop it (that is, flatten it out into a plane). It then presents the appearance shown in Fig. 33.

If we take on the horizontal line equal distances at 1, 2, 3, 4, 5 . . . , and draw perpendiculars at their extremities to meet the curve, it is evident that the points thus found are those which were traced by the pencil when the cylinder had turned through the distances 1, 2, 3, 4, 5. . . . The corresponding verticals represent the spaces traversed in the times 1, 2, 3, 4, 5. . . . Now we find, as the figure shows, that these spaces are represented by the numbers 1, 4, 9, 16, 25 . . . , thus verifying the principle that the spaces described are proportional to the squares of the times employed in their description.

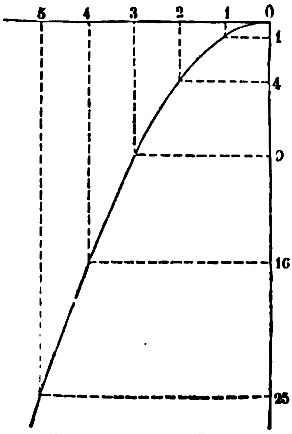

Fig. 33.—Parabolic Trace.

We may remark that the proportionality of the vertical lines to the squares of the horizontal lines shows that the curve is a parabola. The parabolic trace is thus the consequence of the law of fall, and from the fact of the trace being parabolic we can infer the proportionality of the spaces to the squares of the times.

The law of velocities might also be verified separately by Morin's apparatus; we shall not describe the method which it would be necessary to employ, but shall content ourselves with remarking that the law of velocities is a logical consequence of the law of spaces.[1]

38. Formulæ relating to Falling Bodies.—The formulæ (1) and (2), §§ 34, 35, will enable us to obtain numerical solutions of questions relating to the fall of bodies, if we can ascertain the space traversed

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[1] Consider, in fact, the space traversed in any time t, this space is given by the formula $s=\mathrm{K}t^2$; during the time $t+\theta$ the space traversed will be $\mathrm{K}(t+\theta)^2=\mathrm{K}t^2+2\mathrm{K}t\theta+\mathrm{K}\theta^2$, whence it follows that the space traversed during the time θ after the time t is $2\mathrm{K}t\theta+\mathrm{K}\theta^2$. The average velocity during this time θ is obtained by dividing the space by θ, and is $2\mathrm{K}t+\mathrm{K}\theta$, which, by making θ very small, can be made to agree as accurately as we please with the value $2\mathrm{K}t$. This limiting value $2\mathrm{K}t$ must therefore be the velocity at the end of time t.—*D.*

$T = \frac{v}{g}$

$S = \frac{1}{2} a \times \left(\frac{v}{a}\right)^2$

by a falling body in one second, or (which will be numerically double of this) the velocity acquired in one second.

The several forms of apparatus which we have been describing furnish the means of making this determination, but not with much precision. A much better method of determination is furnished by the pendulum, and will be described in the next chapter.

The result obtained is that in Great Britain the velocity acquired by a body falling in vacuo for one second is 32·2 feet per second, or 9·81 metres per second. The velocity in question is usually denoted by the letter g, and is sometimes called the intensity of gravity, because its value for different localities varies in the same ratio as the force exerted by gravity on one and the same mass, and may therefore be taken as the numerical representative of the force of gravity on unit mass.[1]

The space traversed during the first second of fall is equal to $\frac{1}{2} g$, that is, to 16·1 feet.

Introducing g into the formulæ (1), (2), they become:

$$s = \tfrac{1}{2} g t^2 \qquad (a)$$

$$v = g t \qquad (b)$$

Eliminating t from the equations (a), (b), we obtain a third formula, which gives the velocity acquired in falling through a given space:

$$v = \sqrt{2 g s}. \qquad (c)$$

39. Applications.—I. To calculate the space traversed by a body which falls during a given number of seconds. This will be found from formula (a) by putting for t the given number of seconds. Performing the calculation as far as 10 seconds, we obtain the following table:—

TIME OF FALL, in Seconds.	SPACE FALLEN, in Feet.	TIME OF FALL, in Seconds.	SPACE FALLEN, in Feet.
1	16·1	6	579·6
2	64·4	7	788·9
3	144·9	8	1030·4
4	257·6	9	1304·1
5	402·5	10	1610·0

II. What is the time occupied by a body in falling from a height of of 750 feet, and what is the velocity with which it strikes the ground?

---

[1] If we adopt the "absolute" unit of force defined in § 42, the force of gravity on unit mass will be numerically equal to g.

Formulæ (*a*) and (*c*) give:

$$t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{1500}{32 \cdot 2}} = 6 \cdot 8 \text{ seconds.}$$

$$v = \sqrt{2\,g\,s} = \sqrt{64 \cdot 4 \times 750} = 219 \cdot 8 \text{ feet per second.}$$

III. From what height must a body fall to acquire a velocity of 1500 feet per second?

Formula (*c*) gives:

$$s = \frac{v^2}{2\,g} = \frac{2250000}{64 \cdot 4} = 34938 \text{ feet.}$$

The velocity of 1500 feet per second is about that of a cannon-ball on leaving the muzzle of the gun. We see that it would be obtained by falling from a height of about 6½ miles. The duration of the fall would be about 47 seconds. It must be borne in mind that the formulæ (*a*), (*b*), (*c*) are only strictly applicable to fall in vacuo. In air they furnish results more and more remote from the truth, as the velocity increases.

In vacuo, a body thrown upwards from the earth would, on its return, strike the ground with a velocity equal to that with which it was thrown, and the velocity at any given point which it traverses both in its upward and downward course, would be the same in the descent as in the ascent. We see then from above that a cannon-ball would ascend to a height of about 6½ miles. In air, the velocity is slower in the descent than in the ascent, both because the height attained by the projectile is less than it would be in vacuo, and because the velocity acquired in falling from this diminished height is still further diminished by friction in the descent.

One notable difference between fall in vacuo and in air is, that in the latter case the velocity, instead of increasing indefinitely, only increases towards a certain limit which it can never exceed; and if a body be projected downwards with a velocity greater than this, its motion will be retarded. The resistance of the air, in fact, increases with the velocity, and the limit in question is that velocity at which the resistance encountered from the air is exactly equal to the weight of the body. The limiting velocity is not the same for all bodies, but depends on their sizes, densities, and forms.

39A. Motion of Projectiles.—When a body is projected in any direction, its subsequent motion (neglecting the resistance of the air) can be determined by means of the following principles:—

1. The horizontal component of motion will remain unchanged.

2. The vertical component of motion will be the sum or difference (according as the body was projected below or above the horizontal direction) of the motion of a body falling freely, and the vertical component of the initial motion.

These principles are sufficient to determine the position, the velocity, and the direction of motion of the body, after the lapse of any given interval, until it strikes an obstacle.

By reference to Fig. 33, it will easily be understood that, if the direction of projection be horizontal, the curve described will be a parabola; for, in virtue of the first principle, the body describes equal horizontal distances in equal times; and in virtue of the second principle (since the vertical component of initial motion is in this case zero), the vertical spaces described are those of a body falling freely; hence the construction of Fig. 33 is precisely applicable to this case.

If the direction of projection be oblique (that is, neither horizontal nor vertical), the path will still be parabolic. For example, if the body be projected obliquely upwards, we may divide its path into two parts, one described in its ascent, the other in its descent. These two parts will be precisely similar, and at the highest point of the path, where they join, the motion is horizontal. We may regard the curve in Fig. 33 as representing one of the two parts, say the descending part; for the motion, after passing the highest point, must evidently be the same as if the body had been projected horizontally from the highest point with the velocity which it actually has at that point. This velocity is, in fact, the horizontal component of the actual velocity of projection.

If a body be projected vertically downwards, its motion will be the same as if it had fallen from a certain height above. If it be projected vertically upwards, the times of ascent and descent will be equal, and the velocity at any one point will be the same in the descent as in the ascent. At the highest point the body is for an instant stationary; its descent is therefore the motion of a body falling freely. Formulæ (*a*), (*b*), (*c*) of § 38 will apply to the ascent as well as the descent, if in the former case we understand that the time denoted by t is the time reckoned backwards from the instant of attaining the highest point.

Whether the motion be in a vertical line, or in a parabola, the following principle will be found to apply, being in fact the mathematical consequence of the two principles already enunciated, viz. :—

The velocity is the same in the ascent and descent at any two points which are on the same level; and the velocities at any two points, not on the same level, are connected by the law that the difference of their squares is equal to the difference of levels multiplied by $2g$.

40. Composition of Motions.—Principle 2 of last section is a particular case of the following general law:—

When a force acts upon a body already in motion, the subsequent motion will be obtained by compounding (in the same manner as forces are compounded by the parallelogram of forces) the motion which the force would have imparted to the body if initially at rest, with the motion which the body would have had in the absence of the force. The law, as thus stated, is applicable without qualification as long as the force continues to act parallel to a definite direction. In the case of forces which do not fulfil this condition, but gradually change their direction in space, the motion may be approximately determined by dividing the time into intervals so short, that the direction of the force does not change by a sensible amount during any one interval. The motion in each interval can then be determined by the above law. It is frequently possible, by the aid of the higher mathematics, to foresee the exact result to which this tedious method would only approximately lead, but the physical principles on which the investigation is conducted are in all cases those which we have above indicated.

41. Uniform Acceleration.—The general law above stated is exemplified in the case of gravity. In fact, if a denote the space traversed in the first second of a falling body's descent, the space in two seconds is $4a$, and consequently the space traversed in the second second is $3a$. Now the velocity at the end of the first second is $2a$, and this velocity, if it remained unchanged, would cause the space $2a$ to be traversed in the next second. Hence the space $3a$ actually described may be divided into two parts, $2a$ and a, of which the latter is due to the continued action of the force of gravity during the second second.

In like manner the space described in the third second is $9a-4a$, that is, $5a$, which may be divided into two parts, $4a$ and a, of which the latter alone is due to the action of gravity during the third second, the former being due to the velocity $4a$, which the body possessed at the end of the preceding second.

So, again, the velocity acquired by the body in the first second being $2a$, the velocity at the end of any subsequent second will be found to be the sum of two parts, one of which is the velocity at the beginning of the second, and the other is $2a$.

Motion possessing these properties is said to be *uniformly accelerated;* and the force which produces it is a uniformly accelerating force, that is to say, a constant force.

The force of gravity, however, is sensibly constant only within moderate limits of distance from the earth's surface; as we ascend from the earth its intensity continually diminishes, being nearly proportional inversely to the square of the distance from the earth's centre. Hence a body falling *in vacuo* from a great height towards the earth, would not be uniformly accelerated, but would experience continually greater acceleration as it descended. On the other hand, if there were a vacuous shaft down which a body could fall to the centre of the earth, it would fall with continually diminishing acceleration, because the force of gravity in the interior of a solid sphere diminishes as we approach the centre, and becomes zero at the centre itself, where the attractions, being equal in all directions, destroy one another. A body so falling would have its velocity continually increased, but the *rate of increase as measured by the difference between the velocity at the beginning of a second and that at the end of it,* would continually become less. The words italicized in last sentence constitute the definition of *acceleration.* It denotes the rate of increase of velocity, just as velocity itself denotes the rate of increase of distance measured along the path described from a fixed point.

42. Proportionality of Acceleration to Force directly and to Mass inversely.—The general law of motion enunciated in § 40 may be extended to the case of several forces acting simultaneously. The actual motion will be obtained by compounding (on the parallelogram principle) the motions due to the separate forces together with the motion (if any) due to the initial velocity. Just as two forces acting on a point in the same direction are equivalent to a single force equal to their sum; so two motions in the same direction constitute, when compounded, a motion equal to their sum; and this is true, both as regards velocity and space described. Two equal forces acting on a body in the same direction, will therefore produce in any given time double the velocity that one of them would have produced alone. We are thus led to the general principle, that the velocities produced in equal bodies by different forces are simply proportional to the forces.

The above reasoning is not offered as an *à priori* proof of the general principle in question, but as a logical deduction of it from the law of composition of motions due to several forces. The proportionality of velocity to the force which produces it can be proved experimentally by Attwood's machine, and in other ways; and the law of composition in question must be regarded as established by the experimental verification of these and other consequences to which it leads.

From the direct proportionality of velocity to the force by which it is generated, when the mass is given, we may infer the inverse proportionality of velocity to the mass which is set in motion, when the force is given. For instance, if we double the mass, leaving the force unchanged, we may resolve the force into two equal parts, acting one on each half of the mass. Doubling the mass has, therefore, the same effect on the motion as halving the force.

The velocity generated in a given time is thus proportional to the moving force divided by the mass moved; from which it follows that force is proportional to the product of mass by velocity generated in a given time.

When force is expressed in terms of the "absolute" or "invariable" unit, first proposed by Gauss, we can assert that the moving force is *equal* to the product of the mass moved, and the velocity generated in a unit of time.[1] For example, since the force of gravity on a body weighing M pounds causes it to acquire a velocity of g feet in a second, this force is numerically equal to Mg, it being understood that the pound is the unit of mass, and the foot and second the units of space and time.

That the pound is really and strictly a standard of mass is obvious from the consideration that the standard pound is a certain piece of platinum, preserved at the office of the Exchequer in London, and that this piece of platinum would remain a true pound if carried to any part of the earth.

At any one place, since g has a given value, the masses of bodies are proportional simply to their weights, but this proportion obviously does not hold in the comparison of masses at places where the values of g are different. When a body is carried about to different parts of space, its mass, or quantity of matter, remains of course the same, but its weight alters in proportion to the greater or less intensity of gravity;—for instance, at the centre of the earth, regarded as a uniform sphere, its weight would be nothing, This annihilation of its weight would in no way affect its resistance to acceleration. The difference between the mass of a ball of cork, and that of a ball of lead of the same diameter, could in such circumstances be readily detected by the different resistances which they would oppose to attempts to set them in rapid motion, or to check their motion when commenced.

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[1] **The absolute unit of force may be defined as that force which, acting on unit mass for unit time, would generate unit velocity. The force of gravity on unit mass contains g such units.**

It must be regarded as a remarkable fact, and one which could only have been established by experiment, that the two modes of comparing masses perfectly coincide; that is to say, two bodies, even though composed of different materials, if their sizes are so proportioned that they oppose equal resistances to acceleration, will also gravitate with equal forces, as tested by their balancing each other in a pair of scales. This principle is established experimentally by the equal velocities of fall of all bodies in vacuo, and, with much greater accuracy, by the equality of the number of vibrations made in the same time by pendulums of the same size and form, but of different materials.

CHAPTER VI.

THE PENDULUM.

43. The Pendulum.—When a body is suspended so that it can turn about a horizontal axis which does not pass through its centre of gravity, its only position of stable equilibrium is that in which its centre of gravity is in the same vertical plane with the axis and below it (§ 28). If the body be turned into any other position, and left to itself, it will oscillate from one side to the other of the position of equilibrium, until the resistance of the air and the friction of the axis gradually bring it to rest. A body thus suspended, whatever be its form, is called a pendulum. It frequently consists of a rod which can turn about an axis O at its upper end, and which carries at its lower end a heavy lens-shaped piece of metal M called the bob; this latter can be raised or lowered by means of the screw V. The applications of the pendulum are very important: it regulates our clocks, and it has enabled us to measure the intensity of gravity and ascertain the differences in its amount at different parts of the world; it is important then to know at least the fundamental points in its theory. For explaining these we shall begin with the consideration of an ideal body called the *simple pendulum.*

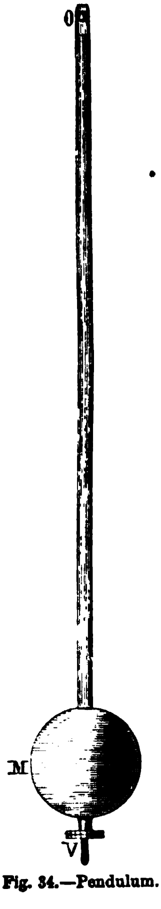

Fig. 34.—Pendulum.

44. Simple Pendulum.—This is the name given to a pendulum consisting of a heavy particle M attached to one end of an inextensible thread without weight, the other end of the thread being fixed at A. When the thread is vertical the weight of the particle acts in the direction of its length, and there is equilibrium. But suppose it is drawn aside into another position, as AM. In this case the weight MG of the particle can be resolved into two forces MC and MH. The former, acting along the prolongation of the thread, is destroyed by the resistance of the thread; the other, acting along the tangent MH, produces the motion of the particle. This effective component is evidently so much the greater as the angle of displacement from the vertical position is greater. The particle will therefore move along an arc of a circle described from A as centre, and the force which urges it forward will continually diminish till it arrives at the lowest point M′. At M′ this force is zero, but, in virtue of the velocity acquired, the particle will ascend on the opposite side, the effective component of gravity being now opposed to the direction of its motion; and, inasmuch as the magnitude of this component goes through the same series of values in this part of the motion as in the former part, but in reversed order, the velocity will, in like manner, retrace its former values, and will become zero when the particle has risen to a point M″ at the same height as M. It then descends again and performs an oscillation from M″ to M precisely similar to the first, but in the reverse direction. It will thus continue to vibrate between the two points M, M″ (friction being supposed excluded), for an indefinite number of times, all the vibrations being of equal extent and performed in equal periods.

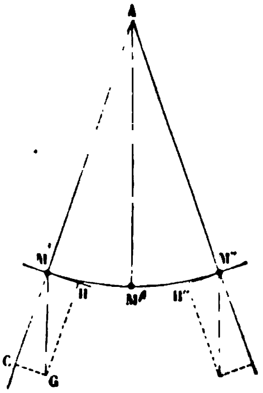

Fig. 35.—Motion of Simple Pendulum.

The distance through which a simple pendulum travels in moving from its lowest position to its furthest position on either side, is called its *amplitude*. It is evidently equal to half the complete arc of vibration, and is commonly expressed, not in linear measure, but in degrees of arc. Its numerical value is of course equal to that of the angle MAM′, which it subtends at the centre of the circle.

The *complete period* of the pendulum's motion is the time which it occupies in moving from M to M″ and back to M, or more generally, is the time from its passing through any given position to its next passing through the same position *in the same direction*.

What is commonly called the time of vibration, or the time of a single vibration, is the half of a complete period, being the time of passing from one of the two extreme positions to the other. Hence what we have above defined as a complete period is often called a double vibration.

When the amplitude changes, the time of vibration changes also, being greater as the amplitude is greater; but the connection between the two elements is very far from being one of simple proportion. The change of time (as measured by a ratio) is much less than the change of amplitude, especially when the amplitude is small; and when the amplitude is less than about 5°, any further diminution of it has little or no sensible effect in diminishing the time. *For small vibrations*, then, *the time of vibration is independent of the amplitude.* This is called the law of *isochronism.*

The time of a single vibration when the amplitude is small is expressed by the formula:

$$T = \pi\sqrt{\frac{l}{g}},$$

l denoting the length of the pendulum, g the intensity of gravity, and π the ratio of the circumference of a circle to the diameter. As regards the units in which T, l, and g are expressed, it must be remarked that if g is expressed in the usual way, as in § 38, l must be expressed in feet, and the value obtained for T will be in seconds.

The formula shows that the time of vibration is proportional to the square root of the length of the pendulum, so that if the pendulum be lengthened four, nine, or sixteen fold, the time will be doubled, trebled, or quadrupled.

45. Experimental Laws of the Motion of the Pendulum.—The preceding laws apply to the simple pendulum; that is to say, to a purely imaginary existence; but they are approximately true for ordinary pendulums, which in contradistinction to the simple pendulum are called compound pendulums. The discovery of the experimental laws of the motion of pendulums was in fact long anterior to the theoretical investigation. It was the earliest and one of the most important discoveries of Galileo, and dates from the year 1582, when he was about twenty years of age. It is related that on one occasion, when in the cathedral of Pisa, he was struck with the regularity of the oscillations of a lamp suspended from the roof, and it appeared to him that these oscillations, though diminishing in extent, preserved the same duration. He tested the fact by repeated trials, which confirmed him in the belief of its perfect exactness. This law of isochronism can be easily verified. It is only necessary to count the vibrations which take place in a given time with different amplitudes. The numbers will be found to be exactly the same. This will be found to hold good even when some of the vibrations compared are so small that they can only be observed with a telescope.

The time of vibration, then, does not depend on the amplitude, and neither does it depend on the material of which the pendulum is composed. From this last fact it follows that gravity acts in precisely the same manner on all substances. It is found, in fact, that balls of the same size, of lead, copper, ivory, &c., suspended by threads of equal length, vibrate in the same time, provided they are large enough to escape sensible retardation from the resistance of the air. This result is virtually identical with that of Galileo's experiment on the fall of bodies (§ 32), and enables us to conclude with certainty that in a vacuum these different pendulums would vibrate in rigorously equal times.

By employing balls suspended by threads of different lengths, Galileo discovered the influence of length on the time of vibration. He ascertained that when the length of the thread increases, the time of vibration increases also; not, however, in proportion to the length simply, but to its square root.

Knowing, then, that the length of the pendulum which beats seconds at Paris is about one metre ($0^m.994$), we see that a pendulum 64 metres long would make a single vibration in eight seconds. This is about the length of the pendulum employed by Foucault at the Pantheon in his celebrated experiments on the rotation of the earth.

This law of lengths experimentally discovered by Galileo, is precisely that which the formula for the simple pendulum gives; an agreement which it was natural to expect, seeing that a small ball suspended by a long string is a practical approximation to the idea of a simple pendulum. When, however, the form of the pendulum departs widely from this, the meaning to be attached to the word *length* ceases to be obvious. It becomes necessary to resort to theoretical investigations, and we shall briefly indicate the results thus obtained.

46. Equivalent Simple Pendulum.—It is demonstrated in treatises on dynamics that a compound pendulum, whatever be its form, always keeps time with a simple pendulum of some determinate length—called the *equivalent simple pendulum;*[1] and whenever the length of a pendulum is mentioned, it is the length of the equivalent simple pendulum that is to be understood.

Any rigid body, oscillating about a fixed horizontal axis, may be regarded as a compound pendulum. That point of the axis which, in the position of equilibrium, is vertically over the centre of gravity of the body, is called the *centre of suspension;* and if we join this point to the centre of gravity, and produce the joining line downwards till its whole length is equal to that of the equivalent simple pendulum, its lower extremity is called the *centre of oscillation.* The body therefore vibrates in the same manner as if its whole mass were collected at the centre of oscillation. This point is always further from the axis than the centre of gravity, and it possesses the following remarkable property:—that if the body were made to vibrate about an axis passing through the centre of oscillation and parallel to the original axis, the time of vibration would be the same as in vibrating about the original axis. In this inverted position, the original centre of suspension becomes the new centre of oscillation, and the original centre of oscillation becomes the new centre of suspension; hence the property in question is commonly called the *convertibility of the centres of oscillation and suspension.*[2]

This important property, which was discovered by Huyghens, furnishes an accurate method of determining the length of the simple pendulum equivalent to a given compound pendulum. This is the principle of Kater's pendulum, which can be made to vibrate about either of two parallel knife-edges, one of which can be adjusted to any distance from the other. The pendulum is swung first upon one of these edges and then upon the other, and, if any difference is detected in the times of vibration, it is corrected by moving the adjustable edge. When the difference has been completely destroyed, the distance between the two edges is the length of the equivalent simple pendulum. It is necessary, in any arrangement of this kind, that the two knife-edges should be in a plane passing through the centre of gravity; also that they should be on opposite sides of the centre of gravity and at unequal distances from it.

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[1] Sometimes the *isochronous* simple pendulum; but it seems better to reserve this adjective and the corresponding noun isochronism for the use in which the latter has been employed in § 44.

[2] The names *centre of suspension* and *centre of oscillation* are not very appropriate, inasmuch as the properties mentioned in the text are properties of two lines rather than of two points: one of the lines being the axis about which the pendulum swings, and the other being a parallel to this axis in the plane containing it and the centre of gravity, and at a distance from it equal to the length of the equivalent simple pendulum. If we call the former the *axis of suspension* and the latter the *axis of oscillation*, we may assert that the axes of suspension and oscillation are convertible.

We see, by what precedes, that the laws of the simple pendulum are applicable to any pendulum, if we understand by its length the length of the equivalent simple pendulum, or, in other words, the distance between the axis and the centre of oscillation.

47. Determination of the Value of *g*.—Returning to the formula for the simple pendulum $T=\pi\sqrt{\frac{l}{g}}$, we easily deduce from it $g=\frac{\pi^2 l}{T^2}$, whence it follows that the value of g can be determined by making a pendulum vibrate and measuring T and l. T is determined by counting the number of vibrations that take place in a given time; l can be calculated, when the pendulum is of regular form, by the aid of formulæ which are given in treatises on rigid dynamics, but its value is more easily obtained by Kater's method, described above, founded on the principle of the convertibility of the centres of suspension and oscillation.

48. Variations in the Intensity of Gravity.—Pendulum observations, which have been taken in great numbers in various places, have established the result that the intensity of gravity varies over the surface of the earth. At London the value of g is 32·182; it increases in approaching the pole, and diminishes in going towards the equator. These variations, however, are not very considerable, as the following table of the values of g shows:—

Value at the equator	32·088.
Value in latitude 45°	32·171.
Value at the poles	32·253.

Its value for any place is approximately given by the formula:—

$$g=32{\cdot}088\,(1+{\cdot}005133\,\sin^2\lambda)\left(1-\frac{2h}{R}\right),$$

λ denoting the latitude of the place, h the height above the level of the sea,[1] and R the earth's radius, which is 20,900,000 feet. Local peculiarities prevent the possibility of laying down any general formula with precision, and the exact value of g for any place can only be ascertained by observations on the spot.

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[1] The correcting factor for elevation $\left(1-\frac{2h}{R}\right)$ is proper to be used in determining the value of g in mid-air; for example, in the positions reached by balloons. On the summit of a mountain, or of an elevated plateau in the interior of a continent, the value of g is greater than at the same level in mid-air above the ocean, owing to the attraction of the excess of land which projects above sea-level. See a paper by Dr. Young in the *Phil. Trans.* for 1819.

49. Centrifugal Force.—There are two distinct causes of these differences in the intensity of gravity: the first is what is called *centrifugal force*. Consider a material point M attached to the extremity of a thread OM, and suppose an impulse to be given it in a direction perpendicular to the thread. At each instant, in virtue of inertia, the point tends to move along a tangent to the circle; but the thread prevents this movement from taking place, by drawing the point, which in its turn reacts on the thread and stretches it with a certain force which has received the name of centrifugal force. It is clear that we may dispense with the thread, if we suppose an attractive force equal to that which the thread would exert directed towards O. This force, which, by combining its effect with that of the initial impulse, would produce circular motion, is called *centripetal force.* It is evidently equal and opposite to centrifugal force.

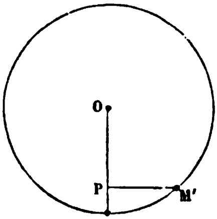


Fig. 36.—Centrifugal Force.

The amount of the centripetal force in any given case of circular motion can be easily calculated. Let MP represent the space which the material point would describe in a certain small time t under the action of the centripetal force alone. This motion, combined with the motion due to the velocity which the particle possessed at M, gives the actual motion in the arc MM′, provided that the time considered and the distance moved in that time are so small that the directions of the centripetal forces at M and M′ are sensibly parallel—in other words, provided the arc MM′ is very small in comparison with the radius OM. In this case, by § 40, PM′, or rather a line equal and parallel to it drawn through M, represents the space due to the velocity which the point had at M, and is therefore equal to vt, v denoting the velocity. Now if we denote by ϕ the acceleration due to the centripetal force, this acceleration may be regarded as uniform during the time t, and we have by § 38, $\mathrm{MP}=\frac{1}{2}\phi t^2$, whence $\phi=\frac{2\mathrm{MP}}{t^2}$. But PM'^2, that is v^2t^2, is (by *Euclid*, iii. 35) equal to $2r.\mathrm{MP}$, r denoting the radius of the circle; therefore $\frac{2\mathrm{MP}}{t^2}$ is equal to $\frac{v^2}{r}$. Consequently $\phi=\frac{v^2}{r}$; that is, the centripetal force upon a particle revolving with velocity v (feet per second) in a circle of radius r (feet) is equal to a force which, acting continuously upon the same particle initially at rest, would in one second give it a velocity of $\frac{v^2}{r}$. The centripetal force upon a mass of m pounds is $\frac{mv^2}{r}$ Gaussian pound units (42), and is equal to the weight of $\frac{mv^2}{gr}$ pounds.

When the circular movement is uniform, we can give the formula for ϕ a more convenient form. In fact if we denote by T the time of revolution, and by π the ratio of circumference to diameter, we have $v=\frac{2\pi r}{T}$, and $\phi=\frac{v^2}{r}=\frac{4\pi^2 r}{T^2}$.

Suppose, for example, a weight of 50 pounds attached to one end of a string 3 feet long, and revolving uniformly in a circle about the other end of the string, at the rate of 40 revolutions per minute. The time of revolution is here $\frac{3}{2}$ of a second, and the force required to be exerted by the string is equal to the weight of

$$\tfrac{50}{32\cdot2}\times 4\pi^2\times\tfrac{4}{9}\times 3=81\cdot7 \text{ pounds};$$

and if the string be not strong enough to bear this weight it will break, and the body will fly off at a tangent.

50. Different Effects of Centrifugal Force.—Several experiments on centrifugal force are exhibited in courses of physics. For example, a rod AB (Fig. 37), passing through two ivory balls M, M′, is set in rotation in a horizontal plane by means of the mechanism shown in the figure. The balls are then seen to move towards the extremities of the rod. If a spring is placed beside one of the balls M′, as shown in the figure, it will be pressed with a force which is precisely equal to the centrifugal force of the ball.

The centrifugal railway (Fig. 38) shows a curious effect of this force. A carriage, starting from A, descends the inclined rails, and by following the course of the rails, which here forms a spiral convolution, to B, rises to C, and descending again on the side next A, passes on the further side of B and finally arrives at D. There is therefore an instant in the motion when the carriage is bottom upwards at the top of the convolution, and remains in contact with the rails in opposition to the force of gravity. The explanation is easy. The carriage attains a certain velocity in descending the incline which forms the first portion of its path. In virtue of its inertia it tends to move with this velocity in a tangential direction, but being compelled to follow the curved rails which lie in its way, it reacts upon them with a force whose amount is given by the formulæ of last section. If the point A is sufficiently elevated above C, this force will prevail over the weight of the carriage and keep it pressed against the rails. It may be easily shown that to secure this result the height of A above B must be to the height of C above B in a greater ratio than 5 to 4.

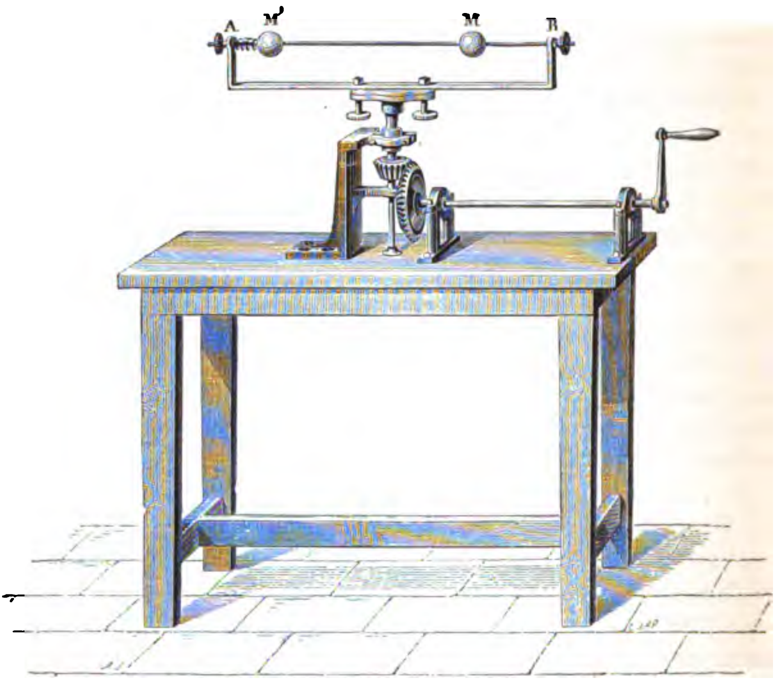


Fig. 37.—Centrifugal-force Apparatus.

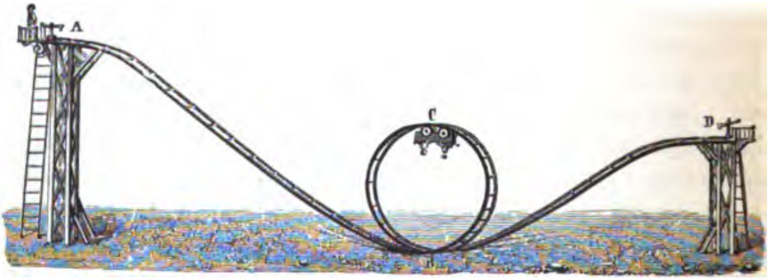
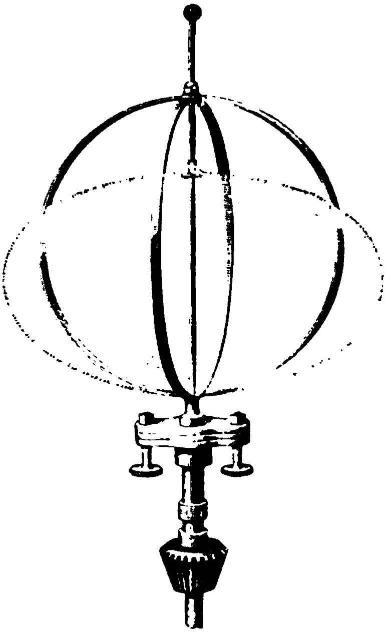


Fig. 38.—Centrifugal Railway.

If the apparatus shown in Fig. 39, consisting of a kind of sphere formed of four flexible springs, be mounted on the whirling table and made to revolve, it will be seen to become flattened, the effect being more decided as the rotation is more rapid. This result is due to centrifugal force, which gives all parts of the springs a tendency to recede from the axis of rotation, and especially those parts which are already most distant from it. This experiment may be regarded as illustrating the manner in which the earth, when in a fluid state, acquired its present form, bulging at the equator and flattened at the poles.



39. Oblateness of the Earth.

51. The influence of centrifugal force in modifying the effect of gravity is easily deduced from what precedes. The various bodies which are on the surface of the earth are retained upon it by gravity, and a certain portion of the force of gravity is expended in constraining them to move in circular paths. The modification thus produced in the apparent force and direction of gravity is the same as if a force equal and opposite to the force thus expended were compounded with the force of gravity proper; that is to say, apparent gravity is the resultant of gravity proper and centrifugal force.

At the equator, centrifugal force is directly opposed to gravity proper, and is therefore to be simply subtracted from it. Let r be the earth's radius, T the length of a sidereal day (or the time of the earth's revolution), then the intensity of centrifugal force at the equator is $\frac{4\pi^2 r}{T^2} = \frac{2\pi r . 2\pi}{T^2}$. Putting for $2\pi r$, or the earth's circumference, its value in metres, viz. 40,000,000, and for T its value in seconds, 86,164, we obtain for the intensity in question the value ·033. Now the intensity of gravity, expressed with reference to the metre and second as units, is about 9·8. Hence centrifugal force at the equator is $\frac{3.39}{980}$ or about $\frac{1}{290}$ of the force of gravity, and apparent gravity is less than gravity proper at the equator by this amount. The exact amount of diminution is more, nearly $\frac{1}{289}$ of gravity proper; and since 289 is the square of 17, and centrifugal force varies as the square of the velocity, we see that if the rate of the earth's revolution were increased seventeenfold, bodies at the equator would lose their weight.

As we recede from the equator, the intensity of centrifugal force diminishes because the distance from the earth's axis diminishes; and, at the same time, the direction of centrifugal force, being always perpendicular to this axis, becomes less directly opposed to gravity, so that only one of its two components is subtractive. For this double reason, the effect of centrifugal force in diminishing the apparent force of gravity becomes continually less as we recede from the equator. As far then as the disturbing effect of centrifugal force is concerned, the apparent intensity of gravity should be least at the equator, and should continually increase towards the poles.

We may add that centrifugal force (except at the equator) affects not only the amount, but also the direction, of apparent gravity, since the resultant of two forces which are not directly opposed does not coincide in direction with either of them. The angle which the actual vertical (indicating the direction of apparent gravity), makes with the direction which the vertical would have if the earth were at rest, varies with the latitude; at Paris, where its value is nearly a maximum, it is between 5 and 6 minutes.

52. Universal Gravitation, Earth's Mean Density.—Terrestrial gravitation is only a particular case of universal gravitation. Newton established, by researches extending over more than twenty-five years, that the movements of the planets around the sun, and of the satellites around the planets, could be explained by assuming the existence of a mutual attraction, which, taken in conjunction with an initial impulse, determines the paths which these bodies describe. This attraction is jointly proportional to the masses of the mutually attracting bodies, and varies inversely as the square of their mutual distance. By the aid of this assumption, astronomers have succeeded not only in explaining all the diversities of motion which the solar system exhibits, but in calculating the positions of the celestial bodies at distant times, both past and future, with marvellous accuracy. The study of the movements of the heavenly bodies, which has thus been reduced to a branch of applied mathematics, is called Physical Astronomy.

It is therefore natural to look upon terrestrial gravity as a particular case of universal gravitation, and to regard the fall of a body as a consequence of the attraction exerted on it by the different parts of the terrestrial globe. This identity is in fact established by numerous proofs. Now it is obvious from the symmetry of the component attractions, that the resultant attraction of a sphere exerted at any point of its surface must be in the direction of the radius, and must therefore be perpendicular to the surface—a result which our every-day experience confirms in the case of the earth's gravitation. Theory also shows that the attraction of an oblate spheroid is greater at the poles than at the equator. In fact, taking into account both the oblateness of the earth and centrifugal force, theory agrees with observation in indicating that apparent gravity increases in going from the equator towards the poles by an amount proportional to the square of the sine of the latitude.

The hypothesis of attraction then explains all the phenomena of gravity: we may add that the existence of attraction at the surface of the earth has been experimentally demonstrated. Maskelyne, Hutton, and Playfair, in the celebrated Schiehallien experiment, proved that the plumb-line experienced, on either side of the mountain of that name, a deviation towards the mountain; and a similar result was established by Sir Henry James at Arthur's Seat, near Edinburgh. Cavendish, by means of a very sensitive torsion-balance, showed that two large spheres of lead exercised a sensible attraction upon two small spheres, and was able to measure the amount of this attraction so precisely as to deduce, from the comparison of it with the earth's attraction, that the mean specific gravity of the earth is 5·5. Cavendish's experiment has been repeated by Reich in Germany and by Baily in this country, with nearly coincident results. The Schiehallien experiment, which was the earliest attempt to determine the earth's mean density, gave 5 for the result, and Mr. Airy's experiment at Harton Colliery gave a result exceeding 6. Baily's result, obtained by Cavendish's method, with some improvements in the details, is generally accepted as the best determination; it is 5·67.

53. Variation of Gravity with Height.—It follows from the identity of attraction and gravity that this force diminishes as we rise above the earth's surface, but the heights with which we commonly have to do in our experiments are so small in comparison with the earth's radius that the resulting differences of force are quite inappreciable. In the case, however, of large differences of height, the variation of force can be detected; for instance, it is easy to establish experimentally that the intensity of gravity is less on the summit of a mountain than at its base. Theory shows that the attraction of a sphere upon external points is the same as if its mass were collected at the centre. Presuming this to be true in the case of the earth, it may be shown that, in ascending to any height above the earth's surface which is small in comparison with the earth's radius, the diminution in the force of gravity is twice as great in comparison with the whole force of gravity as that height is in comparison with the earth's radius. This explains the origin of the factor $1 - \frac{2h}{R}$ in the formula of § 48.

In penetrating into the interior of the earth, the law of the variation of gravity is more complex. If the earth were homogeneous, its attraction would continually diminish in penetrating towards the centre, and would, if the earth were also truly spherical, be simply proportional to the distance from the centre. But in fact the density is greater in the central than in the superficial parts of the earth, the mean density being, as we have seen, about 5·7, while the density of the superficial beds is only about 3. This augmentation of density tends to increase the attractive force as we descend; and it will depend upon the law according to which the density varies which of these two opposite tendencies will prevail. Mr. Airy, in his experiment at Harton Colliery, found the intensity of gravity at the bottom of the mine, 1256 feet below the surface, to be greater than at the surface by about one part in 19,000. Supposing that similar results would be obtained in other places, it follows that, in penetrating the earth, gravity must go on increasing from the surface down to a certain depth, where it becomes a maximum, and from which it decreases down to the earth's centre, where it becomes zero, the equal and opposite attractions there destroying one another.

53 A. Simple Vibrations.—The motion of a pendulum vibrating in a small arc, may be taken as the type of a class of motions which very extensively prevail in nature, and are of great importance in many departments of physical science. They have been called by different writers *simple vibrations, pendulum-like vibrations, sine-like vibrations,* and *simple harmonic motions.* We shall employ the first of these designations, and define a simple vibration to be the movement of a point to and fro along one line (not necessarily straight), with acceleration simply proportional to the distance of the point from the bisection of this line. If the line of motion be curved, the distance is to be measured along the curve, and in this case the acceleration referred to is the tangential acceleration along this curve. In every case, and in all parts of the motion, the acceleration of the moving point urges it *towards* the middle point of its path, becoming zero only at the instant of passing this middle point. In the majority of cases that require to be considered, the line of motion is straight, or approximately straight.

As examples of simple vibrations, we may instance, in addition to that of a pendulum above quoted, the motion of a point on either prong of a tuning-fork, or of a point in a musical string, when vibrating so as to produce a pure note. In these cases, the force urging any point of the fork or string towards the position of equilibrium, varies directly as the distance of the point at any time from this position, and by the second law of motion acceleration is proportional to force.

In the case of the simple pendulum (§ 43), the acceleration of the heavy particle when the thread makes an angle θ with the vertical is $g \sin \theta$ (see § 53D), which when the arc of vibration is small may be taken as equal to $g\theta$, that is, $g\frac{x}{l}$, x denoting the length of arc measured from lowest point, and l the length of the string. It is therefore proportional to x, the distance of the particle from the position of equilibrium.

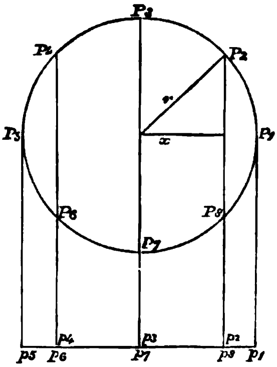


Fig. 39A. Projection of Circular Motion.

53 B. When simple vibration is executed in a straight line, it corresponds to the projection of uniform circular motion (Fig. 39A); that is to say, if a point P move with uniform velocity round the circumference of a circle, and if Pp be the perpendicular let fall from P upon a fixed straight line in the plane of the circle, then p, the foot of this perpendicular, will execute simple vibration.

To prove this proposition, remark that by § 49 the point P is to be regarded as constantly falling away from a tangent in obedience to acceleration directed towards the centre of the circle, the amount of the acceleration being $\frac{4\pi^2}{T^2}.r$, where r denotes the radius of the circle. This acceleration, being directed from P towards the centre of the circle, may be resolved into two components, one parallel to the line of motion of p, and the other perpendicular to it. The former of these, which is obviously the acceleration of p, is $\frac{x}{r}$ of the whole acceleration, x denoting the distance of p from its mean position, that is, from the foot of a perpendicular let fall from the centre of the circle. The acceleration of p is therefore $\frac{4\pi^2}{T^2}x$, and is proportional to x.

53 C. We may hence prove that simple vibrations are isochronous. For if the acceleration be expressed by μx, x denoting displacement from mean position, and μ any constant, we see, by the preceding paragraph, that the vibration keeps time with an imaginary point moving uniformly round a circle, a complete vibration being performed in the same period as a complete revolution; and if T denote this period, we have $\mu=\frac{4\pi^2}{T^2}$; whence $T=\frac{2\pi}{\sqrt{\mu}}$, an expression which is independent of the amplitude of vibration.

Applying this result to a pendulum vibrating in a small arc, we have $\mu=\frac{g}{l}$ (see § 53A); hence $T=\frac{2\pi}{\sqrt{\mu}}=2\pi\sqrt{\frac{l}{g}}$, which is the time of a complete (or double) vibration.[1]

To understand the reason of the isochronism of simple vibrations, we have only to remark that, if the amplitude be changed, the velocity at corresponding points (that is, points whose distances from the middle point are the same fractions of the amplitudes) will be changed in the same ratio. For example, compare two simple vibrations in which the values of μ are the same, but let the amplitude of one be double that of the other. Then if we divide the paths of both into the same number of small equal parts, these parts will be twice as great for the one as for the other; but if we suppose the two points to start simultaneously from their extreme positions, the one will constantly be moving twice as fast as the other. The number of parts described in any given time will therefore be the same for both.

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[1] The mathematical reader will remark that our definition of simple vibration corresponds to the differential equation $\frac{d^2x}{dt^2}=-\mu x$, and that the property proved in § 53B corresponds to the solution of this equation, which is $x=a\cos(t\sqrt{\mu}-e)$, a and e being arbitrary constants, the former called the *amplitude* and the latter the *epoch* of the motion. The quantity $(t\sqrt{\mu}-e)$ is called the argument, and it is obvious that x and its successive differential coefficients will all remain unaltered if the argument be increased by 2π, that is, if t be increased by any multiple of $\frac{2\pi}{\sqrt{\mu}}$. Hence the motion always repeats itself after the interval $\frac{2\pi}{\sqrt{\mu}}$, which is therefore the *period* of a complete vibration.

In the case of vibrations which are not simple, it is easy to see (from comparison with simple vibration) that if the acceleration increases in a greater ratio than the distance from the mean position, the period of vibration will be shortened by increasing the amplitude; but if the acceleration increases in a less ratio than the distance, as in the case of the common pendulum vibrating in an arc of moderate extent, the period is increased by increasing the amplitude.

53 D. Cycloidal Pendulum.—We saw in § 33 that the effective component of gravity upon a particle resting on a smooth inclined plane was proportional to the sine of the inclination. The acceleration of a particle so situated is in fact $g \sin \alpha$, if α denote the inclination of the plane. When a particle is guided along a smooth curve its acceleration is expressed by the same formula, α now denoting the inclination of the curve at any point to the horizon. This inclination varies from point to point of the curve, so that the acceleration $g \sin \alpha$ is no longer a constant quantity. The motion of a common pendulum corresponds to the motion of a particle which is guided to move in a circular arc; and if x denote distance from the lowest point, measured along the arc, and r the radius of the circle (or the length of the pendulum), the acceleration at any point is $g \sin \frac{x}{r}$. This is sensibly proportional to x so long as x is a small fraction of r; but in general it is not proportional to x, and hence the vibrations are not in general isochronous.

To obtain strictly isochronous vibrations we must substitute for the circular arc a curve which possesses the property of having an inclination whose sine is simply proportional to distance measured along the curve from the lowest point. The curve which possesses this property is the cycloid. It is the curve which is traced by a point in the circumference of a circle which rolls along a straight line. The cycloidal pendulum is constructed by suspending an ivory ball or some other small heavy body by a thread between two cheeks (Fig. 39B), on which the thread winds as the ball swings to either side. The cheeks must themselves be the two halves of a cycloid whose length is double that of the thread, so that each cheek has the same length as the thread. It can be demonstrated[1] that under these circumstances the path of the ball will be a cycloid identical with that to which the cheeks belong. Neglecting friction and the rigidity of the thread, the acceleration in this case is proportional to distance measured along the cycloid from its lowest point, and hence, by last section, the time of vibration will be strictly the same for large as for small amplitudes. It will, in fact, be the same as that of a simple pendulum having the same length as the cycloidal pendulum and vibrating in a small arc.

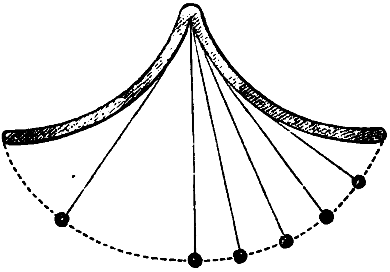

Fig. 39B.—Cycloidal Pendulum.

Attempts have been made to adapt the cycloidal pendulum to clocks, but it has been found that, owing to the greater amount of friction, its rate was less regular than that of the common pendulum. It may be remarked, that the spring by which pendulums are often suspended has the effect of guiding the pendulum bob in a curve which is approximately cycloidal, and thus of diminishing the irregularity of rate resulting from differences of amplitude.

53 E. Centre of Mass, or Centre of Inertia.—The point which has been mentioned in Chapter iv., under the name of centre of gravity, possesses important properties besides those which depend upon its being the point through which the resultant force of gravity passes.

It may be demonstrated by geometry that if a body be divided into a number of equal elements (equal in mass, not necessarily in size), each of them being so small in all its dimensions that it may be treated as a material point, there is a certain point in the body such that its distance from any plane whatever is equal to the mean distance of all the elements from the same plane—that is, to the sum of all the distances divided by the number of elements.[2] This point is called the *centre of mass*.

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[1] Since the evolute of the cycloid is an equal cycloid.

[2] If the plane cuts the body, distances on one side of the plane must be reckoned positive and on the other negative, and the sum in question must be the algebraical sum.

If a set of equal and parallel forces act one on each element in the same direction, their resultant will pass through the centre of mass. Inasmuch then as the force of gravity upon a body is made up of such a set of equal and parallel forces, the centre of mass is also the *centre of gravity.*

Again, if a body at rest be set in motion in such a manner that all its points move in parallel straight lines with equal velocities, the resistance which inertia opposes is composed of a set of such equal and parallel resistances; its resultant therefore passes through the centre of mass, which is hence called the *centre of inertia.* It is evident that the force requisite for producing such motion in a body must be equivalent to a single force applied at this point, and the same remark applies to the force necessary for destroying such motion and bringing the body to rest.

Conversely, if a force be applied to a body at its centre of mass, or in a line passing through the centre of mass, the body will be set in motion in such a way that all its points will have equal and parallel velocities, their common direction being parallel to the line of action of the force.

A force applied to a free body in a direction not passing through the centre of mass will produce movement of the centre of mass combined with rotation of the body about the centre of mass. Of these two components of motion, the former will be the same as would be produced by the given force if it acted in a direction passing through the centre of mass; and the latter—the rotation—will be the same as if the centre of mass were fixed.

A couple applied to a free body will produce rotation of the body about the centre of mass, but will not produce any motion of the centre of mass.

When a body moves so that all its points are at every instant travelling in the same direction (that is in parallel directions and towards the same parts) and with equal velocities, it is said to have a movement of translation. All straight lines in a body so moving remain always parallel to their original positions, and conversely; hence this property may be taken as the definition of movement of translation. Every possible motion of a rigid body can be resolved into motion of translation accompanied by motion of rotation, and the resolution can always be so effected that the axis of rotation at any instant shall be parallel to the direction of the movement of translation. It is always possible, and is generally convenient, to regard the motion of a rigid body under the action of any forces as compounded of a motion of translation of the body as a whole, and a rotation of the body about an axis passing through its centre of mass.

53 F. When two or more bodies, or parts of the same body, are free to move, it is impossible for any action exerted between them to alter the motion of their common centre of mass. It is also impossible for such action to alter the total angular momentum about the centre of mass. For example, when an animal is either jumping or falling, no movement that it can make in mid-air without touching other bodies can either alter the motion of its centre of gravity, or cause part of its body to rotate in one direction without causing the remainder to rotate in the opposite direction.

The recoil of fire-arms depends on the same principle. Whatever force the gases which are produced by the explosion of the powder exert in propelling themselves and the ball forwards, they must always exert the same force for the same time in urging the gun backwards. If a shell explodes at an elevation in the air, then, neglecting the effect of the wind, the common centre of gravity of the fragments of the shell and the products of explosion will describe the same path and with the same velocity which the centre of gravity of the shell would have had if there had been no explosion.

This principle is of great importance in the movement of the heavenly bodies. For example, neglecting any general movement which the solar system as a whole may have in space, we are entitled to assert that in whatever direction the common centre of gravity of the planets may be moving at any time, the centre of gravity of the sun must be moving in a parallel and opposite direction; inasmuch as the centre of gravity of the whole system, consisting of sun and planets, remains always at rest.

53 G. Moment of Inertia.—When a body is capable of turning about a definite axis, its inertia opposes resistance to any force which may be applied to set it in rotation, and, if it has once been set in rotation, its inertia gives it a tendency to continue rotating with constant velocity, so that it can only be brought to rest by the action of opposing force.

The power of a force as regards its tendency to produce rotation about an axis is called the *moment of the force about the axis,* and is measured by the *product of the force and the arm at which it acts.* If the body is acted on by more forces than one, the sum of the moments of the several forces about the axis is the measure of the total tendency to produce rotation, and is called the total moment of all the forces. It is to be understood that if some of the forces tend to make the body turn in one direction and others in the opposite direction, the moments of the one set must be reckoned positive and ot the other negative, and the sum in question must be the algebraical sum.

On the other hand, the resistance which the inertia of the rotating body opposes to the action of forces tending to accelerate or retard its rotation is called its *moment of inertia.* The rate at which the angular velocity changes is equal to the total moment of the forces divided by the moment of inertia of the body.

The moment of inertia of a body about an axis is the sum of all the terms which are obtained by multiplying each element by the square of its distance from the axis.

The *angular momentum* of a rotating body is a name given to the product of the moment of inertia and the angular velocity. Equal forces acting at equal arms for the same time upon different bodies produce equal angular momenta.

The *energy of rotation* of a rotating body is half the product of its moment of inertia and the square of its angular velocity. Equal amounts of work spent upon different bodies in producing rotation yield equal amounts of energy of rotation.

These ideas may be illustrated by a reference to the use of fly-wheels in machinery. A fly-wheel is a wheel which, by means of its inertia, acts as an equalizer of the motion of the machine to which it is attached, resisting, to an extent measured by its moment of inertia, all sudden changes of velocity. It is chiefly employed in cases where either the driving power or the resistance to be overcome is liable to rapid alternations of magnitude. When the power is in excess of the resistance the motion of the fly-wheel is accelerated, and the energy thus accumulated is given out again when the resistance is in excess of the power, the inertia of the fly-wheel then assisting to overcome the resistance, while at the same time the velocity of the wheel is diminished.

Fly-wheels are always made with heavy rims, the rest of the wheel being usually as light as is compatible with the requisite strength. This arrangement is adopted with the view of obtaining the greatest possible moment of inertia; for if all the matter of the wheel were collected at its rim, the moment of inertia would be equal to the mass multiplied by the square of the radius.

53 H. Centre of Percussion.—We have already seen that when a force acts upon a rigid body in a direction not passing through the centre of mass, it tends to produce a motion consisting partly of translation and partly of rotation of the body about the centre of mass. This principle remains true when the force is applied in the shape of a blow, and may easily be tested experimentally in a rough way by suspending a straight rod by a long string attached to one end and striking it with a hammer in different points. If the rod be struck in a horizontal direction near its top, its bottom will at the instant of the blow move in the opposite direction, and if it be struck near the bottom the top will fly back. In each case there is some intermediate line at right angles to the direction of the blow, which neither moves forwards nor backwards at the instant of the blow, while points on opposite sides of it move in opposite directions. With reference to this line, regarded as an instantaneous axis of rotation, the point at which the body was struck is called the centre of percussion. It admits of proof that the centre of percussion with respect to any axis is the same as the centre of oscillation.

When a body is suspended so that it can rotate about an axis, if we desire to strike it without jarring the axis, it is necessary that the blow should be administered at the centre of percussion, and this remark is equally true if the body in question be the striking instead of the struck body. For example, the proper point of a bat for striking a ball so as not to jar the hands is the centre of percussion of the bat with respect to an axis passing through the hands.

53 I. Momentum, Energy of Motion.—The product of the mass and velocity of a body is called the momentum of the body. If equal forces act upon unequal masses *for the same time*, the momenta generated are equal. This principle applies to the recoil of fire-arms, supposing the gun to be free to move.

On the other hand, if equal forces act upon unequal masses originally at rest, *through equal distances* (and therefore do equal amounts of work upon them), the momenta generated will be unequal; the greater mass will receive the greater momentum. Equal products will however be obtained in this case, if we multiply each mass by the square of its velocity. In the case of a falling body, we have seen that the velocity acquired in falling through a height s is $v = \sqrt{2gs}$, whence $\frac{1}{2}v^2 = gs$, and if the mass of the body (in lbs.) is m, we have $\frac{1}{2}mv^2 = gms$. Now, the force which produces the descent is the weight of m lbs., which is equivalent to gm Gaussian units of force, and as the space through which the force works is s, the work done is gms, which is the second member of the above equation, and is equal to $\frac{1}{2}mv^2$. We see, then, that in this case the work done, expressed in Gaussian units of work (of which a foot-pound contains g), is equal to half the product of the mass (in pounds) and the square of the velocity. This principle is perfectly general, and may be extended to bodies already in motion as well as to bodies initially at rest by substituting for "$\frac{1}{2}mv^2$," "the change produced in the value of $\frac{1}{2}mv^2$." Conversely, since the height to which a body will rise when thrown upwards with a given velocity is the same as the height from which it must fall to acquire that velocity, it follows from the foregoing equations that the value of $\frac{1}{2}mv^2$ at the commencement of the ascent is equal to the work which gravity would do upon the body during its descent from the height to which it rises to the point from which its ascent commenced; and if we denote the product of force and distance moved in the case when the direction of the motion is opposite to that of the force, by the name *negative work*, we may assert that the diminution which occurs in the value of $\frac{1}{2}mv^2$ during the whole ascent or during any part of it is equal to the negative work done upon the body by gravity during that part of the motion.

It is in this sense that work and motion are said to be convertible, and the product $\frac{1}{2}mv^2$, whose changes of value are always equal to the work done upon the body, is called the *energy of motion*, or the *kinetic energy* of the body. This equality subsists not only for the case of gravity, but for all forces whatever: we may assert universally (neglecting for the present the effects of friction and molecular changes), that when a body of mass m moves at one time with a velocity v_1, and at a subsequent time with velocity v_2, the whole amount of work done upon the body during the interval (the algebraic sum being taken if any of the work is negative) is equal to $\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$.

The product $\frac{1}{2}mv^2$ has sometimes been called the *accumulated work* in a body, or the work stored up in the body, inasmuch as a moving body is able in virtue of its motion to overcome resistance through such a distance that the work done (or product of resistance and distance through which it is overcome) will be equal to $\frac{1}{2}mv^2$. We have seen one example of this in the case of a body thrown upwards, which overcomes the resistance mg of gravity through a height s such that $mgs = \frac{1}{2}mv^2$.

53 J. Energy of Position, or Potential Energy.—We have now to introduce a new idea, which is of comparatively recent origin, and plays an important part in modern dynamics. When a body of mass m is at the height s above the ground, which we will suppose level, we can cause it to acquire a certain velocity v such that $\frac{1}{2}mv^2 = mgs$ by simply allowing it to fall to the earth. The position of a body in this instance confers the power to obtain motion, and therefore kinetic energy; and as we have just seen, kinetic energy can be made to yield work. A body in an elevated position may therefore be regarded as a reservoir of work: the water in a mill-dam is, in fact, a case in point; and for this reason such a body is said to possess *energy of position*, or, as it is more commonly called, *potential energy*. In contradistinction from this latter name, the energy which a moving body possesses in virtue of its motion is sometimes called *actual energy*.

It should be remarked that energy of position is essentially relative, depending on the position of one body with reference to one or more others. In the case just considered the other body is the earth. In order to be philosophically correct in our language, we should speak not of the potential energy of a body, but rather of the potential energy of two or more bodies with reference to each other in a given relative position; or more briefly, of the potential energy of a certain relative position of the bodies.

It must also be remarked that while we can speak with precision of the *difference* between the potential energies of two specified positions, we cannot in strictness assign a definite value to the potential energy of one specified position unless we know the limits to the possible motion of the bodies in obedience to their mutual forces. For example, in the case just considered—that of a body at a certain height above level ground—the present position of the body is compared with that which it will occupy when it lies upon the ground. But a shaft might be sunk in the ground, and with reference to the bottom of this shaft a body lying on the surface of the ground would possess a certain amount of potential energy, which must be added to that above considered to obtain the potential energy of the body in its first position as compared with the position which it would occupy when lying at the bottom of the shaft.

Whenever motion takes place in obedience to natural forces, the increase or diminution of potential energy which takes place in passing from one position to another is always exactly compensated by an opposite change in the total amount of kinetic energy; from which it follows that the sum of potential and kinetic energies remains unchanged. Whenever kinetic energy is increased at the expense of potential energy, the forces concerned do an amount of positive work equal to the amount by which the former is increased or the latter diminished. On the other hand, whenever potential energy is increased at the expense of kinetic energy, the forces do negative work equal in absolute value to the energy thus transferred. Instances of the former kind of transfer are furnished by the motion of a falling body and the motion of a planet from aphelion to perihelion; instances of the latter kind are furnished by the motion of a body thrown upwards, and the motion of a planet from perihelion to aphelion.

53 x. Effect of Friction upon Transformation of Energy.—Thus far we have been supposing that frictional resistances are neglected. Friction, in fact, causes an apparent loss of energy, but this loss is accompanied by a generation of heat which is itself a form of energy, and a definite amount of heat is produced by each unit of work thus apparently wasted. Conversely, whenever heat is employed as a motive power (in the steam-engine, for example), a quantity of heat is destroyed equivalent, on the same scale, to the work produced.

Another kind of energy is developed when friction is employed as a means of generating electricity. In this case the potential energy of electrical attraction which is called into existence is the precise equivalent of the work spent in producing it.

Similar principles apply to all other cases in which energy is apparently destroyed. *Any particular form of energy may be destroyed, but only on condition of an equivalent amount of energy in some other shape coming into existence.* The whole amount of energy in the universe cannot undergo either increase or diminution. This great natural law is called the *principle of the conservation of energy.*

The exact nature of the various forms of molecular energy, such as heat, light, electricity, magnetism, and chemical affinity, is not at present known, but we run little risk of error in affirming that they all consist either of peculiar kinds of molecular motion or of peculiar arrangements of molecules as regards relative position. They must therefore fall under one or other of the two heads "energy of position" and "energy of motion."

CHAPTER VII.

THE BALANCE.

54. The object of the balance is the measurement of the weights of bodies. It consists essentially of a rigid lever AB called the beam, movable about an axis O at the centre of its length. This axis rests upon two planes, and as it is a little above the centre of gravity, the beam takes a position of stable equilibrium. An index needle attached to the beam traverses a graduated arc, and indicates the position of equilibrium of the beam by pointing to zero.

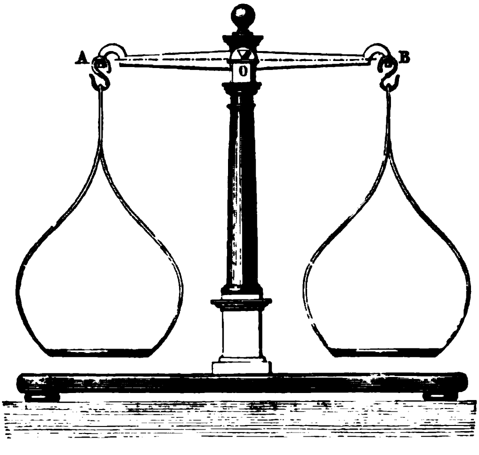

Fig. 40.—Balance.

This equilibrium will not be disturbed if we suspend from the extremities of the beam two scale-pans of the same substance, form, and dimensions. Neither will it be disturbed if in these scale-pans we place bodies of equal weight. And conversely, if two bodies placed in the two scale-pans equilibrate each other, their weights are equal. This, then, is the principle of the well-known use of the balance.

55. Correctness of the Balance.—It is necessary to the validity of the preceding reasoning that the scale-pans should be suspended at exactly the same distance from the axis, or, in other words, that the arms of the balance should be rigorously equal in length. This is known to be the case if the needle points to zero both when the scale-pans are empty and when they are loaded with two bodies of equal weight. If we have not two weights exactly equal, it is sufficient to place any body whatever in one of the scale-pans, and equilibrate it by placing so much matter in the other scale as will bring the index to zero; if we then interchange the contents of the two scale-pans, the needle should still point to zero. If it does not, the reason is that the arms are not of equal length. Easy as it is, however, to make the arms of approximately equal length, it is exceedingly difficult to make them rigorously equal; and accordingly, whenever great accuracy is required, the method of double weighing is employed, which enables us to obtain the exact weight, even when the arms of the balance are slightly unequal. This method consists in first counterpoising the body to be weighed with any substance—as, for example, shot or sand—and then replacing the body by weights sufficient to produce equilibrium. It is evident that these latter, as they produce the same effect as the body under the same circumstances, must have the same weight.

56. Sensibility of the Balance.—A balance is said to be more or less sensitive when the beam, supposed to be originally horizontal, is more or less inclined for a given difference of weights. The sensibility depends, in the first place, on the friction of the axis against its supports. In carefully constructed balances this axis is formed by the edge of a triangular prism of very hard steel, called a knife-edge, which rests upon a plane of steel or agate. In this way, as rotation takes place about a very fine axis, and as, besides, the materials employed are very hard, the friction is rendered exceedingly small.

Supposing friction to be eliminated, the sensibility of the balance depends upon the weight of the beam, its length, and the distance between its centre of gravity and the axis of suspension. We shall proceed to investigate the influence of these different elements.

Let A and B be the points from which the scale-pans are suspended, O the axis about which the beam turns, and G the centre of gravity of the beam. If when the scale-pans are loaded with equal weights, we put into one of them an excess of weight p, the beam will become inclined, and will take a position such as A'B', turning through an angle which we will call α, and which is easily calculated.

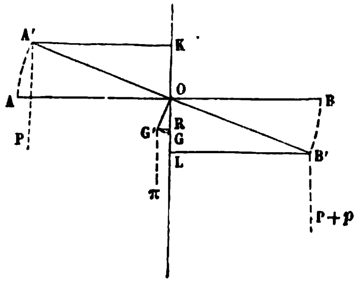

Fig. 41.

In fact, let the two forces P and P + p act at A' and B' respectively, where P denotes the less of the two weights, including the weight of the pan. Then the two forces P destroy each other in consequence of the resistance of the axis O; there is left only the force p applied at B', and the weight π of the beam applied at G', the new position of the centre of gravity. These two forces are parallel, and are in equilibrium about the axis O, that is, their resultant passes through the point O. The distances of the points of application of the forces from a vertical through O are therefore inversely proportional to the forces themselves, which gives the relation

$$\pi . \, G'R = p . \, B'L.$$

But if we call half the length of the beam l, and the distance OG r, we have

$$G'R = r \sin \alpha, \quad B'L = l \cos \alpha,$$

whence $\pi r \sin \alpha = pl \cos \alpha$, and consequently

$$\tan \alpha = \frac{pl}{\pi r}. \qquad (a)$$

The formula (a) contains the entire theory of the sensibility of the balance when properly constructed. We see, in the first place, that $\tan \alpha$ increases with the excess of weight p, which was evident beforehand. We see also that the sensibility increases as l increases and as π diminishes, or, in other words, as the beam becomes longer and lighter. At the same time it is obviously desirable that, under the action of the weights employed, the beam should be stiff enough to undergo no sensible change of shape. The problem of the balance then consists in constructing a beam of the greatest possible length and lightness, which shall be capable of supporting the action of given forces without bending.

Fortin, whose balances are justly esteemed, employed for his beams bars of steel placed edgewise; he thus obtained great rigidity, but certainly not all the lightness possible. At present the makers of balances employ in preference beams of copper or steel made in the form of a frame, as shown in Fig. 42. They generally give them the shape of a very elongated lozenge, the sides of which are connected by bars variously arranged. The determination of the best shape is, in fact, a special problem, and is an application on a small scale of that principle of applied mechanics which teaches us that hollow pieces have greater resisting power in proportion to their weight than solid pieces, and consequently, for equal resisting power, the former are lighter than the latter. Aluminium, which with a rigidity nearly equal to that of copper, has less than one-fourth of its density, seems naturally marked out as adapted to the construction of beams. It has as yet, however, been little used.

The formula (*a*) shows us, in the second place, that the sensibility increases as r diminishes; that is, as the centre of gravity approaches the centre of suspension. These two points, however, must not coincide, for in that case for any excess of weight, however small, the beam would deviate from the horizontal as far as the mechanism would permit, and would afford no indication of approach to equality in the weights. With equal weights it would remain in equilibrium in any position. In virtue of possessing this last property, such a balance is called *indifferent*. Practically the distance between the centre of gravity and the point of suspension must not be less than a certain amount depending on the use for which the balance is designed. The proper distance is determined by observing what difference of weights corresponds to a division of the graduated arc along which the needle moves. If, for example, there are 20 divisions on each side of zero, and if 2 milligrammes are necessary for the total displacement of the needle, each division will correspond to an excess of weight of $\frac{2}{20}$ or $\frac{1}{10}$ of a milligramme. That this may be the case we must evidently have a suitable value of r, and the maker is enabled to regulate this value with precision by means of the screw which is shown in the figure above the beam, and which enables him slightly to vary the position of the centre of gravity.

In the above analysis we have supposed that the three points of suspension of the beam and of the two scale-pans are in one straight line; in which case the value of $\tan \alpha$ does not include P, that is, the sensibility is independent of the weight in the pans. This follows from the fact that the resultant of the two forces P passes through O, and is thus destroyed, because the axis is fixed. This would not be the case if, for example, the points of suspension of the pans were above that of the beam; in this case the point of application of the common load is above the point O, and, when the beam is inclined, acts in the same direction as the excess of weight; whence the sensibility increases with the load up to a certain limit, beyond which the equilibrium becomes unstable.[1] On the other hand, when the points of suspension of the pans are below that of the beam, the sensibility increases as the load diminishes, and, as the centre of gravity of the beam may in this case be above the axis, equilibrium may become unstable when the load is less than a certain amount. This variation of the sensibility with the load is a serious disadvantage; for, as we have just shown, the displacement of the needle is used as the means of estimating weights, and for this purpose we must have the same displacement corresponding to the same excess of weight. If we wish to employ either of the two above arrangements, we should weigh with a constant load. The method of doing so, which constitutes a kind of double weighing, consists in retaining in one of the pans a weight equal to the maximum load. In the other pan is placed the same load subdivided into a number of marked weights. When the body to be weighed is placed in this latter pan, we must, in order to maintain equilibrium, remove a certain number of weights, which evidently represent the weight of the body.

We may also remark that, strictly speaking, the sensibility always depends upon the load, which necessarily produces a variation in the friction of the axis of suspension. Besides, it follows from the nature of bodies that there is no system that does not yield somewhat even to the most feeble action. For these reasons, there is a decided advantage in operating with constant load.

57. Suspension of the Scale-pans.—A fundamental condition of the correctness of the balance is, that the weight of each pan and of the load which it contains should always act exactly at the same point, and therefore at the same distance from the axis of suspension. This important result is attained by different methods. The arrangement represented in Fig. 42 is one of the most effectual. At the extremities of the beam are two knife-edges, parallel to the axis of rotation, and facing upwards. On these knife-edges rests, by a hard plane surface of agate or steel, a stirrup, the front of which has been taken away in the figure. On the lower part of the stirrup rests another knife-edge, at right angles to the former, the two being together equivalent to a universal joint supporting the scale-pan and its contents. By this arrangement, whatever may be the position of the weights, their action is always reduced to a vertical force acting on the upper knife-edge.

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[1] This is an illustration of the general principle, applicable to a great variety of philosophical apparatus, that a maximum of sensibility involves a minimum of stability; that is, a very near approach to instability. This near approach is usually indicated by excessive slowness in the oscillations which take place about the position of equilibrium.

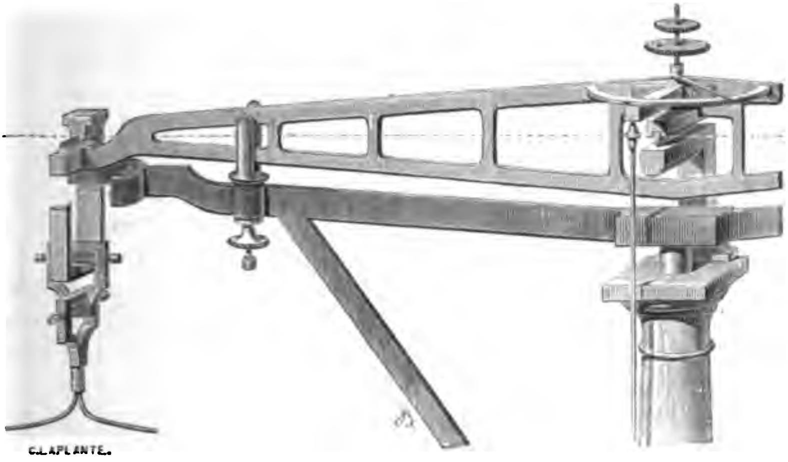

Fig. 42.—Beam of Balance.

Fig. 43 represents a balance of great delicacy, with the glass case that contains it. At the bottom is seen the extremity of a lever, which enables us to raise the beam, and thus avoid wearing the knife-edge when not in use. At the top may be remarked an arrangement employed by some makers, consisting of a horizontal graduated circle, on which a small metallic index can be made to travel; its different displacements, whose value can be determined once for all, are used for the final adjustment to produce exact equilibrium.

58. Densities.—If we weigh equal volumes of different bodies in nature, we find that they have different weights. Thus, a litre of water weighs 1 kilogramme, a litre of mercury weighs 13·6 kilos., a litre of alcohol 0·79 kilos. This we express by saying that different bodies have different densities. It is evidently important to know the density of the different substances with which we have to deal; for this is, in fact, a fundamental element of their physical constitution. Attempts have therefore been made to form a list containing the weight of a given volume of each of the substances

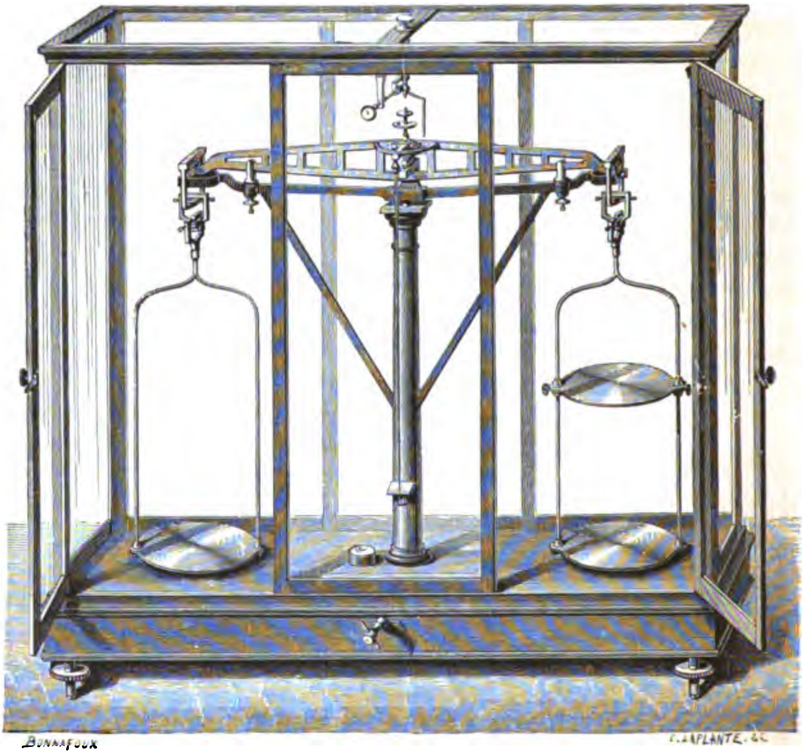

Fig. 43.—Balance for Purposes of Accuracy.

known in nature. The simplest way of determining the density of a substance is to weigh a certain known volume of it, and to divide the weight obtained by the volume; we shall thus obtain the weight of unit volume.

The same object is attained indirectly by determining the *specific gravity* of the substance; that is to say, the ratio of its density to that of some standard substance whose density is known. The standard substance commonly employed for this comparison is distilled water at the temperature of maximum density (about 39°·1 Fahrenheit). The weight of a cubic foot of such water is 62·425 lbs. avoirdupois; hence the specific gravity of a substance multiplied by 62·425 is the weight of a cubic foot of the substance in lbs., which may be called the *density of the substance in lbs. per cubic foot.*

In the metrical system the conversion of specific gravities into densities is much simpler; for since the gramme, kilogramme, and tonne are the weights respectively of a cubic centimetre, cubic decimetre, and cubic metre of distilled water at the temperature of maximum density, it follows that the same number which denotes the specific gravity of a substance, also denotes the weight of a cubic centimetre of the substance in grammes, the weight of a cubic decimetre in kilogrammes, or the weight of a cubic metre in tonnes. In other words, the specific gravity is equal to the density, whether the latter be expressed in grammes per cubic centimetre, in kilogrammes per cubic decimetre, or in tonnes per cubic metre.

If V denote the volume of a body, P its weight, and D its density, or weight per unit volume, we have

$$D=\frac{P}{V}, \qquad P=VD, \qquad V=\frac{P}{D};$$

so that if any two of these three elements are given, the third can be computed.

EXAMPLE I.—What is the weight of a mass of granite of 84 cubic metres, the density of granite being 2·75? The formula gives $P = 84 \times 2·75 = 231$ tonnes.

EXAMPLE II.—What is the volume of 1000 kilos. of mercury, the density of mercury being 13·6?

$V = \frac{1000}{13·6} = 73·5$ litres, since the litre is equal to the cubic decimetre.

59. Experimental Determination of Densities.—The following is one of the simplest methods for the practical determination of densities. We begin by weighing the body. Suppose, for example, its weight to be 10 grammes. It is then placed upon one of the scale-pans of a balance, along with a flask with a wide neck, of a form such as is shown in Fig. 44, and exactly full of water; these are balanced in the other scale by weights of any kind. The body is then introduced into the flask, and evidently displaces a volume of water equal to its own volume. If now we close the flask, taking care that it is filled to the same level as before, wipe it, and put it back on the scale-pan, there will not be equilibrium. In order to re-establish equilibrium we must add, suppose, 2·5 grammes; this is the weight of a volume of water equal to that of the body; the specific gravity of the latter then is $\frac{10}{2\cdot5} = 4$. When we wish to determine the density of a liquid, we employ a flask (Fig. 45), the upper extremity of which terminates in a narrow tube on which there is a mark. After having weighed the flask empty, we fill it successively with the liquid and with water, as far as the mark. This gives us the weights of equal volumes of water and of the liquid whose density is to be determined; and the quotient obtained by dividing the former by the latter is the specific gravity of the liquid.

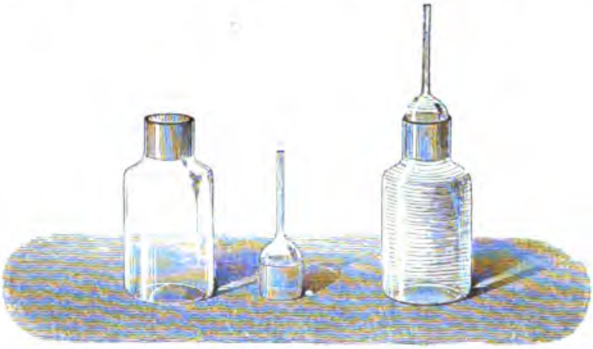
Fig. 44.—Specific Gravity Flask for Solids.

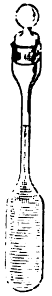
Fig. 45.—Specific Gravity Flask for Liquids.

There is generally some difficulty in filling the flask, on account of the very small diameter of the tube. The usual way is to put a little liquid into the cup at the upper extremity of the tube, and press it with the stopper; this pressure is generally sufficient to make the liquid pass into the flask. These two methods for determining densities are susceptible of great precision.

Other methods will be described in Chapter x.

In the following table we give the specific gravities of some liquids and solids.

Liquids, at Temperature of Freezing Water.

Water, sea, ordinary,	1·026	Oil, linseed,	·940
Alcohol, pure,	·791	„ olive,	·915
„ proof spirit,	·916	„ whale,	·923
Ether,	·716	„ of turpentine,	·870
Mercury,	13·596	Blood, human,	1·055
Naphtha,	·848	Milk, of cow,	1·03

SOLIDS.

Brass, cast,	7·8 to 8·4
„ wire,	8·54
Bronze,	8·4
Copper, cast,	8·6
„ sheet,	8·8
„ hammered,	8·9
Gold,	19 to 19·6
Iron, cast,	6·95 to 7·3
„ wrought,	7·6 to 7·8
Lead,	11·4
Platinum,	21 to 22
Silver,	10·5
Steel,	7·8 to 7·9
Tin,	7·3 to 7·5
Zinc,	6·8 to 7·2
Ice,	·92
Basalt,	3·00
Brick,	2 to 2·17
Brickwork,	1·8
Chalk,	1·8 to 2·8
Clay,	1·92
Glass, crown,	2·5
„ flint,	3·0
Quartz (rock-crystal),	2·65
Sand,	1·42
Fir, spruce,	·48 to ·7
Oak, European,	·69 to ·99
Lignum-vitæ,	·65 to 1·33
Sulphur, octahedral,	2·05
„ prismatic,	1·98

The unit of specific gravity is the specific gravity of pure water at the temperature of maximum density (39°·1 Fahr.)

The weight of a cubic foot of any substance is equal to 62·425 lbs. avoirdupois, multiplied by its specific gravity.

The weight of a cubic centimetre of any substance, in grammes, is equal to its specific gravity.

The weight of a litre (or cubic decimetre) of any substance, in kilogrammes, is equal to its specific gravity.

The weight of a gallon of any liquid, in lbs. avoirdupois, is equal to its specific gravity multiplied by 10.

CHAPTER VIII.

HYDROSTATICS.

60. Transmission of Pressure.—The peculiar constitution of liquids (§ 19) involves some important properties as regards pressure and the transmission of pressure. If we suppose that in a vessel A (Fig. 46), full of liquid, an opening is made at P, and a certain pressure applied there by means of a piston, the effect of this pressure will be to bring the molecules closer together, and, consequently, to create a repulsive action between them. As this result takes place throughout the whole extent of the mass, it is evident that each point in the sides of the vessel will be pressed, and that thus the effect of the single pressure will be transmitted in an infinite number of different directions. This kind of irradiation of pressure in fluids is a distinctive characteristic, and has most important applications.

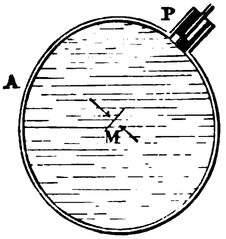

Fig. 46.—Transmission of Pressures.

The pressure exerted at P takes effect not only upon the sides of the vessel, but also at every point in the liquid. Thus a small plane lamina, which we will suppose placed at M, will be subjected to two equal and opposite pressures upon its two faces. It is also very important to remark, that on account of the uniform nature of the liquid, these pressures will not change in magnitude if we suppose the lamina turned round so as to take different directions in the liquid mass, for there is evidently no reason why the pressure should be greater in one direction than in another.

61. Direction of Pressure.—The same reason of symmetry shows us that, at each of their points of application, these pressures are normal or perpendicular to the surface; for if any reason were assigned for their inclination in a certain direction, a similar reason could also be assigned for their inclination in any other direction. This important truth may also be inferred from observing that if at any point M in the side of a vessel (Fig. 47) the pressure PM was not normal, it could be decomposed into two: one, MN, along the normal to the surface, which would be destroyed by the resistance of the surface; the other, MA, along the surface itself, which latter force would cause a sliding motion in the liquid molecule at M, which serves as the medium of transmitting the pressure.

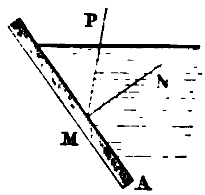

Fig. 47.

Experiment enables us, if not rigorously to demonstrate this principle, at least to show that the direction of transmitted pressure is sensibly normal. For example, if a sphere is taken, pierced with several holes, and containing a liquid, which is compressed by means of a piston in a tube communicating with the sphere, the liquid is seen to spout out in jets which take a curvilinear form under the action of gravity, but which at their origin appear perpendicular to the spherical surface. The effect is the more striking the greater the pressure exerted on the piston.

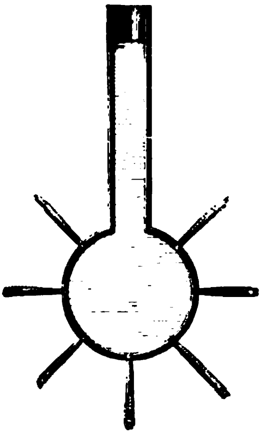

Fig. 48.

62. Pascal's Principle, or the Equal Transmission of Pressure in all Directions.—If we have a vessel A full of a liquid (Fig. 49), and if at a certain point P a certain pressure be exerted by means of a piston of the area of a square inch, suppose, each square inch of the sides of the vessel will be subjected to an equal pressure. If then at any point an opening is made of the area of a square inch, and closed with a piston, it will be necessary, in order to prevent the piston from moving, to apply to it from outside a pressure equal to that which is directly applied to the piston P. A lamina of the same area placed in any direction in the liquid will also be subjected to an equal pressure on each of its two faces.

Hence it follows that if we suppose a piston of the area of two square inches closing a corresponding opening, since each of these two inches of area receives a pressure equal to that which acts upon

P, the whole piston will receive a double pressure; whence we see that in general the transmitted pressure should vary as the area of the surface pressed.

This is the form in which Pascal enunciated the principle in his celebrated treatise on the *Equilibrium of Liquids*. "If a vessel full of water, closed on all sides, has two openings, the one a hundred times as large as the other, and if each be supplied with a piston which fits exactly, a man pushing the small piston will exert a force which will equilibrate that of a hundred men pushing the piston which is a hundred times as large, and will overcome that of ninety-nine. And whatever may be the proportion of these openings, if the forces applied to the pistons are to each other as the openings, they will be in equilibrium."

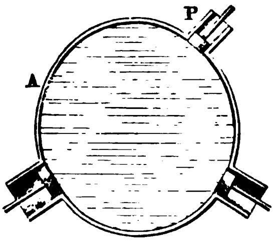

Fig. 49.—Pascal's Principle.

In general, let P be the pressure exerted upon a liquid by the aid of a piston of superficial extent S, each unit of surface of this piston will be subjected to a pressure $\frac{P}{S}$, and, as a consequence, on each unit of surface of the sides of the vessel a similar pressure will be produced. If then openings of areas S′, S″.... are made at different points, and closed with pistons, we must, in order to prevent the pistons from moving, apply to them forces P′, P″.... equal respectively to $S'\frac{P}{S}$, $S''\frac{P}{S}$.... which gives the following equations:—

$$P' = S'\frac{P}{S}, \quad P'' = S''\frac{P}{S}, \quad \text{or} \quad \frac{P}{S} = \frac{P'}{S'} = \frac{P''}{S''}.$$

63. Pascal's principle leads to a consequence which we may verify by experiment. If into a system of two tubes in communication with each other, and of unequal sectional area, we introduce a liquid, it will stand at the same height in both branches. If then we place a piston on the liquid in the narrow tube, and subject it to a certain pressure P, this pressure will be transmitted to the liquid, which will be forced back into the large tube; to hinder this motion we must place a piston in the large tube, and apply to it a force which has the same ratio to the force P as the area of the larger piston to that of the smaller. If, for example, the former has an area 16 times that of the latter, a pressure of 1 pound exerted at one of the extremities of the liquid column will produce a pressure of 16 pounds at the other extremity. We thus see that a small force may be made to produce a very great one. This is the principle of the hydraulic press, a machine which we shall describe further on.

We must remark, however, that if work is to be done, the one piston must displace the other; and it is very evident that, on account of the difference of section, if the small piston moves through a certain length, the large piston will move through one-sixteenth of that length; so that in this apparatus we have a direct verification of this general principle of mechanics, that *what is gained in force is lost in velocity.*

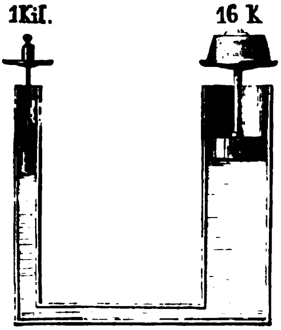

Fig. 50.—Principle of the Hydraulic Press.

This twofold observation has been clearly enunciated by Pascal, who expresses himself in the following manner at the end of the passage which we have already quoted:—"Whence it appears that a vessel full of water is a new principle of mechanics, and a new machine for the multiplication of force to any required degree, since one man will by this means be able to raise any given weight.

"It is, besides, worthy of admiration that in this new machine we find that constant rule which is met with in all the old ones, such as the lever, wheel and axle, screw, &c., which is that the distance is increased in proportion to the force; for it is evident that as one of these openings is a hundred times as large as the other, if the man who pushes the small piston drives it forward one inch, he will drive the large piston backward only one-hundredth part of that length."

If we endeavoured to perform the preceding experiment in order to demonstrate experimentally the principle of Pascal, we should arrive at only an approximate verification; for to obtain an accurate experiment it would be necessary that the pistons should fit their openings with great exactness, and this would involve a large amount of friction.

The verification would be still more difficult if we endeavoured to perform the experiment described in § 62, for in this case, besides the cause of error which we have just mentioned, the phenomena would be complicated by the action of gravity, which of itself produces variable pressures on the different openings, according to their depth below the surface of the liquid.[1] In fact, the principle of Pascal is an abstract principle, a kind of general synthesis of phenomena which cannot be made the subject of direct demonstration. It is by the constant agreement of its consequences with our observation that the authority and legitimacy of the principle are established; we shall see from all that follows how complete and invariable is that agreement.

64. Fundamental Principle of Equilibrium in Heavy Liquids. Surfaces of Equal Pressure.—Pascal's principle is a general consequence of the constitution of liquids, and is independent of the action of gravity. By introducing this latter force we arrive at special results, which we shall describe in succession. The most important, which may be considered as the fundamental rule in hydrostatics, consists in the fact *that the different points in a horizontal layer of a heavy liquid are subject to the same pressure.* Let us consider two points, A and B (Fig. 51), situated in the same horizontal plane in a heavy liquid. If we suppose that A and B are the centres of two small plane surfaces which are vertical and parallel, we may consider these surfaces as the bases of a very narrow horizontal cylinder of liquid. As this cylinder is in separate equilibrium in the general mass, we may conclude that its bases A and B are subject to equal and contrary pressures in the direction of the arrows shown in the figure; for the remaining pressures due to the surrounding liquid act in directions perpendicular to the axis of the cylinder, and thus cannot influence the equilibrium in the direction of the axis. The two elements A and B are therefore subject to the same pressure in one direction; but the pressure at a point is equal in all directions (§ 60); and as A and B are any points in the same horizontal layer, it follows that the pressures at all points in the same horizontal layer are equal. We may add, as a consequence of this, that the density is also the same at all points of a horizontal layer. On account of the slight compressibility of liquids, the variation of their density with the depth

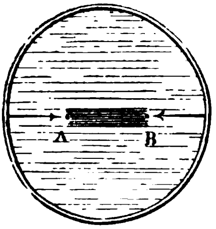

Fig. 51.

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[1] The pressures on the different pistons (supposed equal in area) would differ from each other by constant amounts, depending on their differences of level; but, on account of this constancy, any *increase of pressure* on one piston would require an *equal increase of pressure* on every other in order to the maintenance of equilibrium.

is scarcely sensible; but the above result is true for all heavy fluids, whether compressible or incompressible.

In proceeding from one surface of equal pressure to another, the pressure increases or diminishes according as the depth increases or diminishes. Thus, for example, if we consider a small horizontal element m of a horizontal layer AB, and conceive the vertical cylinder mm' reaching to the horizontal layer CD, it is quite clear that, independently of the pressure at m', which is transmitted undiminished to m, this latter element is subject to a pressure equal to the weight of the liquid contained in the cylinder mm'.

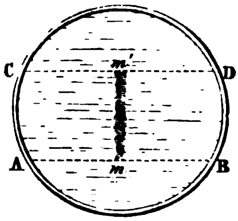

Fig. 52.

If we call s the area of the element m, h the distance between the two horizontal layers, and d the weight of unit volume of the liquid, the volume of the cylinder is expressed by sh, and its weight by shd. This last expression, therefore, represents the variation of pressure for an element of area s, when its depth below the surface varies by a quantity equal to h; and dividing by the area s we see that hd is the expression for the difference of pressure per unit area, corresponding to a difference of depth h.

65. Free Surface.—It follows from the foregoing principles that the free surface of a heavy liquid must be horizontal. We have already given an experimental demonstration of this important fact.

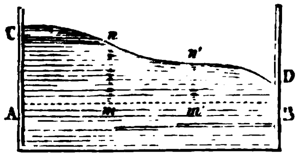

Fig. 53.

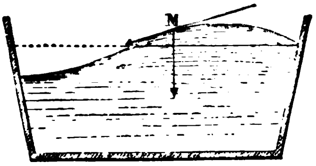

Fig. 54.

It could also have been predicted from *à priori* considerations. Let CD (Fig. 53) be the free surface, and m, m' two small equal elements of surface, in the horizontal layer AB. These two elements must be subject to equal pressures, which are evidently represented by the weights of the cylinders mn, $m'n'$; these cylinders must, therefore, have the same height, or the points n and n' must be in the same horizontal plane.

The same conclusion is arrived at by observing that if at any point whatever of the surface M (Fig. 54) the liquid did not assume a horizontal position, the weight of the liquid molecule at M could be decomposed into two forces, one perpendicular to and one along the surface of the liquid. The effect of the first would merely be to compress the liquid, and it would be destroyed by the reaction of the liquid; but the second would produce a displacement of the molecule. Equilibrium, therefore, can only exist on condition of the annihilation of this second component; that is, the surface must be horizontal at all points.

This mode of reasoning shows us that, in general, when the liquid mass is subjected to the action of any number of forces, it is necessary for equilibrium that the free surface be, at all points, perpendicular to the resultant of the acting forces. If, for instance, we place upon the whirling table a glass containing a liquid (Fig. 55), and give it a rotatory motion, we shall see the surface become hollowed, and assume a curvilinear form. In fact, each of the molecules is subjected simultaneously to the action of gravity and of centrifugal force; and it is the resultant of these two forces which must be everywhere perpendicular to the free surface. It is easily shown that this surface must be a paraboloid of revolution, so that the section represented in the figure is a parabola.

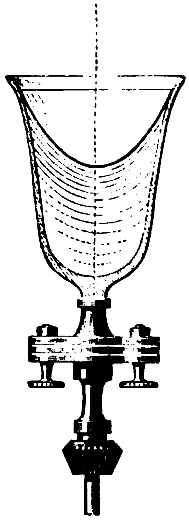

Fig. 55.—Rotating Vessel of Liquid.

66. Pressure upon the Bottom of Vessels.—If we consider a heavy liquid placed in a vessel the bottom of which is formed by a *plane horizontal surface*, it is easy to determine the pressure exerted by the liquid upon this plane. Let ABMN (Fig. 56) be a vessel filled with liquid to the level MN, and *m* an element of surface on the bottom AB. On *m* suppose a small vertical cylinder to stand, meeting the horizontal layer LL′ in *m*′. On the element *n*, equal to *m*, and in the horizontal layer LL′, suppose a vertical cylinder to stand, cutting the horizontal layer RR′ in *n*′. Suppose another similar vertical cylinder on *r*, an element equal to *m*, and let this cylinder meet SS′ in *r*′; it is evident that, by continuing this construction, we shall finally arrive, whatever be the shape of the vessel, at a cylinder *ss*′, which will extend upwards to the free surface

MN. Now, the element *m* is subject to a pressure greater than that at *m′* by a quantity equal to the weight of the cylinder *mm′*. Similarly *m′* is subject to a pressure which is equal to that at *n*, and greater than that at *n′* by a quantity equal to the weight of the cylinder *nn′*; whence it is evident, by pursuing this reasoning, that the element *m* supports a pressure equal to the sum of the weights of the cylinders *mm′*, *nn′*, *rr′*, *ss′*; that is, to the weight of a cylinder of liquid standing on the base *m*, and extending vertically upwards to the free surface. As all points in the bottom AB are subject to the same pressure, it follows that the entire pressure on the bottom of the vessel is equal to the weight of a liquid column whose base is the bottom of the vessel, and height the vertical distance between the bottom and the free surface, or what is called the height of the liquid in the vessel.[1]

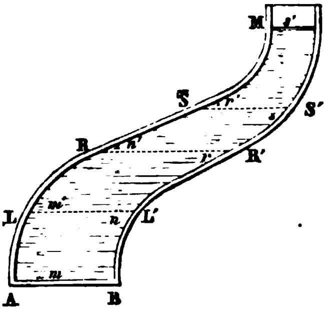

Fig. 56.—Pressure on the Bottom of Vessels.

Let B be the area of the bottom of the vessel, H the height of the liquid, and D its weight per unit volume, then the pressure is expressed by the formula BHD.

If, for instance, in a vessel whose bottom is two square decimetres in area, there is a column of mercury of the height of 5½ decimetres, the volume of this column, which measures the pressure, is $2 \times 5\frac{1}{2} = 11$ cubic decimetres, and its weight[2] is $11 \times 13{\cdot}59 = 149{\cdot}49$ kilogrammes.

67. Experiment of Pascal's Vases.—The preceding proposition shows that the pressure on the bottom of a vessel depends only upon the area of the bottom and the height of the liquid, the form of the vessel being quite immaterial. In order to verify this fact, Pascal contrived an experiment which, with some modifications, is now commonly introduced in courses of physics. The apparatus employed is a tripod (Fig. 57) supporting a ring, into which can successively be screwed three vessels of different shapes, one widened upwards, another cylindrical, and the third tapering upwards. At the lower part of the ring is a disc, supported by a thread fixed to one of the scales of a balance. Weights placed in the other scale keep the disc pressed against the ring with a certain force. Let the cylindrical vase be placed upon the tripod, and filled with water until the pressure exerted on the disc detaches it from the ring. An indicator marks the level of the liquid when this takes place. Let the experiment be repeated with the two other vases, and the disc will be detached when the water has reached the same height; showing plainly that the pressure on the bottom of a vessel is independent of the shape of the vessel.

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[1] In these remarks we neglect the atmospheric pressure which is exerted upon the free surface and transmitted to the bottom of the vessel.

[2] This example illustrates the convenience of the metrical system. The weight of a cubic decim. of water is 1 kilo., and the sp. gr. of mercury is 13·59; hence the weight of a cubic decim. of mercury is 13·59 kilos.

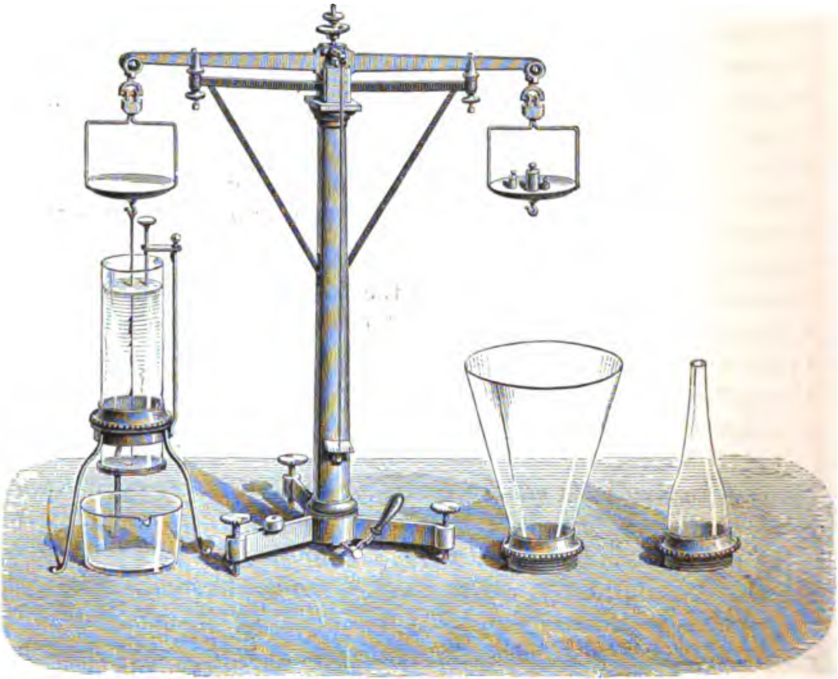


Fig. 57.—Experiment of Pascal's Vases.

But we may go further; for in the case of the cylindrical vessel it is evident that the pressure on the bottom is equal to the weight of the contained liquid. Now this weight is necessarily equal to that which counterpoised it in the other scale of the balance; hence in all three cases the pressure on the bottom of the vessel is equal to the weight of a liquid column with the bottom as base and of the same height as the liquid in the vessel.

68. Upward Pressure.—The pressure exerted at any point of a liquid mass being the same in all directions, a horizontal surface facing downwards should be subjected to an upward pressure equal to the downward pressure which would be exerted if the liquid were acting in the other direction. Suppose we take a tube open at both ends (Fig. 58), and apply to the lower end a flat cover. If we plunge the tube into a liquid, the liquid will press up the cover against the bottom of the tube with a force which increases as the tube is plunged deeper in the liquid. If we then pour liquid into the tube, this will produce a downward pressure upon the cover, and when the level of the liquid is the same inside the tube as outside the cover will be detached, the upward pressure being destroyed by the pressure exerted by the liquid inside the tube in the contrary direction.

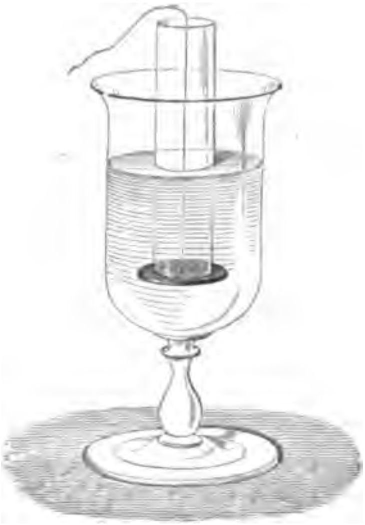


Fig. 58.—Upward Pressure.

69. Total Pressure. Resultant Pressure on Vessel. Hydrostatic Paradox.—When a vessel of any shape is filled with a liquid, normal pressures are exerted against all points of its sides, increasing with the depth, and equal in each case to the pressure in the corresponding horizontal layer of the liquid. We may suppose a summation of all the pressures thus exerted upon the different superficial elements of the sides; this gives what is called the *total pressure* exerted by the liquid (see § 72).

This total pressure must not be confounded with the resultant pressure which is transmitted to the stand on which the vessel rests. In fact we see that, of the elements of pressure, some are transmitted *entire* to the stand; these are the vertical pressures exerted upon the bottom AB; others,

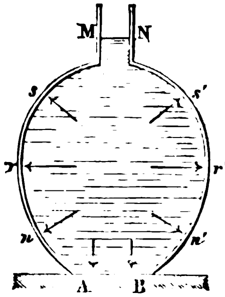


Fig. 59.—Total Pressure.

those for instance at n and n', are only transmitted in part, because their direction is oblique; the horizontal pressures at r and r' are evidently without influence; and the pressures exerted at s and s' tend to raise the vessel. It is by the combination and *composition* of these pressures of different intensity and direction that the resultant pressure upon the stand on which the vessel rests is obtained.

Hydrostatic Paradox.—It has been thought paradoxical that vessels whose bottoms were subjected to equal pressures did not transmit equal pressures to the stand on which they were placed. In reality, nothing is less paradoxical; the pressure on the bottom of the vessel is only one of the elements which combine to produce the resultant pressure transmitted to the stand.

70. Composition of Pressures.—It may be regarded as quite evident that this latter pressure is in all cases equal to the weight of the liquid; which amounts to saying that if we place a vessel containing liquid in one scale of a balance, we must, in order to equilibrate it, place in the other scale weights equal to the sum of the weights of the liquid and the vessel. This result is also easily shown from *à priori* considerations in some simple cases.

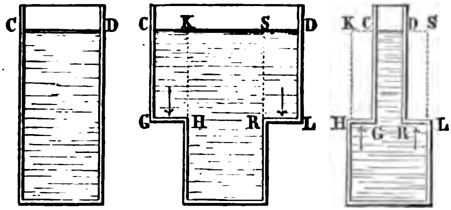


Fig. 60.—Hydrostatic Paradox.

Thus in the case of a cylindrical vessel ABDC (Fig. 60) it is evident that the only pressure transmitted to the stand is that exerted upon the bottom, which is equal to the weight of the liquid. In the case of the vessel which is wider at the top, the stand is subjected to the weight of the liquid column ABSK, which presses on the bottom AB, together with the columns GHKC, RLDS, pressing on GH and RL; the sum of which weights composes the total weight of liquid contained in the vessel. Finally, in the third case, the pressure on the bottom AB, which is equal to the weight of a liquid column ABSK, must be diminished by the pressures in the opposite direction on HG and RL. These last being represented by liquid columns HGCK, RLSD, there is only left to be transmitted to the stand a pressure equal to the weight of the water in the vessel. By the application of the ordinary rules for the composition of forces, it can easily be shown that this result is perfectly general. Here, then, we have Pascal's principle leading to a conclusion which is obviously true, and which therefore furnishes a confirmation of the principle itself.

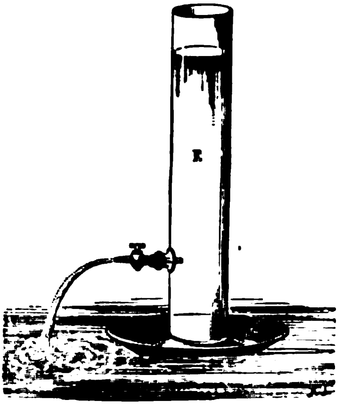


Fig. 61.—Backward Movement of Discharging Vessel.

71. Motion produced by the Flowing of a Liquid.—The demonstration to which we have just referred amounts to showing, by the analysis of the different pressures, that the horizontal components of these pressures equilibrate each other, and that the vertical components are equivalent to a single force equal to the weight of the liquid. The evidence of experiment is as strong for the first part of this proposition as for the second. If we place a vessel (Fig. 61) in such a position as to be very susceptible of motion in a horizontal direction, whether by suspending it by a thread, or by floating it in water, and under these circumstances fill it with liquid, however great may be the mobility of the apparatus, not the slightest displacement is observed. This proves that the horizontal components of pressure equilibrate one another. This equilibrium is effected through the medium of the vessel; but if we suppose an opening made at any point of the vessel, the liquid will run out, and the pressure exerted at the point diametrically opposite will propel the vessel in the opposite direction to that in which the liquid is escaping.

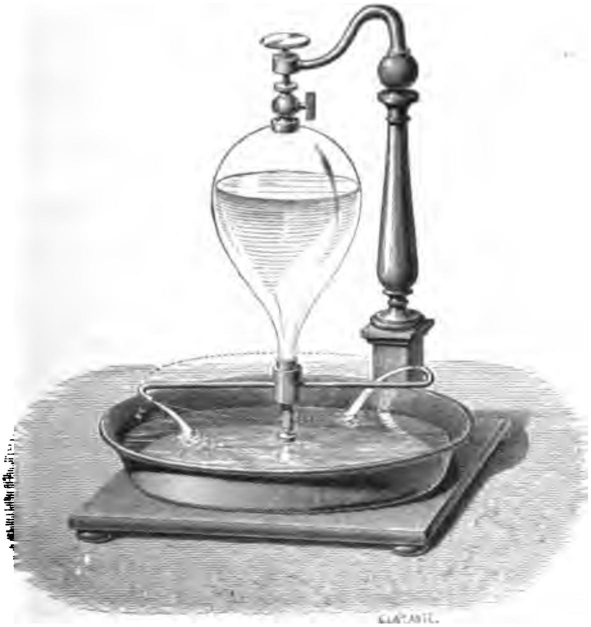


Fig. 62.—Hydraulic Tourniquet.

This observation explains the motion of the apparatus called the *hydraulic tourniquet.* It consists (Fig. 62) of a vessel capable of rotation about a vertical axis, having at its lower extremity a tube, the two portions of which are bent in opposite directions, and left open at the ends to allow the liquid to escape. The reaction at the points opposite to the openings causes the rotatory motion of the apparatus.

When the velocity of efflux is sufficiently great, the motion can be employed for practical purposes, and hydraulic engines based on this principle have frequently been proposed and tried; Barker's mill is one of the best known.

72. Centre of Pressure.—When we consider the particular case of the pressure exerted by a liquid upon a *plane* surface, the different elements of pressure being all parallel, it may be required to determine the point of application of the resultant pressure. This point is called the *centre of pressure.* The centre of pressure does not coincide with the centre of gravity; it is always situated below this latter, since the elementary forces which must be combined in order to obtain it, instead of being uniformly distributed over the surface, increase with the depth.

The investigation of the centre of pressure constitutes a separate chapter of mathematical physics, and we shall not enter upon it here; we shall simply examine a particular case, fitted to give accurate ideas with regard to this point.

Let a rectangular surface RB be immersed in a liquid, which extends as far as R; we may suppose it, for instance, a flood-gate or the side of a dam for holding water. The pressure goes on increasing from the point R, where it is zero, to the point B, where it attains its maximum value; it has the same value for all points in the same horizontal line, and is at each point proportional to the distance from the surface of the water. If, then, at the point B we draw a perpendicular B*b* equal to RB, and join R*b*, the different parallel lines D*d*, H*h*, L*l*, in the triangle RB*b* will be proportional to the pressures at the points D, H, L. The composition of these pressures, then, amounts to finding the centre of gravity of the triangle R*b*B; but the height of this from the base is one-third of the height of the triangle; the centre of pressure thus

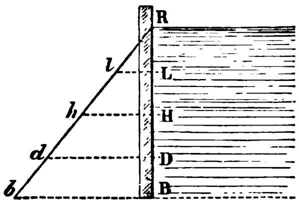


Fig. 63.—Centre of Pressure.

lies at one-third of the height RB. It is further evident by symmetry that it will lie on the line joining the middle points of the upper and lower sides of the rectangle.

As for the total pressure on RB, it may be obtained in the special case under consideration by observing, that since the pressure increases uniformly from R to B, its average intensity is equal to the pressure at the middle point. The total pressure is therefore the same in amount (though not in distribution) as if the surface were pressed at all points by a body of water of half the height of RB.

Suppose the height of $RB = 3$ metres, and its breadth $= 5$ metres, the total pressure will be equal to the weight of $5 \times 1{\cdot}5 = 7{\cdot}5$ cubic metres of water, that is, to 7500 kilos., and will have for its resultant a single force of this amount applied not at the centre of gravity, but at the centre of pressure.

We may remark that the middle point of the height of the rectangle exactly corresponds to the centre of gravity of the figure, and it may be demonstrated in general that *the total pressure on any surface, whether plane or curved, is equal to the weight of a liquid column having that surface for base, and for height the distance of the centre of gravity from the surface of the water.*

CHAPTER IX.

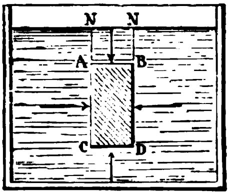
PRINCIPLE OF ARCHIMEDES.

73. Pressure of Liquids on Bodies immersed.—When a body is immersed in a liquid, the different points of its surface are subjected to pressures which obey the rules laid down in the preceding chapter. As these pressures increase with the depth, it is evident that those which tend to raise the body overcome those which tend to sink it, so that the resultant effect is a force in the direction opposite to that of gravity.

By means of an analysis, similar to that in § 70, it may be shown that this resultant upward force is exactly equal to the weight of the liquid displaced by the body.

This conclusion can very readily be verified in some simple cases: suppose, for example (Fig. 64), a right cylinder plunged vertically in a liquid, and let us examine the effect of the different pressures exerted by the liquid upon its surface. It is evident, in the first place, that if we consider any point on the sides of the cylinder, the normal and horizontal pressure on that point is destroyed by the equal and contrary pressure at the point diametrically opposite; and, as the same is the case for all similar points, we see that the horizontal pressures destroy each other. As regards the vertical pressures on the ends, one of them, that on the upper end AB, is in a downward direction, and equal to the weight of the liquid column ABNN; the other, that on the lower end CD, is in an upward direction, and equal to the weight of the liquid column CNND; this latter pressure exceeds the former by the weight of the liquid cylinder ABCD, so that the resultant effect of the pressure is to raise the body with a force equal to the weight of the liquid displaced.

By a synthetic process of reasoning, we may, without having recourse to the analysis of the different pressures, show that this conclusion is perfectly general. Suppose we have a liquid mass in equilibrium, and that we consider specially the portion M (Fig. 65); this portion is likewise in equilibrium. If we suppose it to become solid, without any change in its weight or volume, equilibrium will still subsist. Now this is a heavy mass, and as it does not fall, we must conclude that the effect of the pressures on its surface is to produce a resultant upward pressure exactly equal to its weight, and acting in a line which passes through its centre of gravity. If we now suppose M replaced by a body exactly occupying its place, the exterior pressures remaining the same their resultant effect will also be the same.



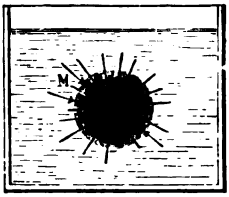


Fig. 64. Principle of Archimedes. Fig. 65.

The name *centre of buoyancy*, or *centre of displacement*, is given to the centre of gravity of the liquid displaced by a body immersed, and we see that we may always suppose that it is in this point that the upward pressure of the liquid is applied. The results of the above explanations may thus be included in the following proposition: *Every body immersed in a liquid is subjected to an upward vertical pressure equal to the weight of the liquid displaced, and applied at the centre of displacement.*

This proposition constitutes the celebrated principle of Archimedes. It is often enunciated in the following form: *Every body immersed in a liquid loses a portion of its weight equal to the weight of the liquid displaced.* This enunciation, though perhaps less correct than the former, is fundamentally identical with it; for if we weigh a body immersed in a liquid, the weight will evidently be diminished by a quantity equal to the upward pressure.

74. Experimental Demonstration of the Principle of Archimedes.—The following experimental demonstration of the principle of Archimedes is commonly exhibited in courses of physics:—

From one of the scales of a hydrostatic balance is suspended a hollow cylinder of copper, and below this a solid cylinder, whose volume is equal to the interior volume of the hollow cylinder; these are balanced by weights in the other scale. A vessel of water is then placed below the cylinders, in such a position that the lower cylinder shall be immersed in it. The equilibrium is immediately destroyed, and the upward pressure of the water causes the scale with the weights to descend. If we now pour water into the hollow cylinder, equilibrium will gradually be re-established; and the beam will be observed to resume its horizontal position when the hollow cylinder is full of water, the other cylinder being at the same time completely immersed. The upward pressure upon this latter is thus equal to the weight of the water added, that is, to the weight of the liquid displaced.

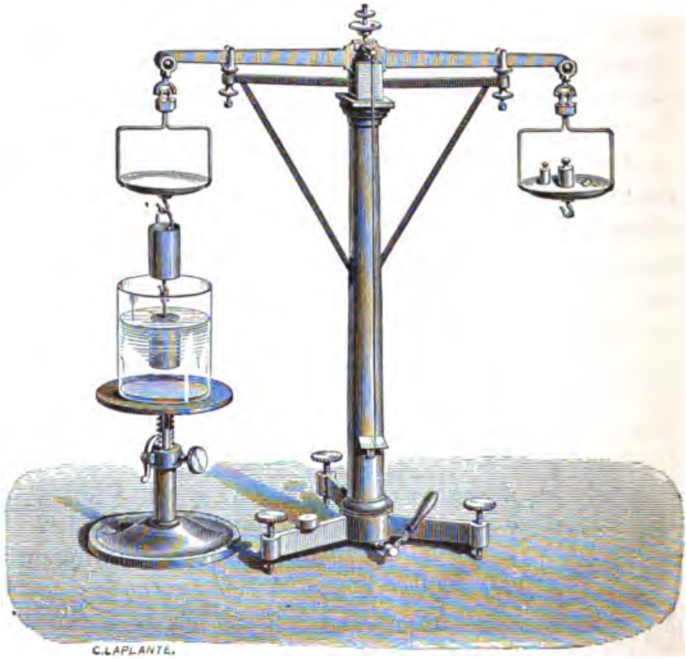


Fig. 66.—Experimental Verification of Principle of Archimedes.

75. Body immersed in a Liquid.—It follows from the principle of Archimedes that when a body is immersed in a liquid, it is subjected to two forces: one equal to its weight and applied at its centre of gravity, tending to make the body descend; the other equal to the weight of the displaced liquid, applied at the centre of buoyancy, and tending to make it rise. There are thus three different cases to be considered:

(1.) The weight of the body may exceed the weight of the liquid displaced, or, in other words, the mean density of the body may be greater than that of the liquid; in this case, the body sinks in the liquid, as, for instance, a piece of lead dropped into water.

(2.) The weight of the body may be less than that of the liquid displaced; in this case the body rises partly out of the liquid, until the weight of the liquid displaced is equal to its own weight. This is what happens, for instance, if we immerse a piece of cork in water and leave it to itself.

(3.) The weight of the body may be equal to the weight of the liquid displaced; in this case, the two opposite forces being equal, the body takes a suitable position (§ 77) and remains in equilibrium.

These three cases are exemplified in the three following experiments (Fig. 67):—

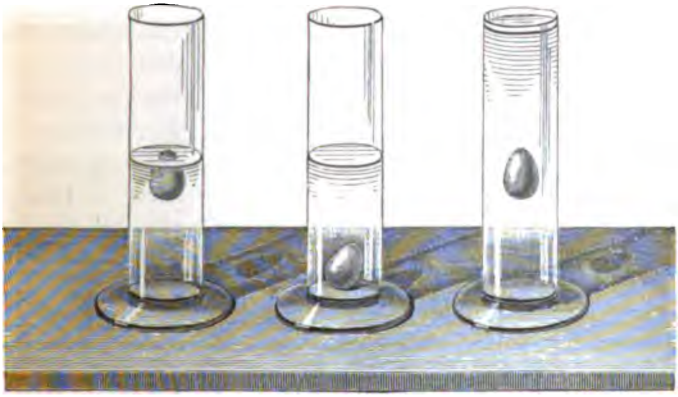


Fig. 67.—Egg Plunged in Fresh and Salt Water.

(1.) An egg is placed in a vessel of water; it sinks to the bottom of the vessel, its mean density being a little greater than that of the liquid.

(2.) Instead of fresh water, salt water is employed; the egg floats at the surface of the liquid, which is a little denser than it.

(3.) Fresh water is carefully poured on the salt water; a mixture of the two liquids takes place where they are in contact; and if the egg is put in the upper part, it will be seen to descend, and, after a few oscillations, remain at rest in a layer of liquid of which it displaces a volume whose weight is equal to its own. We may remark that, in this position the egg is in stable equilibrium; for, if it rises, the upward pressure diminishing, its weight tends to make it descend again; if, on the contrary, it sinks, the pressure increases and tends to make it reascend.

76. Cartesian Diver.—The experiment of the *Cartesian diver*, which is described in old treatises on physics, shows each of the different cases that can present themselves when a body is immersed. The diver (Fig. 68) consists of a hollow ball, at the bottom of which is a small opening O; a little porcelain figure is attached to the ball, and the whole floats upon water contained in a glass vessel, the mouth of which is closed by a strip of caoutchouc or a bladder. If we press with the hand on the bladder, the air is compressed, and the pressure, transmitted through the different horizontal layers, condenses the air in the ball, and causes the entrance of a portion of the liquid by the opening O; the floating body becomes heavier, and in consequence of this increase of weight the diver descends. When we cease to press upon the bladder, the pressure becomes what it was before, some water flows out and the diver ascends. It must be observed, however, that as the diver continues to descend more and more water enters the ball, in consequence of the increase of pressure, so that if the depth of the water exceeded a certain limit, the diver would not be able to rise again from the bottom.

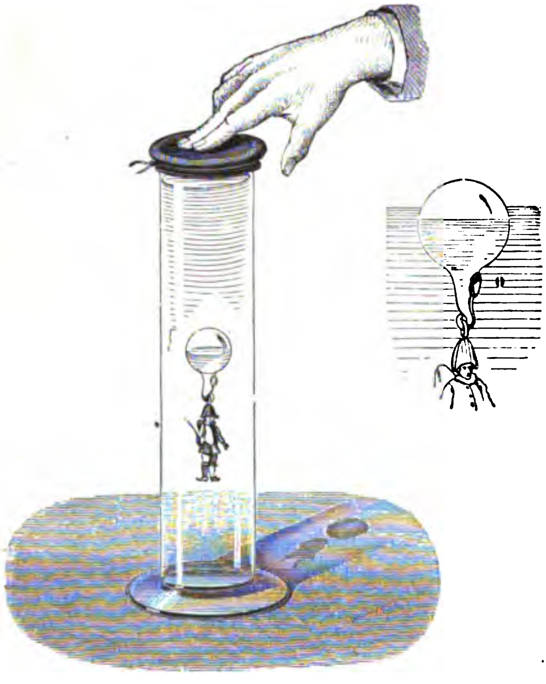


Fig. 68.—Cartesian Diver.

If we suppose that at a certain moment the weight of the diver becomes exactly equal to the weight of an equal volume of the liquid, there will be equilibrium; but, unlike the equilibrium in the experiment in § 75, this will evidently be *unstable*, for a slight movement either upwards or downwards will alter the resultant force so as to produce further movement in the same direction.

77. Relative Positions of the Centre of Gravity and Centre of Buoyancy.—In order that a floating body, wholly or partially immersed in a liquid, may be in equilibrium, it is evidently necessary that its weight be équal to the weight of the liquid displaced.

This condition, which is absolutely necessary, is, however, not sufficient; we require, in addition, that the action of the upward pressure should be exactly opposite to that of the weight; that is, that the centre of gravity and the centre of buoyancy be in the same vertical line; for if this were not the case, the two contrary forces would compose a couple, the effect of which would evidently be to cause the body to turn.

In the case of a body completely immersed, it is further necessary for stable equilibrium that *the centre of gravity should be below* the centre of buoyancy; in fact we see, by Fig. 69, that in any other position than that of equilibrium, the effect of the two forces applied at the two points G and O would be to turn the body, so as to bring the centre of gravity lower. But this is not the case when the body is only partially immersed, as most frequently happens. In this case it may indeed happen that, with stable equilibrium, the centre of gravity is below the centre of pressure; but this is not necessary, and in the majority of instances is not the case. Let Fig. 70 represent the lower part of a floating body—a boat, for instance. The centre of pressure is at O, the centre of gravity at G, considerably above; if the body is displaced, and takes the position shown in the figure, it will be seen that the effect of the two forces acting at O and at G is to restore the body to its former position. This difference from

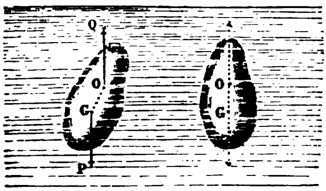


Fig. 69.

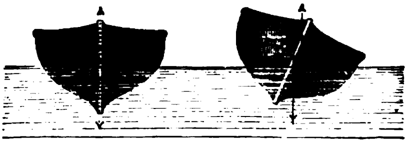


Fig. 70.

Relative Positions of Centre of Gravity and Centre of Pressure.

what takes place when the body is completely immersed, depends upon the fact that, in the case of the floating body, the figure of the liquid displaced changes with the motions of the body, and the centre of buoyancy moves towards the side on which the body is more deeply immersed. It will depend upon the form of the body whether this lateral movement of the centre of buoyancy is sufficient to carry it beyond the vertical through the centre of gravity. The two equal forces which act on the body will evidently turn it to or from the original position of equilibrium, according as the new centre of buoyancy lies beyond or falls short of this vertical.[1]

78. Advantage of Lowering the Centre of Gravity.—Although stable equilibrium may subsist with the centre of gravity above the centre of buoyancy, yet for a body of given form the stability is always increased by lowering the centre of gravity; as we thus lengthen the arm of the couple which tends to right the body when displaced. It is on this principle that the use of ballast depends.

79. Phenomena in apparent Contradiction to the Principle of Archimedes.—A body cannot float in a liquid unless it have a density less than that of the liquid. This natural consequence of the principle of Archimedes seems at first sight to be contradicted by some well-known facts. Thus, for instance, if small needles are placed carefully on the surface of water, they will remain there in equilibrium (Fig. 71). It is on a similar principle that several insects *walk* on water (Fig. 72), that a great number of bodies of various natures, provided they be *very minute*, can, if we may so say, be placed on the surface of a liquid without penetrating into its interior. These curious facts depend on the circumstance that the small bodies in

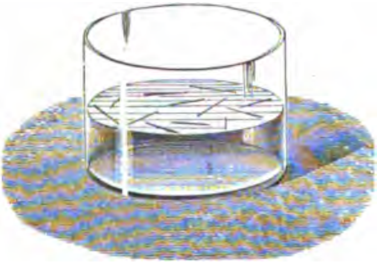


Fig 71.—Steel Needles Floating on Water.

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[1] If a vertical through the new centre of buoyancy be drawn upwards to meet that line in the body which in the position of equilibrium was a vertical through the centre of gravity, the point of intersection is called the *metacentre*. Evidently when the forces tend to restore the body to the position of equilibrium, the metacentre is above the centre of gravity; when they tend to increase the displacement, it is below. In ships the distance between these two points is usually nearly the same for all amounts of heeling, and this distance is a measure of the stability of the ship.

We have defined the metacentre as the intersection of two lines. When these lines lie question are not wetted by the liquid, and hence, in virtue of principles which will be explained in connection with capillarity (Chap. xi.), depressions are formed around them on the liquid surface, as represented in Fig. 73. The curvature of the liquid surface in the neighbourhood of the body is very distinctly shown by observing the shadow cast by the floating body, when it is illumined by the sun; it is seen to be bordered by luminous bands, which are owing to the refraction of the rays of light in the portion of the liquid bounded by a curvilinear surface.

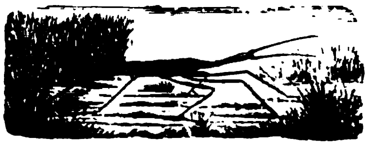

Fig. 72.—Insect Walking on Water

The existence of the depression about the floating body enables us to bring the condition of equilibrium in this special case under the general enunciation of the principle of Archimedes. Let M be a section of the body, CD the distance to which the depression extends, and AB the corresponding portion of any horizontal layer; since the pressure at each of the points AB must be the same as in the other parts of the layer, the liquid acts in exactly the same way as if M did not exist, and the cavity were filled by the liquid itself.

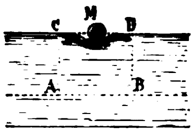

Fig. 73.

We may thus say in this case also that the weight of the floating body is equal to the weight of the *liquid displaced*, understanding by these words the liquid which would occupy the whole extent of the depression due to the presence of the body.

80. Liquids in Superposition.—When liquids of different densities, which do not readily mix, are placed in the same vessel, the particles of the denser liquids unite and fall to the bottom, just as a solid body sinks in a liquid of less density; finally, the liquids arrange themselves in the order of their respective densities, the surfaces of separation being horizontal. This fact is verified by means of the phial called the *phial of the four elements.* It is a flask (Fig. 74) containing mercury, water, and oil. In the state of equilibrium the mercury is at the bottom, the oil at the top, and the water in the middle; if the

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in different planes, and do not intersect each other, there is no metacentre. This indeed is the case for most of the displacements to which a floating body of irregular shape can be subjected. There are in general only two directions of heeling to which metacentres correspond, and these two directions are at right angles to each other. For an investigation of the conditions of stability in floating bodies, see Thomson and Tait's *Natural Philosophy,* §§ 763-768.

flask is shaken, the liquids are for the moment mixed, but in returning to repose do not fail to resume their former positions.

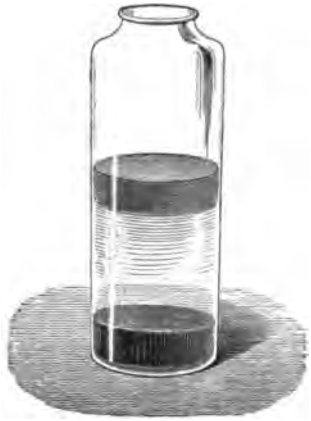

Fig. 74.
Phial of the Four Elements.

It is easily seen from the ordinary rules of hydrostatics, that the surface of separation of two different liquids must be horizontal. Let there be two liquids in a vessel (Fig. 75); the free surface is necessarily horizontal. If now we take two equal superficial elements *n* and *n'* in a horizontal layer of the lower liquid, they must be subjected to equal pressures; these pressures are measured by the weights of the liquid cylinders *nrs n'tl*; and these latter cannot be equal unless there be the same height of the lower liquid above the elements *n* and *n'*. This reasoning holds for all points in the horizontal layer, which must therefore be at a constant distance from the surface of separation; in other words, this surface must be horizontal.

This property is liable to considerable modification in the case of liquids which can dissolve each other or act chemically upon each other. Thus, if alcohol be carefully poured upon water in a glass, the two liquids will be seen to have for their surface of junction a horizontal plane; but on agitation a single liquid will be formed by their mutual action, and the separation will not again take place.

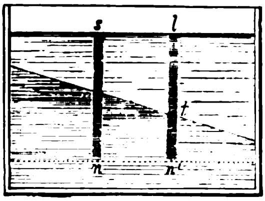

Fig. 75.

If the agitation is not sufficiently great, this intimate mixture will only partially ensue, and will be confined to the neighbourhood of the surface of contact. Two uniform layers of liquid will thus be formed, separated by an intermediate zone of unequal density. This is the case at the mouth of a river, where the fresh water forms on the surface of the sea a layer, the base of which is a compound of fresh and salt water.

CHAPTER X.

APPLICATION OF THE PRINCIPLE OF ARCHIMEDES TO THE DETERMINATION OF DENSITIES.—HYDROMETERS.

81. Determination of Densities.—We have seen in Chap. vii. that in order to determine the density of a body it is only necessary to measure the ratio existing between the weight of a certain volume of the body and the weight of an equal volume of water. The principle of Archimedes enables us to effect this measurement very easily, and the process which it suggests is sometimes more convenient than that which has been described in the chapter mentioned above.

(1.) *Solid bodies.*—Suppose that the object whose density we wish to determine is a piece of copper. It is suspended by a very fine thread to one of the scales of a balance (Fig. 76), its weight is determined, and found to be, say $125{\cdot}35^{\text{gr.}}$ The body is then immersed in water; the equilibrium is destroyed on account of the upward pressure of the water, and in order to re-establish it, we must add a weight of $14{\cdot}24^{\text{gr.}}$ to the scale supporting the body. This additional weight, according to the principle of Archimedes, represents the weight of a volume of water equal to the volume of the body. The density of copper is thus $\frac{125{\cdot}35}{14{\cdot}24}=8{\cdot}8$.

(2.) *Liquid bodies.*—From one of the scales of the balance is suspended (Fig. 77) any body whatever, which must, however, not be capable of being attacked by the liquids in which it is to be immersed; a ball of glass weighted inside with mercury is very well adapted to this purpose. The exact weight of this is obtained; it is then immersed in the liquid whose density is sought—alcohol, for example; an upward pressure is thus produced, and in order to re-establish equilibrium, a weight of $35{\cdot}43^{\text{gr.}}$ must be added to the scale.

bodies or for finding their specific gravities. To find the weight of a body (which we shall suppose to weigh less than 1000 grains), it must be placed in the dish at the top, together with weights just sufficient to make the instrument sink in water as far as the mark. Obviously these weights are the difference between the weight of the body and 1000 grains.

To find the specific gravity of a solid, we first ascertain its weight by the method just described; we then transfer it from the dish above to the basket below, so that it shall be under water during the observation, and observe what additional weights must now be placed in the dish. These additional weights represent the weight of the water displaced by the solid; and the weight of the solid itself divided by this weight is the specific gravity required.

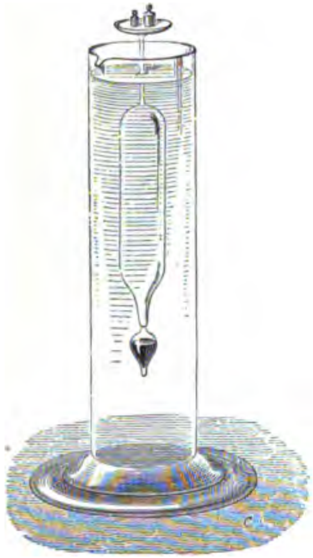

Fig. 79.—Fahrenheit's Hydrometer

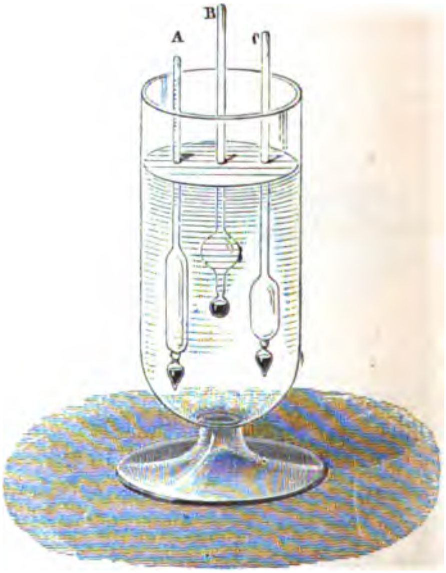

Fig. 80.—Forms of Hydrometers.

84. Fahrenheit's Hydrometer.—This instrument, which is represented in Fig. 79, is generally constructed of glass, and differs from Nicholson's in having at its lower extremity a ball weighted with mercury instead of the basket. It resembles it in having a dish at the top, in which weights are to be placed sufficient to sink the instrument to a definite mark on the stem.

85. Hydrometers of Variable Immersion.—These instruments are usually of the forms represented at A, B, C, Fig. 80. The lower end is weighted with mercury in order to make the instrument sink to a convenient depth and preserve an upright position. The stem is cylindrical, and is graduated, the divisions being frequently marked upon a piece of paper inclosed within the stem, which must in this case be of glass. It is evident that the instrument will sink the deeper the less is the specific gravity of the liquid, since the weight of the liquid displaced must be equal to that of the instrument. Hence if any uniform system of graduation be adopted, so that all the instruments give the same readings in liquids of the same densities, the density of a liquid can be obtained by a mere immersion of the hydrometer—an operation not indeed very precise, but very easy of execution. These instruments have thus come into general use for commercial purposes.

86. General Theory of Hydrometers of Variable Immersion.—Let V be the volume of a hydrometer which is immersed when the instrument floats freely in a liquid whose density (that is, weight per unit volume) is d, then Vd represents the weight of liquid displaced, which by the principle of Archimedes is the same as the weight of the hydrometer itself. If V', d' be the corresponding values for another liquid, we have therefore

$$Vd = V'd', \text{ or } d : d' :: V' : V,$$

that is, the density varies inversely as the volume immersed. Let d_1, d_2, d_3...be a series of densities in diminishing order, and V_1, V_2, V_3...the corresponding volumes immersed, which will be in ascending order; then we have

$$d_1, \; d_2, \; d_3 \ldots \text{ proportional to } \frac{1}{V_1}, \frac{1}{V_2}, \frac{1}{V_3} \ldots$$

$$\text{and } V_1, \; V_2, \; V_3 \ldots \text{ proportional to } \frac{1}{d_1}, \frac{1}{d_2}, \frac{1}{d_3} \ldots$$

Hence, if we wish the divisions to indicate equal differences of density, we must place them so that the corresponding volumes immersed form a harmonical progression. This implies that the divisions must approach nearer together for increasing densities. This is of course on the assumption that the stem is of equal sectional area in all parts as far as the divisions extend.

The following investigation shows how the density of a liquid may be computed from observations made with a hydrometer graduated with equal divisions. It is necessary first to know the divisions to which the instrument sinks in two liquids of known density. Let these divisions be numbered n_1, n_2, reckoning from the top downwards, and let the corresponding densities be d_1, d_2. Now if we take for our unit of volume one of the equal parts on the stem, and if we take c to denote the volume which is immersed when the instrument sinks to the division marked zero, it is obvious that when the instrument sinks to the nth division (reckoned downwards on the stem from zero) the volume immersed is $c - n$, and if the corresponding density be called d, then $(c - n)\, d$ is the weight of the hydrometer. We have therefore

$$(c - n_1)\, d_1 = (c - n_2)\, d_2, \text{ whence } c = \frac{n_1 d_1 - n_2 d_2}{d_1 - d_2}.$$

This value of c can be computed once for all.

Then the density D corresponding to any other division N can be found from the equation

$$(c - \mathrm{N})\, \mathrm{D} = (c - n_1)\, d_1 \text{ which gives } \mathrm{D} = \frac{c - n_1}{c - \mathrm{N}},$$

if d_1, which we may suppose to be the density of water, be called unity.

87. Beaumé's Hydrometers.—In these instruments the divisions are equidistant. There are two distinct modes of graduation, according as the instrument is to be used for determining densities greater or less than that of water. In the former case the instrument is called a salimeter, and is so constructed that when immersed in pure water of the temperature 12° Cent. it sinks nearly to the top of the stem, and the point thus determined is the zero of the scale. It is then immersed in a solution of 15 parts of salt to 85 of water, the density of which is about 1·116, and the point to which it sinks is marked 15. The interval is divided into 15 equal parts, and the graduation is continued to the bottom of the stem, the length of which varies according to circumstances; it generally terminates at the degree 66, which corresponds to sulphuric acid, whose density is commonly the greatest that it is required to determine. Referring to the formulæ of last section, we have here

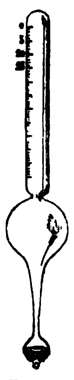

Fig. 81. Beaumé's Salimeter.

$$n_1 = 0,\ d_1 = 1,\ n_2 = 15,\ d_2 = 1{\cdot}116;$$

whence

$$c = \frac{15 \times 1{\cdot}116}{{\cdot}116} = 144,\ \mathrm{D} = \frac{144}{144 - \mathrm{N}}.$$

When the instrument is intended for liquids lighter than water, it is called an alcoholimeter. . In this case the point to which it sinks in water is near the bottom of the stem, and is marked 10; the zero of the scale is the point to which it sinks in a solution of 10 parts of salt to 90 of water, the density of which is about 1·085, the divisions in this case being numbered upward from zero.

In order to adapt the formulæ of last section to the case of graduations numbered upwards, it is merely necessary to reverse the signs of n_1, n_2, and N; that is we must put

$$c=\frac{n_2d_2-n_1d_1}{d_1-d_2},\ D=\frac{c+n_1}{c+N};$$

and as we have now $n_1=10$, $d_1=1$, $n_2=o$, $d_2=1·085$ the formulæ give[1]

$$c=\frac{10}{·085}=118,\ D=\frac{128}{118+N}.$$

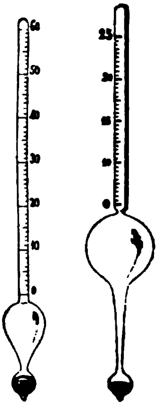

Fig. 82. Fig. 83.
Beaumé's Alcoholimeters.

87A. Twaddell's Hydrometer.—In this instrument the divisions are placed not as in Beaumé's, at equal distances, but at distances corresponding to equal differences of density. In fact the specific gravity of a liquid is found by multiplying the reading by 5, cutting off three decimal places, and prefixing unity. Thus the degree 1 indicates specific gravity 1·005, 2 indicates 1·010, &c.

88. Gay Lussac's Centesimal Alcoholimeter.—When a hydrometer is to be used for a special purpose it may be convenient to adopt a mode of graduation different in principle from any that we have described above, and adapted to give a direct indication of the proportion in which two ingredients are mixed in the fluid to be examined. It may indicate, for example, the quantity of salt in sea-water, or the quantity of alcohol in a spirit consisting of alcohol and water. Where there are three or more ingredients of different specific gravities the method fails. Gay-Lussac's alcoholimeter is graduated to indicate, at the temperature of 15° Cent.,

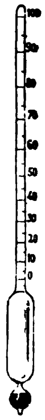

Fig. 87.
Centesimal Alcoholimeter.

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[1] On comparing the two formulæ for D in this section with the tables in the Appendix to Miller's *Chemical Physics*, I find that as regards the salimeter they agree to two places of decimals and very nearly to three. As regards the alcoholimeter, the table in *Miller* implies that c is about 136, which would make the density corresponding to the zero of the scale about 1·074.

the percentage of pure alcohol in a specimen of spirit. At the top of the stem is 100, the point to which the instrument sinks in pure alcohol, and at the bottom is 0, to which it sinks in water. The position of the intermediate degrees must be determined empirically, by placing the instrument in mixtures of alcohol and water in known proportions, at the temperature of 15°. The law of density, as depending on the proportion of alcohol present, is complicated by the fact that, when alcohol is mixed with water, a diminution of volume (accompanied by rise of temperature) takes place.

88A. Specific Gravity of Mixtures.—When two or more substances are mixed without either shrinkage or expansion (that is, when the volume of the mixture is equal to the sum of the volumes of the components), the density of the mixture can easily be expressed in terms of the quantities and densities of the components.

First, let the volumes v_1, v_2, v_3 . . . of the components be given, together with their densities d_1, d_2, d_3 . . .
Then their masses (or weights) are v_1d_1, v_2d_2, v_3d_3 . . .
The mass of the mixture is the sum of these masses, and its volume is the sum of the volumes v_1, v_2, v_3 . . . ; hence its density is

$$\frac{v_1d_1+v_2d_2+\ldots}{v_1+v_2+\ldots}.$$

Secondly, let the weights or masses m_1, m_2, m_3 . . . of the components be given, together with their densities d_1, d_2, d_3 . . .
Then their volumes are $\frac{m_1}{d_1}$, $\frac{m_2}{d_2}$, $\frac{m_3}{d_3}$ . . .
The volume of the mixture is the sum of these volumes, and its mass is $m_1+m_2+m_3+$ . . . ; hence its density is

$$\frac{m_1+m_2+\ldots}{\frac{m_1}{d_1}+\frac{m_2}{d_2}+\ldots}.$$

88B. Graphical Method of Graduation.—When the points on the stem which correspond to some five or six known densities, nearly equidifferent, have been determined, the intermediate graduations can be inserted with tolerable accuracy by the graphical method of interpolation, a method which has many applications in physics besides that which we are now considering. Suppose A and B (Fig. 85) to represent the extreme points, and I, K, L, R intermediate points, all of which correspond to known densities. Erect ordinates (that is to say, perpendiculars) at these points, proportional to the respective densities, or (which will serve our purpose equally well) erect ordinates II′, KK′, LL′, RR′, BC proportional to the excesses of the densities at I, K, L, R, B above the density at A. Any scale of equal parts can be employed for laying off the ordinates, but it is convenient to adopt a scale which will make the greatest ordinate BC not much greater nor much less than the base line AB. In the figure, the density at B is supposed to be 1·80, the density at A being 1. The difference of density is therefore ·80, as indicated by the figures 80 on the scale of equal parts. Having erected the ordinates, we must draw through their extremities the curve AI′K′L′R′C, making it as free from sudden turns as possible, as it is upon the regularity of this curve that the accuracy of the interpolation depends. Then to find the point on the stem AB at which any other density is to be marked—say 1·60, we must draw through the 60th division, on the line of equal parts, a horizontal line to meet the curve, and, through the point thus found on the curve, draw an ordinate. This ordinate will meet the base line AB in the required point, which is accordingly marked 1·6 in the figure. The curve also affords the means of solving the converse problem, that is, of finding the density corresponding to any given point on the stem. At the given point in AB, which represents the stem, we must draw an ordinate, and through the point where this meets the curve we must draw a horizontal line to meet the scale of equal parts. The point thus determined on the scale of equal parts indicates the density required, or rather the excess of this density above the density of A.

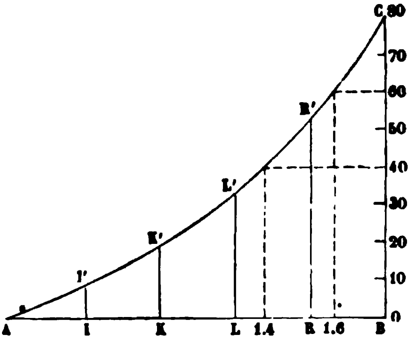

Fig. 85.—Graphical Method of Graduation.

CHAPTER XI.

VESSELS IN COMMUNICATION.—CAPILLARITY.

89. Equilibrium in Vessels in Communication.—When a liquid is contained in vessels communicating with each other, and is in equilibrium, it stands at the same height in the different parts of the system, so that the free surfaces all lie in the same horizontal plane.

This is an immediate consequence of the fact that layers of equal pressure in a liquid are always horizontal (§ 64); for if we take any such layer at the bottom of the system, we must proceed upwards through the same vertical height in all parts of the system in order to reach the free surface which corresponds to the pressure. Thus, in the system represented by Fig. 89, the liquid is seen to stand at the same height in the principal vessel and in the variously shaped tubes communicating with it. If one of these tubes is cut off at a height less than that of the liquid in the principal vessel, and if it

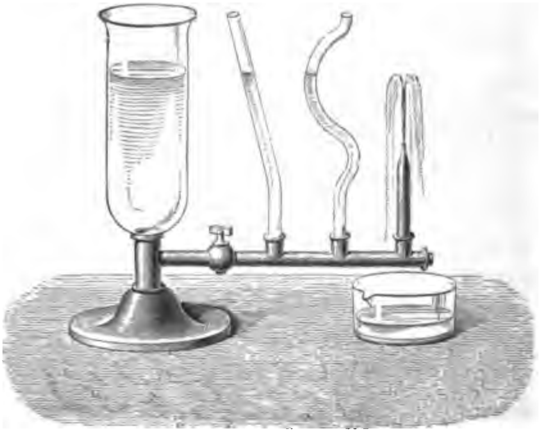

Fig. 89.—Vessels in Communication.

be made to terminate in a narrow mouth, the liquid will be seen to spout up nearly to the level of that in the principal vessel.

This tendency of liquids to find their own level is very important, and of continual application. Thus, a reservoir of water may have different pipes issuing from it and spreading out in all possible directions with any number of turns and windings; provided that the ends of these pipes lie below the level of the reservoir, the water will flow through the pipes and run out at their extremities. The velocity of exit, however, will depend on the form and arrangement of the pipes, as well as on the difference of level. This velocity must of course be taken into account in calculating the quantity of water that will flow in a given time; and in forming plans for the proper distribution of public supplies of water. It also determines the height to which a jet of water can be discharged from an opening at the end of the pipe.

90. Water-level.—The well-known instrument called the water-level depends upon the property just mentioned. It consists of a metal tube *bb*, bent at right angles at its extremities. These carry two glass tubes *aa*, very narrow at the top, and of the same diameter. The tube rests on a tripod stand, at the top of which is a joint that enables the observer to turn the apparatus and set it in any direction. The tube is placed in a position *nearly* horizontal, and water, generally coloured a little, is poured in until it stands at about three-fourths of the height of each of the glass tubes.

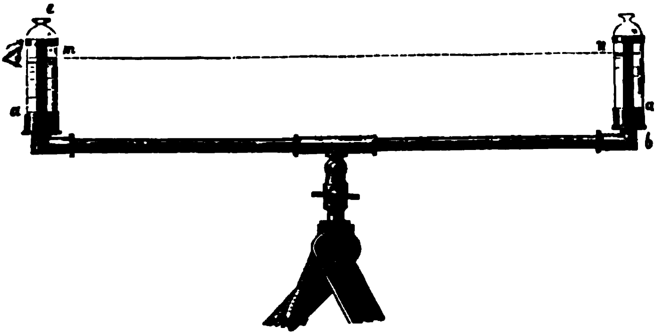

Fig. 90.—Water-level.

By the principle of equilibrium in vessels communicating with each other, the surfaces of the liquid in the two branches are in the same horizontal plane, so that if the line of the observer's sight just grazes the two surfaces, it will be horizontal.

This is the principle of the operation called *levelling*, the object of which is to determine the difference of vertical height, or *difference of level*, between two given points. Suppose A and B to be the two points (Fig. 91). At each of these points is fixed a levelling-staff, that is, an upright rod divided into parts of equal length, on which slides a small square board whose centre serves as a mark for the observer.

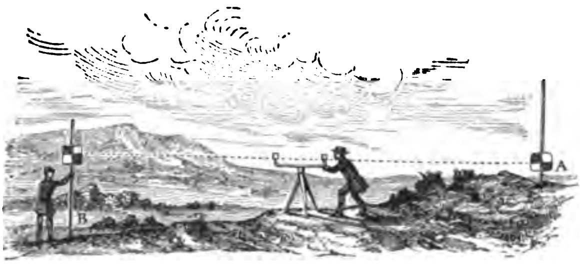

Fig. 91.—Levelling.

The level being placed at an intermediate station, the observer directs the line of sight towards each levelling-staff, and the mark is raised or lowered till the line of sight passes through its centre. The marks on the two staves are in this way brought to the same level. The staff in the rear is then carried in advance of the other, the level is again placed between the two, and another observation taken. In this way, by noting the division of the staff at which the sliding mark stands in each case, the difference of levels of two distant stations can be deduced from observations at a number of intermediate points.

91. Spirit-level.—These observations can be made in a much more exact and convenient manner by means of the spirit-level. This instrument is composed of a glass tube slightly curved, containing a liquid, which is generally alcohol, and which fills the whole extent of the tube, except a small space occupied by an air-bubble. This tube is inclosed in a mounting which is firmly supported on a stand.

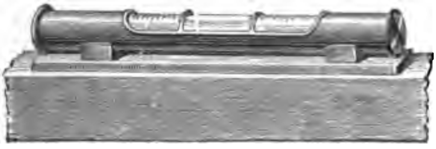

Fig. 92.—Spirit-level.

Suppose the tube to have been so constructed that a vertical section of its upper surface is an arc of a circle, and suppose the instrument placed upon a horizontal plane (Fig. 93). The air-bubble will take up a position MN at the highest part of the tube, such that the arcs MA and NB are equal. Hence it follows that if the level be reversed end for end, the bubble will occupy the same position, the point N coming to M, and *vice versa*. This will not be the case if AB is inclined to the horizon (Fig. 94), for then the distance MA being different from NB, after the apparatus has been turned, the bubble will assume a symmetrical position at the opposite end of the tube. The condition, therefore, that the line on which the spirit-level rests should be horizontal is, that after this operation of reversal the bubble should remain within the same limits. In order to avoid the trouble of turning the instrument, the maker marks these limits by reference-marks on the tube or its mounting, and in order to determine that a line is horizontal it is only necessary to make sure that, when the level is placed upon it, the bubble lies exactly between these reference-marks.

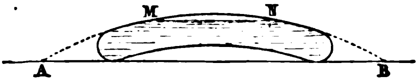

Fig. 93.

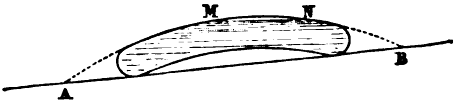

Fig. 94.

In order that a plane surface may be horizontal, we must have two lines in it horizontal. This result is practically attained in the following manner:—The surface is made to rest on three levelling screws which form the three vertices of an isosceles triangle; the level is first placed parallel to the base of the triangle, and, by means of one of the screws, the bubble is brought between the reference-marks. The instrument is then placed perpendicularly to its first position, and the bubble is brought between the marks by means ot

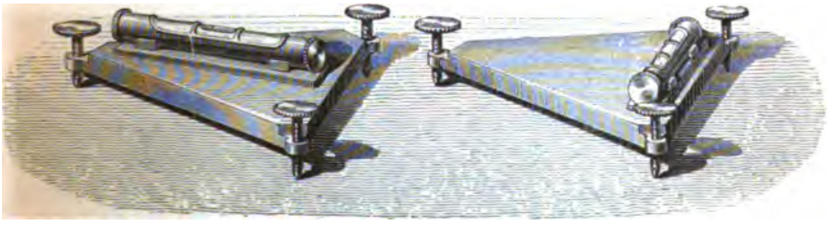

Fig. 95.—Testing the Horizontality of a Surface.

the third screw; this second operation cannot disturb the result of the first, since the plane has only been turned about a horizontal line as hinge.

92. Level furnished with Telescope.—In order to apply the spirit-level to land-surveying, an apparatus such as that represented in the figure is employed. Upon a frame AA, movable about a vertical axis B, are placed a spirit-level *nn*, and a telescope LL, in positions parallel to each other. The telescope is furnished at its focus with two fine wires crossing one another, whose point of intersection determines the line of sight with great precision. The apparatus, which is provided with levelling screws H, rests on a tripod stand, and the observer is able, by turning it about its axis, to command the different points of the horizon. By a process of adjustment which need not here be described, it is known that when the bubble is between the marks the line of sight is horizontal; so that we may proceed to find the difference of level between two points in the same way as with the water-level; but the operation is much more precise, and the range of vision much more extensive. By furnishing the instrument with a graduated horizontal circle P, we may obtain the azimuths of the points observed, and thus map out contour lines.

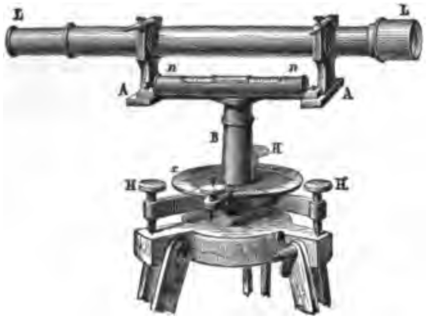

Fig. 96.—Spirit-level with Telescope.

On each side of the reference-marks of the bubble are divisions for measuring small deviations from horizontality. It is, in fact, easy to see, by reference to Fig. 93, that by tilting the level through any small angle, the bubble is displaced by a quantity proportional to this angle, at least when the curvature of the instrument is that of a circle.

For determining the angular value corresponding to each division of the tube, it is usual to employ an apparatus opening like a pair of compasses by a hinge C, on one of the legs of which rests, by two V-shaped supports, the tube T of the level. The compass is opened by means of a micrometer screw V, of very regular action; and as the distance of the screw from the hinge is known, as well as the distance between the threads of the screw, it is easy to calculate beforehand the value of the divisions on the micrometer head. The levelling screws of the instrument serve to bring the bubble between its reference-marks, so that the micrometer screw is only used to determine the value of the divisions on the tube.

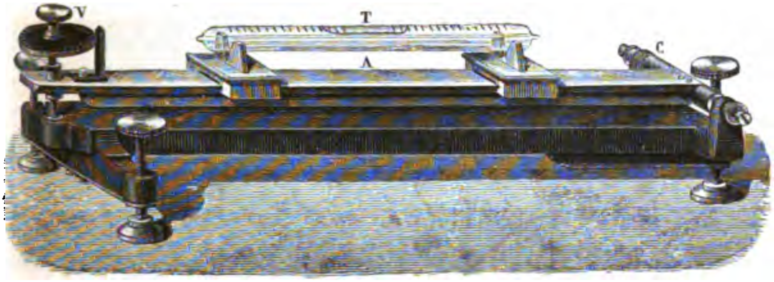

Fig. 97.—Graduation of Spirit-level.

93. Equilibrium of Two Different Liquids in Communicating Vessels.—If into one of two tubes in communication we pour a liquid, say mercury, this liquid will rise to the same height in both branches. If we now pour water into one of them, the mercury will be pushed back in the other branch; and when equilibrium has been established, the *heights of the two liquids above the surface of separation* will be very unequal, as shown in the figure. In general, these heights, since they correspond to the same pressure upon the surface of separation, *will be inversely proportional to the densities.*

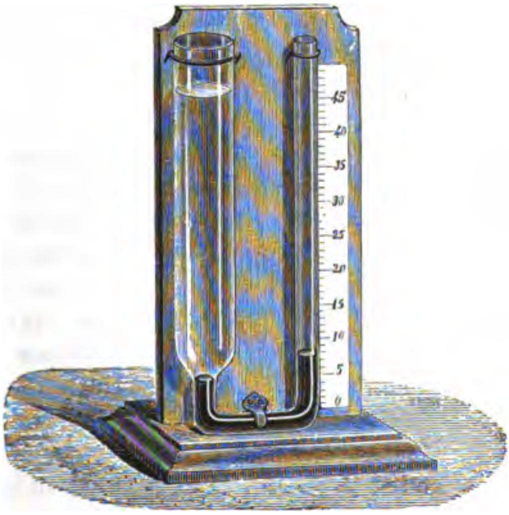

Fig. 98.—Equilibrium of Two Fluids in Communicating Vessels.

94. Capillarity—General Phenomena.—The different principles of equilibrium which have been explained in the preceding paragraphs, are subject to remarkable exceptions when the vessels in which the liquids are contained are very narrow, or, as they are called, capillary (*capillus*, a hair); and also in the case of vessels of any size, when we consider the portion of the liquid which is in close proximity to the sides.

1. *Free Surface.*—The surface of a liquid is not horizontal in the neighbourhood of the sides of the vessel, but presents a very decided curvature. When the liquid wets the vessel, as in the case of water in a glass vessel (Fig. 99), the surface is concave; on the contrary when the liquid does not wet the vessel, as in the case of mercury in a glass vessel (Fig. 100), the surface is, generally speaking, convex.

2. *Capillary Elevation and Depression.*—If a very narrow tube of glass be plunged in water, or any other liquid that will wet it (Fig. 101), it will be observed that the level of the liquid, instead of remaining at the same height inside and outside of the tube, stands perceptibly higher in the tube; a *capillary ascension* takes place, which varies in amount according to the nature of the liquid and the diameter of the tube. It will also be seen that the liquid column thus raised terminates in a concave surface. If a glass tube be dipped in mercury, which does not wet it, it will be seen, by bringing the tube to the side of the vessel, that the mercury is depressed in its interior, and that it terminates in a convex surface (Fig. 102).

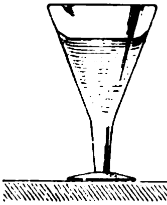
Fig. 99.

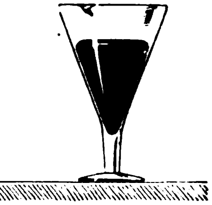
Fig. 100.

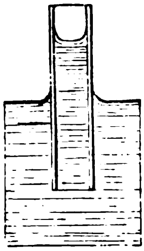
Fig. 101.

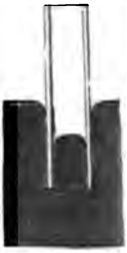
Fig. 102.

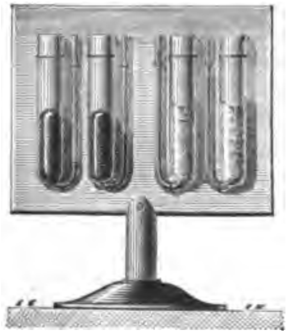
Fig. 103.

3. *Capillary Vessels in Communication with Others.*—If we take two bent tubes, each having one branch of a considerable diameter and the other extremely narrow, and pour into one of them a liquid which

wets it, and into the other mercury, the liquid will be observed in the former case to stand higher in the capillary than in the principal branch, and in the latter case to stand lower; the free surfaces being at the same time concave in the case of the liquid which wets the tubes, and convex in the case of the mercury.

95. Circumstances which influence Capillary Elevation and Depression.—In wetted tubes the elevation depends upon the nature of the liquid; thus, at the temperature of 18° Cent., water rises $29{\cdot}79^{\mathrm{mm}}$ (1·2 inch) in a tube 1 millimetre ($\frac{1}{25}$ inch) in diameter, alcohol rises $12{\cdot}18^{\mathrm{mm}}$, nitric acid $22{\cdot}57^{\mathrm{mm}}$, essence of lavender $4{\cdot}28^{\mathrm{mm}}$, &c. The nature of the tube is almost entirely immaterial, provided the precaution be first taken of wetting it with the liquid to be employed in the experiment, so as to leave a film of the liquid adhering to the sides of the tube.

Capillary depression, on the other hand, depends both on the nature of the liquid and on that of the tube. Both ascension and depression diminish as the temperature increases; for example, the elevation of water, which in a tube of a certain diameter is equal to 132^{mm}. at 0° Cent., is only 106^{mm} at 100°.

96. Law of Diameters.—*Capillary elevations and depressions, when all other circumstances are the same, are inversely proportional to the diameters of the tubes.* As this law is a consequence of the mathematical theories which are generally accepted as explaining capillary phenomena, its verification has been regarded as of great importance.

The experiments of Gay-Lussac, which confirmed this law, have been repeated, with slight modifications, by several observers. The method employed consists essentially in measuring the capillary elevation of a liquid by means of a cathetometer (Fig. 104). The telescope ll is directed first to the top n of the column in the tube, and then to the end of a pointer b, which touches the surface of the liquid at a point where it is horizontal. In observing the depression of mercury, since the opacity of the metal prevents us from seeing the tube, we must bring the tube close to the side of the vessel e.

The diameter of the tube can be measured directly by observing its section through a microscope, or we may proceed by the method employed by Gay-Lussac. He weighed the quantity of mercury which filled a known length l of the tube; this weight w is that of a cylinder of mercury whose radius x is determined by the equation $13{\cdot}59\ \pi x^2 l = w$.*

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* This formula is only true for the metrical system, 13·59 being the specific gravity of mercury. If x and l are in centimetres, w will be in grammes.

The result of these different experiments is, that in the case of wetted tubes the law is exactly fulfilled, provided that they be previously washed with the greatest care, so as to remove all foreign matters, and that the liquid on which the experiment is to be performed be first passed through them. When the liquid does not wet the tube, various causes combine to affect the form of the surface in which the liquid column terminates; and we cannot infer the depression from knowing the diameter, unless we also take into consideration some element connected with the form of the terminal surface, such as the length of the sagitta, or the angle made with the sides of the tube by the extremities of the curved surface, which is called the *angle of contact.*

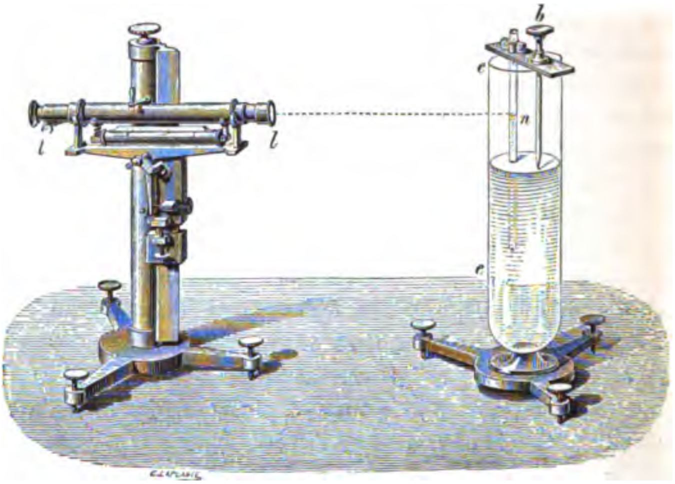

Fig. 104.—Verification of Law of Diameters.

97. Cause of Capillary Phenomena.—Capillary phenomena, as they take place alike in air and in vacuo, cannot be attributed to the action of the atmosphere. They depend upon molecular actions which take place between the particles of the liquid itself, and between the liquid and the solid containing it, the actions in question being purely superficial—that is to say, being confined to an extremely thin layer forming the external boundary of the liquid, and to an extremely thin superficial layer of the solid in contact with the liquid. For example, it is found in the case of glass tubes, that the amount of capillary elevation or depression is not at all affected by the thickness of the sides of the tube. The following are some of the principles which govern capillary phenomena.

1. For a given liquid in contact with a given solid, with a definite intimateness of contact (this last element being dependent upon the cleanness of the surface, upon whether the surface of the solid has been recently washed by the liquid, and perhaps upon some other particulars), there is (at any specified temperature) a definite angle of contact, which is independent of the directions of the surfaces with regard to the vertical.

2. Every liquid behaves as if a thin film, forming its external layer, were in a state of tension, and exerting a constant effort to contract. This tension, or contractile force, is exhibited over the whole of the free surface (that is, the surface which is exposed to air), but wherever the liquid is in contact with a solid, its existence is masked by other molecular actions. It is uniform in all directions in the free surface, and at all points in this surface, being dependent only on the nature and temperature of the liquid. Its intensity for several specified liquids is given in tabular form further on (§ 97F) upon the authority of Van der Mensbrugghe. Tension of this kind must of course be stated in units of force per linear unit, because by doubling the width of a band we double the force required to keep it stretched. Mensbrugghe considers that such tension really exists in the superficial layer; but the majority of authors (and we think with more justice) regard it rather as a convenient fiction, which accurately represents the effects of the real cause. Two of the most eminent writers on the cause of capillary phenomena are Laplace and Dr. Thomas Young. The subject presents difficulties which have not yet been fully surmounted.

The law of diameters is a direct consequence of the two preceding principles; for if α denote the external angle of contact (which is acute in the case of mercury against glass), T the tension per unit length, and r the radius of the tube, then $2\pi r\text{T}$ will be the whole amount of force exerted at the margin of the surface; and as this force is exerted in a direction making an angle α with the vertical, its vertical component will be $2\pi r\text{T}\cos\alpha$, which is exerted in pulling the tube upwards and the liquid downwards.

If w be the weight of unit volume of the liquid, then $\pi r^2 w$ is the weight of as much as would occupy unit length of the tube; and if h denote the height of a column whose weight is equal to the force tending to depress the liquid, we have

$$\pi r^2 h w = 2\pi r \text{T} \cos\alpha;$$

whence $h = \frac{2\mathrm{T}\cos a}{r \,.\, w}$, which, when the other elements are given, varies inversely as r, the radius of the tube.

Having regard to the fact that the surface is not of the same height in the centre as at the edges, it is obvious that h denotes the mean height.

If a be obtuse, h will be negative—that is to say, there will be elevation instead of depression. In the case of water against a tube which has been well wetted with that liquid, a is 180°—that is to say, the tube is tangential to the surface. For this case the formula for h gives

$$\text{elevation} = \frac{2\mathrm{T}}{rw}.$$

Again, for two parallel vertical plates at distance u, the vertical force of capillarity for a unit of length is $2\mathrm{T}\cos a$, which must be equal to whu, being the weight of a sheet of liquid of height h, thickness u, and length unity. We have therefore

$$h = \frac{2\mathrm{T}\cos a}{uw},$$

which agrees with the expression for the depression or elevation in a circular tube whose radius is equal to the distance between these parallel plates.

The surface tension always tends to reduce the surface to the smallest area which can be inclosed by its actual boundary; and therefore always produces a normal force directed *towards* the concave side of the superficial film. Hence, wherever there is capillary elevation the free surface must be concave; wherever there is depression it must be convex.

97A. It follows from a well-known proposition in statics (Todhunter's *Statics*, § 194), that if a *cylindrical* film be stretched with a uniform tension T (so that the force tending to pull the film asunder across any short line drawn on the film, is T times the length of the line), the resultant normal pressure (which the film exerts, for example, against the surface of a solid internal cylinder over which it is stretched) is T divided by the radius of the cylinder.

It can be proved that a film of any form, stretched with uniform tension T, exerts at each point a normal pressure equal to the sum of the pressures which would be exerted by two overlapping cylindrical films, whose axes are at right angles to one another, and whose cross sections are circles of curvature of normal sections at the point. That is to say, if P be the normal force per unit area, and rr' the radii of curvature in two mutually perpendicular normal sections at the point, then

$$P = T\left(\frac{1}{r} + \frac{1}{r'}\right).$$

At any point on a curved surface, the normal sections of greatest and least curvature are mutually perpendicular, and are called the principal normal sections at the point. If the corresponding radii of curvature be R, R′, we have

$$P = T\left(\frac{1}{R} + \frac{1}{R'}\right); \qquad (1)$$

or *the normal force per unit area is equal to the tension per unit length multiplied by the sum of the principal curvatures.*

In the case of capillary depressions and elevations, the superficial film at the free surface is to be regarded as pressing the liquid inwards, or pulling it outwards, according as this surface is convex or concave, with a force P given by the above formula. The value of P at any point of the free surface is equal to the pressure due to the height of a column of liquid extending from that point to the level of the general horizontal surface. It is therefore greatest at the edges of the elevated or depressed column in a tube, and least in the centre; and the curvature, as measured by $\frac{1}{R} + \frac{1}{R'}$, must vary in the same proportion. If the tube is so large that there is no sensible elevation or depression in the centre of the column, the centre of the free surface must be sensibly plane.

97 B. Another consequence of the formula is, that in circumstances where there can be no normal pressure towards either side of the surface,

$$\frac{1}{R} + \frac{1}{R'} = 0; \qquad (2)$$

which implies that either the surface is plane, in which case each of the two terms is separately equal to zero, or else

$$R = -R'; \qquad (3)$$

that is, the principal radii of curvature are equal, and lie on opposite sides of the surface. The formulæ (2), (3) apply to a film of soapy water attached to a loop of wire. If the loop be in one plane, the film will be in the same plane. If the loop be not in one plane, the film cannot be in one plane, and will in fact assume that form which gives the least area consistent with having the loop for its boundary. At every point it will be observed to be, if we may so say, concave towards both sides, and convex towards both sides, the concavity being precisely equal to the convexity—that is to say, equation (3) is satisfied at every point of the film.

In this case both sides of the film are exposed to atmospheric pressure. In the case of a common soap-bubble the outside is exposed to atmospheric pressure, and the inside to a pressure somewhat greater, the difference of the pressures being compensated by the tendency of the film to contract. Formula (1) becomes for either the outer or inner surface of a spherical bubble

$$P = \frac{2T}{R};$$

but this result must be doubled, because there are two free surfaces; hence the excess of pressure of the inclosed above the external air is $\frac{4T}{R}$, R denoting the radius of the bubble.

The value of T for soapy water is about 1 grain per linear inch; hence, if we divide 4 by the radius of the bubble expressed in inches, we shall obtain the excess of internal over external pressure *in grains per square inch.*

The value of T for any liquid may be obtained by observing the amount of elevation or depression in a tube of given diameter, and employing the formula

$$T = \frac{whr}{2\cos\alpha}, \qquad (4)$$

which follows immediately from the formula for h in § 97.

97c. It is this uniform surface tension, of which we have been speaking, which causes a drop of a liquid falling through the air either to assume the spherical form, or to oscillate about the spherical form. The phenomena of drops can be imitated on an enlarged scale, under circumstances which permit us to observe the actual motions, by a method devised by Professor Plateau of Ghent. Olive-oil is intermediate in density between water and alcohol. Let a mixture of alcohol and water be prepared, having precisely the density of olive-oil, and let about a cubic inch of the latter be gently introduced into it with the aid of a funnel or pipette. It will assume a spherical form, and if forced out of this form and then left free, will slowly oscillate about it; for example, if it has been compelled to assume the form of a prolate spheroid, it will pass to the oblate form, will then become prolate again, and so on alternately, becoming however more nearly spherical every time, because its movements are hindered by friction, until at last it comes to rest as a sphere.

97 D. Capillarity furnishes no exception to the principle that the pressure in a liquid is the same at all points at the same depth. When the free surface within a tube is convex, and is consequently depressed below the general level of the external surface, the pressure becomes suddenly greater on passing downwards through the superficial layer, by the amount due to the curvature. Below this it increases regularly by the amount due to the depth of liquid passed through. The pressure at any point vertically under the convex meniscus[1] may be computed, either by taking the depth of the point below the general free surface, and adding atmospheric pressure to the pressure due to this depth, according to the ordinary principles of hydrostatics, or by taking the depth of the point below that point of the meniscus which is vertically over it, adding the pressure due to the curvature at this point, and also adding atmospheric pressure.

When the free surface of the liquid within a tube is concave, the pressure suddenly diminishes on passing downwards through the superficial layer, by the amount due to the curvature as given by formula (1); that is to say, the pressure at a very small depth is less than atmospheric pressure by this amount. Below this depth it goes on increasing according to the usual law, and becomes equal to atmospheric pressure at that depth which corresponds with the level of the general external surface. The pressure at any point in the liquid within the tube can therefore be obtained either by subtracting from atmospheric pressure the pressure due to the elevation of the point above the general surface, or by adding to atmospheric pressure the pressure due to the depth below that point of the meniscus which is on the same vertical, and subtracting the pressure due to the curvature at this point.

These rules imply, as has been already remarked, that the curvature is different at different points of the meniscus, being greatest where the elevation or depression is greatest, namely at the edges of the meniscus; and least at the point of least elevation or depression, which in a cylindrical tube is the middle point.

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[1] The convex or concave surface of the liquid in a tube is usually denoted by the name *meniscus* (*μηνίσκος*, a crescent), which denotes a form approximately resembling that of a watch-glass.

The principles just stated apply to all cases of capillary elevation and depression.

They enable us to calculate the force with which two parallel vertical plates, partially immersed in a liquid which wets them, are urged towards each other by capillary action. The portion of liquid elevated between them is at less than atmospheric pressure, and therefore is insufficient to resist the atmospheric pressure which is exerted on the outer faces of the plates. The average pressure in the elevated portion of liquid is that which exists half-way up it, and is less than atmospheric pressure by the pressure of a column of liquid whose height is half the elevation.

Even if the liquid be one which does not wet the plates, they will still be urged towards each other by capillary action; for the inner faces of the plates are exposed to merely atmospheric pressure over that portion of their areas which corresponds to the depression, while the corresponding portions of the external faces are exposed to atmospheric pressure increased by the weight of a portion of the liquid.

These principles explain the apparent attraction exhibited by bodies floating on a liquid which either wets them both or wets neither of them. When the two bodies are near each other they behave somewhat like parallel plates, the elevation or depression of the liquid between them being greater than on their remote sides.

If two floating bodies, one of which is wetted and the other unwetted by the liquid, come near together, the elevation and depression of the liquid will be less on the near than on the remote sides, and apparent repulsion will be exhibited.

In all cases of capillary elevation or depression, the solid is pulled downwards or upwards with a force equal to that by which the liquid is raised or depressed. In applying the principle of Archimedes to a solid partially immersed in a liquid, it is therefore necessary (as we have seen in § 79), when the solid produces capillary depression, to reckon the void space thus created as part of the displacement; and when the solid produces capillary elevation, the fluid raised above the general level must be reckoned as *negative* displacement, tending to *increase* the apparent weight of the solid.

97E. Thus far all the effects of capillary action which we have mentioned are connected with the curvature of the superficial film, and depend upon the principle that a convex surface increases and a concave surface diminishes the pressure in the interior of the liquid.

But there is good reason for maintaining that whatever be the form of the free surface there is always a certain amount of pressure in the interior due to the molecular action at this surface, and that the pressure due to the curvature of the surface is to be added to or subtracted from a definite amount of pressure which is independent of the curvature and depends only on the nature and condition of the liquid. This indeed follows at once from the fact that capillary elevation can take place in vacuo. As far as the principles of the preceding paragraphs are concerned, we should have, at points within the elevated column, a pressure less than that existing in the vacuum. This, however, cannot be; we cannot conceive of negative pressure existing in the interior of a liquid, and we are driven to conclude that the elevation is owing to the excess of the pressure caused by the plane surface in the containing vessel above the pressure caused by the concave surface in the capillary tube.

There are some other facts which seem only explicable on the same general principle of interior pressure due to surface action,—facts which attracted the notice of some of the earliest writers on pneumatics, namely, that siphons will work in vacuo, and that a column of mercury at least 75 inches in length can be sustained—as if by atmospheric pressure—in a barometer tube, the mercury being boiled and completely filling the tube.

97F. We have now to notice certain phenomena which depend on the difference in the surface tensions of different liquids, or of the same liquid in different states.

Let a thin layer of oil be spread over the upper surface of a thin sheet of brass, and let a lamp be placed underneath. The oil will be observed to run away from the spot directly over the flame, even though this spot be somewhat lower than the rest of the sheet. This effect is attributable to the excess of surface tension in the cold oil above the hot.

In like manner, if a drop of alcohol be introduced into a thin layer of water spread over a nearly horizontal surface, it will be drawn away in all directions by the surrounding water, leaving a nearly dry spot in the space which it occupied. In this experiment the water should be coloured in order to distinguish it from the alcohol.

Again, let a very small fragment of camphor be placed on the surface of hot water. It will be observed to rush to and fro with frequent rotations on its own axis, sometimes in one direction and sometimes in the opposite. These effects, which have been a frequent subject of discussion, are now known to be due to the diminution of the surface tension of the water by the camphor which it takes up. Superficial currents are thus created, radiating from the fragment of camphor in all directions; and as the camphor dissolves more quickly in some parts than in others, the currents which are formed are not equal in all directions, and those which are most powerful prevail over the others and give motion to the fragment.

The values of T, the apparent surface tension, for several liquids, are given in the following table, on the authority of Van der Mensbrugghe, in milligrammes (or thousandth parts of a gramme) per millimetre of length. They can be reduced to grains per inch of length by multiplying them by ·392; for example, the surface tension of distilled water is 7·3 × ·392 = 2·86 grains per inch.

Distilled water at 20° Cent.,	7·3
Sulphuric ether,	1·88
Absolute alcohol,	2·5
Olive-oil,	3·5
Mercury,	49·1
Bisulphide of carbon,	3·57
Solution of Marseilles soap, 1 part of soap to 40 of water,	2·83
Solution of saponine,	4·67
Saturated solution of carbonate of soda,	4·28
Water impregnated with camphor,	4·5

98. Endosmose.—Capillary phenomena have undoubtedly some connection with a very important property discovered by Dutrochet, and called by him *endosmose*.

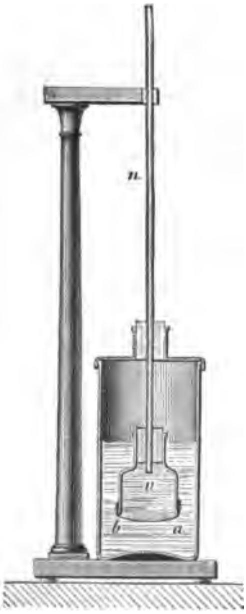

Fig. 105.—Endosmometer.

The *endosmometer* invented by him to illustrate this phenomenon consists of a reservoir *v* closed below by a membrane *ba*, and terminating above in a tube of considerable length. This reservoir is filled, suppose, with a solution of gum in water, and is kept immersed in water. At the end of some time the level of the liquid in the tube will be observed to have risen to *n*, suppose, and at the same time traces of gum will be found in the water in which the reservoir is immersed. Hence we conclude that the two liquids have penetrated through the membrane, but in different proportions; and this is what is called endosmose.

If instead of a solution of gum we employed water containing albumen, sugar, or gelatine in solution, a similar result would ensue. The membrane may be replaced by a slab of wood or of porous clay. Physiologists have justly attached very great importance to this discovery of Dutrochet. It explains, in fact, the interchange of liquids which is continually taking place in the tissues and vessels of the animal system, as well as the absorption of water by the spongioles of roots, and several similar phenomena.

As regards the power of passing through porous diaphragms, Graham has divided substances into two classes—*crystalloids* and *colloids* (κόλλη, glue). The former are susceptible of crystallization, form solutions free from viscosity, are sapid, and possess great powers of diffusion through porous septa. The latter, including gum, starch, albumen, &c., are characterized by a remarkable sluggishness and indisposition both to diffusion and to crystallization, and when pure are nearly tasteless.

CHAPTER XII.

THE BAROMETER.

99. Weight of the Air and of Gases.—Gaseous bodies possess a number of properties in common with liquids; like them, they transmit pressures entire and in all directions, according to the principle of Pascal; but they differ essentially from liquids in the permanent repulsive force exerted between their molecules, in virtue of which a mass of gas always tends to expand.

The opinion was long held that the air was without weight; or, to speak more precisely, it never occurred to any of the philosophers who preceded Galileo to attribute any influence in natural phenomena to the weight of the air. And as this influence is really of the first importance, and comes into play in many of the commonest phenomena, it very naturally happened that the discovery of the weight of air formed the commencement of the modern revival of physical science.

It appears, however, that Aristotle conceived the idea of the possibility of air having weight, and, in order to convince himself on this point, he weighed a skin inflated and collapsed. As he obtained the same weight in both cases, he relinquished the idea which he had for the moment entertained. In fact, the experiment, as he performed it, could only give a negative result; for if the weight of the skin was increased, on the one hand, by the introduction of a fresh quantity of air, it was diminished, on the other, by the corresponding increase in the upward pressure of the air displaced. In order to draw a certain conclusion, the experiment should be performed with a vessel which could receive within it air of different degrees of density, without changing its own volume.

Galileo is said to have devised the experiment of weighing a globe filled alternately with ordinary air and with compressed air.

As the weight is greater in the latter case, Galileo should have drawn the inference that air is heavy. It does not appear, however, that the importance of this conclusion made much impression on him, for he did not give it any of those developments which might have been expected to present themselves to a mind like his.

100. Experiment of Otto Guericke.—Otto Guericke, the illustrious inventor of the air-pump, in 1650 performed the following experiment, which is decisive:—

A globe of glass, furnished with a stop-cock, and of a sufficient capacity (about twelve litres), is exhausted of air. It is then suspended from one of the scales of a balance, and a weight sufficient to produce equilibrium is placed in the other scale. The stop-cock is then opened, the air rushes into the globe, and the beam is observed gradually to incline, so that an additional weight is required in the other scale, in order to re-establish equilibrium. If the capacity of the globe is 12 litres, about 15·5 grammes will be needed, which gives 1·3 gramme as the approximate weight of a litre of air.[1]

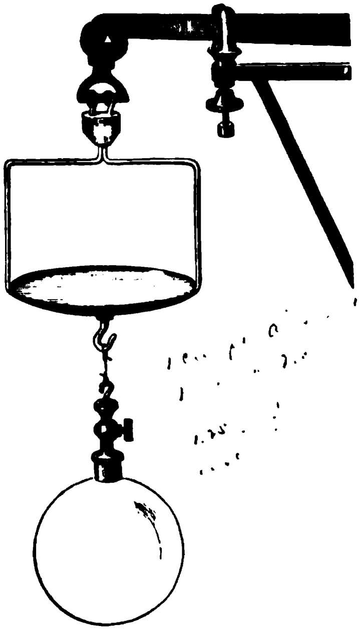

Fig. 106.— Weight of Air.

If, in performing this experiment, we take particular precautions to insure its precision, as we shall explain in the book on heat, it will be found that, at the temperature of freezing water, and under the pressure of one atmosphere, a litre of air weighs 1·293 gramme.[2] Under these circumstances, the ratio of the weight of a volume of air to that of an equal volume of water is $\frac{1 \cdot 293}{1000} = \frac{1}{773}$. Air is thus 773 times lighter than water.

By repeating this experiment with other gases, we may determine their weight as compared with that of air, and the absolute weight of a litre of each of them. Thus it is found that a litre of oxygen weighs 1·43 gramme, a litre of carbonic acid 1·97 gramme, a litre of hydrogen 0·089 gramme, &c.

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[1] A cubic foot of air in ordinary circumstances weighs about an ounce and a quarter.

[2] In strictness, the weight in grammes of a litre of air under the pressure of 760 millimetres of mercury is different in different localities, being proportional to the intensity of gravity—not because the force of gravity on the litre of air is different, for though this

101. Atmospheric Pressure.—The atmosphere encircles the earth with a layer some 50 or 100 miles in thickness; this heavy fluid mass exerts on the surface of all bodies a pressure entirely analogous both in nature and origin to that sustained by a body wholly immersed in a liquid. It is subject to the fundamental law mentioned in § 64. The pressure should therefore diminish as we ascend from the surface of the earth, but should have the same value for all points in the same horizontal layer, provided that the air is in a state of equilibrium. On account of the great compressibility of gas, the lower layers are much more dense than the upper ones; but the density, like the pressure, is constant in value for the same horizontal layer, throughout any portion of air in a state of equilibrium. Whenever there is an inequality either of density or pressure at a given level, wind must ensue.

We owe to Torricelli an experiment which plainly shows the pressure of the atmosphere, and enables us to estimate its intensity with great precision. This experiment, which was performed in 1643, one year after the death of Galileo, at a time when the weight and pressure of the air were scarcely even suspected, has immortalized the name of its author, and has exercised a most important influence upon the progress of natural philosophy.

102. Torricelli's Experiment.—A tube of about a quarter or a third of an inch in diameter, and about a yard in length, is completely filled with mercury; the extremity is then stopped with the finger, and the tube is inverted in a vessel containing mercury. If the finger is now removed, the mercury will descend in the tube, and after a few

is true, it does not affect the numerical value of the weight when stated in grammes, but because the pressure of 760 millimetres of mercury varies as the intensity of gravity, so that more air is compressed into the space of a litre as gravity increases. (§ 107, 6.)

The *weight in grammes* is another name for the *mass*. The force of gravity on a litre of air under the pressure of 760 millimetres is proportional to the square of the intensity of gravity.

This is an excellent example of the ambiguity of the word *weight*, which sometimes denotes a mass, sometimes a force; and though the distinction is of no practical importance so long as we confine our attention to one locality, it cannot be neglected when different localities are compared.

Regnault's determination of the weight of a litre of dry air at 0° Cent. under the pressure of 760 millimetres at Paris is 1·293187 gramme. Gravity at Paris is to gravity at Greenwich as 3456 to 3457. The corresponding number for Greenwich is therefore 1·293561.

oscillations will remain stationary at a height which varies according to circumstances, but which is generally about 30 inches.

The column of mercury is maintained at this height by the pressure of the atmosphere upon the surface of the mercury in the vessel. In fact, the pressure at the level ABCD must be the same within as without the tube; so that the column of mercury BE exerts a pressure equal to that of the atmosphere.

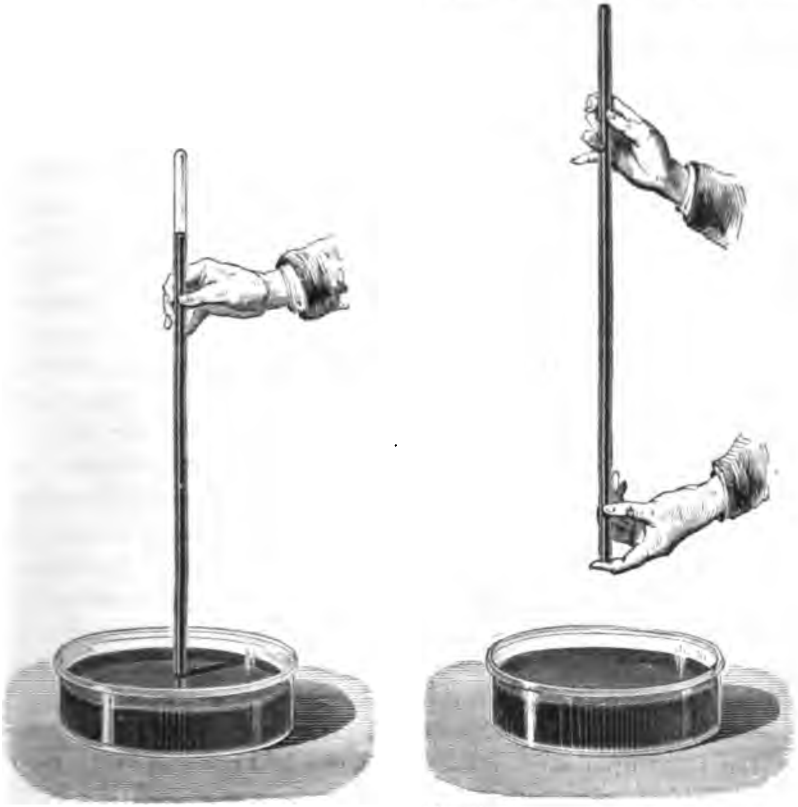

Fig. 107.—Torricellian Experiment.

Accordingly, we conclude from this experiment of Torricelli that *every surface exposed to the atmosphere sustains a normal pressure equal, on an average, to the weight of a column of mercury whose base is this surface, and whose height is 30 inches.*

It is evident that if we performed a similar experiment with water, whose density is to that of mercury as 1 : 13·59, the height of the column sustained would be 13·59 times as much; that is, 30 × 13·59 inches, or about 34 feet. This is the maximum height to which water can be raised in a pump; as was observed by Galileo.

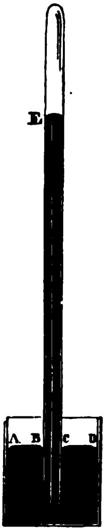

Fig. 108.

In general the heights of columns of different liquids equal in weight to a column of air on the same base, are inversely proportional to their densities.

103. Pressure of One Atmosphere.—We can easily calculate the amount of this pressure for a given surface, for example, a square inch. It is the weight of a column of mercury whose base is a square inch and height 30 inches, that is, the weight of 30 cubic inches of mercury; and as a cubic inch of mercury weighs about half a pound, the atmospheric pressure on a square inch is about 15 pounds. This pressure of 15 pounds[1] to the square inch is called the pressure *of one atmosphere;* it is exerted in a normal direction at all points on the surface of a body, and in consequence, as in the case of a body wholly immersed in a liquid, the resultant of the different elementary pressures is a vertical upward pressure equal to the weight of the air displaced. The effect of the air, therefore, is not, as was formerly supposed, to press bodies to the surface of the earth; on the contrary, it tends to raise them, as in a liquid, but with comparatively small force, owing to its small density. It is upon this principle that the ascent of balloons depends, as we shall see hereafter.

104. Pascal's Experiments.—It is supposed, though without any decisive proof, that Torricelli derived from Galileo the definite conception of atmospheric pressure.[2] However this may be, when the experiment of the Italian philosopher became known in France in 1644, no one was capable of giving the correct explanation of it, and the famous doctrine that "nature abhors a vacuum," by which the rising of water in a pump was accounted for, was generally accepted. Pascal was the first to prove incontestably the falsity of this old doctrine, and to introduce a more rational belief. For this purpose he proposed or executed a series of ingenious experiments, and discussed minutely all the phenomena which were attributed to nature's abhorrence of a vacuum, showing that they were necessary consequences of the pressure of the atmosphere.

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[1] As the weight of a cubic centimetre of mercury at zero is 13·596 grammes, the pressure of 760 millimetres is 13·596 × 76 = 1033·3 grammes per square centimetre = 14·70 pounds per square inch. 760 millimetres are 29·922 inches.

[2] In the fountains of the Grand-duke of Tuscany some pumps were required to raise water from a depth of from 40 to 50 feet. When these were worked, it was found that they would not draw. Galileo determined the height to which the water rose in their tubes, and found it to be about 32 feet; and as he had observed and proved that air has weight, he readily conceived that it was the weight of a column of the atmosphere which maintained the water at this height in the pumps. No very useful results, however, were expected from this discovery, until, at a later date, Torricelli adopted and greatly extended it. Desiring to repeat the experiment in a more convenient form, he conceived the idea of substituting for water a liquid that is 14 times as heavy, namely, mercury, rightly imagining that a column of one-fourteenth of the length would balance the force which sustained 32 feet of water (Biot, *Biographie Universelle,* article "Torricelli").—*D.*

We may cite in particular the observation, made at his suggestion, that the height of the mercurial column decreases in proportion as we ascend. This beautiful and decisive experiment, which is repeated as often as heights are measured by the barometer, and which leaves no doubt as to the nature of the force which sustains the mercurial column, was performed for the first time at Clermont, and on the top of the Puy-de-Dôme, on the 19th September, 1648.

105. The Barometer.—By fixing the Torricellian tube in a permanent position, we have a means of measuring the amount of the atmospheric pressure at any moment; and this pressure may be expressed by the height of the column of mercury which it supports. Such an instrument is called a *barometer*. In order that its indications may be accurate, several precautions must be observed. In the first place, the liquid used in different barometers must be identical, for the height of the column supported naturally depends upon the density of the liquid employed, and if this varies, the observations made with different instruments will not be comparable.

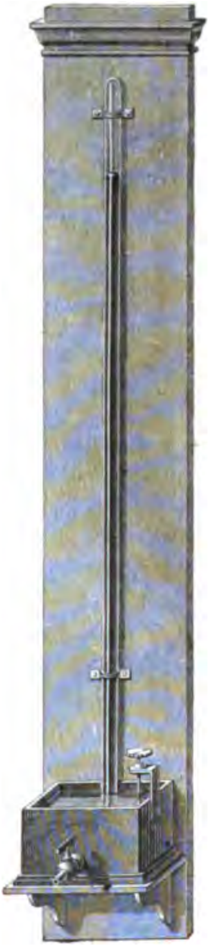

Fig. 109.—Barometer in its simplest form.

The mercury employed is chemically pure, being generally made so by washing with a dilute acid and by subsequent distillation. The barometric tube is filled nearly full, and is then placed upon a sloping furnace, and heated till the mercury boils. The object of this process is to expel the air and moisture which may be contained in the mercurial column, and which, without this precaution, would gradually ascend into the vacuum above, and cause a downward pressure of unknown amount, which would prevent the mercury from rising to the proper height.

The next step is to fill up the tube with pure mercury, taking care not to introduce any bubble of air. The tube is then inverted in a cistern likewise containing pure mercury recently boiled, and is firmly fixed in a vertical position, as shown in Fig. 109.

We have thus a fixed barometer; and in order to ascertain the atmospheric pressure at any moment, it is only necessary to measure the height of the top of the column of mercury above the surface of the mercury in the cistern. For this purpose an iron rod, working in a screw, is fixed vertically above the surface of the mercury in the dish. The extremities of this rod are pointed, and the lower extremity being brought down to touch the surface of the liquid below, the distance of the upper extremity from the top of the column of mercury is measured. Adding to this the length of the rod, which has previously been determined once for all, we have the barometric height. This measurement may be effected with great precision by means of the cathetometer.

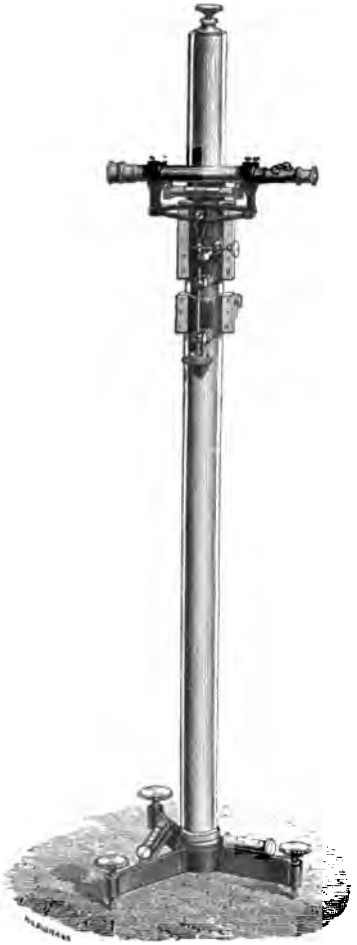

Fig. 110.—Cathetometer.

105 A. Cathetometer.—This instrument, which is so frequently employed in physics to measure the vertical distance between two points, was invented by Dulong and Petit. It consists essentially (Fig. 110) of a vertical scale divided usually into half millimetres. This scale forms part of a brass cylinder capable of turning very easily about a strong steel axis. This axis is fixed on a pedestal provided with three levelling screws, and with two spirit-levels at right angles to each other. Along the scale moves a sliding frame carrying a telescope furnished with crosswires, that is, with two very fine threads, usually spider lines, in the focus of the eye-piece, whose point of intersection serves to determine the line of vision. By means of a clamp and slow-motion screw, the telescope can be fixed with great precision at any required height. The telescope is also provided with a spirit-level and adjusting screw. When the apparatus is in correct adjustment, the line of vision of the telescope is horizontal, and the graduated scale is vertical. If then we wish to measure the difference of level between two points, we have only to sight them successively, and measure the distance passed over on the scale, which is done by means of a vernier attached to the sliding frame.

106. Fortin's Barometer.—The barometer just described is intended to be fixed; when portability is required the barometer invented by Fortin is employed. It is also perfectly adapted to general use. The cistern, which is formed of a tube of boxwood, surmounted by a tube of glass, is closed below by a piece of leather, which can be raised or lowered by means of a screw. This screw works in the bottom of a copper case, which incloses the cistern except at the middle, where it is cut away in front and at the back, so as to leave the surface of the mercury open to view. The barometric tube is encased in a tube of copper, with two slits at opposite sides (Fig. 112); and it is on this tube that the divisions are engraved, the zero point from which they are reckoned being the lower extremity of an ivory point fixed in the covering of the cistern. The temperature of the mercury, which is required for one of the corrections mentioned in next section, is given by a thermometer with its bulb resting against the tube. A sliding

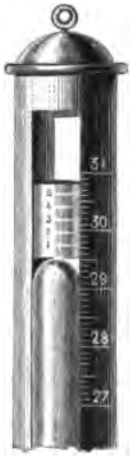

Fig. 112.
Upper portion of Barometer.

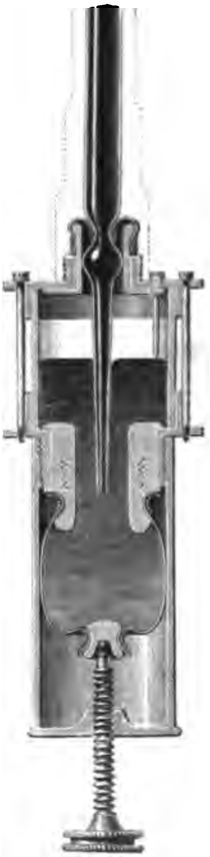

Fig. 111.
Cistern of Fortin's Barometer.

piece, furnished with a vernier,[1] moves along the tube by means of a screw, and enables us to determine the height with great precision. Its lower edge is the zero of the vernier. The way in which the barometric tube is fixed upon the cistern is worth notice. In the centre of the upper surface of the copper casing there is an opening, from which rises a short tube of the same metal, lined with a tube of boxwood. The barometric tube is pushed inside, and fitted in with a piece of chamois leather, which prevents the mercury from issuing; but does not exclude the air, which, passing through the pores of the leather, penetrates into the cistern, and so transmits its pressure.

Before taking an observation, the surface of the mercury is adjusted, by means of the lower screw, to touch the ivory point. The observer knows when this condition is fulfilled by seeing the extremity of the point touch its image in the mercury. The vernier is then raised or lowered, until the horizontal plane in which its zero lies is tangential

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[1] The vernier is an instrument very largely employed for measuring the fractions of a unit of length on any scale. Suppose we have a scale divided into inches, and another scale containing nine inches divided into ten equal parts. If now we make the end of this latter scale, which is called the vernier, coincide with one of the divisions in the scale of inches, as each division of the vernier is $\frac{9}{10}$ of an inch, it is evident that the first division on the scale will be $\frac{1}{10}$ of an inch beyond the first division on the vernier, the second on the scale $\frac{2}{10}$ beyond the second on the vernier, and so on until the ninth on the scale, which will exactly coincide with the tenth on the vernier. Suppose next that in measuring any length we find that its extremity lies between the degrees 5 and 6 on the scale; we bring the zero of the vernier opposite the extremity of the length to be measured, and observe what division on the vernier coincides with one of the divisions on the scale. We see in the figure that it is the seventh, and thus we conclude that the fraction required is $\frac{7}{10}$ of an inch.

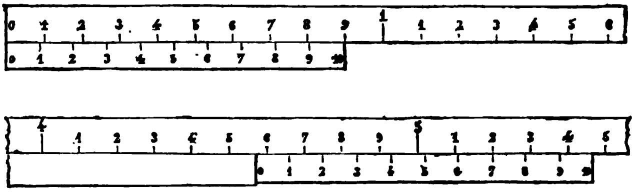

Fig. 113.—Vernier.

If the vernier consisted of 19 inches divided into 20 equal parts, it would read to the $\frac{1}{20}$ of an inch; but there is a limit to the precision that can thus be obtained. An exact coincidence of a division on the vernier with one on the scale seldom or never takes place, and we merely take the division which approaches nearest to this coincidence; so that when the difference between the degrees on the vernier and those on the scale is very small, there may be so much uncertainty in this selection as to nullify the theoretical precision of the instrument. Verniers are also employed to measure angles; when a circle is divided into half degrees, a vernier is used which gives $\frac{1}{30}$ of a division on the circle, that is, $\frac{1}{30}$ of a half degree, or one minute.—*D.*

to the upper surface of the mercurial column, as shown in Fig. 112. In making this adjustment, the back of the instrument should be turned towards a good light, in order that the observer may be certain of the position in which the light is just cut off at the summit of the convexity.

When the instrument is to be moved, the screw at the bottom is turned until the tube is filled. The cistern will then be full also, and the barometer should be inverted, as an additional safeguard both against the introduction of air and the escape of mercury. In making observations upon the surface of the ground, the instrument is suspended from a tripod stand by gimbals,[1] so that it always takes a vertical position; or it may be fixed permanently against a wall.

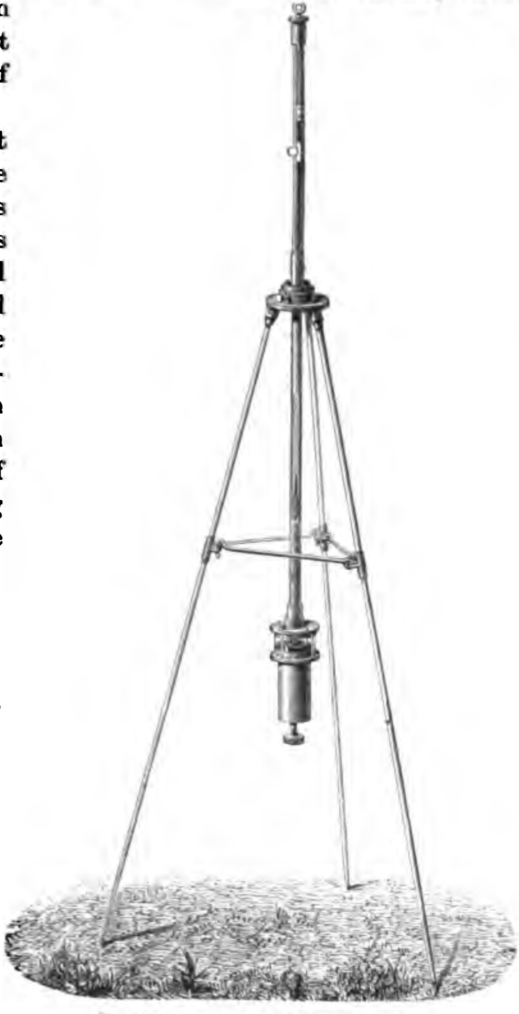

Fig. 114.—Barometer with Tripod Stand.

106 A. Float Adjustment.—In some barometers the ivory point for indicating the proper level of the mercury in the cistern is replaced by a float. F (Fig. 115) is a small ivory piston, having the float attached to its foot, and moving freely up and down between the two ivory guides I. A horizontal line (interrupted by the piston) is engraved on the two guides, and another is engraved on the piston, at such a height that the three lines form one straight line when the surface of the mercury in the cistern stands at the zero point of the scale.

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[1] A kind of universal joint, in common use on board ship for the suspension of compasses, lamps, &c. It is seen in Fig. 115, at the top of the tripod stand.

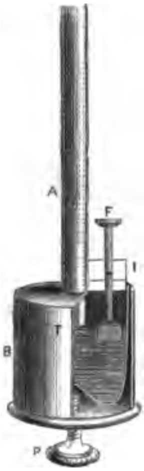

Fig. 115.
Float Adjustment.

107. Barometric Corrections.—In order that barometric heights may be comparable as measures of atmospheric pressure, certain corrections must be applied.

1. *Correction for Temperature.* As mercury expands with heat, it follows that a column of warm mercury exerts less pressure than a column of the same height at a lower temperature; and it is usual to reduce the actual height of the column to the height of a column at the temperature of freezing water, which would exert the same pressure.

Let h be the observed height at temperature $t°$ Centigrade, and h_0 the height reduced to freezing-point. Then, if m be the coefficient of expansion of mercury per degree Cent., we have

$$h_0 (1 + m t) = h, \text{ whence } h_0 = h - h m t \text{ nearly.}$$

The value of m (Chap. xxii.) is $\frac{1}{5550} = \cdot00018018$. For temperatures Fahrenheit, we have

$$h_0 \left\{ 1 + m (t - 32) \right\} = h, \; h_0 = h - h m (t - 32),$$

where m denotes $\frac{1}{9990} = \cdot0001001$.

But temperature also affects the length of the divisions on the scale by which the height of the mercurial column is measured. If these divisions be true inches at 0° Cent., then at $t°$ the length of n divisions will be $n\ (1 + l\ t)$ inches, l denoting the coefficient of linear expansion of the scale, the value of which for brass, the usual material, is ·00001878. If then the observed height h amounts to n divisions of the scale, we have

$$h_0 (1 + m t) = h = n\ (1 + l t);$$

whence

$$h_0 = \frac{n\ (1 + l\ t)}{1 + m t} = n - n\ t\ (m - l), \text{ nearly;}$$

that is to say, if n be the height read off on the scale, it must be diminished by the correction $n\ t\ (m-l)$, t denoting the temperature of the mercury in degrees Centigrade. The value of $m-l$ is ·0001614.

For temperatures Fahrenheit, assuming the scale to be of the correct length at 32° Fahr., the formula for the correction (which is still subtractive), is $n\ (t-32)\ (m-l)$, where $m-l$ has the value ·00008967.[1]

2. *Correction for Capillarity.*—In the preceding chapter we have seen that mercury in a glass tube undergoes a capillary depression, whence it follows that the observed barometric height is too small, and that we must add to it the amount of this depression. In all tubes of internal diameter less than about $\frac{3}{4}$ of an inch this correction is sensible; and its amount, for which no simple formula can be given, has been computed, from theoretical considerations, for various sizes of tube, by several eminent mathematicians, and recorded in tables, from which that given below is abridged. These values are applicable on the assumption that the meniscus which forms the summit of the mercurial column is decidedly convex, as it always is when the mercury is rising. When the meniscus is too flat, the mercury must be lowered by the foot-screw, and then screwed up again.

It is found by experiment, that the amount of capillary depression is only half as great when the mercury has been boiled in the tube, as when this precaution has been neglected.

For purposes of special accuracy, tables have been computed,

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[1] The correction for temperature is usually made by the help of tables, which give its amount for all ordinary temperatures and heights. These tables, when intended for English barometers, are generally constructed on the assumption that the scale is of the correct length not at 32° Fahr., but at 62° Fahr., which is (by act of Parliament) the temperature at which the British standard yard (preserved in the office of the Exchequer) is correct. On this supposition, the length of n divisions of the scale at temperature $t°$ Fahr., is

$$n\left\{1+l\ (t-62)\right\};$$

and by equating this expression to

$$h_0\left\{1+m\ (t-32)\right\}$$

we find

$$\begin{aligned} h_0 &= n\left\{1-m\ (t-32)+l\ (t-62)\right\} \\ &= n\left\{1-\ (m-l)\ t+\ (32\,m-62\,l)\right\} \\ &= n\left\{1-{\cdot}00008967\ t+{\cdot}00255654\right\}; \end{aligned}$$

which, omitting superfluous decimals, may conveniently be put in the form—

$$n-\frac{n}{1000}\ ({\cdot}09\ t-2{\cdot}56).$$

The correction vanishes when

$${\cdot}09\ t-2{\cdot}56=0;$$

that is, when $t=\frac{256}{9}=28{\cdot}5$.

For all temperatures higher than this the correction is subtractive.

giving the amount of capillary depression for different degrees of convexity, as determined by the sagitta (or height) of the meniscus, taken in conjunction with the diameter of the tube. Such tables, however, are seldom used in this country.[1]

TABLE OF CAPILLARY DEPRESSIONS IN UNBOILED TUBES.

(To be halved for Boiled Tubes.)

Diameter of tube in inches.	Depression.	Diameter.	Depression.	Diameter.	Depression.
·10	·140	·20	·058	·40	·015
·11	·126	·22	·050	·42	·013
·12	·114	·24	·044	·44	·011
·13	·104	·26	·038	·46	·009
·14	·094	·28	·033	·48	·008
·15	·086	·30	·029	·50	·007
·16	·079	·32	·026	·55	·005
·17	·073	·34	·023	·60	·004
·18	·068	·36	·020	·65	·003
·19	·063	·38	·017	·70	·002

3. *Correction for Capacity.*—When there is no provision for adjusting the level of the mercury in the cistern to the zero point of the scale, another correction must be applied. It is called the correction for *capacity*. In barometers of this construction, which were formerly much more common than they are at present, there is a certain point in the scale at which the mercurial column stands when the mercury in the cistern is at the correct level. This is called the neutral point. If A be the interior area of the tube, and C the area of the cistern (exclusive of the space occupied by the tube and its contents), when the mercury in the tube rises by the amount x, the mercury in the cistern falls by an amount $y=\frac{A}{C}x$, for the volume of the mercury which has passed from the cistern into the tube is $Cy=Ax$. The change of atmospheric pressure is correctly measured by $x+y=\left(1+\frac{A}{C}\right)x$, and if we now take x to denote the distance of the summit of the mercurial column from the neutral point, the corrected distance will be $\left(1+\frac{A}{C}\right)x$, and the correction to be applied to the observed reading will be $\frac{A}{C}x$, which is additive if the observed reading be above the neutral point, subtractive if below.

It is worthy of remark that the neutral point depends upon the volume of mercury. It will be altered if any mercury be lost or added; and as temperature affects the volume, a special temperature-correction must be applied to barometers of this class. The investigation will be found in a paper by Professor Swan in the *Philosophical Magazine* for 1861.

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[1] The most complete collection of meteorological and physical tables, is that edited by Professor Guyot, and published under the auspices of the Smithsonian Institution, Washington.

In some modern instruments the correction for capacity is avoided, by making the divisions on the scale less than true inches, in the ratio $\frac{\text{C}}{\text{A}+\text{C}}$, and the effect of capillarity is at the same time compensated by lowering the zero point of the scale. Such instruments, if correctly made, simply require to be corrected for temperature.

4. *Index Errors.*—Under this name are included errors of graduation, and errors in the position of the zero of the graduations. An error of zero makes all readings too high or too low by the same amount. Errors of graduation (which are generally exceedingly small) are different for different parts of the scale.

Barometers intended for accurate observation are now usually examined at Kew Observatory before being sent out; and a table is furnished with each, showing its index error at every half inch of the scale, errors of capillarity and capacity (if any) being included as part of the index error. We may make a remark here once for all respecting the signs attached to errors and corrections. The sign of an error is always opposite to that of its correction. When a reading is too high the index error is one of excess, and is therefore positive; whereas the correction needed to make the reading true is subtractive, and is therefore negative.

5. *Reduction to Sea-level.*—In comparing barometric observations taken over an extensive district for meteorological purposes, it is usual to apply a correction for difference of level. Atmospheric pressure, as we have seen, diminishes as we ascend; and it is usual to add to the observed height the difference of pressure due to the elevation of the place above sea-level. The amount of this correction is proportional to the observed pressure. The law according to which it increases with the height will be discussed in the next chapter.

6. *Correction for Unequal Intensity of Gravity.*—When two barometers indicate the same height, at places where the intensity of gravity is different (for example, at the pole and the equator), the same mass of air is superincumbent over both; but the pressures are unequal, being proportional to the intensity of gravity as measured by the values of g (Chaps. v. vi.) at the two places. When intensity

of pressure is to be expressed in absolute measure, it should be stated in absolute units of force (§ 42) per unit area. If we adopt as our absolute unit of force, that force which, acting on a pound of matter for a second, would generate a velocity of a foot per second, it is necessary that the square foot should be made the unit of area.

Since the force of gravity on a pound contains g absolute units of force, and the weight of 144 cubic inches of mercury at 0° Centigrade is 70·7275 lbs., we have the following rules for reducing pressure per unit area to absolute measure:—

To reduce lbs. per sq. foot to absolute measure, multiply by g.
" lbs. per sq. inch " " $144\,g$.
" inches of mercury " " $g \times 70{\cdot}7275$.

108. Other kinds of Mercurial Barometer.—The *Siphon Barometer*, which is represented in Fig. 116, consists of a bent tube, generally of uniform bore, having two unequal legs. The longer leg, which must be more than 30 inches long, is closed, while the shorter leg is open. A sufficient quantity of mercury having been introduced to fill the longer leg, the instrument is set upright (after boiling to expel air), and the mercury takes such a position, that the difference of levels in the two legs represents the pressure of the atmosphere.

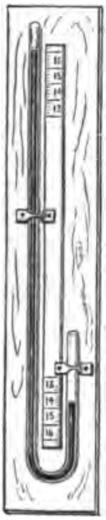

Fig. 116. Siphon Barometer.

Supposing the tube to be of uniform section, the mercury will always fall as much in one leg as it rises in the other. Each end of the mercurial column therefore rises or falls through only half the height corresponding to the change of atmospheric pressure.

In the best siphon barometers there are two scales, one for each leg, as indicated in the figure, the divisions on one being reckoned upwards, and on the other downwards, from an intermediate zero point, so that the sum of the two readings is the difference of levels of the mercury in the two branches.

Inasmuch as capillarity tends to depress both extremities of the mercurial column, its effect is generally neglected in siphon barometers; but practically it causes great difficulty in obtaining accurate observations, for according as the mercury is rising or falling its extremity is more or less convex, and a great deal of tapping is usually required to make both ends of the column assume the same form, which is the condition necessary for annihilating the effect of capillary action.

Wheel Barometer.—The wheel barometer, which is in more general use than its merits deserve, consists of a siphon barometer, the two branches of which have usually the same diameter. On the surface of the mercury of the open branch floats a small piece of iron or glass suspended by a thread, the other extremity of which is fixed to a pulley, on which the thread is partly rolled. Another thread, rolled parallel to the first, supports a weight which balances the float. To the axis of the pulley is fixed a needle which moves on a dial. When the level of the mercury varies in either direction, the float follows its movement through the same distance; by the action of the counterpoise the pulley turns, and with it the needle, the extremity of which points to the figures on the dial, marking the baro-

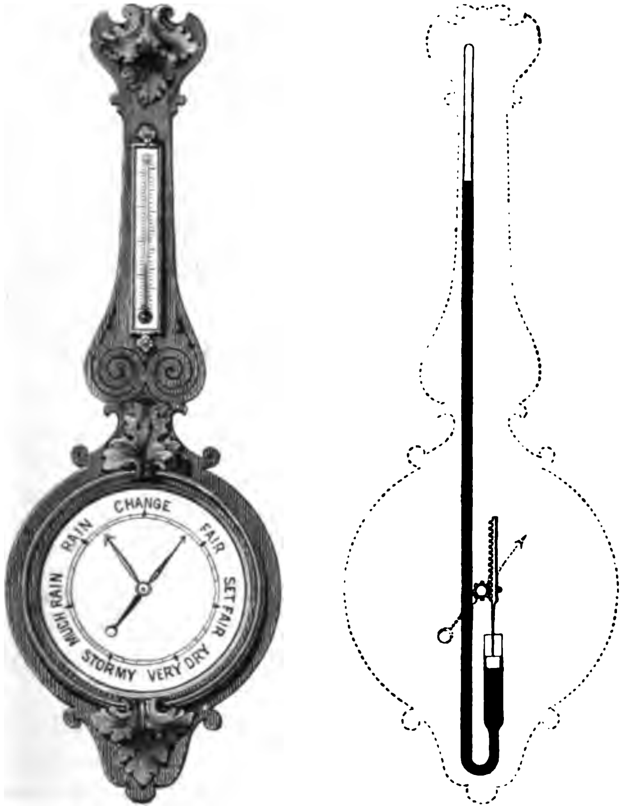

Fig. 117.—Wheel Barometer.

metric heights. The mounting of the dial is usually placed in front of the tube, so as to conceal its presence. The wheel barometer is a very old invention, and was introduced by the celebrated Hooke in 1683. The pulley and strings are sometimes replaced by a rack and pinion, as represented in the figure (Fig. 117).

Besides the faults incidental to the siphon barometer, the wheel barometer is encumbered in its movements by the friction of the additional apparatus. It is quite unsuitable for measuring the exact amount of atmospheric pressure, and is slow in indicating changes.

Marine Barometer.—The ordinary mercurial barometer cannot be used at sea, on account of the violent oscillations which the mercury would experience from the motion of the vessel. In order to meet this difficulty, the tube is contracted in its middle portion nearly to capillary dimensions, so that the motion of the mercury in either direction is hindered. An instrument thus constructed is called a marine barometer. When such an instrument is used on land it is always too slow in its indications.

Adie's Barometer.—A very convenient form of barometer, which is extensively used under the direction of the Board of Trade, is constructed by Adie of London. The error of capillarity is allowed for in fixing the zero point of the scale. The error of capacity is obviated by making the divisions of the scale less than true inches, in such a ratio as exactly to correct for capacity. The observer, therefore, has merely to read the height of the top of the mercurial column, and correct for temperature. The tube is generally contracted in its middle part, to diminish the "pumping" (*i.e.* oscillation), which occurs when it is carried from place to place; but the contraction is much less than in the marine barometer.

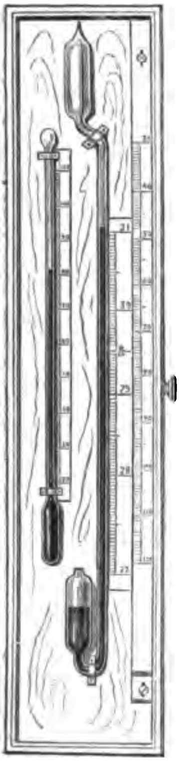

Fig. 118.
Sympiesometer.

108 A. Sympiesometer (*συν, πιέζω*).—Adie's sympiesometer (Fig. 118) consists of a glass tube 18 inches in length and ¾ inch in diameter, with a small chamber at the top, and an open cistern below. In the original construction the upper part of the tube was filled with hydrogen, and the lower part and cistern with oil of almonds. In the construction now employed these materials are replaced by common air and glycerine.

When the pressure of the atmosphere increases, the air in the upper part of the tube is compressed, and the fluid rises; when it diminishes, the fluid falls. The instrument is graduated by comparison with a mercurial barometer. The intervals corresponding to inches of mercurial pressure are much longer than inches, and are of unequal length, becoming shorter as we ascend on the tube. To obviate error from the increased pressure of the inclosed air when its temperature is raised, a thermometer and sliding scale are added to the instrument, so that it may be adjusted for temperature at each observation. The sympiesometer is very quick in its indications, and from its portability is well adapted for being used at sea, but it is not suited for exact observation.

As originally made it was liable to gradual change, from absorption of the hydrogen by the oil of almonds. In the present construction absorption is less liable to occur, at least if the glycerine be of the proper consistency.

109. Aneroid Barometer (α, νηρος).—This barometer depends upon the changes in the form of a thin metallic vessel, partially exhausted of air, as the atmospheric pressure varies. M. Vidie was the first to overcome the numerous difficulties which were presented in the con-

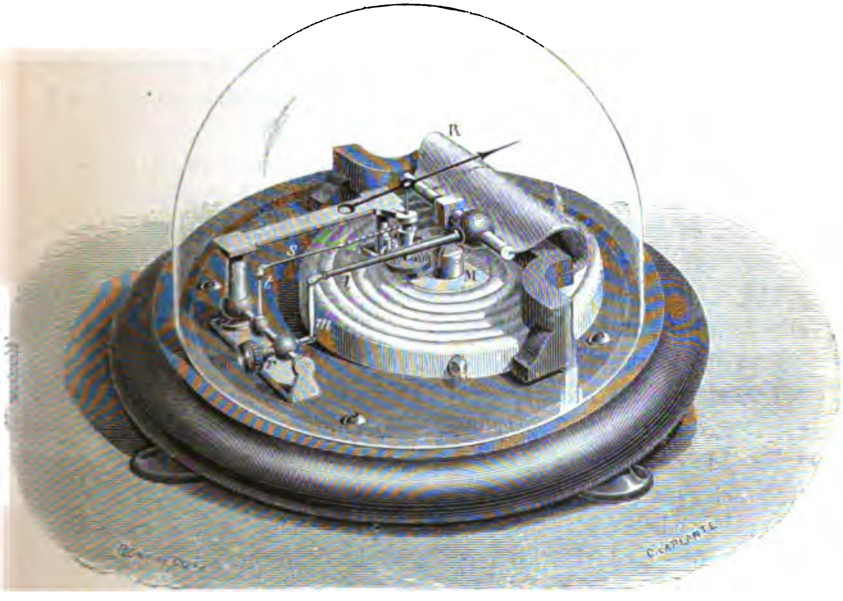

Fig. 119.—Aneroid Barometer.

struction of these instruments. We subjoin a figure of the model which he finally adopted.

The essential part is a cylindrical box partially exhausted of air, the upper surface of which is corrugated in order to make it yield more easily to external pressure. At the centre of the top of the box is a small metallic pillar M, which acts upon a powerful steel spring R. As the pressure varies, the top of the box rises or falls, transmitting its movement to the spring, and thence, by means of two levers l and m, to a metallic axis r. This latter carries a third lever t, the extremity of which is attached to a chain s which turns a drum, the axis of which bears the index needle. A spiral spring keeps the chain constantly stretched, and thus makes the needle always take a position corresponding to the shape of the box at the time. The graduation is performed empirically by comparison with a mercurial barometer. The aneroid barometer is very sensitive, and is much more portable than any form of mercurial barometer, being both lighter and less liable to injury. It is sometimes made small enough for the waistcoat pocket. It has the drawback of being affected by temperature to an extent which must be determined for each instrument separately, and of being liable to gradual changes which can only be checked by occasional comparison with a good mercurial barometer.

In the *metallic barometer*, which is a modification of the aneroid, the exhausted box is crescent-shaped, and the horns of the crescent separate or approach according as the external pressure diminishes or increases.

110. Old Forms Revived.—There are two ingenious modifications of the form of the barometer, which, after long neglect, have recently been revived for special purposes.

Counterpoised Barometer.—The invention of this instrument is attributed to Samuel Morland, who constructed it about the year 1680. It depends upon the following principle:—If the barometric tube is suspended from one of the scales of a balance, there will be required to balance it in the other scale a weight equal to the weight of the tube and the mercury contained in it, minus the upward pressure of the liquid against the bottom of the tube and its contents.[1]

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[1] It may be shown that if a be the (annular) area of a section of the tube itself, and A the area of the inclosed space (which is filled with mercury), the resultant force to which the tube is subjected from atmospheric and liquid pressure combined is a downward force

$$PA - pa,$$

If the atmospheric pressure increases, the mercury will rise in the tube, and consequently the weight of the floating body will increase, while the upward pressure will be slightly diminished on account of the sinking of the mercury in the cistern. The beam will thus incline to the side of the barometric tube, and the reverse would be the case if the pressure diminished. For the balance may be substituted, as in the figure, a lever carrying a counterpoise; the variations of pressure will be indicated by the movements of this lever.

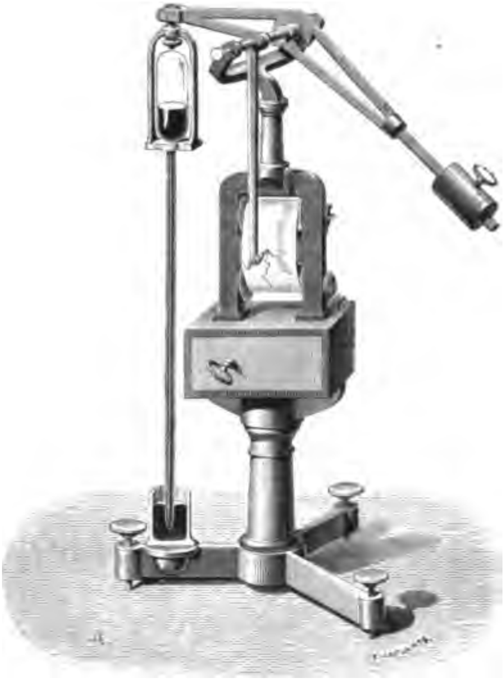

Fig. 120.—Counterpoised Barometer.

Such an instrument may very well be used as a *barograph* or recording barometer; for this purpose we have only to attach to the lever an arm with a pencil, which is constantly in contact with a sheet of paper moved uniformly by clockwork. The result will be a continuous trace, whose form corresponds to the variations of pressure. It is very easy to determine, either by calculation or by comparison with a standard barometer, the pressure corresponding to a given position of the pencil on the paper;

P denoting atmospheric pressure, and p the fluid pressure due to the depth of immersion (exclusive of the transmitted atmospheric pressure). This resultant force together with the weight of the tube must be equal to the supporting force at the point of suspension. If the latter be constant, $PA - pa$ must be constant, and the changes in P and p must be inversely as the areas A and a. If these areas are equal P and p will be equal; that is, the tube will descend through the same distance as the mercury in a common barometer would rise; and if A is greater than a, the movement will be proportionately magnified. For great sensitiveness, therefore, the tube should be large and thin.

We have here neglected the changes of level in the mercury in which the tube is immersed. These changes tend to increase the distance moved by the tube, and must be added to the movements as above calculated.

thus, if the paper is ruled with twenty-four equidistant lines, corresponding to the twenty-four hours of the day, we can see at a glance what was the pressure at any given time. An arrangement of this kind has been adopted by the Abbé Secchi for the meteorograph of the observatory at Rome. The first successful employment of this kind of barograph appears to be due to Mr. Alfred King, a gas engineer of Liverpool, who invented and constructed such an instrument in 1853, for the use of the Liverpool Observatory, and subsequently designed a larger one, which is still in use, furnishing a very perfect record, magnified five-and-a-half times.

Fahrenheit's Barometer.—Fahrenheit's barometer consists of a tube bent several times, the lower portions of which contain mercury; the upper portions are filled with water, or any other liquid, usually coloured. It is evident that the atmospheric pressure is balanced by the sum of the differences of level of the columns of mercury, diminished by the sum of the corresponding differences for the columns of water; whence it follows that, by employing a considerable number of tubes, we may greatly reduce the height of the barometric column. This circumstance renders the instrument interesting as a scientific curiosity, but at the same time diminishes its sensitiveness, and renders it unfit for purposes of precision. It is therefore never used for the measurement of atmospheric pressure; but an instrument upon the same principle has recently been employed for the measurement of very high pressures, as will be explained in Chap. xiv.

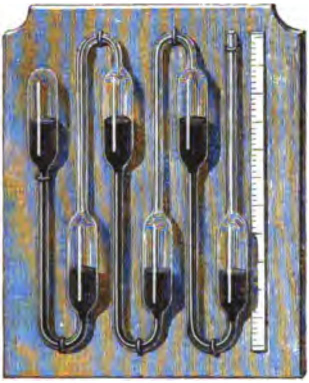

Fig. 121.—Fahrenheit's Barometer.

110 A. Photographic Registration.—Since the year 1847 various meteorological instruments at the Royal Observatory, Greenwich, have been made to yield continuous traces of their indications by the aid of photography, and the method is now generally employed at meteorological observatories in this country. The Greenwich system is fully described in the *Greenwich Magnetical and Meteorological Observations* for 1847, pp. lxiii.–xc. (published in 1849).

The general principle adopted for all the instruments is the same. The photographic paper is wrapped round a glass cylinder, and the axis of the cylinder is made parallel to the direction of the movement which is to be registered. The cylinder is turned by clockwork, with uniform velocity. The spot of light (for the magnets and barometer), or the boundary of the line of light (for the thermometers), moves, with the movements which are to be registered, backwards and forwards in the direction of the axis of the cylinder, while the cylinder itself is turned round. Consequently (as in Morin's machine, Chap. v.), when the paper is unwrapped from its cylindrical form, there is traced upon it a curve of which the abscissa is proportional to the time, while the ordinate is proportional to the movement which is the subject of measure.

The barometer employed in connection with this system is a large siphon barometer, the bore of the upper and lower extremities of its arms being about 1·1 inch. A glass float in the quicksilver of the lower extremity is partially supported by a counterpoise acting on a light lever (which turns on delicate pivots), so that the wire supporting the float is constantly stretched, leaving a definite part of the weight of the float to be supported by the quicksilver. This lever is lengthened to carry a vertical plate of opaque mica with a small aperture, whose distance from the fulcrum is eight times the distance of the point of attachment of the float-wire, and whose movement, therefore (§ 108), is four times the movement of the column of a cistern barometer. Through this hole the light of a lamp, collected by a cylindrical lens, shines upon the photographic paper.

Every part of the cylinder, except that on which the spot of light falls, is covered with a case of blackened zinc, having a slit parallel to the axis of the cylinder; and by means of a second lamp shining through a small fixed aperture, and a second cylindrical lens, a base line is traced upon the paper, which serves for reference in subsequent measurements.

The whole apparatus, or any other apparatus which serves to give a continuous trace of barometric indications, is called a *barograph;* and the names *thermograph, magnetograph, anemograph,* &c., are similarly applied to other instruments for automatic registration.

CHAPTER XIII.

VARIATIONS OF THE BAROMETER.

111. Measurement of Heights by the Barometer.—As the height of the barometric column diminishes when we ascend in the atmosphere, it is natural to seek in this phenomenon a means of measuring heights. The problem would be extremely simple, if the air had everywhere the same density as at the surface of the earth. In fact, the density of the air at sea-level being about 10,500 times less than that of mercury, it follows that, on the hypothesis of uniform density, the mercurial column would fall an inch for every 10,500 inches, or 875 feet, that we ascend. This result, however, is far from being in exact accordance with fact, inasmuch as the density of the air diminishes very rapidly as we ascend, on account of its great compressibility.

111 A. Height of Homogeneous Atmosphere.—If the atmosphere were of uniform and constant density, its height would be approximately obtained by multiplying 30 inches by 10,500, which gives 26,250 feet, or about 5 miles.

More accurately, if we denote by H the height of the atmosphere at a given time and place, on the assumption that the density throughout is the same as the observed density D at the base, and if we denote by P the observed pressure at the base, expressed in absolute units of force per unit area (§ 107, 6), then since the pressure P must be equal to the weight of a column of volume H and of mass HD, we have

$$P = g\,HD \qquad H = \frac{P}{g\,D} \qquad (1)$$

The height H, computed on this imaginary assumption, is called the *height of the homogeneous atmosphere*, corresponding to the pressure P, density D, and intensity of gravity g, and is frequently introduced in physical formulæ.

The expression for H shows that its value is not affected if P and D vary in the same ratio, as is the case in barometric fluctuations when the temperature is constant; but that increase of temperature and increase of moisture increase H, since warm air and moist air are less dense than cold and dry air at the same pressure.

It is not necessary that the height H should be reckoned from the surface of the earth. It may be reckoned upwards from any point in the atmosphere, and denotes the height which the air above this point would have, if reduced to the density D which exists at the point.

Neglecting differences of temperature and moisture, and the trifling diminution of gravity as we ascend, the value of H is the same for all points in the same vertical column, because, as we ascend, P and D diminish in the same ratio.

112. Principles of Hypsometry.—Supposing the temperature, moisture, and intensity of gravity to be uniform in a vertical column of air, it is easy to state the law according to which the pressure would diminish as we ascend. Consider, for example, three layers of equal thickness, which is so small that we may regard the density as constant within the limits of each layer, though varying from each layer to the next. Let D, D′, D″ be their densities, and P, P′, P″ the pressures at their lower faces, the weights of the two lower layers are P−P′ and P′−P″, and these must be proportional to their densities; hence we have

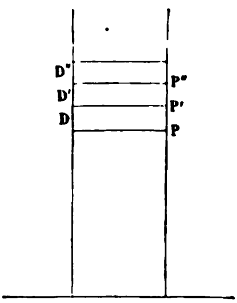

Fig. 122.

$$\frac{P-P'}{P'-P''}=\frac{D}{D'},$$

but by Boyle's law $\frac{D}{D'}=\frac{P}{P'}$; consequently we have $\frac{P-P'}{P'-P''}=\frac{P}{P'}$, whence it easily follows that $\frac{P}{P'}=\frac{P'}{P''}$; that is to say, the ratio of the density of the first layer to that of the second, is the same as of the second to the third. Applying this principle to any number of consecutive layers of equal thickness, we see that the ratio of the density of each to that of the next will be the same for the whole series. It follows that, *as the heights increase in arithmetical progression, the pressures diminish in geometrical progression.*

This proposition may be put into the algebraical form:

$$x_2-x_1=H\log\frac{P_1}{P_2},$$

where x_1, x_2 are the heights of two stations above a fixed level, P_1, P_2 the pressures at the two stations, and H some constant. The proof given in the note[1] shows, that if the logarithms are Napierian, H is equal to the height of the homogeneous atmosphere. If the logarithms be of the common kind, H is equal to the height of the homogeneous atmosphere multiplied by 2·3026, the value of which product for the latitude of Great Britain, and for the temperature of freezing water, is about 60,360 feet.

This formula has been deduced on the supposition that the temperature and the intensity of gravity are uniform through the whole extent of the air between the two stations. If these two elements vary, they cause the value of H to vary; and it would be necessary for accuracy to employ in the formula the mean value of H for the stratum of air which intervenes between the two stations. The variation in the intensity of gravity is usually insignificant, and it is customary to assume as the mean temperature, the arithmetical mean of the temperatures t_1 and t_2 of the two stations. On these assumptions the value of H, if the temperatures be expressed in degrees Fahrenheit, will (by the law of expansion of air, Chap. xxiii.) be $60{,}360\left(1+\frac{t_1+t_2-64}{986}\right)$.

It is proved in treatises on logarithms that if $\frac{P_1}{P_2}$ be but little greater than unity,

$$\text{Nap. log}\ \frac{P_1}{P_2} = 2\ \frac{P_1-P_2}{P_1+P_2}\ \text{nearly;}$$

and since the height of the homogeneous atmosphere at freezing temperature in these latitudes is about 26,214 feet, we obtain the formula—

$$\text{Difference of level in feet} = 52428\ \frac{P_1-P_2}{P_1+P_2}\left(1+\frac{t_1+t_2-64}{986}\right),$$

which may be used for differences of level not exceeding about 3000 feet.

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[1] Let x denote distance measured upwards from a fixed level, then, using the notation of § 111A, the pressure due to the weight of a layer of thickness dx is $g\,D\,dx$; but this is the amount by which the pressure diminishes as x is increased by the amount dx; we have therefore

$$-d\,P = g\,D\,dx = \frac{P}{H}dx,$$

since by § 111A, $g\,D = \frac{P}{H}$. We have therefore

$$-\frac{d\,P}{P} = \frac{dx}{H},\ \text{whence}\ \log P_1 - \log P_2 = \frac{x_2-x_1}{H}.$$

The determination of heights by means of atmospheric pressure, whether the pressure be observed directly by the barometer or indirectly by the boiling-point thermometer (see Chap. xxvi.), is called *hypsometry* (ὕψος, height).

As a rough rule, it may be stated that, in ordinary circumstances, the barometer falls an inch in ascending 900 feet.

113. Diurnal Oscillation of the Barometer.—In these latitudes, the mercurial column is in a continual state of irregular oscillation; but in the tropics it rises and falls with great regularity according to the hour of the day, attaining two maxima in the twenty-four hours.

It generally rises from 4 A.M. to 10 A.M., when it attains its first maximum; it then falls till 4 P.M., when it attains its first minimum; a second maximum is observed at 10 P.M., and a second minimum at 4 A.M. The hours of maxima and minima are called the tropical hours (τρέπω, to turn), and vary a little with the season of the year. The difference between the highest maximum and lowest minimum is called the diurnal[1] *range*, and the half of this is called the *amplitude* of the diurnal oscillation. The amount of the former does not exceed about a tenth of an inch.

The character of this diurnal oscillation is represented in Fig. 123. The vertical lines correspond to the hours of the day; lengths have been measured upwards upon them proportional to the barometric heights at the respective hours, diminished by a constant quantity; and the points thus determined have been connected by a continuous curve. It will be observed that the two lower curves, one of which relates to Cumana, a town of Venezuela, situated in about 10° north latitude, show strongly marked oscillations corresponding to the maxima and minima. In our own country the regular diurnal oscillation is marked by irregular fluctuations, so that a single day's observations give no clue to its existence. Nevertheless, on taking observations at regular hours for a number of consecutive days, and comparing the mean heights for the different hours, some indications of the law will be found. A month's observations will be sufficient for an approximate indication of the law; but observations, extending over some years, will be required to establish with anything like precision the hours of maxima and the amplitude of the oscillation.

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[1] The epithets *annual* and *diurnal*, when prefixed to the words *variation*, *range*, *amplitude*, denote the *period* of the variation in question; that is, the time of a complete oscillation. Diurnal variation does not denote variation from one day to another, but the variation which goes through its cycle of values in one day of twenty-four hours. Annual range denotes the range that occurs within a year. This rule is universally observed by writers of high scientific authority.

A table, exhibiting the values of an element for each month in the year, is a table of annual (not monthly) variation; or it may be more particularly described as a table of variations from month to month.

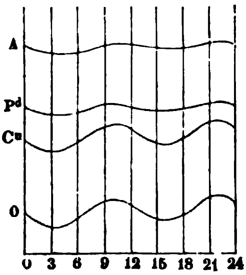

Fig. 123.
Curves of Diurnal Variation.

The two upper curves represent the diurnal variation of the barometer at Padua (lat. 45° 24′) and Abo (lat. 60° 56′), the data having been extracted from Kaemtz's *Meteorology*. We see, by inspection of the figure, that the oscillation in question becomes less strongly marked as the latitude increases. The range at Abo is less than half a millimetre. At about the 70th degree of north latitude it becomes insensible; and in approaching still nearer to the pole, it appears from observations, which however need further confirmation, that the oscillation is reversed; that is to say, that the maxima here are contemporaneous with the minima in lower latitudes.

There can be little doubt that the diurnal oscillation of the barometer is in some way attributable to the heat received from the sun, which produces expansion of the air, both directly, as a mere consequence of heating, and indirectly, by promoting evaporation, and thus increasing the volume of the air (as well as diminishing its sp. gravity) by the addition of aqueous vapour. The precise nature of the connection between this cause and the diurnal barometric oscillation has not, however, as yet been satisfactorily established.

114. Irregular Variations of the Barometer.—The height of the barometer, at least in the temperate zones, depends on the state of the atmosphere, and its variations often serve to predict the changes of weather with more or less certainty. In this country the barometer generally falls for rain or S.W. wind, and rises for fine weather or N.E. wind.

Barometers for popular use have generally the words—

Set fair.	Fair.	Change.	Rain.	Much rain.	Stormy.
30·5	30	29·5	29	28·5	28 inches.

marked at the respective heights. These words must not, however, be understood as absolute predictions. A low barometer rising is generally a sign of fine, and a high barometer falling of wet weather. Moreover, it is to be borne in mind that the barometer stands about a tenth of an inch lower for every hundred feet that we ascend above sea-level.

The connection between a low or falling barometer and wet weather is to be found in the fact that moist air is specifically lighter than dry, even at the same temperature, and still more when, as usually happens, moist air is warmer than dry.

115. Inverse March of Barometer and Thermometer.—It is impossible to lay down universal rules for the connection between the indications of the barometer and the state of the weather, since rules which would usually hold true in one place might be quite inapplicable at another. We may, however, state a principle which is of very extensive application, namely, that warm winds, especially when they have passed over considerable masses of water, are likely to be accompanied by rain; for they are charged with vapour which is liable to be condensed as its temperature falls. Cold winds, on the contrary, contain vapour which was taken up at a lower temperature than it now has, and is therefore far from a state of saturation. They are therefore unlikely to produce rain unless it be when they first begin to blow, when they may condense vapour previously existing in the air.

These characteristics are very marked in our own country, where the warm winds from the south-west have passed over the Atlantic, while the cold winds from the north-east have for the most part traversed dry land.

Again, the march of the barometer is in general opposite to that of the thermometer; that is to say, *the barometer usually falls when the thermometer rises*, and *vice versâ*. This law is one of the most general in meteorology, and is easily explained; in fact, when the temperature rises at any place, it produces a dilatation of the air, and consequently an overflow into neighbouring regions; the weight of air over the place is thus diminished. On the contrary, a fall of temperature produces an inflow of air and an increase of pressure.

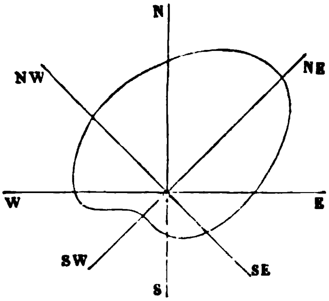

Fig. 124.
Barometric Rose of Winds at Paris.

It is therefore to be expected that the mean barometric height should be lower during warm and rainy winds than during cold and dry winds; and that this is the case is rendered extremely evident by the annexed figure, which represents the mean barometric rose of the winds at Paris. Upon each of the eight lines which represent the principal directions of the wind, have been laid off lengths proportional to the corresponding barometric pressures diminished by a constant. The figure shows a very sudden increase in passing from S.W. to W. It is, in fact, during the change of the wind from one of these quarters to the other, that the greatest atmospheric perturbations occur.

116. Synoptic Weather Charts. Isobaric Lines.—The extension of telegraphic communication over Europe has led to the establishment of a system of correspondence by which the barometric pressures, at a given moment, at a number of stations which have been selected for meteorological observation, are known at one or more stations appointed for receiving the reports. From the information thus furnished, curves (called isobaric lines) are drawn upon a chart through those places at which the pressure is the same. The barometric condition of an extensive region is thus rendered intelligible at a glance. Plate I. is a specimen of these synoptic charts,[1] which are prepared every day at the observatory of Paris; it refers to the 22d of January, 1868. Besides the isobaric lines, these charts indicate, by the system of notation explained at the left of the figure, the general state of the weather, the strength of wind, and state of the sea. The isobaric curves correspond to differences of five millimetres (about 0·2 inch) of pressure, and according as they are near together, or far apart, the variation of pressure in passing from one to another is more or less sudden (or to use a very expressive modern phrase, the barometric gradient is more or less steep), just as the contour lines on a map of hilly ground approach each other most nearly where the ground is steepest. Generally speaking, the wind blows from regions of high to regions of low barometer, and with greater force as the barometric gradient is steeper.

The isobaric lines frequently, as in the example here selected, form closed curves encircling a region of barometric depression. Two such centres are here exhibited—one in the south of England and the other in the west of Russia. Such centres of depression always accompany great atmospheric disturbances. The air, in fact, rushes

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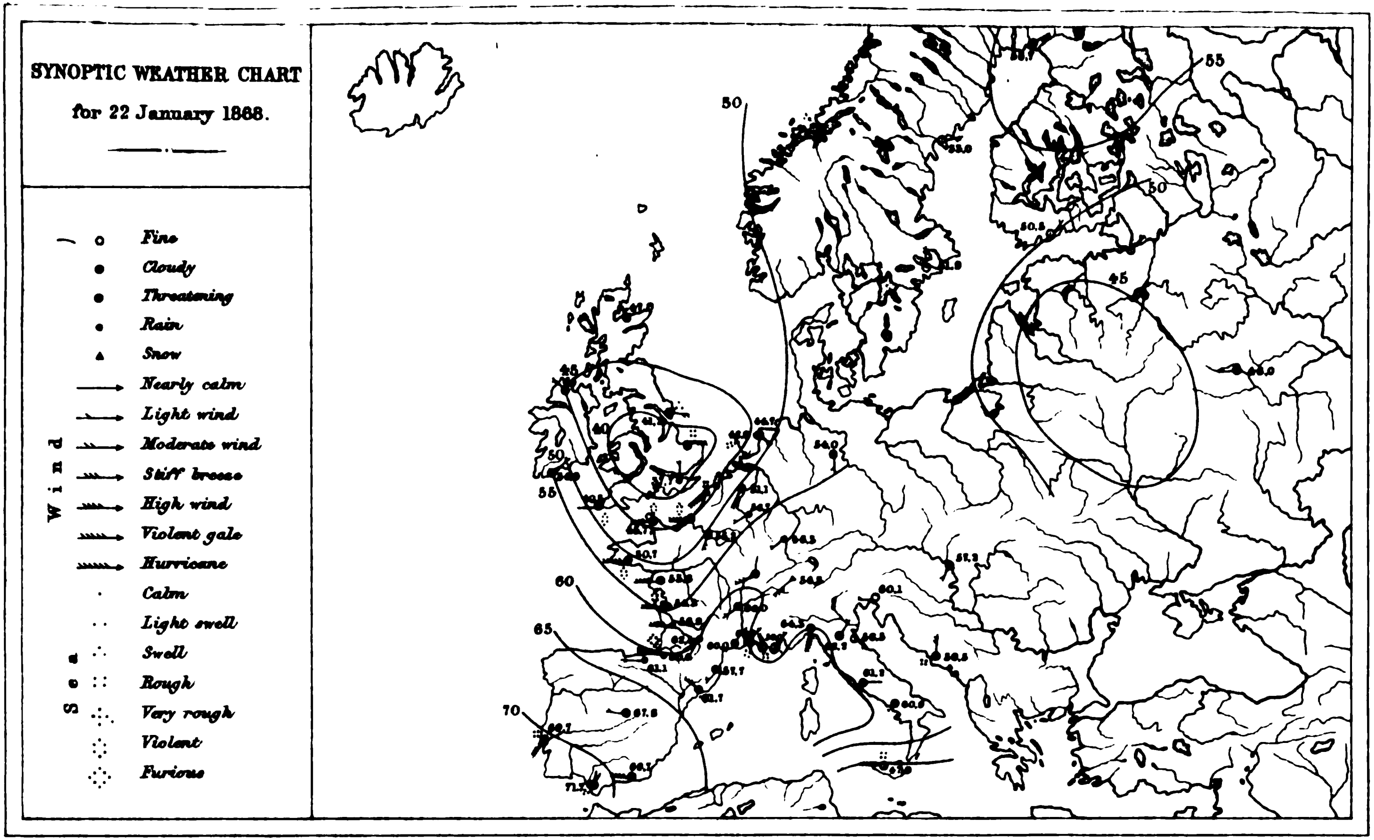
[1] The curves drawn upon this chart are isobaric lines, each corresponding to a particular barometric pressure, which is indicated by the numerals marked against it. These denote the pressure in millimetres diminished by 700. For example, the line which passes through the south of Spain corresponds to the pressure 770 millimetres; that through the north of Spain to 765 millimetres. The curves are drawn for every fifth millimetre. The smaller numerals, which are given to one place of decimals, indicate the pressures actually observed at the different stations, from which the isobaric lines are drawn by estimation.

The other symbols refer to cloud, wind, and sea, and are explained at the left of the chart.

SYNOPTIC WEATHER CHART
for 22 January 1868.



in from all sides, usually with a spiral motion, towards these centres of depression, the direction of rotation in the spiral being, for the northern hemisphere, opposite to the motion of the hands of a watch with its face upwards. The centrifugal force due to this rotation tends to increase the central depression, and thus protracts the duration of the phenomenon.

These revolving storms are called cyclones. They attain their greatest violence in tropical regions, the West Indies being especially noted for their destructive effect. They frequently proceed from the Gulf of Mexico in a north-easterly direction, increasing in diameter as they proceed, but diminishing in violence. Their velocity of translation is usually from ten to twenty miles an hour.

The storm-warnings inaugurated by the late Admiral Fitzroy are based partly upon information received by telegraph of storms that have actually commenced at some distant locality, and partly upon a comparison of barometric pressures at different localities.[1]

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[1] For fuller information respecting the laws of storms, which is a purely modern subject, and is continually receiving fresh developments, we would refer to Mr. Buchan's *Handy Book of Meteorology*. See also § 406 A in Part II. of the present Work.

It will be observed, by the arrows in the annexed chart, that the direction of the wind, instead of being coincident with the line of steepest descent from each isobaric curve to the next below it, generally makes a large angle (considerably exceeding 45°) to the right of it. This law (known as Buys Ballot's) is general for the northern hemisphere, and is dependent on the earth's rotation (§ 406 A). The influence of the earth's rotation in modifying the direction of winds, is discussed in a paper "On the General Circulation and Distribution of the Atmosphere," by the Editor of this Work, in the *Philosophical Magazine* for September, 1871.

CHAPTER XIV.

BOYLE'S (OR MARIOTTE'S) LAW.[1]

117. Boyle's Law.—As gases are composed of molecules in a state of permanent repulsion, they may be compared to springs constantly bent, and making constant efforts to free themselves. The amount of pressure which they exert against the sides of the vessels which contain them, depends upon the volume which they occupy, increasing as this volume diminishes. By a number of careful experiments upon this point, Boyle and Mariotte independently established the law that this volume varies inversely as the pressure, provided that the temperature remain constant. As the density evidently varies inversely as the volume, we may express the law in other words by saying that at the same temperature the density varies directly as the pressure.

If V and V′ be the volumes of the same quantity of gas, P and P′, D and D′, the corresponding pressures and densities, Boyle's law will be expressed by the equations

$$\frac{V}{V'} = \frac{P'}{P} = \frac{D'}{D}.$$

118. Mariotte's Tube.—The correctness of this law may be verified by means of the following apparatus, which was employed by both the experimenters above named. It consists (Fig. 125) of a bent tube with branches of unequal length; the long branch is open, and the short branch closed. The tube is fastened to a board provided with two scales; one by the side of the long branch, divided into parts of equal length; the other by the side of the short branch, having divisions which correspond to parts of equal volume. The graduation of both scales begins from the same horizontal line through 0, 0. Mercury is first poured in at the extremity of the long branch, and by inclining the apparatus to either side, and cautiously adding more of the liquid if required, the mercury can be made to stand at the same level in both branches, and at the zero of both scales. Thus we have, in the short branch, a quantity of air separated from the external air, and at the same pressure. Mercury is then poured into the long branch, so as to reduce the volume of this inclosed air by one-half; it will then be found that the difference of level of the mercury in the two branches is equal to the height of the barometer at the time of the experiment; the compressed air therefore exerts a pressure equal to that of two atmospheres. If more mercury be poured in so as to reduce the volume of the air to one-third or one-fourth of the original volume, it will be found that the difference of level is respectively two or three times the height of the barometer; that is, that the compressed air exerts a pressure equal respectively to that of three or four atmospheres. This experiment therefore shows that if the volume of the gas becomes two, three, four times as small, the pressure becomes two, three, four times as great. This is the principle expressed in Boyle's law.

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[1] Boyle, in his *Defence of the Doctrine touching the Spring and Weight of the Air against the Objections of Franciscus Linus,* appended to *New Experiments, Physico-mechanical, &c.* (second edition, 4to, Oxford, 1662), describes the two kinds of apparatus represented in Figs. 125, 126 as having been employed by him, and gives in tabular form the lengths of tube occupied by a body of air at various pressures. These observed lengths he compares with the theoretical lengths computed on the assumption that volume varies reciprocally as pressure, and points out that they agree within the limits of experimental error.

Mariotte's treatise, *De la Nature de l'Air,* is stated in the *Biographie Universelle* to have been published in 1679. (See Preface to Tait's *Thermodynamics,* p. iv.)

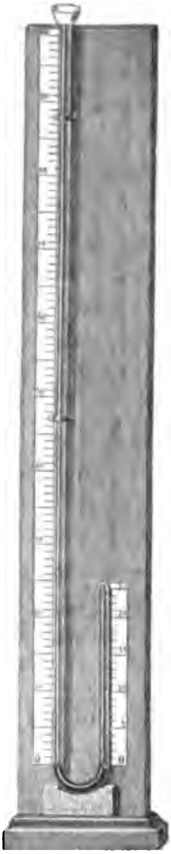

Fig. 125.
Mariotte's Tube.

The law may also be verified in the case where the gas expands, and where its pressure consequently diminishes. For this purpose a barometric tube (Fig. 126), partially filled with mercury, is inverted in a tall vessel, containing mercury also, and is held in such a position that the level of the liquid is the same in the tube and in the vessel. The volume occupied by the gas is marked, and the tube is raised; the gas expands, its pressure diminishes, and, in virtue of the excess of the atmospheric pressure, a column of mercury *ab* rises in the tube, so that its height, added to the pressure of the expanded air, is equal to the atmospheric pressure. It will then be seen that if the volume of air becomes double what it was before, the height of the column raised is one-half that of the barometer; that is, the expanded air exerts a pressure equal to half that of the atmosphere. If the volume is trebled, the height of the column is two-thirds that of the barometer; that is, the pressure of the expanded air is one-third that of the atmosphere, a result which is in accordance with Boyle's law.

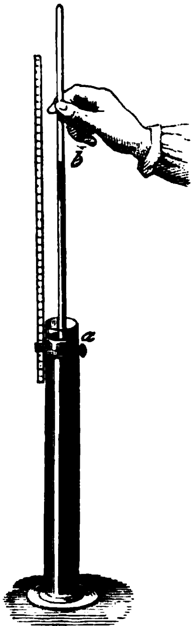

Fig. 126.—Proof of Boyle's Law for Expanding Air.

119. Despretz's Experiments.—The simplicity of Boyle's law, taken in conjunction with its apparent agreement with facts, led to its general acceptance as a rigorous truth of nature, until in 1825 Despretz published an account of experiments, showing that different gases are unequally compressible. He inverted in a cistern of mercury several cylindrical tubes of equal height, and filled them with different gases. The whole apparatus was then inclosed in a strong glass vessel filled with water, and having a screw-piston as in Œrsted's piesometer (§ 22). On pressure being applied, the mercury rose to unequal heights in the different tubes, carbonic acid for example being more reduced in volume than air. These experiments proved that though Boyle's law might possibly be true for one of the gases employed, it could not be rigorously true for more than one.

In 1829 Dulong and Arago undertook a laborious series of experiments with the view of testing the accuracy of the law as applied to air; and the results which they obtained, even when the pressure was increased to twenty-seven atmospheres, agreed so nearly with it as to confirm them in the conviction that, for air at least, it was rigorously true. When re-examined, in the light of later researches, the results obtained by Dulong and Arago seem to point to a different conclusion.

120. Unequal Compressibility of Different Gases.—The unequal compressibility of different gases, which was first established by Despretz's experiments above described, is now usually exhibited by the aid of the following apparatus designed by Pouillet. A is a cast-iron reservoir, containing mercury surmounted by oil. In this latter liquid dips a bronze plunger P, the upper part of which has a thread cut upon it, and works in a nut, so that the plunger can be screwed up or down by means of the lever L. The reservoir A communicates by an iron tube with another cast-iron vessel, into which are firmly fastened two tubes T T about six feet in length and $\frac{1}{10}$th of an inch in internal diameter, very carefully calibrated (§ 180). Equal volumes of two gases, perfectly dry, are introduced into these tubes through their upper ends, which are then hermetically sealed. The plunger is then made to descend, and a gradually increasing pressure is exerted, the volumes occupied by the gases are measured, and it is ascertained that no two gases follow precisely the same law of compression. The difference, however, is almost insensible when the gases employed are non-liquefiable, as air, oxygen, hydrogen, nitrogen, nitric oxide, and marsh-gas. But when we compare any one of these with a liquefiable gas, such as carbonic acid, cyanogen, or ammonia, the difference is rapidly and distinctly manifested. Thus, under a pressure of twenty-five atmospheres, carbonic acid occupies a volume which is only $\frac{4}{5}$ths of that occupied by air.

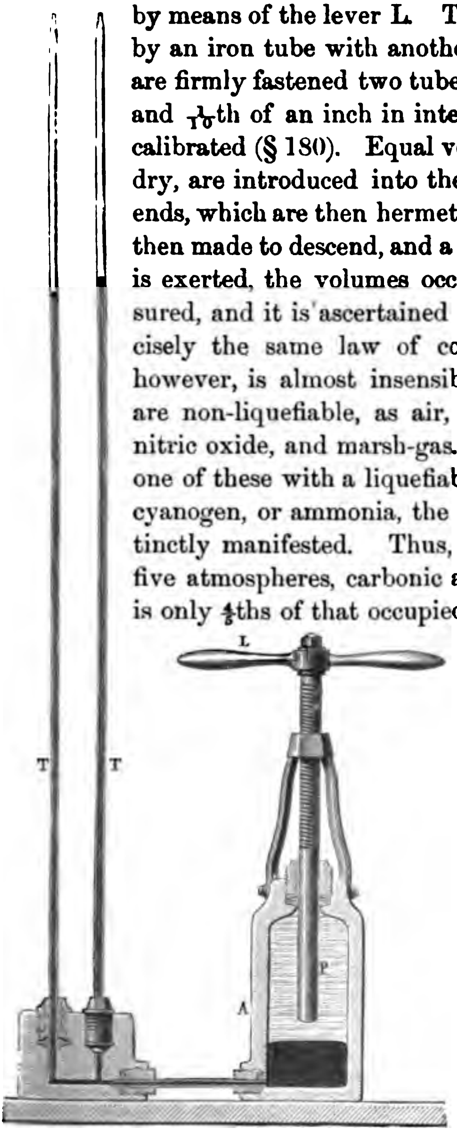

Fig. 127.—Pouillet's Apparatus for showing Unequal Compressibility of Different Gases.

121. Regnault's Experiments.—Boyle's law, therefore, is not to be considered as rigorously exact; but it is certainly a very close approximation to the truth, except for gases near their point of liquefaction. In order to demonstrate the inaccuracy of the law for air, or any gas that is not liquefiable, and more especially if it is required to determine the law of deviation for each particular gas, it is necessary to employ very precise methods of measurement. In ordinary experiments on compression, and even in the elaborate investigations of Dulong and Arago, a definite portion of gas is taken and successively diminished in volume by the application of continually increasing pressure. Now it is evident that, in experiments of this kind, in proportion as the pressure increases, the variations in volume become smaller, and the precision with which they can be determined consequently diminishes. Regnault adopted the plan of operating in all cases upon the same volume of gas, which, being initially at different pressures, was always reduced to one-half. The pressure was observed before and after this operation, and, if Boyle's law were true, its value should be found to be doubled. In this way the same precision of measurement is obtained at high as at low pressures.

A general view of Regnault's apparatus is given in Fig. 128. It consists of an iron reservoir containing mercury, furnished at the top with a force-pump for water. The lower part of this reservoir communicates with a cylinder which is also of iron, and in which are two openings to admit tubes. Communication between the reservoir and the cylinder can be established or interrupted by means of a stop-cock R, of very exact workmanship. Into one

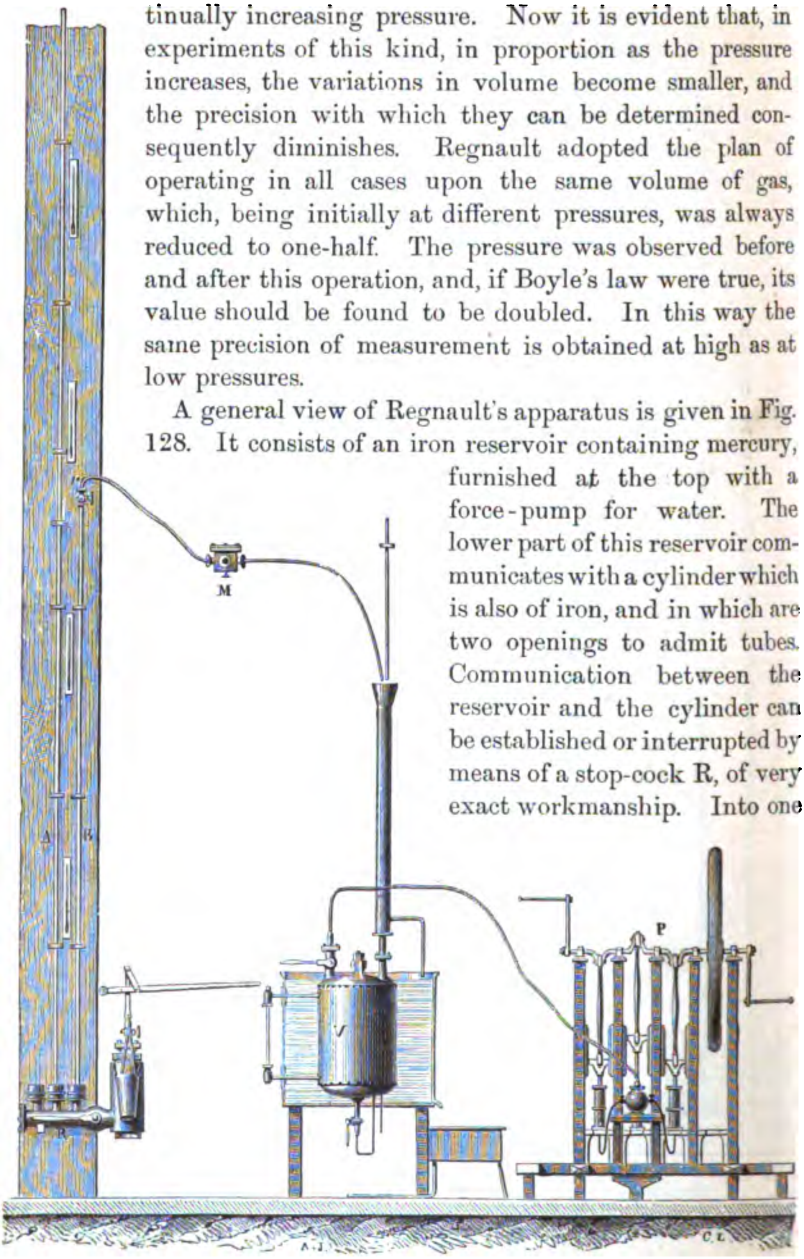

Fig. 128.—Regnault's Apparatus for Testing Boyle's Law.

of the openings is fitted the lowest of a series of glass tubes A, which are placed end to end, and firmly joined to each other by metal fittings, so as to form a vertical column of about twenty-five metres in height.

The height of the mercurial column in this long manometric tube could be exactly determined by means of reference marks placed at distances of about ·95 of a metre, and by the graduation on the tubes forming the upper part of the column. The mean temperature of the mercurial column was given by thermometers placed at different heights. Into the second opening in the cylinder fits the lower extremity of the tube B, which is divided into millimetres, and also gauged with great accuracy. This tube has at its upper end a stop-cock *r* which can open communication with the reservoir V, into which the gas to be operated on is forced and compressed by means of the pump P.

An outer tube, which is not shown in the figure, envelops the tube B, and, being kept full of water, which is continually renewed, enables the operator to maintain the tube at a temperature sensibly constant, which is indicated by a very delicate thermometer. Before fixing the tube in its place, the point corresponding to the middle of its volume is carefully ascertained, and after the tube has been permanently fixed, the distance of this point from the nearest of the reference marks is observed.[1]

After these explanatory remarks we may describe the mode of conducting the experiments. The gas to be operated on, after being first thoroughly dried, is introduced at the upper part of the tube B, the stop-cock of the pump being kept open, so as to enable the gas to expel the mercury and occupy the entire length of the tube. The force-pump is then brought into play, and the gas is reduced to about half of its former volume; the pressure in both cases being ascertained by observing the height of the mercury in the long tube above the nearest mark. It is important to remark that it is not at all necessary to operate always upon exactly the same initial volume, and reduce it exactly to one-half, which would be a very tedious operation; these two conditions are approximately fulfilled, and the graduation of the tube enables the observer always to ascertain the actual volumes.

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[1] Regnault's apparatus was fixed in a small square tower of about fifteen metres in height, forming part of the buildings of the Collége de France, and which had formerly been built by Savart for experiments in hydraulics. The tower could therefore contain only the lower part of the manometric column; the upper part rose above the platform at the top of the tower, resting against a sort of mast which could be ascended by the observer. The readings inside the tower could be made by means of a cathetometer, but this was impossible in the upper portion of the column, and for this reason the tubes forming this portion were graduated.—*D.*

122. Results.—The general result of the investigations of Regnault is, that Boyle's law does not exactly represent the compressibility of even non-liquefiable gases, such as air, hydrogen, nitrogen, which, with carbonic acid, were the gases operated on by him. But the resulting differences are so small that they would not be detected by a mere inspection of the numbers which represent volumes and pressures. They may, however, be clearly exhibited by submitting the results to the following test:—Suppose we take a certain quantity of gas which, under the pressure P, occupies the volume V, and that we reduce it to a volume V′, when the pressure becomes P′, and, if Boyle's law were accurate, we should have the equation

$$\frac{V}{V'} = \frac{P'}{P}, \text{ or } VP = V'P'$$

Or, in another form, $$\frac{VP}{V'P'} - 1 = 0.$$

Now it is found that for all gases except hydrogen this difference, instead of being always zero, is constantly positive, and has not only a sensible value, but, which is of especial importance, it increases regularly with the pressure, which shows that it cannot be attributed to the inevitable errors of observation.

If we measure off upon any line lengths proportional to the different pressures, and raise perpendiculars proportional to the differences $\frac{VP}{V'P'} - 1$, by joining the extremities of these perpendiculars by a continuous line an uninterrupted curve is obtained, which evidently is a graphic representation of the departure of the gas in question from Boyle's law. These curves have been very carefully traced by Regnault; and their algebraic equations can be found by the ordinary methods of interpolation. These equations are employed when it is required to calculate rigorously the change of volume corresponding to a very high pressure.

Since the difference $\frac{VP}{V'P'} - 1$ is positive, V′P′ must be less than VP. and consequently the volume V′ corresponding to the pressure P′ is less than that given by Boyle's law. We thus see that, in general, gases are more compressible than Boyle's law would indicate; and in the case of gases that are liquefiable this difference of compressibility is, as we have said, considerable.

In this respect hydrogen is a remarkable exception, as was originally shown by Despretz in the experiments which we have mentioned; it is less compressible than it should be by Boyle's law, the difference $\frac{VP}{V'P'}-1$ being negative.

This singular peculiarity of hydrogen is quite in harmony with the views which are entertained as to the nature of this gas. It has been observed, from several comparative experiments performed upon carbonic acid, that at the temperature of 100° Cent. the law of compressibility of this gas differs from Boyle's law much less than at ordinary temperatures. We may thus fairly suppose that if we were to operate at a still higher temperature, we should approach still more nearly to the law, which would doubtless be verified at a particular temperature, beyond which the error would be in the opposite direction. It would thus appear that for each gas there is a sort of normal temperature, at which the compressibility is exactly represented by Boyle's law.

The compressibility of a gas should also increase as the temperature decreases, as is proved by the experiment on carbonic acid. With the exception of hydrogen, all gases under ordinary conditions are below this normal temperature. But if, as chemical phenomena tend to prove, hydrogen is a kind of metal, we must suppose it to be relatively in a high state of rarefaction, which accounts for the peculiarity presented by its compressibility.

123. Manometers or Pressure-gauges.—Manometers are instruments for measuring the elastic force of a gas or vapour contained in the interior of a closed space. This elastic force is generally expressed in units called atmospheres (§103), and is often measured by means of a column of mercury.

When the column of mercury moves freely in an open tube, the manometer is said to be open [*à air libre*]; it was a manometer of this kind that Regnault employed to measure the successive pressures to which the volume of gas was subjected.

The open mercurial pressure-gauge is often used in the arts to measure pressures that are not very considerable. The figure represents one of the simplest forms. The apparatus consists of a box, generally of iron, at the top of which is an opening closed by a screw stopper, which is traversed by the tube *b*, open at both ends, and dipping into the mercury in the box. The air or vapour whose elastic force is to be measured enters by the tube *a*, and presses upon the mercury. It is evident that if the level of the liquid in the box is the same as in the tube, the pressure in the box must be exactly equal to that of the atmosphere. If the mercury in the tube rises above that in the box, the pressure of the air in the box must exceed that of the atmosphere by a pressure corresponding to the height of the column raised. The pressures are generally marked in atmospheres upon a scale beside the tube.

124. Multiple Branch Manometer.—When the pressures to be measured are considerable, as in the boiler of a high-pressure steam-engine, the above instrument, if employed at all, must be of a length corresponding to the pressure. If, for instance, the pressure in question is eight atmospheres, the length of the tube must be at least 8 × 30 inches= 20 feet. Such an arrangement is inconvenient even for stationary machines, and is entirely inapplicable to movable machines.

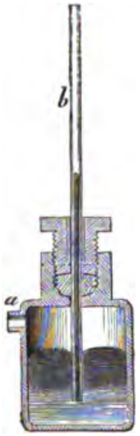

Fig. 129.—Open Mercurial Manometer.

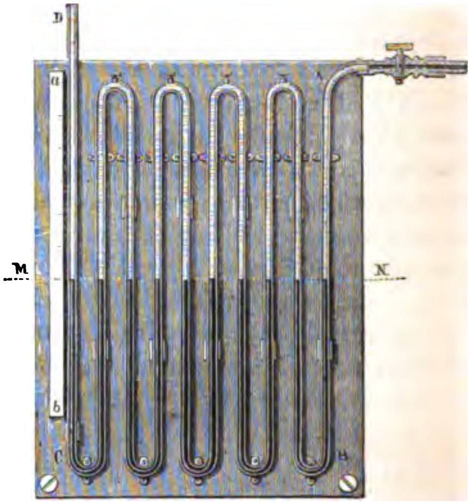

Fig. 130.—Multiple Branch Manometer.

Without departing from the principle of the open mercurial pressure-gauge, namely, the balancing of the pressure to be observed against the weight of a liquid increased by one atmosphere, we may reduce the length of the instrument by an artifice already employed by Fahrenheit in his barometer (§110).

The apparatus for this purpose consists of an iron tube ABCD bent back upon itself a certain number of times. The extremity A communicates with the boiler by a stop-cock, and the last branch CD is of glass, and has a scale by its side.

The first step is to fill the tube with mercury as far as the level MN. At this height are holes by which the mercury escapes when it reaches them, and which are afterwards hermetically sealed. The upper portions are filled with water through openings which are also stopped after the tube has been filled. If the mercury in the first tube, which is in communication with the reservoir of gas, falls through a certain distance h, it will alternately fall and rise through the same distance in each of the tubes, and will consequently rise through the same distance in the last tube; now this distance corresponds to an effective pressure represented by a column of mercury of height $10h$, diminished by ten times the same height of water; that is, to a height of mercury equal to $10h\ (1-\frac{1}{13{\cdot}59})$. It will thus be seen that a very considerable pressure will be indicated by a comparatively small variation of the mercurial column. If, for instance, beginning with the atmospheric pressure, an additional pressure of five atmospheres is exerted, that is, an effective pressure of six atmospheres, the quantity h will be given in metres by the equation

$$5 \times {\cdot}76 = 10h\left(1-\frac{1}{13{\cdot}59}\right),$$

whence

$$5 \times {\cdot}76 \times 13{\cdot}59 = 10h \times 12{\cdot}59,$$
$$h = {\cdot}40m.$$

125. Compressed-air Manometer.—This instrument, which may assume different forms, sometimes consists, as in Fig. 131, of a bent tube AB closed at one end a, and containing within the space Aa a quantity of air, which is cut off from external communication by a column of mercury. The apparatus has been so constructed, that when the pressure on B is equal to that of the atmosphere, the mercury stands at the same height in both branches; so that, under these circumstances, the inclosed air is exactly at atmospheric pressure. But if the pressure increases, the mercury is forced into the left branch, so that the air in that branch is compressed, and its tension gradually increases until equilibrium is established. The pressure of the gas exerted at B is then equal to the pressure of the compressed air, together with that of a column of mercury equal to the difference of level of the liquid in the two branches. This pressure is expressed in atmospheres on the scale ab.

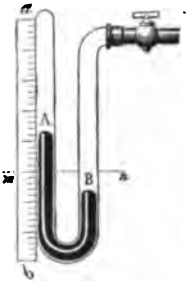

Fig. 131.—Compressed-air Manometer.

The graduation of this scale is effected directly in practice, by placing the manometer in communication with a reservoir of compressed air whose pressure is given by an open mercurial gauge, or by a standard manometer of any kind.

If the tube AB be supposed cylindrical, the graduation can be previously effected by an application of Boyle's law.

Let l be the length of the tube occupied by the inclosed air when its pressure is equal to that of one atmosphere; at the point to which the level of the mercury rises is marked the number 1. It is required to find to what point the end of the liquid column should reach when a pressure of n atmospheres is exerted at B. Let x be the height of this point above 1; then the volume of the air, which was originally l, has become $l-x$, and its pressure is therefore equal to $H\frac{l}{l-x}$, H being the mean height of the barometer. This pressure, together with that due to the difference of level $2x$, is equivalent to n atmospheres. We have thus the equation—

$$H\frac{l}{l-x}+2x=nH,$$

whence

$$2x^2-(nH+2l)x+(n-1)Hl=0.$$

$$x=\frac{nH+2l\pm\sqrt{(nH+2l)^2-8(n-1)Hl.}}{4}$$

We thus find two values of x; but that given by taking the positive sign of the radical is inadmissible; for if we put $n=1$, we ought to have $x=o$, which cannot be the case unless the sign of the radical is negative.

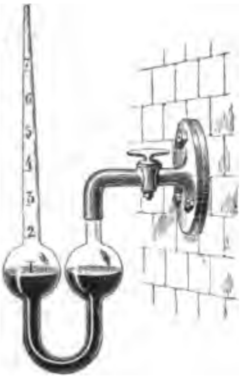

Fig. 132.
Compressed-air Manometer.

By giving n the successive values of $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, &c., in this expression for x, we have the points on the scale corresponding to pressures of one atmosphere and a half, two atmospheres, &c.

As we have before remarked, the sensibility of the instrument decreases as the pressure increases, and the distance traversed by the mercury for an increment of pressure equal to one atmosphere becomes less and less. This inconvenience is partly avoided by the arrangement shown in Fig. 132. The branch containing the air is of a conical form; in this way, as the mercury rises, equal changes of volume correspond to increasing lengths. The effect of this arrangement is seen by an inspection of the scale, on which the numbers corresponding to successive atmospheres of pressure are nearly equidistant, whereas when the tube is cylindrical they rapidly approach each other.

126. **Metallic Manometers.**—The fragility of glass tubes, and the fact that they are liable to become opaque by the mercury clinging to their sides, are serious drawbacks to their use, especially in machines in motion. Accordingly, metallic manometers are often employed, depending upon the changes of form effected by the pressure of gas on its containing vessel, when suitably constructed. We shall here mention only Bourdon's gauge (Fig. 133). It consists essentially of a copper tube of elliptic section, which is bent through about 540° as represented in Fig. 133. One of the extremities communicates by a stop-cock with the reservoir of steam or compressed gas; to the other extremity is attached a steel needle which traverses a scale. When the stop-cock permits communication with the atmosphere, the end of the needle stands at the mark 1; but if the pressure increases the curvature diminishes, the free extremity of the tube moves away from the fixed extremity, and the needle traverses the scale.

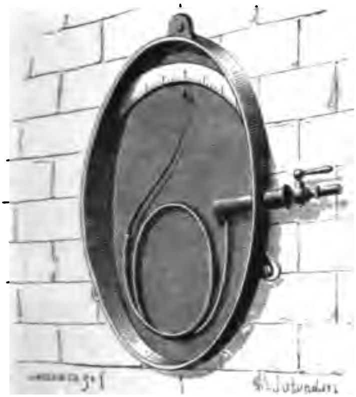

Fig. 133.—Bourdon's Pressure-guage.

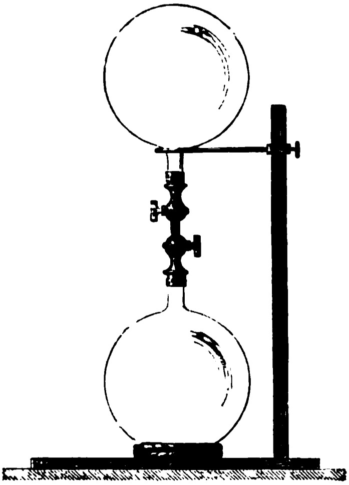

Fig. 134.—Mixture of Gases.

127. **Mixture of Gases.**—When gases of different densities are inclosed in the same space, experiment shows that, even under the most unfavourable circumstances, an intimate mixture takes place, so that each gas becomes uniformly diffused through the entire space. This

fact has been shown by a decisive experiment due to Berthollet. He took two globes (Fig. 134) which could be screwed together, and placed them in a cellar. The lower globe was filled with carbonic acid, the upper globe with hydrogen. Communication was established between them, and at the end of a certain time it was ascertained that the gases had become intimately mixed; in fact, the proportion of carbonic acid and of hydrogen was exactly the same in both globes. The fact that the composition of the air is the same at all heights is another striking proof.

If several gases are inclosed in the same space, each of them exerts the same pressure as if the others were absent, and consequently the pressure exerted by the mixture is equal to the sum of the pressures due to each gas separately. These separate pressures can easily be calculated by Boyle's law, when the original pressure and volume of each gas are known.

For example, let V and P, V' and P', V'' and P'' be the volumes and pressures of the gases which are made to pass into a vessel of volume U. The first gas exerts, when in this vessel, a pressure equal to $\frac{VP}{U}$, the second a pressure equal to $\frac{V'P'}{U}$, the third a pressure equal to $\frac{V''P''}{U}$, and so on, so that the total pressure M is equal to $\frac{VP}{U} + \frac{V'P'}{U} + \frac{V''P''}{U}$, whence $MU = VP + V'P' + V''P''$.

This formula expresses the law of pressure for a mixture of gases; it may easily be verified by passing different volumes of gas into a graduated glass jar inverted over mercury, after having first measured their volumes and pressures.

128. Absorption of Gases by Liquids and Solids.—All gases are to a greater or less extent soluble in water. This property is of considerable importance in the economy of nature; thus the life of aquatic animals and plants is sustained by the oxygen of the air which the water holds in solution. The *volume* of a given gas that can be dissolved in water at a given temperature is found to be in general the same at all pressures,[1] and the ratio of this volume to that of the water which dissolves it is called the *coefficient of solubility or of absorption*. At the temperature 0° Cent. the coefficient of solubility for carbonic acid is 1, for oxygen ·04, and for ammonia 1150.

If a mixture of two or more gases be placed in contact with water, each gas will be dissolved to the same extent as if it were the only gas present.

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[1] Hence the *weight* of gas absorbed is directly as the pressure.

Other liquids as well as water possess the power of absorbing gases, according to the same laws, but with coefficients of solubility which are different for each liquid.

Increase of temperature diminishes the coefficient of solubility, which is reduced to zero when the liquid boils.

Some solids, especially charcoal, possess the power of absorbing gases. Boxwood charcoal absorbs about nine times its volume of oxygen, and about ninety times its volume of ammonia. When saturated with one gas, if put into a different gas, it gives up a portion of that which it first absorbed, and takes up in its place a quantity of the second. Finely-divided platinum condenses on the surface of its particles a large quantity of many gases, amounting in the case of oxygen to many times its own volume. If a jet of hydrogen gas be allowed to fall, in air, upon a ball of spongy platinum, the gas combines rapidly, in the pores of the metal, with the oxygen of the air, giving out an amount of heat which renders the platinum incandescent and usually sets fire to the jet of hydrogen.

Most solids have in ordinary circumstances a film of air adhering to their surfaces. Hence iron filings, if carefully sprinkled on water, will not be wetted, but will float on the surface, and hence also the power which many insects have of running on the surface of water without wetting their feet. The film of air in these cases prevents wetting, and hence, by the principles of capillarity, produces increased buoyancy.

CHAPTER XV.

AIR-PUMP.

129. Air-pump.—The air-pump was invented by Otto Guericke about 1650, and has since undergone some improvements in detail which have not altered the essential parts of its construction.

It consists of a glass or metal cylinder called the barrel, in which a piston works. This piston has an opening through it which is closed at the lower end by a valve S opening upwards. The barrel communicates with a passage leading to the centre of a brass surface carefully polished, which is called the plate of the air-pump. The entrance to the passage is closed by a conical stopper S′, at the extremity of a metal rod which passes through the piston-head, and works in it tightly, so as to be carried up and down with the motion of the piston. A catch at the upper part of the rod confines its motion within very narrow limits, and only permits the stopper to rise a small distance above the opening.

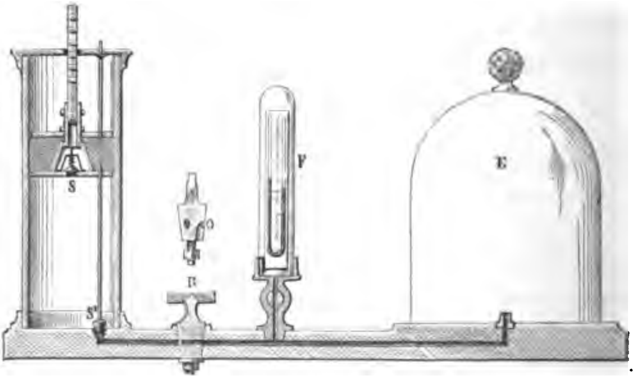


Fig. 135.—Air-pump.

Suppose now that the piston is at the bottom of the cylinder, and is raised. The valve S′ is opened, and the air of the receiver E rushes into the cylinder. On lowering the piston, the valve S′ closes its opening, the air which has entered the cylinder cannot return into the receiver, and, on being compressed, raises the valve in the piston, and escapes into the air outside. On raising the piston again, a portion of the air remaining in the receiver will pass into the cylinder, whence it will escape on pushing down the piston, and so on.

We see, then, that if this motion be continued, a fresh portion of the air in the receiver will be removed at each successive stroke. But as the quantity of air removed at each stroke is only a fraction of the quantity remaining, we can never produce a perfect vacuum, though we might approach as near to it as we pleased if this were the only obstacle.

130. Calculation of the Degree of Exhaustion.—It is easy to calculate the quantity of air left in the receiver after a given number of strokes of the piston. Let V be the volume of the cylinder, V′ that of the receiver, and M the mass of air in the receiver at first. On raising the piston, the air which occupied the volume V′ occupies a volume $V'+V$; of the air thus expanded the volume V is removed, and the volume V′ left, being $\frac{V'}{V'+V}$ of the whole quantity or mass M. The quantity remaining after the second stroke is $\frac{V'}{V'+V}$ of that after the first, or is $\left(\frac{V'}{V'+V}\right)^2 M$; and after n strokes $\left(\frac{V'}{V'+V}\right)^n M$. Hence the density and (by Boyle's law) the pressure are each reduced by n strokes to $\left(\frac{V'}{V'+V}\right)^n$ of their original values.

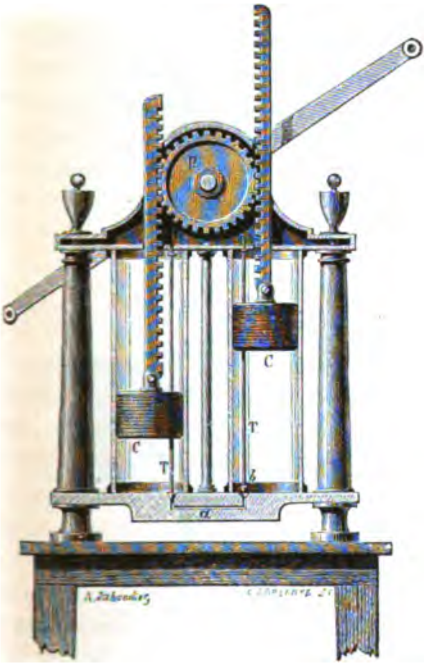
We see, then, that the pressure goes on decreasing indefinitely, and that, consequently, the elasticity of the air may, theoretically at least, be rendered less than any assigned quantity.

131. Mercurial Gauge.—In order to follow the steps of the operation, and to observe at each instant the elastic force of the air in the receiver, the instrument is provided with a siphon-barometer, called the mercurial gauge, inclosed in a bell-shaped vessel of glass F, and communicating by a stop-cock with the receiver. This barometer consists of a bent tube, the branches of which are about a foot in length; one of these is closed and filled with mercury, the other is open. When the pressure of the air in the receiver becomes less than that represented by a column of mercury equal in length to the closed branch of the gauge, the mercury falls, and the elastic force of the air at any moment is given by the difference of level of the mercury in the two branches; this difference can be measured on a graduated scale. The mercurial gauge serves to show whether the instrument is working properly; for instance, in the case of air getting in anywhere, this would be shown by the fluctuations of the mercurial column. It also shows when the greatest possible effect has been attained, by the level of the mercury remaining stationary notwithstanding the motion of the piston. In theory, as we have said above, there is no limit to the action of the machine, and at each stroke of the piston the elastic force of the air should decrease; but in reality this is not the case, on account of the inevitable imperfections of the apparatus; there is always a limit, extending further in proportion to the excellence of the machine, and the barometer shows the moment when this limit is reached. Instead of a siphon-barometer, we might have an ordinary barometer in connection with the receiver, and thus observe the progress of the vacuum from the first strokes of the piston.

132. Admission Stop-cock.—After the receiver has been exhausted of air, if it was required to raise it from the plate, a very considerable force would be necessary, amounting to as many times fifteen pounds as the area of the plate contained square inches. It would, therefore, be in general impossible to raise the receiver. This is, however, rendered possible by means of the stop-cock R, which is shown in section above. It is perforated by a straight channel, which, when the machine is being worked, forms part of the communicating passage. At 90° from the extremities of this channel is another opening O, forming the mouth of a bent passage, leading to the external air. When we wish to admit the air into the receiver, we have only to turn the stop-cock so as to bring the opening O to the side next the receiver; if, on the contrary, we turn it towards the pump-barrel, all communication between the pump and the receiver is stopped, the risk of air entering is diminished, and the vacuum remains good for a greater length of time. This precaution is taken when we wish to leave bodies in a vacuum for a considerable time. Another method is to employ a separate plate, which can be detached so as to leave the machine available for other purposes.

133. Double-barrelled Air-pump.—The machine just described has only a single pump-barrel; air-pumps of this kind are sometimes employed, and are usually worked by a lever like a pump-handle. With this arrangement, it is evidently necessary that the piston, after having ascended, should descend again to expel the air from the pump-barrel, and it is only after this double stroke that the operation can begin anew.

Double-barrelled pumps are more frequently used. An idea of their general arrangement may be formed from Figs. 136, 137, and 138. Fig. 138 gives the machine in perspective, Fig. 136 is a section through the axes of the pump-barrels, and Fig. 137 shows the manner in which communication is established between the receiver and the two barrels. It will be observed that the two passages from the barrels unite in a single passage to the centre of the plate *p*.



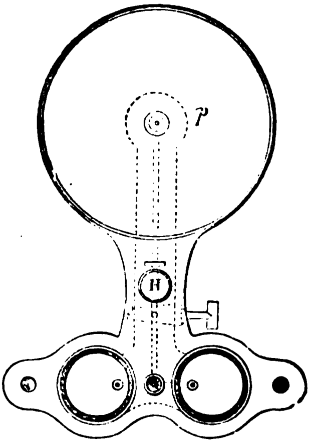


Fig. 136. Double-barrelled Air-pump. Fig. 137.

The piston-rods C are two racks working with the pinion P. This pinion is turned by a double-handed lever, which is worked alternately in opposite directions. In this arrangement, when one piston ascends the other descends, and consequently in each single stroke the air of the receiver passes into one or other pump-barrel. A vacuum is thus produced by half the number of strokes which would be required with a single-barrelled pump. It has besides another advantage. In the single-barrelled pump the force required to raise the piston increases as the exhaustion proceeds, and when it is nearly completed there is the resistance of almost an atmosphere to be overcome; that is, nearly 15 pounds to the square inch. In the double-barrelled pump, at the moment when one piston is at the top, and the other at the bottom, the force opposing the ascent of the one is precisely equal to that assisting the descent of the other. We must observe, however, that this equality exists only at the beginning of the stroke; for when one of the pistons descends, the air below it is compressed, its tension becoming greater and greater, until it reaches that of the atmosphere and raises the piston-valve. At this moment the resistance to the ascent of the other piston is entirely uncompensated, and up to this point the compensation has been gradually diminishing. But the more nearly we approach to a perfect vacuum, the more slowly does the tension of the air compressed beneath the piston increase, so that, unlike the single-barrelled pump, it becomes easier to work as the exhaustion proceeds.

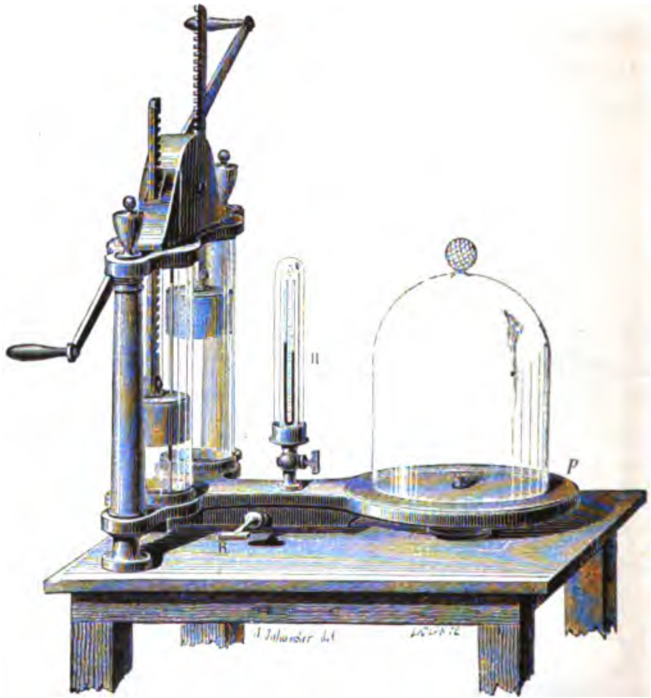


Fig. 138.—Air pump.

134. Single-barrelled Pumps with Double Action.—We do not, however, require two pump-barrels in order to obtain double action, as the same effect may be obtained with a single barrel. An arrangement for this purpose was long ago suggested by Delahire for water-pumps; but the principle has only lately been applied to the construction of air-pumps.

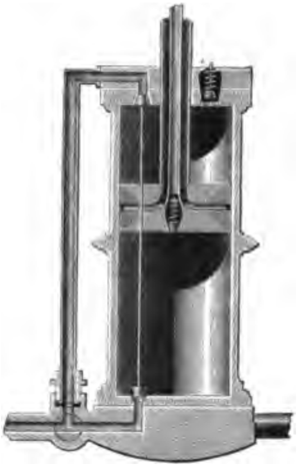
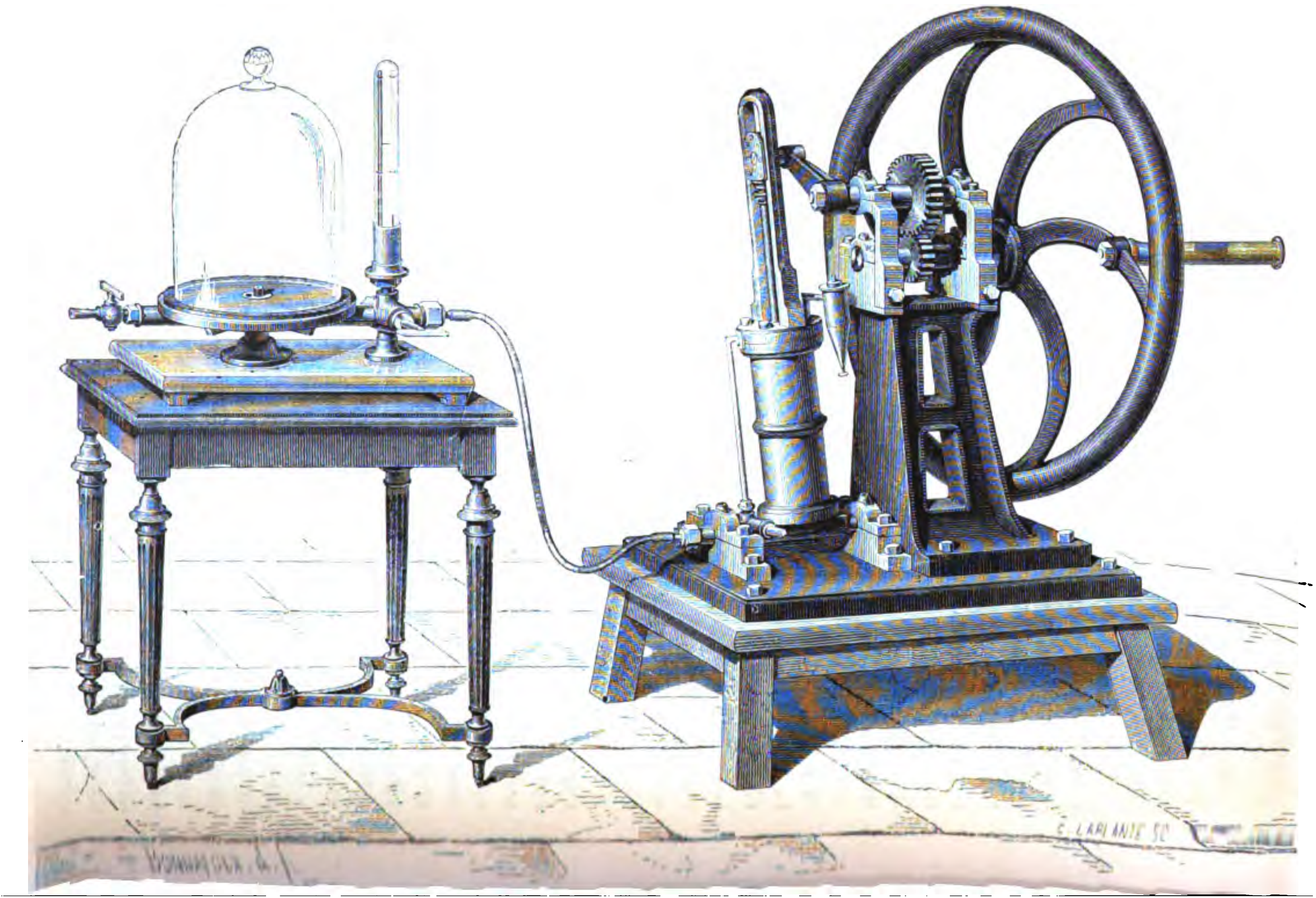


Fig. 139.
Barrel of Bianchi's Air-pump.

Fig. 139 represents the single barrel of the double-acting pump of Bianchi. It will be seen that the piston-valve opens into the hollow piston-rod; a second valve, also opening upward, is placed at the top of the pump-barrel. Two other openings, one above, the other below, serve to establish communication by means of a bent vertical tube between the pump-barrel and the passage to the plate. These openings are closed alternately by two conical stoppers at the two extremities of a metal rod passing through the piston, and carried with it in its vertical movement by means of friction. When the piston ascends, as in the figure, the upper opening is closed and the lower one is open; when the piston begins to descend, the opposite effect is immediately produced. Accordingly we see *that*, whichever be the direction in which the piston is moving, the receiver is being exhausted of air. In fact, when the piston ascends, air from the receiver will enter by the lower opening, and the air above the piston will be gradually compressed, and will finally escape by the valve above. In the descending movement, air will enter by the upper opening, and the compressed air beneath the piston will escape by the piston-valve. The movement of the piston is produced by a peculiar arrangement shown in Fig. 140, which gives a general view of the apparatus.

The pump-barrel, which is composed entirely of cast-iron, oscillates about an axis passing through its base. On the top are guides in which the end of a crank travels. The pump is worked by turning a heavy fly-wheel of cast-iron, on the axis of which is a pinion which drives a toothed wheel on the axis of the crank. The end of the crank is attached to the extremity of the piston-rod. It is evident that on turning the fly-wheel the pump-barrel will oscillate from side to side, following the motions of the crank, and the piston will alternately ascend and descend in the barrel, the length of which should be equal to the diameter of the circle described by the end of the crank.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135. Various Experiments with the Air-pump.—At the time when the air-pump was invented, several experiments were devised to show the effects of a vacuum, some of which have become classical, and are usually repeated in courses of experimental physics.

Experiment of the Burst Bladder.—On the plate of an air-pump is placed a glass cylinder open at the bottom, and having a piece of bladder tightly stretched over the top. As the exhaustion proceeds, the bladder bends inwards in consequence of the atmospheric pressure above it, and finally bursts with a loud report.

It often happens that, notwithstanding the strong exterior pressure, the bladder does not give way, its molecules preserving their equilibrium of cohesion. But this equilibrium is, so to speak, unstable, and a few taps are sufficient to destroy it and cause the bladder to burst.

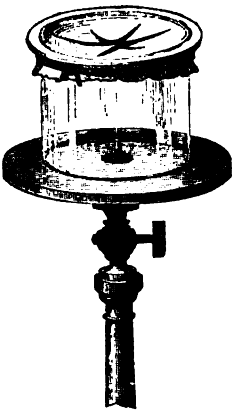

Fig. 141.
Burst Bladder.

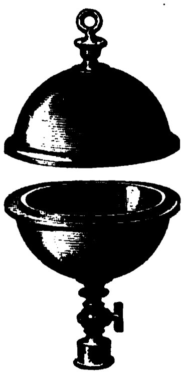

Fig. 142.
Magdeburg Hemispheres.

Magdeburg Hemispheres.—We take two hemispheres (Fig 142), which can be exactly fitted on each other; their exact adjustment is further assisted by a projecting internal rim, which is smeared with lard. The apparatus is exhausted of air through the medium of the stop-cock attached to one of the hemispheres, and when a vacuum has been produced, it will be found that a considerable force is required to separate the two parts, and this force increases with the size of the hemispheres.

This resistance to a force of separation is due to the normal exterior pressure of the air on every point of the surface, a pressure which is counterbalanced by only a very feeble pressure from the interior. In order to estimate the resultant effect of these different pressures, let us suppose that the external surface, instead of being spherical, is formed of a series of steps; that is to say, of alternate vertical and horizontal elements. It is evident that the pressures exerted upon these latter will have no influence upon the adhesion of the hemispheres; the first alone produce this effect, and the sum of these is evidently equal to the pressure of the atmosphere upon the circular area forming the common base of the hemispheres. For example, if this area is ten square inches, the hemispheres will be pressed against each other with a force of 150 pounds.

Fountain in Vacuo.—The apparatus for this experiment consists of a bell-shaped vessel of glass (Fig. 143), the base of which is pierced by a tube fitted with a stop-cock which enables us to exhaust the vessel of air. If, after a vacuum has been produced, we place the lower end of the tube in a vessel of water, and open the stop-cock, the liquid, being pressed externally by the atmosphere, mounts up the tube and ascends in a jet into the interior of the vessel. This experiment is often made in the opposite manner. Under the receiver of the air-pump is placed a vial partly filled with water, and having its cork pierced by a tube open at both ends, the lower end being beneath the surface of the water. As the exhaustion proceeds, the air in the vial, by its excess of pressure, acts upon the liquid and makes it issue in a jet.

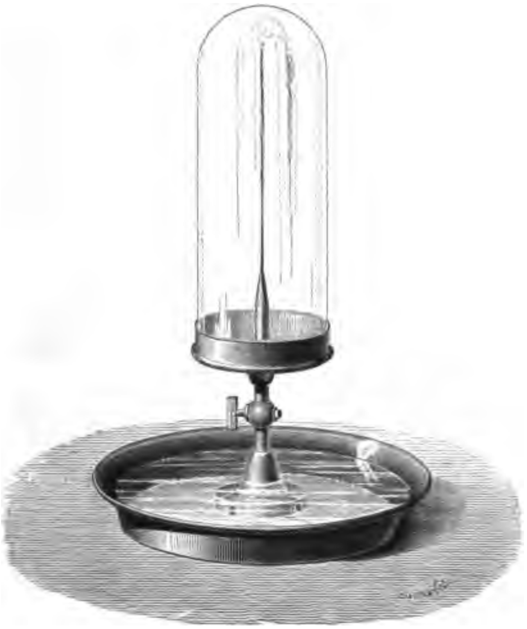
Fig. 143.—Fountain in Vacuo.

136. Limit to the Action of the Air-pump.—We have said above (§ 131) that the air-pump does not continue the process of rarefaction indefinitely, but that at a certain stage its effect ceases, and the tension of the air in the receiver undergoes no further diminution. If the pump is very badly made, this tension is considerable; but even with the most perfect machines it is always sensible. A pump such as we have described may be considered as very good if it reduces the tension of the air in the receiver to one-fiftieth of an inch of mercury: it is very rarely that a lower limit is reached.

LEAKAGE.—This limit to the action of the machine is due to various causes. One of these is evidently the leakage at different parts of the apparatus. It is impossible to prevent the air from getting in at several points; and although at the beginning of the operation the quantity of air which thus enters is small in comparison with that which is pumped out, still, as the exhaustion proceeds, the air enters faster, on account of the diminished internal pressure, and at the same time the quantity expelled at each stroke becomes less, so that at length a point is reached at which the inflow and outflow are equal.

SPACE UNTRAVERSED BY PISTON.—Another reason of imperfect exhaustion is that, after all possible precautions, a space is still left between the bottom of the pump-barrel and the lower surface of the piston when the latter is at the end of its downward stroke. It is evident that at this moment the air contained in this *untraversed space* is of the same tension as the atmosphere. On raising the piston, this air is indeed rarefied; but it still preserves a certain tension, and it is evident that when the air in the receiver has been brought to this stage of rarefaction, the machine will cease to produce any effect.

If v is the volume of this space, V the volume of the pump-barrel, the air, which at volume v has a tension H equal to that of the atmosphere, will have, at volume V, a tension equal to $H\frac{v}{V}$. This gives the limit to the action of the machine as deduced from the consideration of the untraversed space.

AIR GIVEN OUT BY OIL.—Finally, perhaps the most important cause, and the most difficult to remedy, is the absorption of air by the oil used for lubricating the pistons. This oil is poured on the top of the piston, but the pressure of the external air forces it between the piston and the barrel, whence it falls in greater or less quantity to the bottom of the barrel, where it absorbs air, and partially yields it up at the moment when the piston begins to rise, thus evidently tending to derange the working of the machine. It has been attempted to get rid of untraversed space by employing a kind of piston of mercury. This has also the advantage of fitting the barrel more accurately, and thus preventing the entrance of air. The use of oil is at the same time avoided, and we thus escape the injurious effects mentioned above. We proceed to describe two machines founded upon this principle.

137. Kravogl's Air-pump.—This contains a hollow glass cylinder AB tapering at the upper end, and surmounted by a kind of funnel. The piston is of the same shape as the cylinder, and is covered with a layer of mercury, whose depth over the point of the piston is about $\frac{1}{10}$th of an inch when the piston is at the bottom of its stroke, but is nearly an inch when the piston rises and fills the funnel-shaped cavity in which the pump-barrel terminates. A small interval, filled by the liquid, is left between the barrel and the piston; but at the

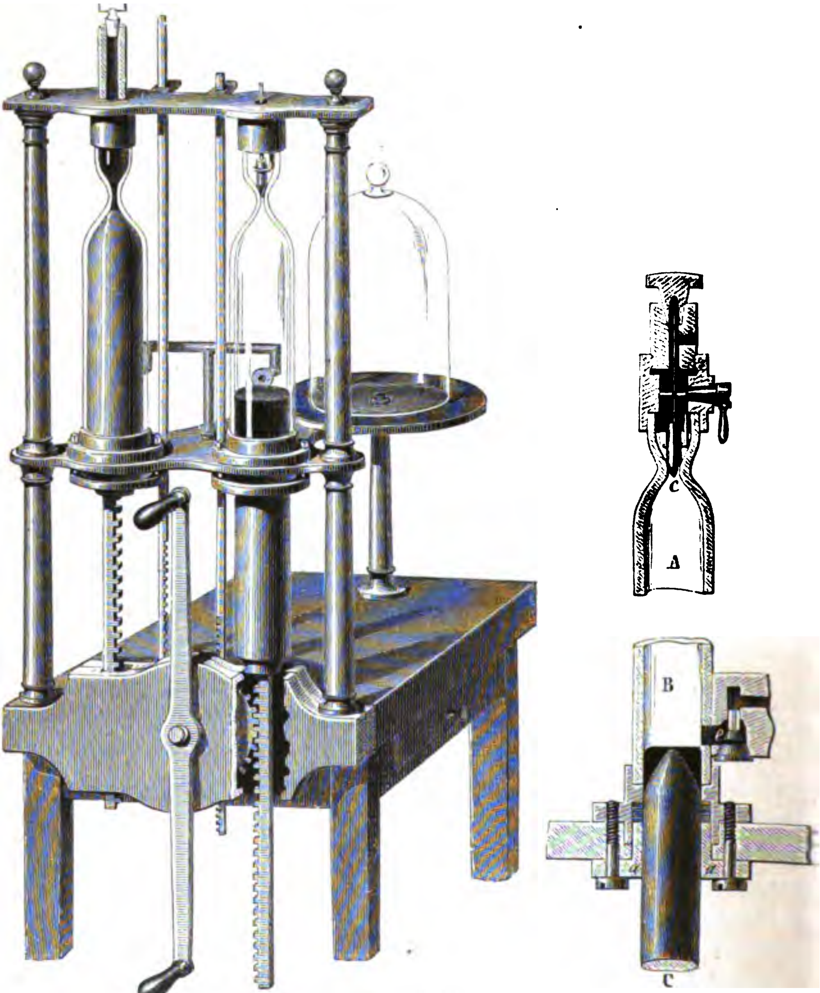

Fig. 145.—Kravogl's Air-pump.

bottom of the barrel the piston passes through a leather box carefully made, so as to be perfectly air-tight.

The air from the receiver passes through the lateral opening *e*; it is driven before the mercury into the funnel above. With the air passes a certain quantity of mercury, which is detained by a steel valve *c* at the narrowest part of the funnel. This valve rises automatically when the surface of the mercury is at a distance of about half an inch from the funnel, and falls back into its former position when the piston is at the end of its upward stroke. In the downward stroke, when the mercury is again half an inch from the funnel, the valve opens again and allows a portion of the mercury to pass.

The effect of this arrangement is easily understood; there is no "untraversed space," the presence of the mercury above and around the piston causes a very complete fit, and excludes the external air; and hence the machine, when well made, is very effective.

When this is the case, and when the mercury used in the apparatus is perfectly dry, a vacuum of about $\frac{1}{250}$th of an inch can be obtained. The dryness of the mercury is a very important condition, for at ordinary temperatures the elastic force of the vapour of water has a very sensible value. If we wish to employ the full powers of the machine, we must have, between the vessel to be exhausted of air and the pump-barrel, a desiccating apparatus.

The arrangement of the valve *e* is peculiar. It is of a conical form, so as, in its lowest position, to permit the passage of air coming from the receiver. Its ascent is produced by the pressure of the mercury, which forces it against the conical extremity of the passage, and the liquid is thus prevented from escaping.

The figure represents a double-barrelled machine analogous to the ordinary air-pump. Besides the pinion working with the racks of the pistons, there is a second smaller pinion, not shown in the figure, which governs the movements of the valves *c*. All the parts of this machine, as the stop-cocks, valves, pipes, &c., must be of steel, to avoid the action which the mercury would have upon any other metal.

138. Geissler's Machine.—Geissler, of Bonn, has invented a mercurial air-pump, in which the vacuum is produced by communication of the receiver with the Torricellian vacuum. Fig. 146 represents this machine as constructed by Alvergniat. It consists of a vertical tube, which serves as a barometric tube, and communicates at the bottom, by means of a caoutchouc tube, with a globe which serves as the cistern.

At the top of the tube is a three-way stop-cock, by which communication can be established either with the receiver to the left, or with a funnel to the right, which latter has an ordinary stop-cock at the bottom. By means of another stop-cock on the left, communication with the receiver can be opened or closed. These stop-cocks are made entirely of glass. The machine works in the following manner: communication being established with the funnel, the globe which serves as cistern is raised, and placed, as shown in the figure, at a higher level than the stop-cock of the funnel. By the law of equilibrium in communicating vessels, the mercury fills the barometric tube, the neck of the funnel, and part of the funnel itself. If the communication between the funnel and tube be now stopped, and the globe lowered, a Torricellian vacuum is produced in the upper part of the ertical tube.

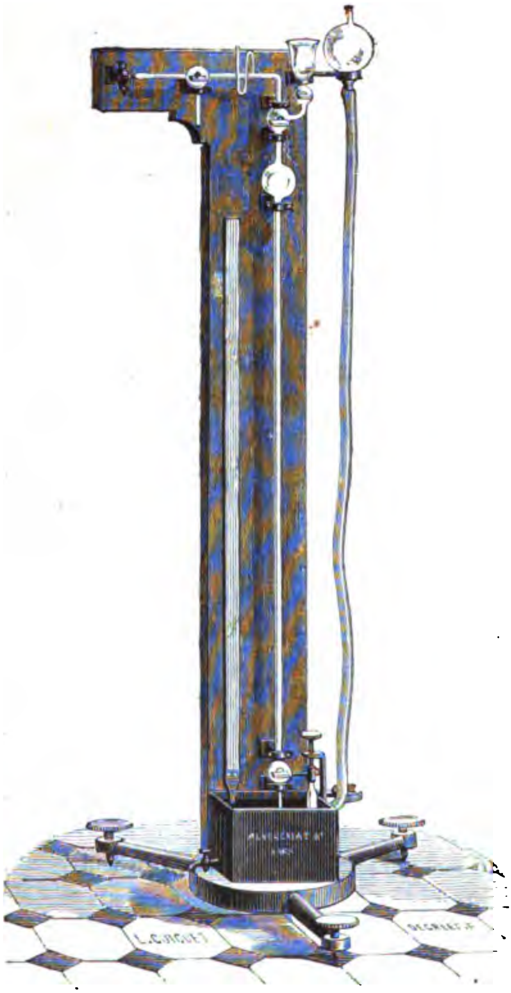

Fig. 146.—Geissler's Machine.

Communication is now opened with the receiver; the air rushes into the vacuum, and the column of mercury falls a little. Communication is now stopped between the tube and receiver, and opened between the tube and the funnel, the simple stop-cock of the funnel being, however, left shut. If at this moment the globe is replaced in the position shown in the figure, the air endeavours to escape by the funnel, and it is easy to allow it to do so. Thus, a part of the air of the receiver has been removed, and the apparatus is in the same position as at the beginning. The operation described is equivalent to a stroke of the piston in the ordinary machine, and this process must be repeated till the receiver is exhausted.

As the only mechanical parts of this machine are glass stop-cocks, which are now executed with great perfection, it is capable of giving very good results. With dry mercury a vacuum of $\frac{1}{250}$th of an inch may very easily be obtained. The working of the machine, however, is inconvenient, and becomes exceedingly laborious when the receiver is large. It is therefore employed directly only for producing a vacuum in very small vessels; when the spaces to be exhausted of air are at all large, the operation is begun with the ordinary machine, and the mercurial air-pump is only employed to render the vacuum thus obtained more perfect.

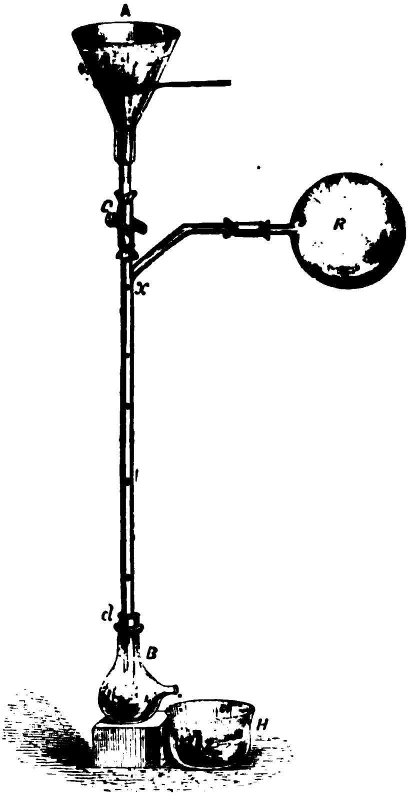

Fig. 146 A.—Sprengel's Air-pump.

138 A. Sprengel's Air-pump.—This instrument, which may be regarded as an improvement upon Geissler's, is represented in its simplest form in Fig. 146 A. *cd* is a glass tube longer than a barometer tube, down which mercury is allowed to fall from the funnel A. Its lower end dips into the glass vessel B, into which it is fixed by means of a cork. This vessel has a spout at its side, a few millimetres higher than the lower end of the tube. The first portions of mercury which run down will consequently close the tube, and prevent the possibility of air entering it from below. The upper part of *cd* branches off at *x* into a lateral tube communicating with the receiver R, which it is required to exhaust. A convenient height for the whole instrument is six feet. The funnel A is supported by a ring as shown in the figure, or by a board with a hole cut in it. The tube *cd* consists of two parts, connected by a piece of india-rubber tubing, which can be compressed by a clamp so as to keep the tube closed when desired. As soon as the mercury is allowed to run down the exhaustion begins, and the whole length of the tube, from *x* to *d*, is seen to be filled with cylinders of mercury separated by cylinders of air, all moving downwards. Air and mercury escape through the spout of the bulb B, which is above the basin H, where the mercury is collected. This has to be poured back from time to time into the funnel A, to pass through the tube again and again until the exhaustion is completed.

As the exhaustion is progressing, it will be noticed that the inclosed air between the mercury cylinders becomes less and less, until the lower part of *cd* presents the aspect of a continuous column of mercury about 30 inches high. Towards this stage of the operation a considerable noise begins to be heard, similar to that of a shaken water-hammer, and common to all liquids shaken in a vacuum. The operation may be considered completed when the column of mercury does not inclose any air, and when a drop of mercury falls upon the top of this column without inclosing the slightest air-bubble. The height of this column now corresponds exactly with the height of the column of mercury in a barometer; or, what is the same, it represents a barometer whose vacuum is the receiver R and connecting tube.

Dr. Sprengel recommends the employment of an auxiliary air-pump of the ordinary kind, to commence the exhaustion when time is an object, as without this from 20 to 30 minutes are required to exhaust a receiver of the capacity of half a litre. As, however, the employment of the auxiliary pump involves additional connections and increased leakage, it should be avoided when the best possible exhaustion is desired. The fall tube must not exceed about a tenth of an inch in diameter, and special precautions must be employed to make the india-rubber connections air-tight. (See *Chemical Journal* for 1865, p. 9.)

By this instrument air has been reduced to $\frac{1}{1800000}$th of atmospheric density, and the average exhaustion attainable by its use is about one-millionth, which is equivalent to ·00003 of an inch of mercury.

139. Double Exhaustion.—In the mercurial machines just described there is no "untraversed space," as the liquid completely expels all the air from the pump-barrel. These machines are of very recent invention. Babinet long before introduced an arrangement for the purpose, not of getting rid of this space, but of exhausting it of air.

For this purpose, when the machine ceases to work with the ordinary arrangement, the communication of the receiver with one of the pump-barrels is shut off, and this barrel is employed to exhaust the air from the other. This change is effected by means of a stop-cock at the point of junction of the passages leading from the two barrels (Fig. 147). The stop-cock has a T-shaped aperture, the point of intersection of the two branches being in constant communication with the receiver. In a different plane from that of the T-shaped aperture is another aperture *mn*, which, by means of the tube *l*, establishes communication between the pump-barrel B and the communicating passage of the pump-barrel A. From this explanation it will be seen that if the stop-cock be turned as shown in the first figure, the two pump-barrels both communicate with the receiver, and the operation proceeds in the ordinary manner. But if the stop-cock be turned through a quarter of a revolution, as shown in the second figure, the pump-barrel B alone communicates with the receiver, while it is itself exhausted of air by the barrel A.

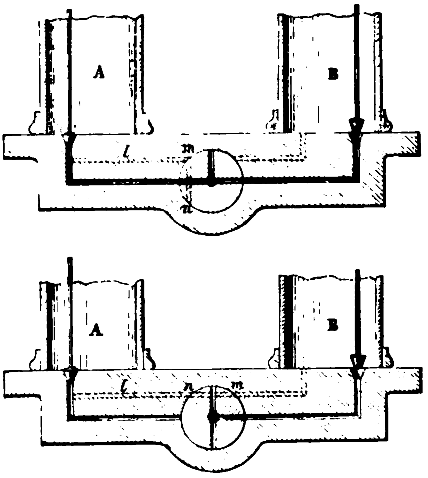

Fig. 147.—Babinet's Doubly-exhausting Stop-cock.

It is easy to express by a formula the effect of this double exhaustion. Suppose the pump to have ceased, under the ordinary method of working, to produce any farther exhaustion, the air in the receiver has therefore reached a tension nearly equal to $H\frac{v}{V}$ (§ 136). At this moment the stop-cock is turned into its second position. When the piston B descends, the piston A rises, and the air of the "untraversed space" in B is drawn into A and rarefied. During the inverse operation the air in A is prevented from returning to B, and thus the rarefied air from B, becoming still further rarefied, will draw a fresh quantity of air from the receiver. This air will then be driven into A, where it will be compressed by the descending movement of the piston, and will find its way into the air outside.[1]

This double exhaustion will itself cease to work when air ceases to pass from the pump-barrel B into the pump-barrel A. Now when the piston in this latter is raised, the elastic force of the air which was contained in its "untraversed space" is equal to $H\frac{v}{V}$, for on the last opening of the valve, the air in this space escaped into the atmosphere. On the other hand, when the piston in B is at the end of its upward stroke, the tension of the air is the same as in the receiver. Let this be denoted by x. When the piston in B descends the air is compressed into the "untraversed space" and the passage leading to A. Let the volume of this passage be l. Then the tension will increase, and become $x\frac{V+l}{v+l}$ When the machine ceases to produce any farther effect, this tension cannot be greater than that in the pump-barrel A, which is $H\frac{v}{V}$; we have thus, to determine the limit to the action of the pump, the equation

$$x\frac{V+l}{v+l}=H\frac{v}{V}, \text{ whence}$$

$$x = H\cdot\frac{v}{V}\cdot\frac{v+l}{V+l}.$$

140. Air-pump with Free Piston.—We shall describe one more air-pump, constructed by Deleuil, and founded upon an interesting principle. We know that gases possess a remarkable power of adhesion for solids, so that a body placed in the atmosphere may be considered as covered with a very thin coat of air, forming, so to speak, a permanent envelope. On account of this circumstance, gases find very great difficulty in moving in very narrow spaces. On these facts depends the principle of what is called the air-pump with free piston.

The piston P (Fig. 149), which is composed entirely of metal, is of a considerable length, and on its outer surface is a series of parallel circular grooves very close together. It does not touch the pump-barrel at any point; but the distance between the two is very small, about ·001 of an inch. This free piston is surrounded by a cushion of gas, which forms its only stuffing, and is sufficient to enable the

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[1] It will be observed that during the process of double exhaustion the piston of B behaves like a solid piston; its valve never opens, because the pressure below it is always less than atmospheric.

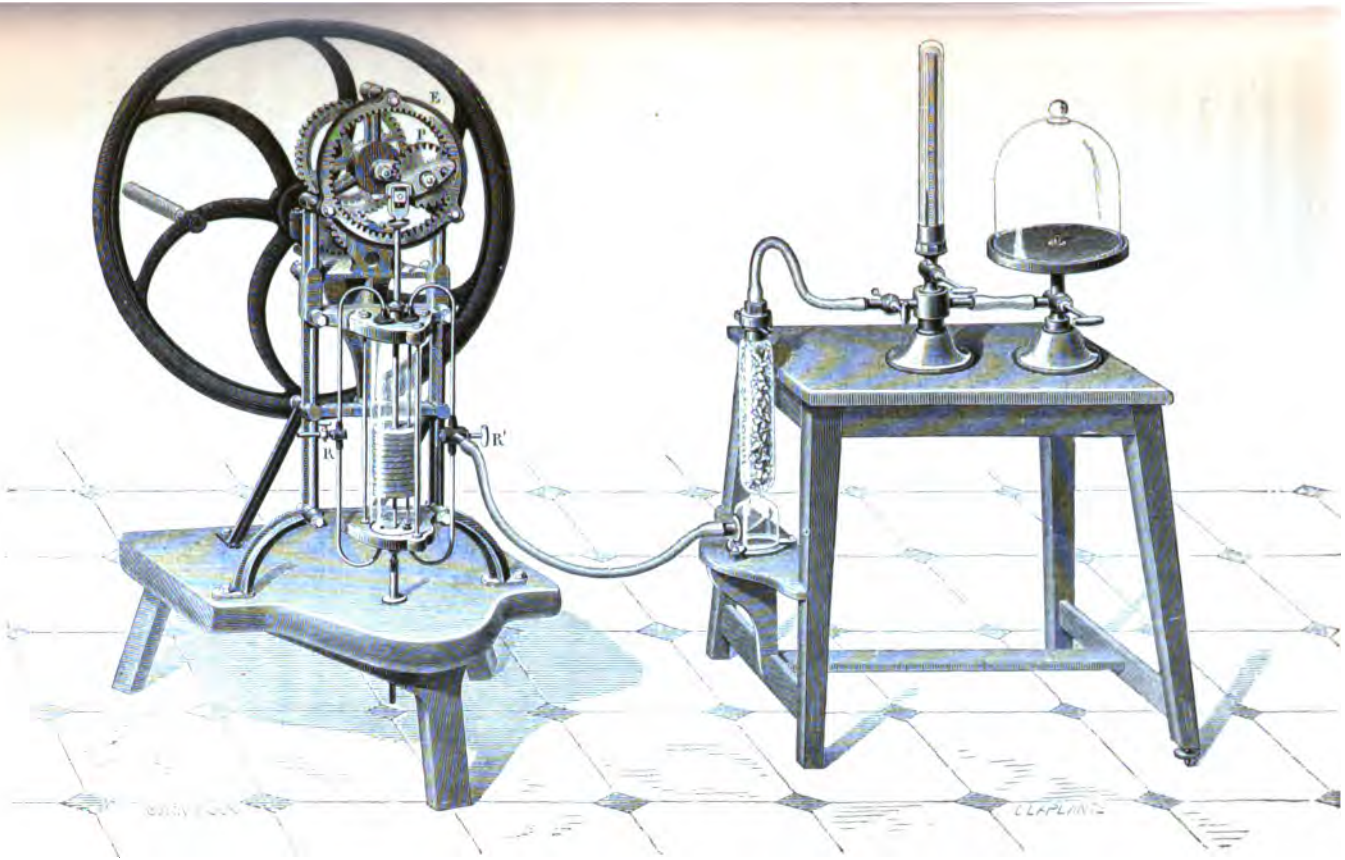

Fig. 148.—Deleuil's Air-Pump.

machine to work in the ordinary manner, notwithstanding the permanent communication between the upper and lower surfaces of the piston. This machine gives a vacuum about as good as is obtainable by ordinary pumps, and it has the important advantages of not requiring oil, and of having less friction. It consequently wears better, and is less liable to the development of heat, which is a frequent source of annoyance in air-pumps. It is single-barrelled with double action, like Bianchi's. The two openings S and S′ are to admit air from the receiver; they are closed and opened alternately by conical stoppers at the ends of the rod T, which passes through the piston, and is carried with it by friction in its movement. They communicate with tubes which unite at R′ with a tube leading from the receiver. A and A′ are valves for the expulsion of the air, which escapes by tubes uniting at R. The alternate movement of the piston is produced by what is called Delahire's gearing. This depends on the principle, that *when a circle rolls without sliding in the interior of another circle of double the diameter, any point on the circumference of the rolling circle describes a diameter of the fixed circle.* In order to utilize this property, the end of the piston-rod is jointed to the extremity of a piece of metal which is rigidly attached to the pinion P, the joint being exactly opposite the circumference of the pinion. This latter is driven by a fly-wheel with suitable gearing, and works with the fixed wheel E, which is toothed on the inside. Thus the piston will freely, and without any lateral effort, describe a vertical line, the length of the stroke being equal to the diameter of the fixed wheel.

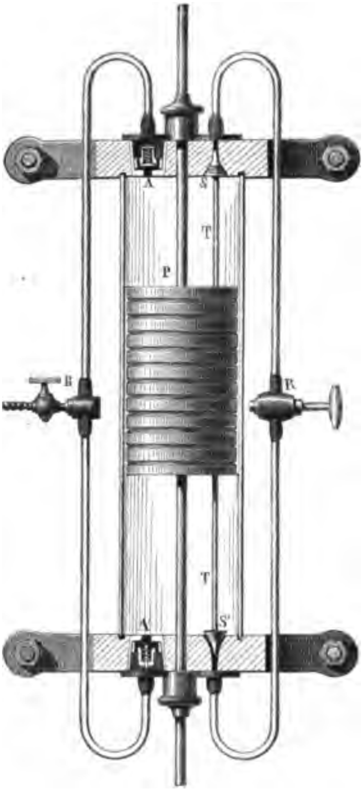

Fig. 149.
Piston and Barrel of Deleuil's Air-pump.

141. Condensing Pump.—It can easily be seen from the description of the air-pump, that if the expulsion-valves were connected with a tube communicating with a reservoir, the air removed by the pump would be forced into this reservoir. This communication is established in the instrument just described. If, therefore, R′ be made to communicate with the external air, this air will be continually drawn in at that point and forced back into the reservoir connected with R, so that the instrument will act as a condensing pump. The condensing pump is thus seen to be the same instrument as the air-pump, the only difference being that the receiver is connected with the expulsion-valves, instead of with the exhaustion-valves; it is thus, so to speak, the air-pump reversed.

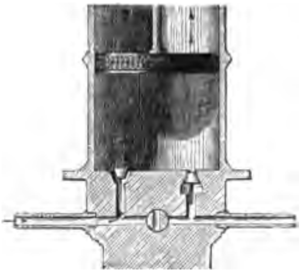

Fig. 150.—Barrel of Condensing Pump.

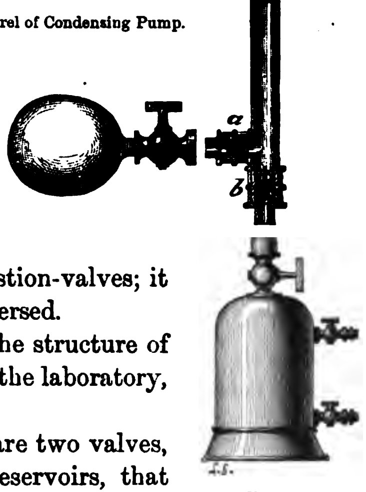

Fig. 151.
Condensing Pump.

This fact can be very well seen in the structure of a small pump frequently employed in the laboratory, and represented in Fig. 150.

At the bottom of the pump-barrel are two valves, communicating with two separate reservoirs, that on the left being an admission-valve, and that on the right an expulsion-valve.

When the piston is now raised, rarefaction is produced in the reservoir to the left; and when it is pushed down, the air in the reservoir to the right is compressed.

142. In Fig. 151 is represented a condensing pump often employed. At the bottom of the pump-barrel is a valve *b* opening downward; in a lateral tube is an admission-valve *a* opening inward. The position of these valves is shown in the figure. They are conical metal stoppers, fitted with a rod passing through a hole in a small plate behind, an arrangement which prevents the valve from overturning. The rod is surrounded by a small spiral spring, which keeps the valve pressed against the opening. If now the lower part of the pump-barrel be screwed upon a reservoir, at each upward stroke of the piston the barrel will be filled with air through the valve *a*, and at every downward stroke this air will be forced into the reservoir.

If the lateral tube be made to communicate with a bladder or gas-holder filled with any gas, this gas will be forced into the reservoir, and compressed.

143. Calculation of the Effect of the Instrument.—The density of the compressed air after a given number of strokes of the piston may easily be calculated. If v be the volume of the pump-barrel, and V that of the reservoir; at each stroke of the piston there is forced into the reservoir a volume of air equal to that of the pump-barrel, which gives a volume nv at the end of n strokes. The air in the reservoir, accordingly, which when at atmospheric pressure had density D, and occupied a volume $V+nv$, will, when the volume is reduced to V, have the density $D\frac{V+nv}{V}$, and the pressure will, by Boyle's law, be $\frac{V+nv}{V}$ atmospheres.

If this formula were rigorously applicable in all cases, there would be no limits to the pressure attainable, except those depending on the strength of the reservoir and the motive power available.

But, in fact, the untraversed space left below the piston when at the end of its downward stroke, sets a limit to the action of the instrument, just as in the common air-pump. For when the air in the barrel is reduced from the volume of the barrel v to that of the untraversed space v', its tension becomes $H\frac{v}{v'}$, and this air cannot pass into the reservoir unless the tension of the air in the reservoir is less than this quantity. This is accordingly the utmost limit of compression that can be attained.

We must, however, carefully distinguish between the effects of untraversed space in the air-pump and in the compression-pump. In the first of these instruments the object aimed at is to rarefy the air to as great a degree as possible, and untraversed space must consequently be regarded as a defect of the most serious importance.

The object of the condensing pump, on the contrary, is to compress the air, not indefinitely, but up to a certain point. Thus, for instance, one pump is intended to give a compression of five atmospheres, another of ten, &c. In each of these cases the maker provides that this limit shall be reached, and accordingly the untraversed space can have no injurious effect beyond increasing the number of strokes required to produce the desired amount of condensation.

144. Various Contrivances for producing Compression.—In order to expedite the process of compression, several pumps such as we have described are combined, which may be done in various ways. Fig. 152 represents the system employed by Regnault in his investigations connected with Boyle's law and the elastic force of vapour. It consists of three pumps, the piston-rods of which are jointed to three cranks on a horizontal axle, by means of three connecting-rods. This axle, which carries a fly-wheel, is turned by means of one or two handles. The different admission-valves are in communication with a single reservoir in connection with the external air, and the compressed gas is forced into another reservoir which is in communication with the experimental apparatus.

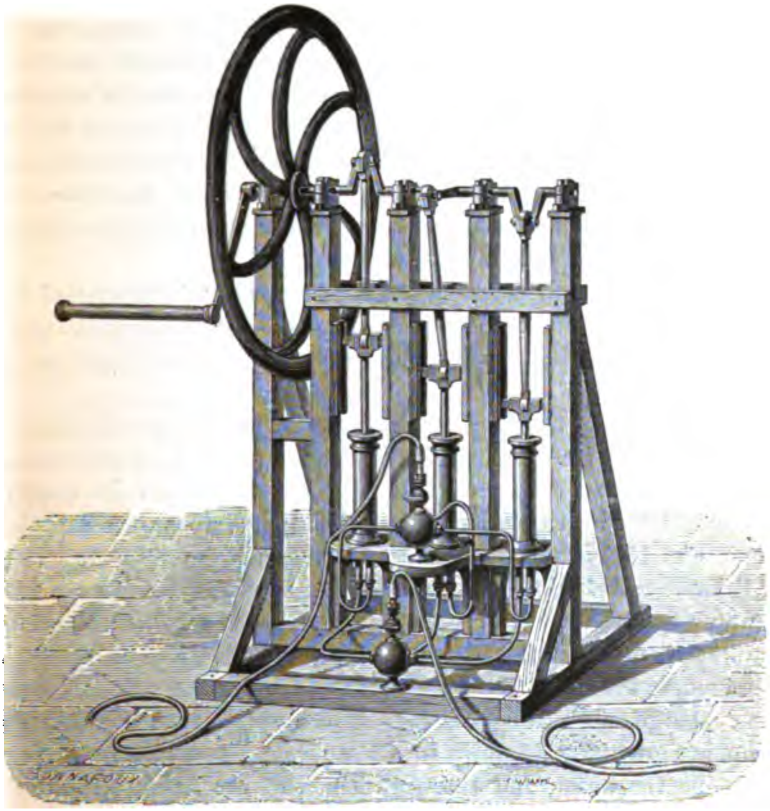
Fig. 152.—Connected Pumps.

A serious obstacle to the working of these instruments is the heat generated by the compression of the air, which expands the different parts of the instrument unequally, and often renders the piston so tight that it can scarcely be driven. In some of these instruments which are employed in the arts, this inconvenience is lessened by keeping the lower valves covered with water, which has the additional advantage of getting rid of "untraversed space." In this way a pressure of forty atmospheres may easily be obtained with air. Air may also be compressed directly, without the intervention of pumps, when a sufficient height of water can be obtained. It is only necessary to lead the liquid in a tube to the bottom of a reservoir containing air. This air will be compressed until its tension exceeds that of the atmosphere by the amount due to the height of the summit of the tube. It is by a contrivance of this kind that the compressed air is obtained which drives the boring-machines employed in the tunnel through Mont Cenis.

145. Practical Applications of the Air-pump and of Compressed Air.—Besides the use made of the air-pump and the compression-pump in the laboratory, these instruments are variously employed in the arts.

The air-pump is employed by sugar-refiners to lower the boiling point of the syrup. Compression-pumps are used by soda-water manufacturers to force the carbonic acid into the reservoirs containing the water which is to be aerated. The small apparatus described above (Fig. 151) is sufficient for this purpose; it is only necessary to fill the side-vessel with carbonic acid, and to pour a certain quantity of water into the reservoir below. Compressed air has for several years been employed to assist in laying the foundations of bridges in rivers where the sandy nature of the soil requires very deep excavations. Large tubes called *caissons*, in connection with a condensing pump, are gradually let down into the river; the air by its pressure keeps out the water, and the workmen, who are admitted into the apparatus by a sort of lock, are thus enabled to walk on dry ground.

In pneumatic despatch tubes, which have recently been established in many places, a kind of train is employed, consisting of a piston preceded by boxes containing the despatches. By exhausting the air at the forward end of the tube, or forcing in compressed air at the other end, the train is blown through the tube with great velocity.

The atmospheric railway, which was for a few years in existence, was worked upon the same principle: an air-tight piston travelled through a fixed tube, and was connected by an ingenious arrangement with the train above.

Excavating machines driven by compressed air are coming into extensive use in mining operations. They have the advantage of assisting ventilation, inasmuch as the compressed air, which at each stroke of the machine escapes into the air of the mine, cools as it expands.

In the air-gun the bullet is projected by a portion of compressed air which, on pulling the trigger, escapes into the barrel from a reservoir in which it has been artificially compressed.

We may add that the large machines employed in iron-works for supplying air to the furnaces, are really compression-pumps.

CHAPTER XVI.

UPWARD PRESSURE OF THE AIR.

146. The Baroscope.—Atmospheric air exerts, as we have already mentioned (§ 101), an upward pressure on bodies surrounded by it. This pressure, according to the principle of Archimedes, which applies to gases as well as to liquids, is equal to the weight of the air displaced. Hence it follows that the weight of a body in the air is not its actual weight, but differs from it by a quantity equal to the upward pressure on the body. This principle is illustrated by the baroscope.

This is a kind of balance, the beam of which supports two balls of very unequal sizes, which balance each other in the air. If the apparatus is placed under the receiver of an air-pump, after a few strokes of the piston, the beam will be seen to incline towards the larger ball, and the inclination will increase as the exhaustion proceeds. The reason is that the air, before it was pumped out, produced an upward pressure, which is now removed. The weight of each ball is thus increased by that of an equal volume of air, whose density is the difference between the densities at the beginning and end of the experiment. This addition is greater for the larger ball, which therefore preponderates.

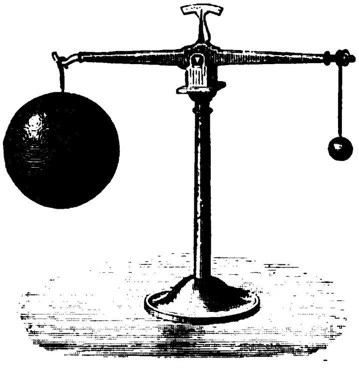

Fig. 153.—Baroscope.

If after exhausting the air, carbonic acid, which is heavier than air, were allowed to enter the receiver, the large ball would be subjected to a greater increase of upward pressure than the small one, and the beam would incline to the side of the latter.

147. Balloons.—Suppose a body to be lighter than an equal volume of air, then this body will rise in the atmosphere. For example, if we fill soap-bubbles with hydrogen, and shake them off from the end of the tube at which they are formed, they will be seen, if sufficiently large, to ascend in the air. This curious experiment is due to the philosopher Cavallo, who announced it in 1782.[1]

The same principle applies to balloons, which may essentially be reduced to an envelope inclosing a gas lighter than air. In consequence of this difference of density, we can always, by taking a sufficiently large volume, make the weight of the gas and containing envelope less than that of the air displaced. In this case the balloon will ascend.

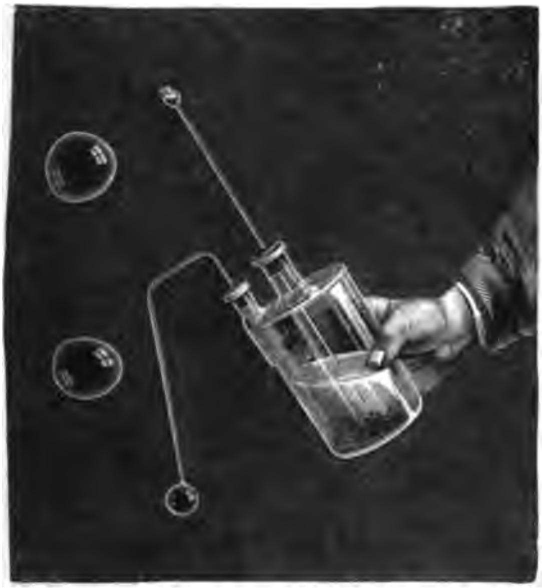

Fig. 154.—Ascent of Soap-bubbles filled with Hydrogen.

The invention of balloons is due to the brothers Joseph and Stephen Montgolfier. The balloons made by them were globe-shaped, and constructed of paper, or of paper covered with cloth, the air inside being rarefied by the action of heat. It is curious to remark that in their first attempts they employed hydrogen gas, and showed that balloons filled with this gas could ascend. But as the hydrogen readily escaped through the paper, the flight of the balloons was short, and thus the use of hydrogen was abandoned, and hot air was alone employed.

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[1] The first idea of a balloon must be attributed to Francisco de Lana, who, about 1670, proposed to exhaust the air in globes of copper of sufficient size and thinness to weigh less, under these conditions, than the air displaced. The experiment was not tried, and would certainly not have succeeded, for the pressure of the atmosphere would have caused the globes to collapse. The theory, however, was thoroughly understood by the author, who made an exact calculation of the amount of force tending to make the globes ascend. —*D.*

The name *montgolfières* is still applied in France to fire-balloons. They generally consist of an envelope with a wide opening below, under which is hung a brazier,[1] in which, at the moment of ascent, combustibles are placed, and the ascending power of the balloon is thus kept up for some time.

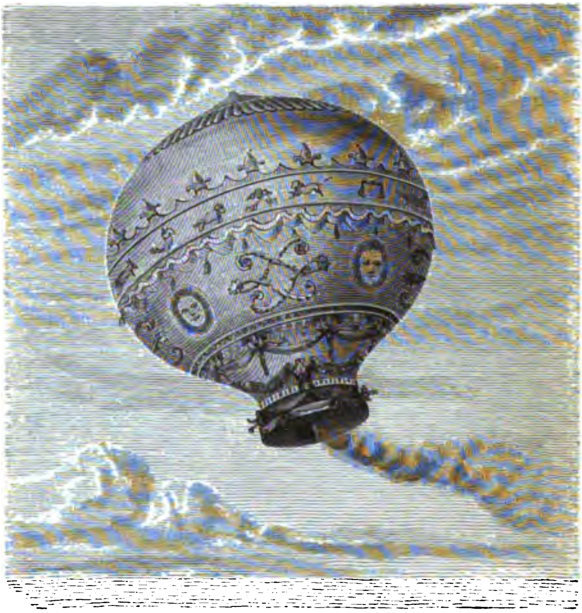

Fig. 155.—Fire-balloon of Pilatre de Rozier.

The first public experiment of the ascent of a balloon was performed at Annonay on the 5th June, 1783. On October 21st of the same year, Pilatre de Rozier and the Marquis d'Arlandes achieved the first aerial voyage in a fire-balloon, represented in our figure.

Charles proposed to reintroduce the use of hydrogen by employing an envelope impermeable to the gas. This is usually made of silk varnished on both sides, or of two sheets of silk with a sheet of india-rubber between. Instead of hydrogen, coal-gas is now generally employed, on account of its cheapness and of the facility with which it can be procured.

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[1] A sponge, dipped in spirits of wine, and ignited, is frequently employed as the sour of heat, and is fixed in its place by a light wire-frame.

148. Buoyancy.—The buoyancy or lifting power of a balloon is the difference between its weight and that of the air displaced. It is easy to compare the three modes of inflation with respect to the buoyancy which they respectively afford.

A cubic metre of air weighs	1·300	grammes.
A cubic metre of hydrogen	·089	,,
A cubic metre of coal-gas	about ·750	,,
A cubic metre of air heated to 200° Cent.	·800	,,

We thus see that the buoyancy per cubic metre with hydrogen is 1·211, with coal-gas ·550, and with hot air about ·500 grammes. If, for instance, the total weight to be raised is estimated at 1500 grammes, the volume of a balloon filled with hydrogen capable of raising the weight will be $\frac{1500}{1\cdot210}=1239$ cubic metres. If coal-gas were employed the required volume would be $\frac{1500}{\cdot550}=2727$ cubic metres.

The car in which the aeronauts sit is usually made of wicker-work or whalebone. It is sustained by cords attached to a net-work covering the entire upper half of the balloon, so as to distribute the weight as evenly as possible. The balloon terminates below in a kind of neck opening freely into the air. At the top there is another opening in the inside, which is closed by a valve held to by a spring. Attached to the valve is a cord which passes through the interior of the balloon, and hangs above the car within reach of the hand of the aeronaut.

When the aeronaut wishes to descend, he opens the valve for a few moments and allows some of the gas to escape. An important part of the equipment consists of sand-bags for ballast, which are gradually emptied to check too rapid descent. In the figure is represented a contrivance called a parachute, by means of which the descent is sometimes effected. This is a kind of large umbrella with a hole at the top, from the circumference of which hang cords supporting a small car. When the parachute is left to itself, it opens out, and the resistance of the air, acting upon a large surface, moderates the rate of descent. The hole at the top is essential to safety, as it affords a regular passage for air which would otherwise escape from time to time from under the edge of the parachute, thus producing oscillations which might prove fatal to the aeronaut.

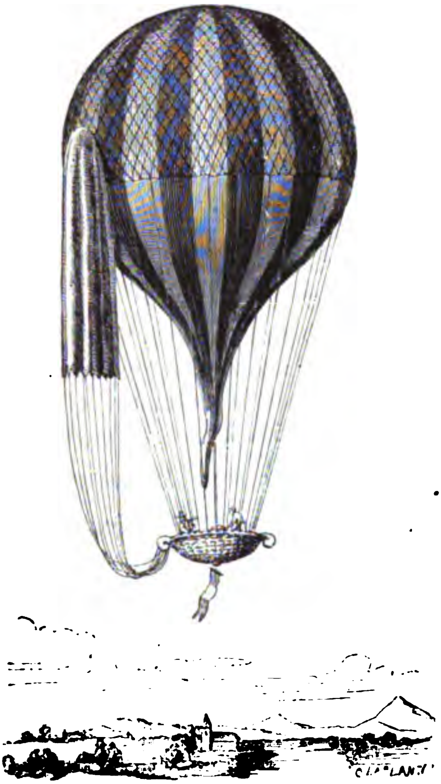

Fig. 156.—Balloon with Car and Parachute.

One very important precaution to be observed is not to inflate the balloon completely at the commencement of the ascent. The reason is, that the atmospheric pressure diminishing as the balloon ascends, the expansive power of the gas contained produces an increasing effect, as in the experiment described in § 18, and the result would probably be the bursting of the balloon. As the balloon ascends, it increases in volume; but until it is completely inflated, the buoyancy remains constant. Suppose, for instance, that the atmospheric pressure is reduced by one-half, the volume of the balloon will be doubled; it will thus displace a volume of air twice as great as before, but the density of this air will only be half as much, so that the buoyancy remains the same. This conclusion, however, is not quite exact, because part of the balloon, as the cords, the car, &c., are of invariable volume; the density of the air displaced by them is constantly diminishing, and consequently the buoyancy diminishes also. If the balloon continues to rise after it is completely inflated, its buoyancy diminishes rapidly, becoming zero when a stratum of air is reached in which the weight of the volume displaced is equal to that of the balloon itself. It is carried past this stratum in the first instance in virtue of the velocity which it has acquired, and finally comes to rest in it after a number of oscillations.

149. Theory of the Balloon.—The tension of the air in this stratum, the radius of the balloon, and the weight of the different parts, are connected by a relation which can be very easily established, if we neglect variations of temperature, and which may serve as a guide in the construction of the balloon. If V be the volume of the balloon in litres, the weight of air displaced in grammes is[1]

$$1{\cdot}293\ \mathrm{V}\frac{h}{\mathrm{H}},$$

h being the pressure in the stratum of equilibrium, and H that at the surface of the earth. If w and v be the weight and volume of the solid parts, including the aeronauts themselves, δ the density of the gas in the balloon, the equation of equilibrium will be $(\mathrm{V}+v)\ 1{\cdot}293\ \frac{h}{\mathrm{H}} = \mathrm{V}{\cdot}\delta\frac{h}{\mathrm{H}}+w$.

In this equation w and v include the weight and volume of the substance composing the balloon and net-work, and therefore are not altogether independent of V, the volume of the balloon. The equation is thus in reality rather complicated, but it may be solved approximately by trial, or by known algebraic methods, and we may find from it the size of balloon necessary for reaching a stratum of a given pressure; and by § 112 we can find the height corresponding to this pressure.

150. Effect of the Air upon the Weight of Bodies.—The upward pressure of the air impairs the exactness of weights obtained even with a perfectly true balance, tending, by the principle of the baroscope, to make the denser of two equal masses preponderate. The stamped weights used in weighing are, strictly speaking, standards of mass, and will equilibrate any equal masses in vacuo; but in air the equilibrium will be destroyed by the greater upward pressure of the air upon the larger and less dense body. When the specific gravities of the weights and of the body weighed are known, it is easy from the apparent weight to deduce the true weight (that is to say, the mass) of the body.

Let x be the real weight of a body which balances a weight marked P pounds.

The apparent weight of the body is $x-\frac{x}{\mathrm{D}}\,a=x\left(1-\frac{a}{\mathrm{D}}\right)$, D being the density of the body, and a that of air.[2] The body marked as weighing P pounds has in air the apparent weight $P-\frac{P}{M}a=P(1-\frac{a}{M})$, M denoting the density of the substance of which it is composed. These two apparent weights balance one another; hence we have the equation

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[1] The weight of a litre (or cubic decimetre = 61·027 cubic inches) of dry air at temperature 0° Cent., and pressure of 760 millimetres of mercury, is 1·293 grammes. Reduced to British measure, this gives 32·7 grains as the weight of 100 cubic inches.

[2] The value of a varies according to the temperature and pressure (see Chap. xxiii.) Its ordinary value is about $\frac{1}{770}$. (See § 100.)

$$x\left(1-\frac{a}{D}\right)=P\left(1-\frac{a}{M}\right),$$

whence

$$x=P\frac{1-\frac{a}{M}}{1-\frac{a}{D}}=P\left\{1+a\left(\frac{1}{D}-\frac{1}{M}\right)\right\}\text{ nearly.}$$

Let us take, for instance, a piece of sulphur whose weight has been found to be 100 grains, the weights being of copper, the density of which is 8·8. The density of sulphur is 2.

We have, by applying the formula,

$$x=100\left\{1+\frac{1}{770}\left(\frac{1}{2}-\frac{1}{8{\cdot}8}\right)\right\}=100{\cdot}05\text{ grains.}$$

We see then that the difference is not altogether insensible. It varies in sign, as the formula shows, according as M or D is the greater. When the density of the body to be weighed is less than that of the weights used, the real weight is greater than the apparent weight; if the contrary, the case is reversed. If the body to be weighed were of the same density as the weights used, the real and apparent weights would be equal. We may remark, that in determining the ratio of the weights of two bodies of the same density, we need not concern ourselves with the effect of the upward pressure of the air, as the correcting factor, which has the same value for both cases, will disappear in the quotient.

N1,5·B

CHAPTER XVII.

PUMPS FOR LIQUIDS.

151. Machines for raising water have been known from very early ages, and the invention of the common pump is pretty generally ascribed to Ctesibius, teacher of the celebrated Hero of Alexandria; but the true theory of its action was not understood till the time of Galileo and Torricelli.

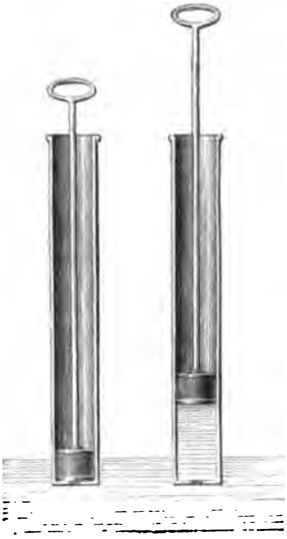
Fig. 157.—Principle of Suction-pump.

152. **Reason of the Rising of Water in Pumps.**—Suppose we take a tube with a piston at the bottom, and immerse the lower end of it in water. The raising of the piston tends to produce a vacuum below it, and the atmospheric pressure, acting upon the external surface of the liquid, compels it to rise in the tube and follow the upward motion of the piston. This upward movement of the water would take place even if some air were interposed between the piston and the water; for on raising the piston this air would be rarefied, and its pressure no longer balancing that of the atmosphere, this latter pressure would cause the liquid to ascend in a column, whose weight, added to the pressure of the air below the piston, would be equal to the atmospheric pressure. This is the principle on which water rises in pumps. These instruments have a considerable variety of forms, of which we shall describe the most important types.

153. **Suction-pump.**—The suction-pump consists of a cylindrical

pump-barrel traversed by a piston, and communicating by means of a smaller tube, called the suction-tube, with the water in the pump-well. At the junction of the pump-barrel and the tube is a valve opening upward, called the suction-valve, and in the piston is an opening closed by another valve, also opening upward.

Suppose now the suction-tube to be filled with air at the atmospheric pressure, and the water consequently to be at the same level inside the tube and in the well. Suppose the piston to be at the end of its downward stroke, and to be now raised. This motion tends to produce a vacuum below the piston, hence the air contained in the suction-tube will open the suction-valve, and rush into the pump-barrel. Its elastic force being thus diminished, the atmospheric pressure will cause the water to rise in the tube to a height such that the pressure due to this height, increased by the pressure of the air inside, will exactly counterbalance the pressure of the atmosphere. If the piston now descends, the suction-valve closes, the water remains at the level to which it has been raised, and the air, being compressed in the barrel, opens the piston-valve and escapes. At the next stroke of the piston the water will rise still further, and a fresh portion of air will escape.

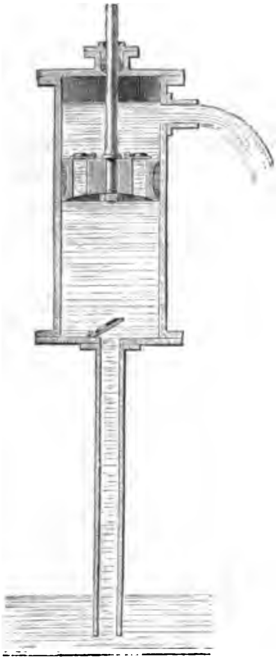

Fig. 158.—Suction-pump.

If, then, the length of the suction-tube is less than about 30 feet, the water will, after a certain number of strokes of the piston, be able to reach the suction-valve and rise into the pump-barrel. When this point has been reached the action changes. The piston in its downward stroke compresses the air, which escapes through it, but the water also passes through, so that the piston when at the bottom of the pump-barrel will have above it all the water which has previously risen into the barrel. If the piston be now raised, supposing the total height to which it is raised to be not more than 34 feet above the level of the water in the well, as should always be the case, the water will follow it in its upward movement, and will fill the pump-barrel. In the downward stroke this water will be forced through the piston-valve, and in the following upward stroke will be discharged at the spout. A fresh quantity of water will by this time have risen into the pump-barrel, and the same operations will be repeated.

We thus see that from the time when the water has entered the pump-barrel, at each upward stroke of the piston a volume of water is ejected equal to the contents of the pump-barrel.

In order that the water may be able to rise into the pump-barrel, the suction-valve must not be more than 34 feet above the level of the water in the well, otherwise the water would stop at a certain point of the tube, and could not be raised higher by any farther motion of the piston.

Moreover, in order that the working of the pump may be such as we have described, that is, that at each upward stroke of the piston a quantity of water may be removed equal to the volume of the pump-barrel, it is necessary that the piston when at the top of its stroke should not be more than 34 feet above the water in the well.

154. Condition that the Water may reach the Pump-barrel.—If the piston does not descend to the bottom of the barrel, the water may fall short of rising to the suction-valve, even though the total height reached by the piston be less than 34 feet. For, if the piston when at the end of its downward stroke leaves below it a space containing air, the tension which this air possesses when the piston is raised diminishes by a corresponding quantity the height to which the water can attain. If, for instance, the length of the suction-tube is 33·5 feet, and the tension of the air remaining above it is, when at its least value, equal to the pressure of 1 foot of water, it is evident that the total height to which the water can rise will be less than 33 feet, and it will, in consequence, be unable to reach the pump-barrel.

Example. The suction-valve of a pump is at a height of 27 feet above the surface of the water, and the piston, the entire length of whose stroke is 7·8 inches, when at the lowest point is 3·1 inches from the fixed valve; find whether the water will be able to rise into the pump-barrel.

When the piston is at the end of its downward stroke, the air which it leaves below it is at the atmospheric pressure; when the piston is raised this air becomes rarefied, and its pressure, by Boyle's law, becomes $\frac{3 \cdot 1}{10 \cdot 9}$ that of the atmosphere; this pressure can therefore balance a column of water whose height is $34 \times \frac{3 \cdot 1}{10 \cdot 9}$ feet, or 9·67 feet. Hence, the maximum height to which the water can attain is 34 —

9·67 feet = 24·33 feet; and consequently, as the suction-tube is 27 feet long, the water will not rise into the pump-barrel, even supposing the pump to be perfectly free from leakage.

Practically, the pump-barrel should not be more than about 25 feet above the surface of the water in the well; but the spout may be more than 34 feet above the barrel, as the water after rising above the piston is simply pushed up by the latter, an operation which is independent of atmospheric pressure. Pumps in which the spout is at a great height above the barrel are commonly called *lift-pumps*, but they are not essentially different from the suction-pump.

155. Force necessary to raise the Piston.—The force which must be expended in order to raise the piston, is equal to the weight of a column of water, whose base is the section of the piston, and whose height is that to which the water is raised. Let S be the section of the piston, P the atmospheric pressure upon this area, h the height of the column of water which is above the piston in its present position, and h' the height of the column of water below it; then the upper surface of the piston is subjected to a pressure equal to $P + Sh$; the lower face is subjected to a pressure in the opposite direction equal to $P - Sh'$, and the entire downward pressure is represented by the difference between these two, that is, by $S(h + h')$.

The same conclusion would be arrived at even if the water had not yet reached the piston. In this case, let l be the height of the column of water raised; then the pressure below the piston is $P - Sl$; the pressure above is simply the atmospheric pressure P, and, consequently, the difference of these pressures acts downward, and its value is Sl.

156. Efficiency of Pumps.—From the results of last section it would appear that the force required to raise the piston, multiplied by the height through which it is raised, is equal to the weight of water discharged multiplied by the height of the spout above the water in the well. This is an illustration of the principle of work (§ 17A). As this result has been obtained from merely statical considerations, and on the hypothesis of no friction, it presents too favourable a view of the actual efficiency of the pump.

Besides the friction of the solid parts of the mechanism, there is work wasted in generating the velocity with which the fluid, as a whole, is discharged at the spout, and also in producing eddies and other internal motions of the fluid. These eddies are especially produced at the sudden enlargements and contractions of the passages through which the fluid flows. To these drawbacks must be added loss from leakage of water, and at the commencement of the operation from leakage of air, through the valves and at the circumference of the piston. In common household pumps, which are generally roughly made, the *efficiency* may be as small as ·25 or ·3; that is to say, the product of the weight of water raised, and the height through which it is raised, may be less than the work done in driving the pump in the ratio of one of these numbers to unity.

In Figs. 159 and 160 are shown the means usually employed for working the piston. The first figure needs no explanation; it will be seen that the upward and downward movement of the piston is effected by means of a lever. The second figure represents an arrangement often employed, in which the alternate motion of the piston is effected by means of a rotatory motion. For this purpose the piston-rod T is joined by means of the connecting-rod

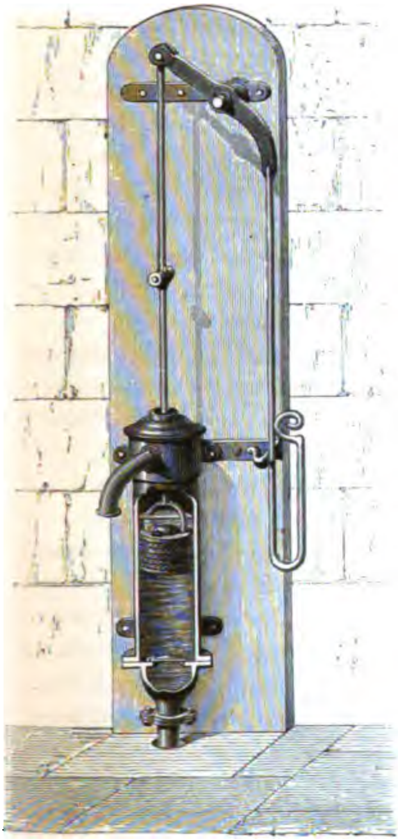
Fig. 159.

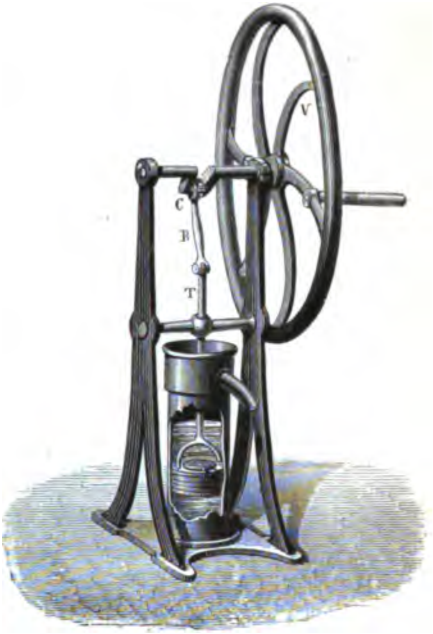

Fig. 160.

Suction-pump.

B to the crank C of an axle turned by a handle attached to the fly-wheel V.

157. Forcing-pump.—The forcing-pump consists of a pump-barrel dipping into water, and having at the bottom a valve opening upward. In communication with the pump-barrel is a side-tube with a valve at the point of junction opening from the barrel into the tube. A solid piston moves up and down the pump-barrel, and it is evident that when this piston is raised, water enters the barrel by the lower valve, and that when the piston descends, this water is forced into the side-tube. The greater the height of this tube, the greater will be the force required to push the piston down, for the resistance to be overcome is the pressure due to the column of water raised.

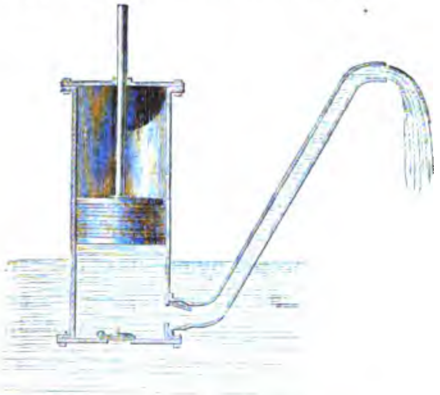

Fig. 161.—Forcing-pump.

The forcing-pump most frequently has a short suction-pipe leading from the reservoir, as represented in Fig. 163. In this case the water is raised from the reservoir into the barrel by atmospheric pressure during the up-stroke, and is forced from the barrel into the ascending pipe in the down-stroke.

158. Plunger.—When the height to which the water is to be forced is very considerable, the different parts of the pump must be very strongly made and fitted together, in order to resist the enormous pressure produced by the column of water, and to prevent leakage. In this case the ordinary piston stuffed with tow or leather washers cannot be used, but is replaced by a solid cylinder of metal called a *plunger*. Fig. 164 represents a section of a pump thus constructed. The plunger is of smaller section than the barrel, and passes through a stuffing-box in which it fits air-tight. The volume of water which enters the barrel at each up-stroke, and is expelled at the down-stroke, is the same as the volume of a length of the plunger equal to the length of stroke; and the hydrostatic pressure to be overcome is proportional to the section of the plunger, not to that of the barrel. As the operation proceeds, air is set free from the water, and would eventually impede the working of the pump were it not permitted to escape.

For this purpose the plunger is pierced with a narrow passage, which is opened from time to time to blow out the air.

The drainage of deep mines is usually effected by a series of pumps. The water is first raised by one pump to a reservoir, into which dips the suction-tube of a second pump, which sends the water up to a second reservoir, and so on. The piston-rods of the different pumps are all joined to a single rod called the *spear*, which receives its motion from a steam-engine.

159. Fire-engine.—The ordinary fire-engine is formed by the union of two forcing-pumps which play into a common reservoir, containing in its upper portion (called the air-chamber) air compressed by the working of the engine.

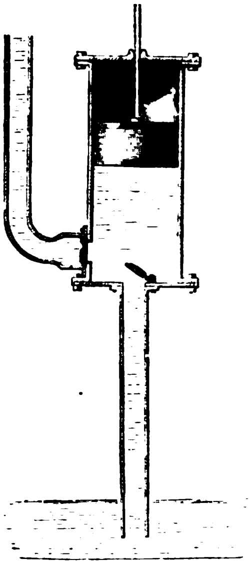
Fig. 163.

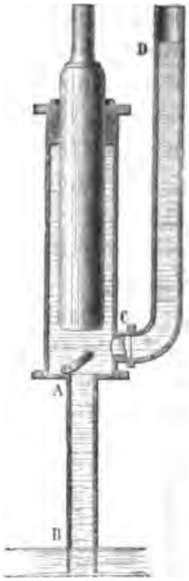

Fig. 161.

Suction and Force Pump.

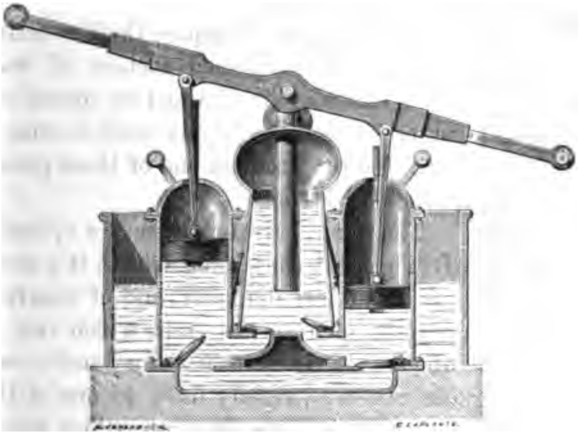
Fig. 162.—Fire engine.

A tube dips into the water in this reservoir, and to the upper end of this tube is screwed the leather hose through which the water is discharged. The piston-rods are jointed to a lever, the ends of which are raised and depressed alternately, so that one piston is ascending while the other is descending. Water is thus continually being forced into the common reservoir except at the instant of reversing stroke, and as the compressed air in the air-chamber performs the part of a reservoir of work (nearly analogous to the fly-wheel), the discharge of water from the nozzle of the hose is very steady.

The engine is sometimes supplied with water by means of an attached cistern (as in Fig. 162) into which water is poured; but it is more usually furnished with a suction-pipe which renders it self-feeding.

160. Double-acting Pumps.—These pumps, the invention of which is due to Delahire, are often employed for household purposes. They consist of a pump-barrel VV (Fig. 165), with four openings in it, A, A′, B, B′. The openings A and B′ are in communication with the suction-tube C; A′ and B are in communication with the ejection-tube C′. The four openings are fitted with four valves opening all in the same direction, that is, from right to left, whence it follows that A and B′ act as suction-valves, and A′ and B as ejection-valves, and, consequently, in whichever direction the piston may be moving, the suction and ejection of water are taking place at the same time.

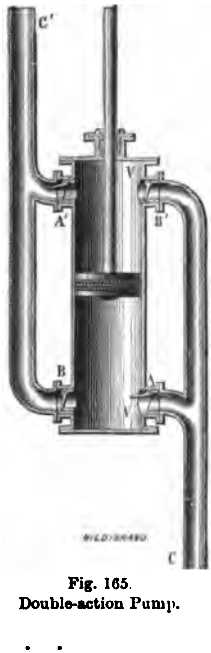

Fig. 165.
Double-action Pump.

161. Rotatory Pump.—Double-acting pumps produce a continuous suction of water. The same end may be attained by means of rotatory pumps, which are largely used in some countries. The figure represents one of these pumps as constructed by Dietz.

The pump-barrel consists of a cylindrical drum B (Fig. 166), containing within it a second cylinder A, of less diameter, and of nearly the same length, open at the ends, which can be turned about its axis by means of a handle; around this axis is a cam or guide *mn m′n′*, rigidly fixed to one of the ends of the drum B. In the box A are four slits, into which fit four blades *p*; these latter are constantly guided by the movement of one of their ends upon the cam, while the other ends move along the interior surface of the drum B, and the broad iron plate *b′a′ ab*, whose distance from the cam is everywhere equal to the length of one of the blades. By means of two holes in this plate, communication is established between the pump-barrel and the suction-tube C, and between the pump-barrel and the ejection-tube C′. From this arrangement it follows, that on causing the box A to rotate in the direction shown by the arrow, a partial vacuum will be produced behind the tongues, and the water will be drawn in at this side, and ejected at the opposite side.

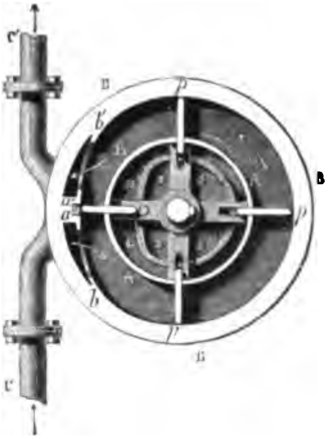

Fig. 166.—Rotatory Pump.

162. Hydraulic Press.—The hydraulic press (Fig. 167) consists of a suction and force pump *aa* worked by means of a lever turning about an axis O. The water drawn from the reservoir BB′ is forced along the tube CC into the cistern V. In the top of the cistern is an opening through which moves a heavy metal plunger AA. This carries on its upper end a large plate BB, upon which are placed the objects to be pressed. Suppose the cistern V to be at first empty, and the piston A to be carried by its own weight to the bottom of the cistern; under these circumstances, suppose the pump to begin to work. The cistern first begins to fill with water; then the pressure exerted by the piston of the pump is transmitted, according to the principles laid down in § 63, to the bottom of the piston A; this piston

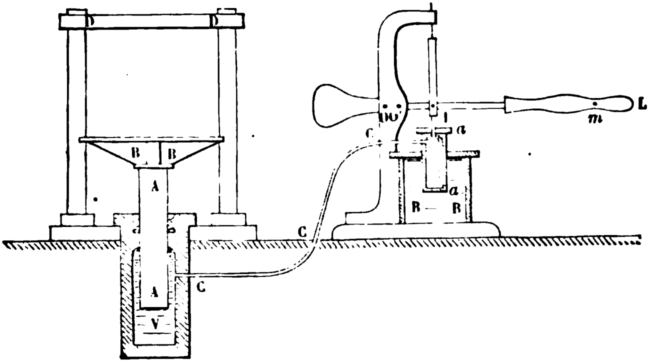

Fig. 167.—Bramah Press.

accordingly rises, and the objects to be pressed, being intercepted between the plate and the top of a fixed frame, are subjected to the transmitted pressure. The amount of this pressure depends both on the ratio of the sections of the pistons and on the length of the lever used to work the force-pump. Suppose, for instance, that the distance of the point *m*, where the hand is applied, from the point O, is equal to twelve times the distance IO, and suppose the force exerted to be equal to fifty pounds. By the principle of the lever this is equivalent to a force of 50 × 12 at the point I; and if the section of the piston A be at the same time 100 times that of the piston of the pump, the pressure transmitted to A will be $50 \times 12 \times 100 = 60{,}000$ pounds. These are the ordinary conditions of the press usually employed in workshops. By drawing out the pin which serves as an axis at O, and introducing it at O′, we can increase the mechanical advantage of the lever.

Two parts essential to the working of the hydraulic press are not represented in the figure. These are a safety-valve, which opens when the pressure attains the limit which is not to be exceeded; and, secondly, a tap in the tube C, which is opened when we wish to put an end to the action of the press. The water then runs off, and the piston A descends again to the bottom of the cistern.

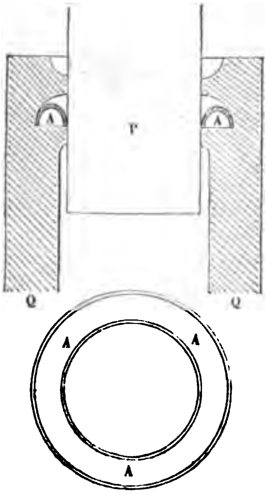

Fig. 168.—Cup—Leather.

The hydraulic press was clearly described by Pascal, and at a still earlier date by Stevinus, but for a long time remained practically useless; because as soon as the pressure began to be at all strong, the water escaped at the surface of the piston A. Bramah invented the *cupped leather collar*, which prevents the liquid from escaping, and thus enables us to utilize all the power of the machine. It consists of a leather ring AA (Fig. 168), bent so as to have a semicircular section. This is fitted into a hollow in the interior of the sides of the cistern, so that water passing between the piston and cylinder will fill the concavity of the cupped leather collar, and by pressing on it will produce a packing that fits more tightly as the pressure on the piston increases.

The hydraulic press is very extensively employed in the arts. It is of great power, and may be constructed to give pressures of two or three hundred tons. It is the instrument generally employed in cases where very great force is required, as in testing anchors or raising very heavy weights. It was used for raising the sections of the Britannia tubular bridge, and for launching the *Great Eastern*.

CHAPTER XVIII.

EFFLUX OF LIQUIDS.—TORRICELLI'S THEOREM.

163. If an opening is made in the side of a vessel containing water, the liquid escapes with a velocity which is greater as the surface of the liquid in the vessel is higher above the orifice, or to employ the usual phrase, as the *head* of liquid is greater. This point in the dynamics of liquids was made the subject of experiments by Torricelli, and the result arrived at by him was that the velocity of efflux is equal to that which would be acquired by a body falling freely from the upper surface of the liquid to the centre of the orifice. If h be this height, the velocity of efflux is given by the formula

$$V = \sqrt{2gh}.$$

This is called Torricelli's theorem; it supposes the sides of the vessel to be thin, and the diameter of the orifice to be very small compared with that of the vessel. It is further assumed that the orifice and the upper surface are under the same conditions as regards atmospheric pressure.

Torricelli's theorem has been regarded as an immediate consequence of the theory of gravitation; according to which, whatever be the path of a heavy body, its velocity depends only on the height of the point of starting above the point finally reached. If this height be h, the velocity is always $\sqrt{2gh}$.

But it is not evident that the molecules of a liquid which is escaping are subjected to no force but that of gravity. Besides, the first portions which escape from the vessel do not come from the free surface, and their velocity is due solely to the pressure exerted by the liquid column. It will thus be seen that the velocity of efflux, owing to the complex nature of the phenomenon, can be rigorously established only by the experimental method. It is very easy to perform a simple experiment upon this point. In fact, the molecules issuing from the orifice are ejected with a certain velocity, and should therefore, by the theory of projectiles, describe parabolic paths. The jet issuing from the vessel should accordingly be parabolic, and by measuring its range, we can calculate the velocity of efflux.

The experiment may easily be made by means of the apparatus represented in Fig. 169. It consists of a cylinder in which are a number of equidistant orifices in the same vertical line. A tap placed above the cylinder supplies the vessel with water, and with the help of an overflow-pipe, maintains the liquid at a constant level, which is as much above the highest orifice as each orifice is above that next below it.

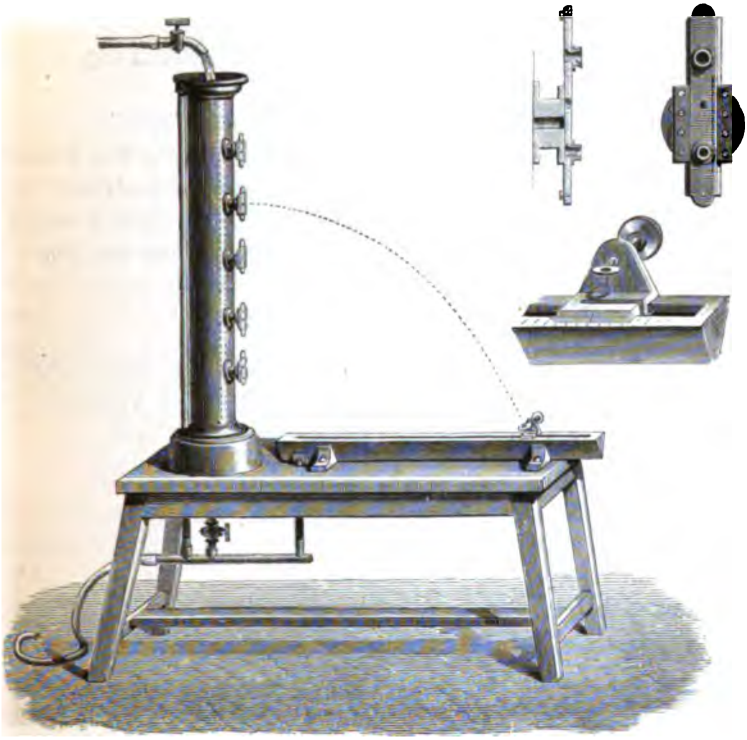

Fig. 169.—Apparatus for verifying Torricelli's Theorem.

The liquid which escapes is received in a trough, the edge of which is graduated. A travelling piece with an index line engraved on it slides along the trough; it carries, as shown in one of the separate figures, a disc pierced with a circular hole, and capable of being turned in any direction about a horizontal axis passing through its centre. In this way the disc can always be placed in such a position that its plane shall be at right angles to the liquid jet, and that the jet shall pass freely and exactly through its centre. The index line then indicates the range of the parabolic jet with considerable precision. This range is reckoned from the vertical plane containing the orifices, and is measured on the horizontal plane passing through the centre of the disc. The distance of this latter plane below the lowest orifice is equal to that between any two consecutive orifices.

The following is the way in which the result of an experiment is estimated. Let b be the height of the orifice above the horizontal plane through the centre of the ring, and let a be the range of the jet. If the liquid molecules were simply falling from a height, b, they would traverse this space in a time given by the formula

$$b = \frac{gt^2}{2}.$$

On the other hand, if they were simply obeying the force of ejection at the orifice, they would, by virtue of their initial velocity x, traverse the distance a in the same time t, whence we have

$$a = xt.$$

Eliminating t between these two equations, we have

$$b = \frac{ga^2}{2x^2};$$

whence

$$x = a\sqrt{\frac{g}{2b}}.$$

On comparing this velocity with that given by Torricelli's theorem, there is generally found to be a small difference between them, as is shown in the subjoined table:—

Head.	Jet.		Velocity.		Ratio of actual to theoretical velocity.
	a.	*b.*	Actual.	Theoretical.	
metres.	metres.	metres.			
2·29	6·28	7·53	6·65	6·70	·993
3·93	4·66	8·45	8·67	8·70	·997
7·17	1·41	6·25	11·67	11·88	·983

164. Intersection of Jets.—If Torricelli's theorem is correct, the value of x just found ought to be equal to $\sqrt{2gh}$; thus, we have the equation

$$a\sqrt{\frac{g}{2b}} = \sqrt{2gh}, \text{ whence } a^2 = 4bh. \qquad (3)$$

From this a curious result may be deduced: it will be seen that if h and b vary in such a manner that their produce remains constant, the value of a will remain unchanged. This law may easily be verified experimentally. It is only necessary to open at the same time the top and bottom orifices, or the second and fourth; the result will be two jets which will intersect one another at the centre of the ring.

We may remark, however, that the verification of this law does not prove Torricelli's theorem, for the result would be the same, if in (3) the constant 4 were replaced by any other constant; it proves, however, that the velocity is proportional to the square root of the head of water, and not to the head itself, as was formerly believed.

165. Quantity of Liquid Discharged.—It would appear at first sight that Torricelli's theorem could be tested by a very simple and decisive experiment. Suppose the level of the liquid in a vessel to be maintained constant, and the volume of the liquid which escapes through an orifice during a certain time to be measured. This can be compared with the volume calculated *a priori*, by assuming that the quantity discharged in a unit of time is equal to a cylinder whose base is the section of the orifice and height the velocity; so that the quantity which escapes in time t will be given by the formula

$$Q = ts\sqrt{2gh};$$

s being the section of the orifice. Now, in all the experiments which have been performed, when the orifice is a simple perforation in a thin plate, the actual discharge has been found to be less than this, being generally about ·6 of it.

This discrepancy arises from neglecting the fact, that the particles of liquid at the margin of the jet have a converging motion, in consequence of which the jet contracts rapidly for a small distance after issuing from the orifice. Beyond this small distance the contraction is very gradual, depending only on the continued action of gravity. The portion which forms the termination of the rapid contraction, is called the contracted vein, or *vena contracta*, and its section appears to be about ·6 of that of the orifice. If, then, in the above formula we make s denote the section of the contracted vein, we shall obtain results agreeing with experiment.

166. Efflux-tubes.—This explanation is confirmed by the effect of efflux-tubes. These are pipes not exceeding a few inches in length, which are fitted to the holes in the side of the vessel. If, for instance, a tube of cylindrical form is employed, the contraction will be prevented by the adherence of the liquid to the sides of the tube; the section of the jet is then the same as that of the tube. In this case, if the velocity be measured, it will be found to be less than when the orifice is a hole in a thin plate; but, on multiplying the velocity thus obtained by the section of the tube, we shall obtain a result agreeing with the actual discharge.

The apparatus above described enables us to estimate the differences in velocity caused by efflux-tubes. For this purpose a sliding plate is used, with one orifice and two efflux-tubes, one cylindrical, the other conical; by sliding the plate the liquid can be made to flow out of either of these tubes.

167. Efflux through Pipes.—When the liquid, instead of escaping through a short spout, flows through a long tube, the velocity is considerably reduced by the friction of the molecules against each other, and against the sides of the tube. This velocity is also not the same at all points in the same section; it is least where the liquid is in contact with the tube, and is greatest in the centre of the liquid column. When a uniform delivery has been established, the quantity of water which passes in each unit of time is constant, and is the same for all sections; and the average velocity across any section will be obtained by dividing the volume discharged in a second by the sectional area. Many experiments have been performed with the view of determining this velocity in a certain number of particular cases, and certain empirical rules have thus been arrived at. It is difficult to treat the subject rationally, or to establish results that shall be perfectly general.

168. Fountains.—If the lower end of a water-pipe be connected with a mouth-piece pointing vertically upwards, the liquid on issuing from the opening will rise in a vertical jet to a height depending on the velocity of efflux. If there were no resistance, this height would be that *due to the velocity*, namely $\frac{v^2}{2g}$, according to the formulæ of § 38; but this is not actually the case. The friction of the liquid against itself, and the weight of the particles which fall back upon the rest, counteract the tendency to ascend. The effect of this last cause can be somewhat diminished by slightly inclining the jet.

169. Efflux of a Liquid in contact with Confined Air.—When the surface of the liquid is in contact with a volume of air whose pressure varies, the velocity of efflux is affected by this variation. Let A B C D be a closed vessel filled with a liquid as far as M N, and let the space above contain air at the atmospheric pressure. If a small orifice be opened below, the liquid will begin to flow out; but the air above will become gradually rarefied, until at length its pressure, together with that due to the depth of liquid, will be equal to that of the atmosphere; when this happens the liquid will cease to flow, unless the circumstances are such that air-bubbles can enter. Let us see what will be the height of the liquid column in the vessel when the flow ceases. Let A C$=l$, A M$=h$, and let p be the height of a column of the liquid which will balance the pressure of the atmosphere. The air, which at the beginning of the experiment had a volume $l-h$, and a pressure p, will now have a volume $l-x$, and consequently a pressure $p\,\frac{l-h}{l-x}$; we have then

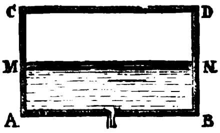

Fig. 170.

$$p\,\frac{l-h}{l-x}+x=p,$$

whence

$$x=\frac{p+l\pm\sqrt{(p+l)^2-4\,p\,h}}{2}.$$

The — sign alone can be taken with the radical, since x must be less than l. This case of equilibrium occurs in several well-known experiments.

Pipette.—This is a glass tube (Fig. 171) open at both ends, and terminating below in a small tapering spout. If a certain quantity of water be introduced into the tube, either by aspiration or by direct immersion in water, and if the upper end be closed with the finger, the efflux of the liquid will cease after a few seconds. On admitting the air above, the efflux will begin again, and can again be stopped at pleasure.

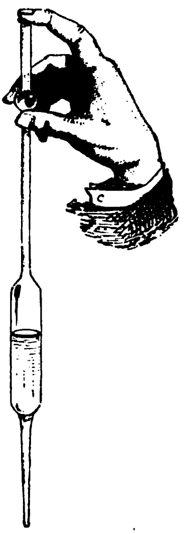

Fig. 171.—Pipette.

The Magic Funnel.—This funnel is double, as is shown in Fig. 172. Near the handle is a small opening by which the space between the two funnels communicates with the external air. Another opening connects this same space with the tube of the inner funnel. If the interval between the two funnels be filled with any liquid, this liquid will run out or will cease to flow according as the upper hole is open or closed. The opening and closing of the hole can be easily effected wit[illegible]humb of the hand holding the funnel without the knowledge of the spectator. This device has been known from very early times.

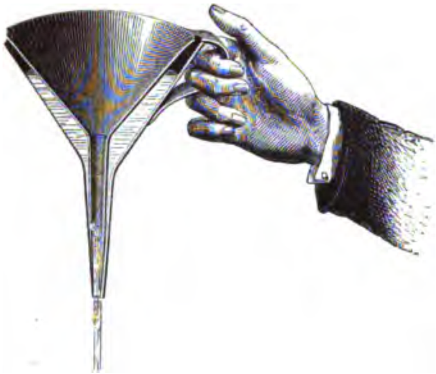

Fig. 172.—Magic Funnel.

The instrument may be used in a still more curious manner. For this purpose the space inside is secretly filled with highly-coloured wine, which is prevented from escaping by closing the opening above.

Water is then poured into the central funnel, and escapes either by itself or mixed with wine, according as the thumb closes or opens the orifice for the admission of air. In the second case, the water being coloured with the wine, it will appear that wine alone is issuing from the funnel; thus the operator will appear to have the

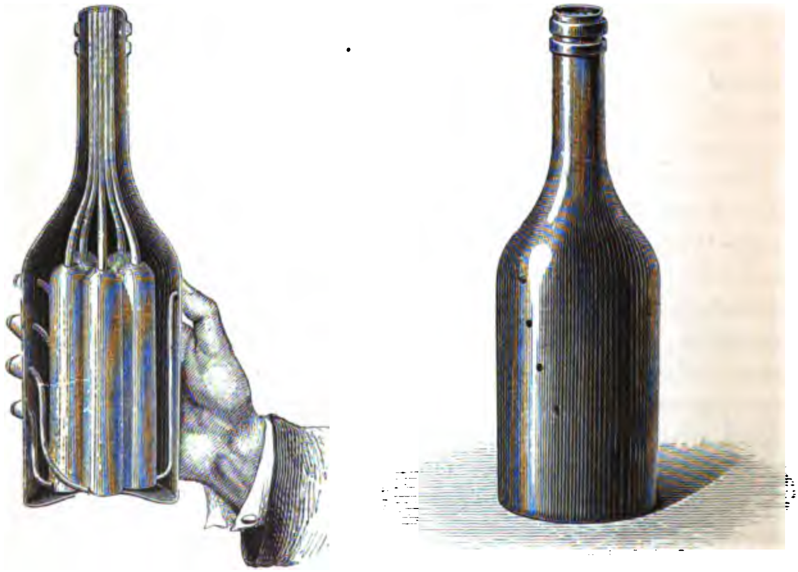

Fig. 173.—Inexhaustible Bottle.

power of making either water or wine flow from the vessel at his pleasure.

The Inexhaustible Bottle.—The inexhaustible bottle is a toy of the same kind. It is an opaque bottle of sheet-iron or gutta-percha, containing within it five small vials. These communicate with the exterior by five small holes, which can be closed by the five fingers of the hand. Each vial has also a small neck which passes up the large neck of the bottle. The five vials are filled with five different liquids, any one of which can be poured out at pleasure by uncovering the corresponding hole.

170. Intermittent Fountain.—The intermittent fountain is an apparatus analogous to the preceding, except that the interruptions in the efflux are produced automatically by the action of the instrument, without the intervention of the operator. It consists of a globe V, which can be hermetically closed by means of a stopper, and can be put in communication with efflux-tubes *a*, by which the water contained in it can escape. A vertical tube *t* rises nearly to the top of the globe, and terminates below at a short distance from the bottom of the basin B. This basin is pierced with a small opening *o*, by which the water contained in it escapes into the lower basin C. Suppose the globe to be filled with water, and communication with the efflux-tubes to be established, then the liquid will flow into the basin B, and thence into C. But the size of the opening *o* is such that it suffers less water to escape than passes out by the efflux-tubes; the liquid will therefore accumulate in B, and will finally cover the bottom of the tube *t*. Communication between the external air and the upper part of the globe will then be cut off, and the liquid will after a few moments cease to flow.

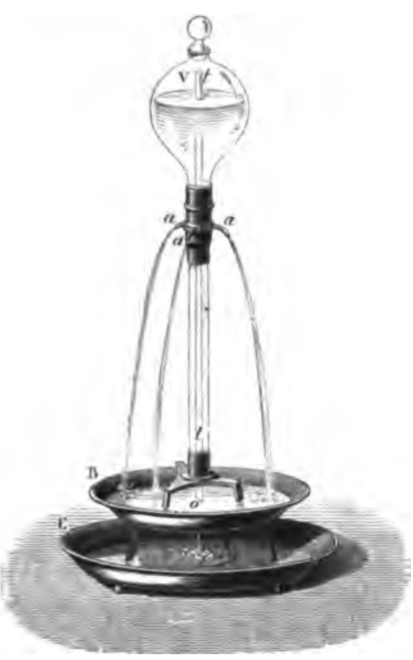

Fig. 174.—Intermittent Fountain.

But as the basin B continues to be emptied by the opening *o*, the liquid in the basin will sink below the end of the tube; air will then enter the globe, the liquid will again begin to issue from the efflux-tubes, and so on.

171. Siphon.—The object of the siphon is the transference of liquid from one vessel to another. It essentially consists of a bent tube (Fig. 175) with branches of unequal length. The short branch dips into the liquid to be transferred, the other opens directly into the air. If we suppose the siphon full of liquid, it is easy to see that the liquid will flow from the short to the long branch.

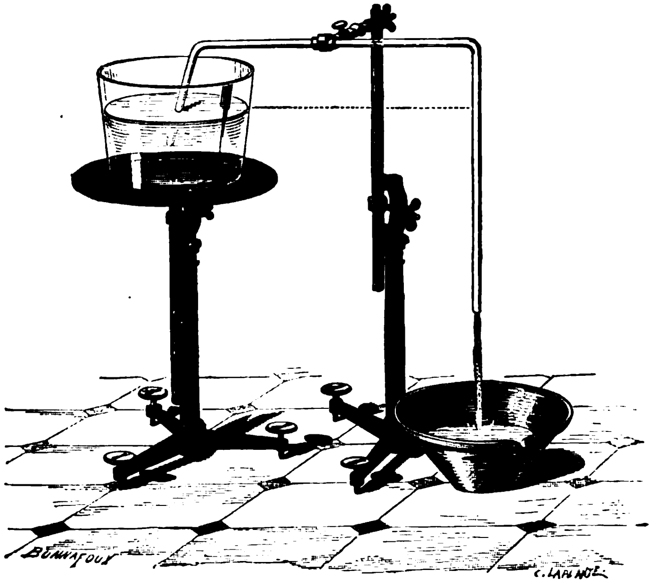

Fig. 175.—Siphon.

For if we consider (Fig. 176) a layer of liquid M, at the highest point of the siphon, this layer will be subjected to a pressure from left to right equal to the atmospheric pressure diminished by the height DC, or MI.

Let this latter height be h, and let H be the external pressure expressed as the height of an equivalent column of the liquid, then the pressure from left to right will be $H - h$. The pressure from right to left will be $H - EF = H - h'$. Now as h' is greater than h, the first pressure will overcome the second, and the layer M will consequently move from left to right. But if the height of the smaller branch be less than H, the liquid cannot separate, for the pressure of the atmosphere would immediately fill up the vacuum which would be formed. Thus the liquid will continue to flow uninterruptedly until the liquid in the vessel AB has fallen below the level of the end of the shorter branch of the siphon.

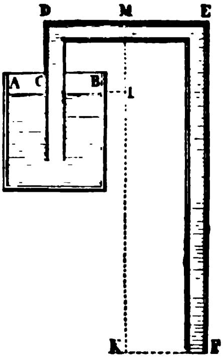

Fig. 176.

The force causing the liquid to flow is the pressure represented by a column of liquid $h'-h$; the velocity of efflux is thus equal to $\sqrt{2g(h'-h)}$, friction being neglected.

In the above reasoning we have supposed the external pressure H to be the same at C and at F. This is evidently the case when the pressure is that of the atmosphere. If we suppose the surrounding medium to be of a density such that the variation of pressure for the levels AB and KF cannot be neglected, the expression for the velocity must be modified. Let d be the density of the liquid, d' that of the surrounding medium, the excess of pressure from left to right is represented by the weight of a liquid column of density d, and of height $h'-h$, diminished by the weight of a column of the same height of density d'; that is, it is given by the expression $(h'-h)\,d-(h'-h)\,d'=(h'-h)\,(d-d')$. Now the height m of the liquid which would produce the same pressure is given by the equation $md=(h'-h)\,(d-d')$. Thus the velocity of efflux will be

$$V=\sqrt{2\,gm}=\sqrt{\frac{2\,g\,(h-h')\,(d-d')}{d}}.$$

In the case (which could scarcely occur in practice), where d' is greater than d, the pressure from left to right will be negative; that is, the excess of pressure will be from right to left. The liquid will then flow from right to left, and with a velocity given by the above formula if $d-d'$ is replaced by $d'-d$.

172. Starting the Siphon.—In order that the siphon should work, it must first be charged with liquid. This is effected in various ways. When the liquid can be taken into the mouth without danger, the charging can be effected (Fig. 177) by sucking at a side-tube attached to the long branch.

This method is inapplicable to liquids which would have an injurious effect upon the mouth. The following method is often employed in the transfer of sulphuric acid from one vessel to another.

The long branch of the siphon (Fig. 178) is first filled with sulphuric acid. This is effected by means of two funnels (which can be plugged at pleasure) at the bend of the tube. One of these admits the liquid, and the other suffers the air to escape. The two funnels above are then closed, and the tap at the lower end of the tube is opened so as to allow the liquid to escape. The air in the short branch follows the acid, and becomes rarefied; the acid behind it rises, and if it passes the bend, the siphon will be charged; for each portion of the liquid which issues from the tube will draw a corresponding portion from the short to the long branch.

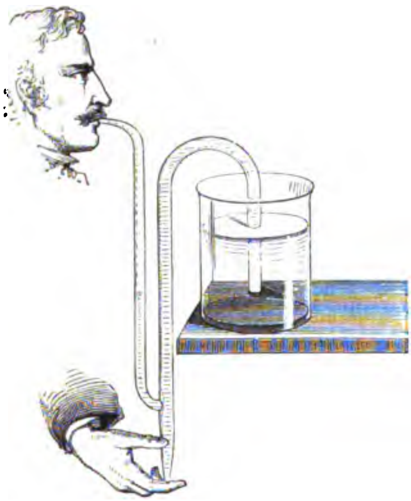
Fig. 177.—Starting the Siphon.

To insure the working of the sulphuric acid siphon, it is not sufficient to have the vertical height of the long branch greater than that of the short branch; it is farther necessary that it should exceed a certain limit, which depends upon the dimensions of the siphon in each particular case. In order to calculate this limit, we must remark that when the liquid begins to flow, its height diminishes in the long and increases in the short branch; if these two heights should become equal, there would be equilibrium. We see, then, that in order that the siphon may work, it is necessary that when the liquid rises to the bend of the tube, there should be in the long branch a column of liquid whose vertical height is at least equal to that of the short branch, which we shall denote by h, and the actual length of the short branch from the surface of the liquid in which it dips to the summit of the bend by h'. Then if a be the

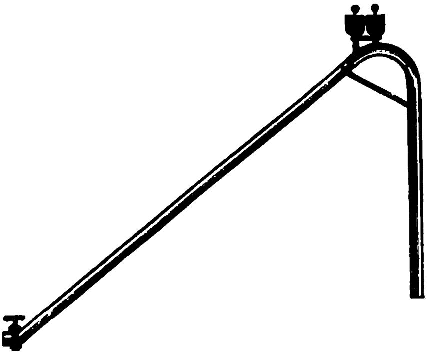
Fig. 178.—Siphon for Sulphuric Acid.

inclination of the long branch to the vertical, and L the length of the long branch, which we suppose barely sufficient, the length of the column of liquid remaining in the long branch will be $h \sec \alpha$. The air which at atmospheric pressure H occupied the length h', now under the pressure $H - h$ occupies a length $L - h \sec \alpha$; hence, by Boyle's law, we have

$$Hh' = (H - h)(L - h \sec \alpha), \text{ whence } L = h \sec \alpha + \frac{Hh'}{H - h}.$$

In this formula H denotes the height of a column of sulphuric acid whose pressure equals that of the atmosphere.

173. Vase of Tantalus.—The siphon may be employed to produce the intermittent flow of a liquid. Suppose, for instance, that we have a vase in which is a bent tube rising to a height *n*, and with the short branch terminating near the bottom of the vase, while the long branch passes through the bottom. If liquid be poured into the vase, the level will gradually rise in the short branch of the bent tube, and will finally reach and pass the point *n*, when the siphon will begin to discharge the liquid. If, then, we suppose the liquid to escape by the siphon faster than it is poured into the vessel, the level of the liquid will gradually fall below the termination of the shorter branch. The siphon will then empty itself, and will not recommence its action till the liquid has again risen to the level of the bend.

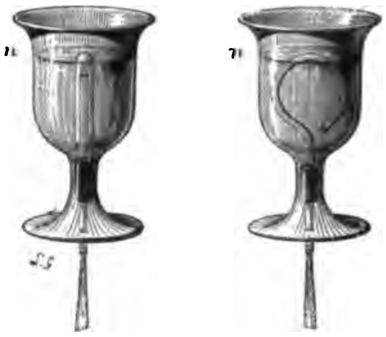

Fig. 179.—Vase of Tantalus.

If the cup is made of metal with the siphon concealed in the thickness of the sides, when a person in lifting it to his lips inclines it to the side in which the siphon is, the siphon will become charged, and will empty the vessel. Hence the name *vase of Tantalus* given to this cup in old treatises on physics. Instead of a bent tube we may employ, as in the first figure, a straight tube covered by a bell-glass left open below; in this case the space between the tube and the bell takes the place of the shorter leg of the siphon.

It is to an action of this kind that natural intermittent springs are generally attributed. Suppose a reservoir (Fig. 180) to communicate with an outlet by a bent tube forming a siphon, and suppose it to be fed by a stream of water at a slower rate than the siphon is able to discharge it. When the water has reached the bend, the siphon will become charged, and the reservoir will be emptied; it will then be filled again as far as the bend, and so on.

174. Mariotte's Bottle.—This is an apparatus often employed to obtain a continuous flow of water. It consists of a flask whose cork is pierced by a straight tube open at both ends, and with the lower extremity descending to *a*. An efflux-tube is placed at *b* near the bottom of the flask. Suppose that the flask is full of water, and that the tube is also full to the upper end. If the tube *b* be now opened, the liquid molecules at the orifice will be pressed inwards with a force equal to the atmospheric pressure, but will be pressed outwards with a force exceeding this pressure by the height of the column of water as far as the upper end of the tube. The liquid will therefore flow out; but no vacuum will be produced in the upper part of the flask, for the pressure of the atmosphere will compel the liquid in the tube to replace that which escapes. The level of the liquid in the tube will thus rapidly fall, and the velocity will gradually decrease, as will be seen by the diminished range of the jet. When the liquid reaches the point *a*, the efflux will continue; but then air will enter

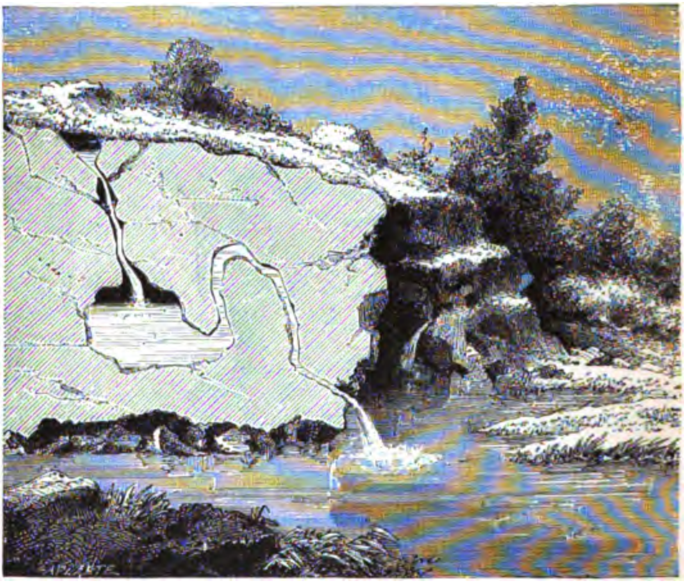

Fig. 180.—Intermittent Spring.

the vase in successive bubbles, and will rise to the upper part of the vase, in such quantity that its pressure, together with that of the height of water above the horizontal plane through *a*, will maintain a pressure on this plane equal to that of the atmosphere. From this time the liquid will flow with a constant velocity due to the height of *a* above *b*. Strictly speaking, inasmuch as the air enters, not in a continuous manner, but in successive bubbles, that is, in jerks, the velocity of discharge oscillates about a constant mean value, but the oscillations are in general almost imperceptible. Instead of the vertical tube, we may use a vase with two openings at different levels; the liquid escapes by the lower orifice *b*, while air enters by the upper orifice *a*. Mariotte's vase is sometimes used in the laboratory to produce the uniform flow of a gas by employing the water which escapes to expel the gas. We may also draw in gas through the tube of Mariotte's bottle; in this case, the flow of the *water* is uniform, but the flow of the *gas* is continually accelerated, since the space occupied by it in the bottle increases uniformly, but the density of the gas in this space continually increases.

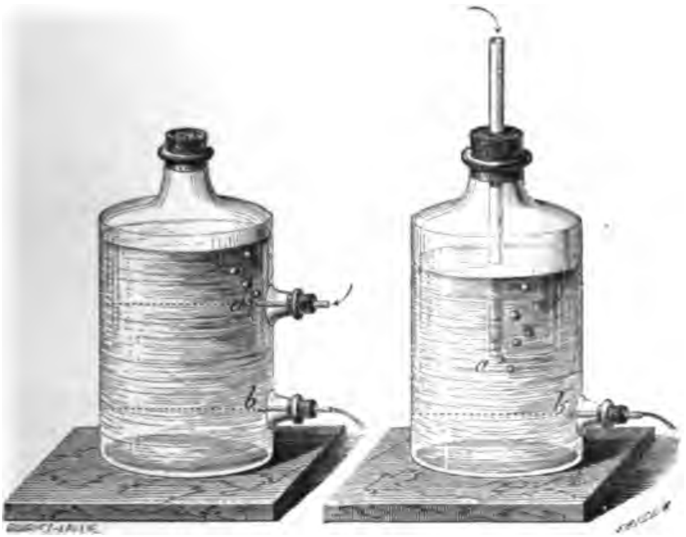

Fig. 181.—Mariotte's Bottle.

HEAT.

CHAPTER XIX.

THERMOMETRY.

175. **Heat—Cold.**—The words *heat* and *cold* express sensations so well known as to need no explanation; but these sensations are modified by subjective causes, and do not furnish an invariable criterion of objective reality. In fact, we may often see one person suffer from heat while another complains of cold. Even for the same person the sensations of heat and cold are comparative. A temperature of 50° Fahr. suddenly occurring amid the heat of summer produces a very decided sensation of cold, whereas the same temperature in winter has exactly the opposite effect. We may mention an old experiment upon this subject, which is at once simple and instructive. If we plunge one hand into water at 32° Fahr., and the other into water at about 100°; and if after having left them some time in this position we immerse them simultaneously in water at 70°, they will experience very different sensations. The hand which was formerly in the cold water now experiences a sensation of heat; that which was in the hot water experiences a sensation of cold, though both are in the same medium. This plainly shows that the sensations of heat and cold are modified by the condition of the observer, and consequently cannot serve as a sure guide in the study of calorific phenomena. Recourse must therefore be had to some more constant standard of reference, and such a standard is furnished by the thermometer.

176. **Temperature.**—If several bodies heated to different degrees are placed in presence of each other, an interchange of heat takes place between them, by which they undergo modifications of opposite kinds; those that are hottest grow cooler, and those that are coldest grow warmer; and after a longer or shorter time these inverse phenomena cease to take place, and the bodies come to a state of mutual equilibrium. They are then said to be at the same *temperature*. If a source of heat is now brought to act upon them, their temperature is said to *rise;* if they are left to themselves in a colder medium, they all grow cold, and their temperature is said to *fall*. *Two bodies are said to have the same temperature if when they are placed in contact no heat passes from the one to the other*. If when two bodies are placed in contact heat passes from one to the other, that which gives heat to the other is said to have the higher temperature. Heat always tends to pass from bodies of higher to those of lower temperature.

177. Expansion.—At the same time that bodies undergo these changes in temperature, which may be verified by the different impressions which they make upon our organs, they are subjected to other modifications which admit of direct measurement, and which serve as a means of estimating the changes of temperature themselves. These modifications are of different kinds, and we shall have occasion to speak of them all in the course of this work; but that which is especially used as the basis of thermometric measurement is change of volume. In general, when a body is heated, it increases in volume; and, on the other hand, when it is cooled its volume diminishes. The expansion of bodies under the action of heat may be illustrated by the following experiments.

1. *Solid Bodies.*—We take a ring through which a metal sphere just passes. This latter is heated by holding it over a spirit-lamp, and it is found that after this operation it will no longer pass through the ring. Its volume has increased. If it is now cooled by immersion in water, it resumes its former volume, and will again pass

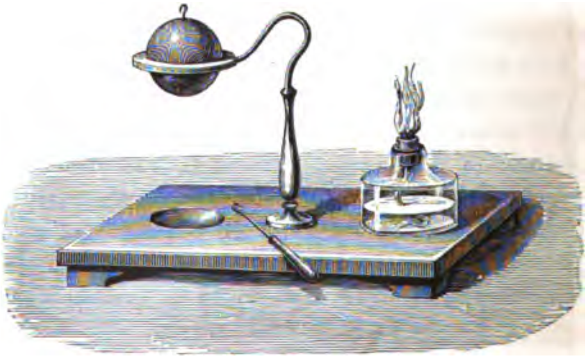
Fig. 182.—Gravesande's Ring.

through the ring. If, while the sphere was hot, we had heated the ring to about the same degree, the ball would still have been able to pass, their relative dimensions being unaltered. This little apparatus is called *Gravesande's Ring.*

2. *Liquids.*—A liquid, as water for instance, is introduced into the apparatus shown in Fig. 183, so as to fill at once the globe and a portion of the tube as far as *a*. The instrument is then immersed in a vessel containing hot water, and at first the extremity of the liquid column descends for an instant to *b*; but when the experiment has continued for some time, the liquid rises to a point *a'* at a considerable height above. This twofold phenomenon is easily explained. The globe, which receives the first impression of heat, increases in volume before any sensible change can take place in the temperature of the liquid. The liquid consequently is unable to fill the entire

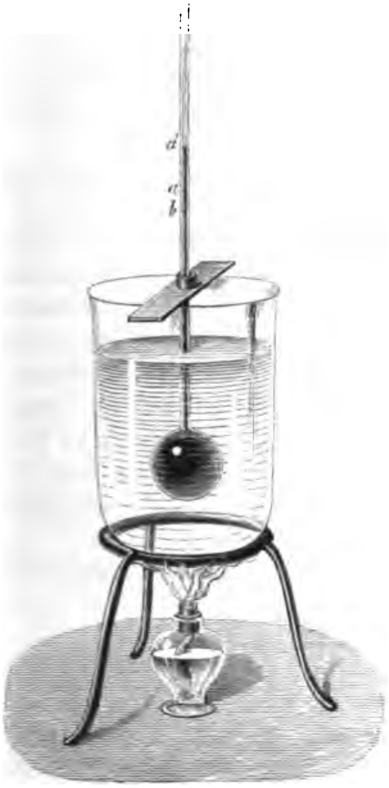

Fig. 183.—Expansion of Liquids.

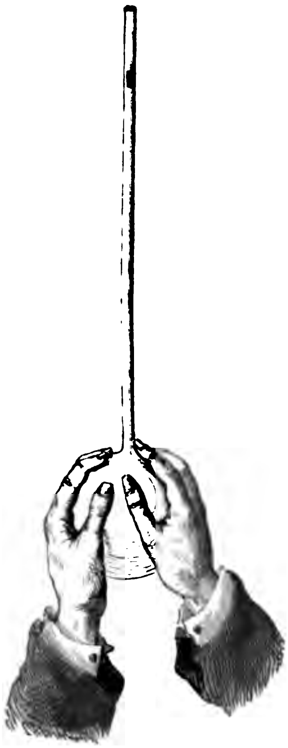

Fig. 184.—Expansion of Gases.

capacity of the globe and tube up to the original mark, and thus the extremity of the liquid column is seen to fall. But the liquid receiving in its turn the impression of heat, expands also, and as it passes the original mark, we may conclude that it not only expands, but expands more than the vessel which contains it.

3. *Gases.*—The globe in Fig. 184 contains air, which is separated from the external air by a small liquid index. We have only to warm the globe with the hands and the index will be seen to be pushed quickly upwards, thus showing that gases are exceedingly expansible.

178. General Idea of the Thermometer.—Since the volume of a body is always changed by heat, it follows that when a body is subjected to variations of temperature, it undergoes at the same time corresponding variations of volume. If we suppose that the different volumes successively assumed by the body can easily be measured, we may indicate the temperature by stating the volume. And the body will not only indicate its own temperature by this means, it will also exhibit the temperature of the bodies by which it is surrounded, and which are in equilibrium with it as regards temperature; that is, which do not experience those inverse changes mentioned in § 176. Such is the most general idea of the thermometer, which may be defined as *a body which, under the action of heat, exhibits changes of volume which can be ascertained and measured.*

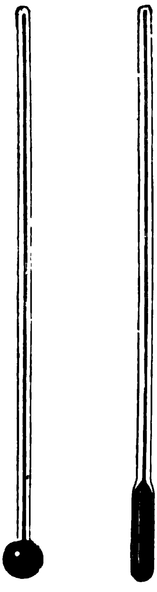

Fig. 185.—Mercurial Thermometers.

179. Choice of the Thermometric Substance.—Any substance whatever will serve as a thermometric substance, and in fact there are several kinds of thermometers, founded upon the expansion of different substances. In order, however, that thermometric indications may be comparable with each other, it is necessary to adopt a standard substance or combination of substances, and physicists have by common consent adopted as the standard of reference the apparent expansion of mercury contained in a glass vessel. The instrument which exhibits this expansion is called the *mercurial thermometer.* It consists essentially, as shown in Fig. 185, of a tub of very small diameter, terminating in a bulb or reservoir of a cylin drical, spherical, or any other form. The reservoir and a portion o the tube are filled with mercury. If the temperature varies, the level of the liquid will rise or fall in the tube, and the points at which it remains stationary can be easily identified by means of a scale attached to or engraved on the tube.

The choice of mercury as a thermometric substance is extremely suitable. It is a liquid which may easily be procured in a state of purity. It is a very good conductor of heat, and consequently soon comes into equilibrium of temperature with the bodies which it touches. Besides, its *calorific capacity* is very small, so that if it be brought into contact with a heated body, for instance, at whose expense it grows hot, this body experiences in consequence only a very slight change of temperature, which may generally be neglected.

180. Construction of the Mercurial Thermometer.—The construction of a mercurial thermometer is an operation of great delicacy, and comprises several different processes, which we shall successively indicate.

1. *Choice of the Tube.*—The first object is to procure a tube of as uniform bore as possible. In order to ascertain whether this condition is fulfilled, a small column of mercury is introduced into the tube, and its length in different parts of the tube is measured. If these lengths are exactly equal, the tube must be of uniform bore. This is not generally the case, and we have to content ourselves with an approximation to this result; but we must reject tubes in which the differences of length observed are too great. When a suitable tube has been obtained, a reservoir is either blown at one end or attached by melting, the former plan being usually preferable.

When a thermometer of great precision is required, the tube is first *calibrated;* that is, divided into parts of equal volume.

2. *Introduction of the Mercury.*—At the upper extremity of the tube a bulb has been blown, drawn out to a point, at which there is a small opening; this bulb is gently heated, and the point is then immersed in a vessel containing mercury (Fig. 186). The air in the tube growing cold, suffers a diminution at once of volume and of pressure, so that mercury is forced into the bulb by the atmospheric pressure. The end of the point is then closed to prevent the escape of mercurial vapours, and the reservoir and tube, still remaining empty, are heated in a gas or charcoal furnace (Fig. 187), so as to rarefy the contained air. The liquid in the bulb is then heated, and on setting the instrument upright, and allowing it to cool, a portion of the mercury enters the reservoir in consequence of the contraction

of the air. The liquid in the reservoir is then heated to ebullition, the air is expelled from the reservoir and tube by the vapour of mercury, and on placing the instrument in an upright position, the mercury during the process of cooling enters the reservoir and completely fills it. If a bubble of air still remains, as is often the case, it may be expelled by repeating the same operation several times. The quantity of mercury to be left is then regulated according to the temperatures which the instrument is intended to indicate; the bulb at the upper end is removed, and the tube, having been drawn out to a point, is hermetically sealed at the moment when the mercury reaches its extremity, so as to leave no air in the interior.

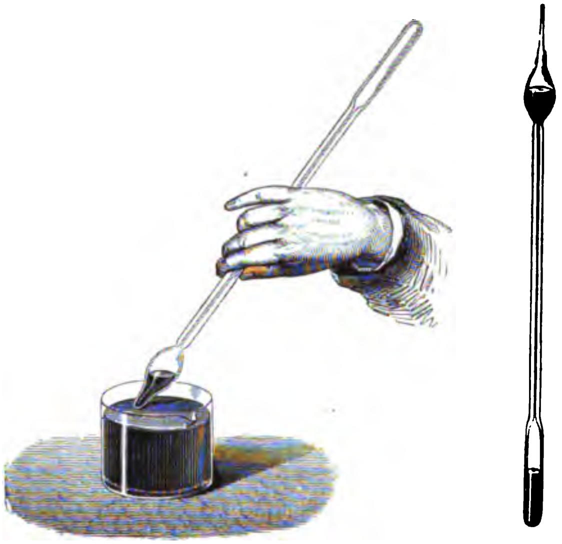

Fig. 186.—Introduction of the Mercury.

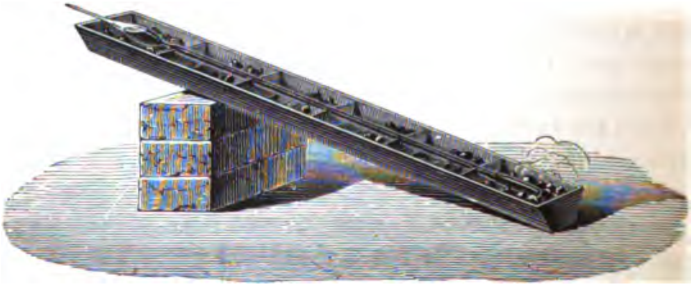

Fig. 187.—Furnace for heating Thermometers.

3. *Determination of the Fixed Points.*—The instrument, under these conditions, and with any scale of equal parts marked on the tube, would of course indicate variations of temperature, but these indications would be arbitrary, and two thermometers so constructed would in general give different indications.

In order to insure that the indications of different thermometers shall be comparable, it has been agreed to adopt two standard temperatures, which can easily be reproduced and maintained for a considerable time, and to denote them by fixed numbers. These two temperatures are the freezing-point and boiling-point of water; or to speak more strictly, the temperature of melting ice, and the temperature of the steam given off by water boiling under average atmospheric pressure. It has been observed that if the thermometer be surrounded with melting ice (or melting snow), the mercury, under whatever circumstances the experiment is performed, invariably stops at the same point, and remains stationary there as long as the melting continues. This then is a fixed temperature. On the Centigrade scale it is called zero, on Fahrenheit's scale 32°.

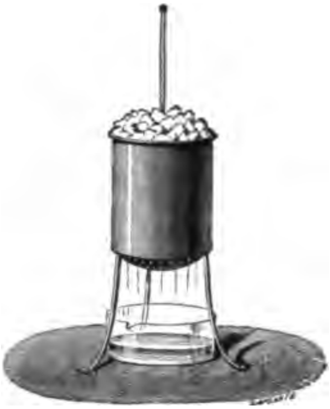

Fig. 188.—Determination of Freezing-point.

In order to mark this point on a thermometer, it is surrounded by melting ice, which is contained in a perforated vessel, so as to allow the water produced by melting to escape. When the level of the mercury ceases to vary, a mark is made on the tube with a fine diamond at the extremity of the mercurial column. This is frequently called for brevity the *freezing-point.*

It has also been observed that if water be made to boil in an open metallic vessel, under average atmospheric pressure (760 millimetres, or 29·922 inches), and if the thermometer be plunged into the steam, the mercury stands at the same point during the entire time of ebullition, provided that the external pressure does not change. This second fixed temperature is called 100° in the Centigrade scale (whence the name), and 212° on Fahrenheit's scale. In order to mark this second point on the thermometer, an apparatus is employed which was devised by Gay-Lussac, and perfected by Regnault. It consists of a copper boiler (Fig. 189) containing water which is raised to ebullition by means of a furnace. The steam circulates through a double casing, and escapes by a tube near the bottom. The thermometer is fixed in the interior casing, and when the mercury has become stationary, a mark is made at the point at which it stops, which denotes what is commonly called for brevity the *boiling-point.* It now only remains to divide the portion of the instrument between the freezing and boiling points into equal parts corresponding to single degrees, and to continue the division beyond the fixed points. Below the zero point are marked the numbers 1, 2, 3, &c. These

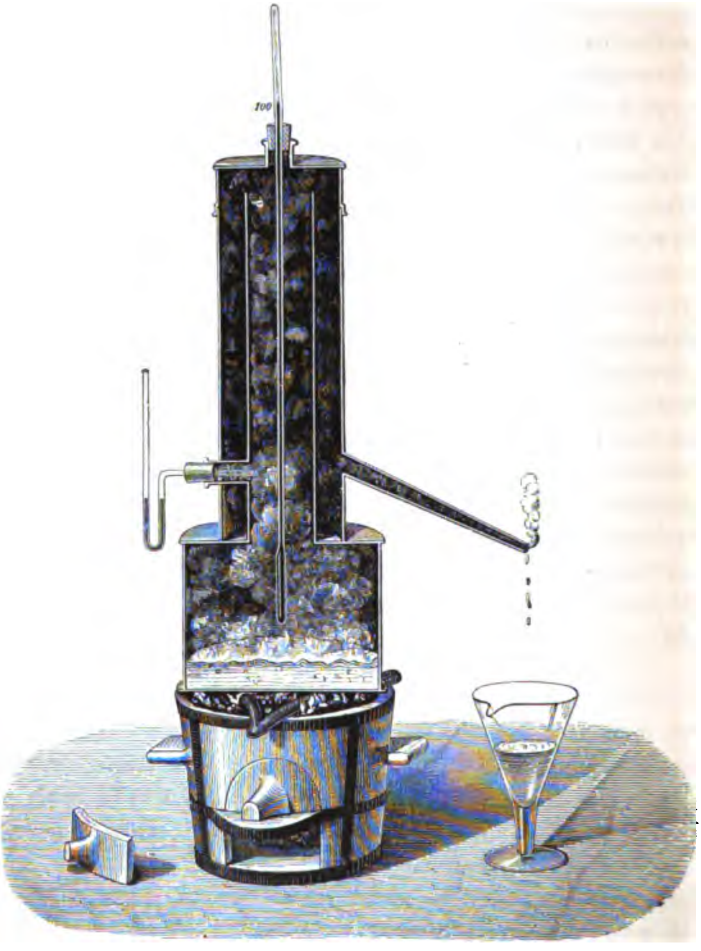

Fig. 189.—Determination of Boiling-point.

temperatures are generally expressed with the sign —. Thus the temperature of 17° below zero is written —17°.

A small manometric tube, open at both ends, serves to show, by the equality of level of the mercury in its two branches, that the ebullition is taking place at a pressure equal to that which prevails externally, and consequently that the steam is escaping with sufficient freedom. It frequently happens that the external pressure is not exactly 760 millimetres, in which case the boiling-point should be placed a little above or a little below the point at which the mercury remains stationary, according as the pressure is less or greater than this standard pressure. When the difference on either side is inconsiderable, the position of the boiling-point is calculated by the rule, that a difference of 20·6 millimetres, either above or below the normal pressure, causes a difference of 1° in the temperature of the steam produced. We shall return to this point in Chap. xxvi.

181. **Adjustment of the Quantity of Mercury.**—In order to avoid complicating the above explanation, we have omitted to consider an operation of great importance, which should precede those which we have just described. This is the determination of the volume which must be given to the reservoir, in order that the instrument may have the required range. When the reservoir is cylindrical, this is very easily effected in the following manner. Suppose we wish the thermometer to indicate temperatures comprised between —20° and 130° Cent., so that the range is to be 150°; the reservoir is left open at O (Fig. 190), and is filled through this opening, which is then hermetically sealed. The instrument is then immersed in two baths whose temperatures differ by 50°, for instance, and the mercury rises through a distance *mm'*. This length, if the quantity of mercury in the reservoir be exactly sufficient, should be the third part of the length of the stem. But the quantity of mercury in the reservoir is always taken too large at first, so that it has only to be reduced, and thus the space traversed by the liquid is at first too great. Suppose it to be equal to $\frac{3}{4}$ths of the length of the stem. The degrees will then be too long, in the ratio $\frac{3}{4} : \frac{1}{3} = \frac{9}{4}$. That is, the reservoir is $\frac{9}{4}$ of what it should be. We therefore measure off $\frac{4}{9}$ths of the length of the reservoir, beginning at the end next the stem; this distance is marked by a line, and the end O is then broken and the mercury suffered to escape. The glass

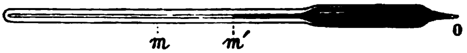

Fig. 190.

is then melted down to the marked line, and the reservoir is thus brought to the proper dimensions. It only remains to regulate the quantity of mercury admitted, by making it fill the tube at the highest temperature which the instrument is intended to indicate.

If the reservoir were spherical, which is a shape generally ill adapted for delicate thermometers, the foregoing process would be inapplicable, and it would be necessary to determine the proper size by trial.

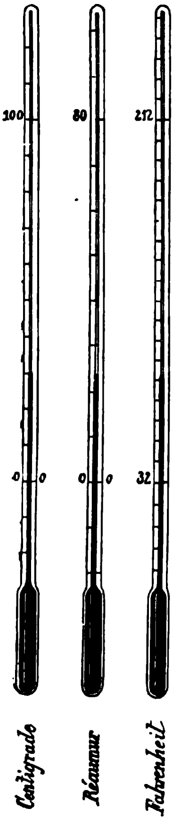

Fig. 191.
Thermometric Scales.

182. Thermometric Scales.—In the *Centigrade* scale the freezing-point is marked 0°, and the boiling-point 100°. In *Réaumur's* scale, which is still sometimes used abroad, the freezing-point is also marked 0°, but the boiling-point is marked 80°. Hence, 5 degrees on the former scale are equal to 4 on the latter, and the reduction of temperatures from one of these scales to the other can be effected by multiplying by $\frac{4}{5}$ or $\frac{5}{4}$.

For example, the temperature 75° Centigrade is the same as 60° Réaumur, since $75 \times \frac{4}{5} = 60$; and the temperature 36° Réaumur is the same as 45° Centigrade, since $36 \times \frac{5}{4} = 45$.

The relation between either of these scales and that of *Fahrenheit* is rather more complicated, inasmuch as Fahrenheit's zero is not at freezing-point, but at 32 of his degrees below it.

As regards intervals of temperature, 180 degrees Fahrenheit are equal to 100 Centigrade, or to 80 Réaumur, and hence, in lower terms, 9 degrees Fahrenheit are equal to 5 Centigrade, or to 4 Réaumur.

The conversion of temperatures themselves (as distinguished from intervals of temperature) will be best explained by a few examples.

Example 1. To find what temperatures on the other two scales are equivalent to the temperature 50° Fahrenheit.

Subtracting 32, we see that this temperature is 18 Fahrenheit degrees above freezing-point, and as this interval is equivalent to $18 \times \frac{5}{9}$, that is 10 Centigrade degrees, or to $18 \times \frac{4}{9}$, that is 8 Réaumur degrees, the equivalent temperatures are respectively 10° Centigrade and 8° Réaumur.

Example 2. To find the degree on Fahrenheit's scale, which is equivalent to the temperature 25° Centigrade.

An interval of 25 Centigrade degrees is equal to $25 \times \frac{9}{5}$, that is 45 Fahrenheit degrees, and the temperature in question is above freezing-point by this amount. The number denoting it on Fahrenheit's scale is therefore $32 + 45$, that is 77°.

The rules for the conversion of the three thermometric scales may be summed up in the following formulæ, in which F, C, and R denote equivalent temperatures expressed in degrees of the three scales:—

$$F = \tfrac{9}{5}C + 32 = \tfrac{9}{4}R + 32.$$
$$C = \tfrac{5}{4}R = \tfrac{5}{9}(F - 32).$$
$$R = \tfrac{4}{5}C = \tfrac{4}{9}(F - 32).$$

It is usual in stating temperatures to indicate the scale referred to by the abbreviations *Fahr.*, *Cent.*, *Réau.*, or more briefly by the initial letters F., C., R.

183. Apparent Expansion of Mercury. Degree of the Mercurial Thermometer.—The indications of the mercurial thermometer depend not upon the real but upon the *apparent* expansion of mercury, that is, upon the difference between the expansion of the mercury and that of the glass in which it is contained.

To understand the physical meaning of a degree, suppose that we have a thermometric tube (Fig. 192) divided into parts of equal capacity, and that by gauging the reservoir we have ascertained its capacity to be equal to N of these parts. Mercury is introduced, and when the instrument is surrounded with melting ice the mercury fills the bulb and n parts of the tube. The instrument is then raised to the temperature of 100° C., and the mercury expands so as to occupy n' parts of the stem. The apparent volumes of the mercury at 0° and 100° Centigrade are therefore $N + n$ and $N + n'$ respectively, and the apparent increase of volume is $n' - n$. The apparent increase per unit volume is therefore $\frac{n' - n}{N + n}$, and as this is the increase for 100° the apparent expansion per degree Centigrade is $\frac{1}{100} \frac{n' - n}{N + n}$. This quantity is found by experiment to be about equal to $\frac{1}{6480}$. Each degree Centigrade represents then a difference of apparent volume of the mercury equal to about $\frac{1}{6480}$ of its volume at freezing-point.

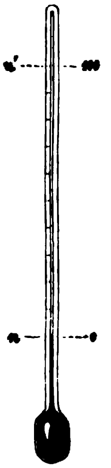

Fig. 192.

184. Comparability of Mercurial Thermometers.—This enables us to examine the important question, whether different thermometers are comparable with each other, that is, whether they will indicate the same temperature under the same conditions. This must evidently be the case where the glass employed for the construction of the tubes is absolutely the same. The agreement will likewise be exact, even when the apparent expansions due to the quality of the glass employed in each case are not the same, provided that they preserve a constant ratio at different temperatures.

Experiment alone can teach us whether this disagreement actually exists, and in what degree. The result of Regnault's investigations upon this point tends to show that up to 300° Cent. this disagreement is almost nothing, or, at any rate, may be neglected with perfect safety; above this point a slight difference is observed, which increases to about 3° or 4° at the temperature of 350°, that is, at the superior limit to the use of the mercurial thermometer.

This defect in the comparability of thermometers, slight as it is, must be attributed to irregularities in the expansion of the glass. The influence of these irregularities would obviously be less sensible with a more expansible material than mercury, and this is the reason that in experiments where great precision is necessary air-thermometers are employed, which at the same time serve to indicate the highest temperatures, whereas mercury cannot be employed beyond 350°, its boiling-point.

185. Displacement of the Zero Point.—A thermometer left to itself after being made, gradually undergoes a contraction of its capacity, leading to a uniform error of excess in its indications. This phenomenon is attributable to molecular change in the glass, which has, so to speak, been tempered in the construction of the instrument, and to atmospheric pressure on the exterior of the bulb, which is unresisted by the internal vacuum. The progress of this change ceases at the end of a certain time, about fifteen or eighteen months, and the displacement is always inconsiderable, never amounting to a degree. In precise experiments, however, it is necessary to verify the position of the zero point in the thermometer employed, and, in the observation of temperatures, to take into consideration the slight displacement which may have occurred.

186. Sensibility of the Thermometer.—The power of the instrument to detect very small differences of temperature may be regarded as measured by the length of the degrees, which is proportional to the capacity of the bulb directly and to the section of the tube inversely. In fact, if l denote the length of one degree, and s the sectional area of the tube, ls will be the volume of one degree, which is $\frac{1}{6480}C$, where C denotes the capacity of the bulb, together with as much of the tube as is below the freezing-point. Hence, $l=\frac{1}{6480}\frac{C}{s}$, which varies directly as C, and inversely as s.

Quickness of action, on the other hand, requires that the bulb be small in at least one of its dimensions, so that no part of the mercury shall be far removed from the exterior, and also that the glass of the bulb be thin.

Quickness of action is important in measuring temperatures which vary rapidly. It should also be observed that, as the thermometer, in coming to the temperature of any body, necessarily causes an inverse change in the temperature of that body, it follows that when the mass of the body to be investigated is very small, the thermometer itself should be of extremely small dimensions, in order that it may not cause a sensible variation in the temperature to be ascertained.

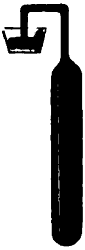
Fig. 193. Weight Thermometer.

187. Weight Thermometer.—In this thermometer, which often takes the place of the ordinary thermometer in physical investigations, the stem is done away with, and the mercury contained in the reservoir overflows into a small cup, where it is collected; and the temperature is deduced from the weight of the mercury which thus escapes. Let P be the weight of the mercury which fills the reservoir at freezing-point, and π the weight which issues when the instrument is placed in the steamer in order to determine the boiling-point. The apparent expansion between these points is evidently represented by the fraction $\frac{\pi}{P-\pi}$, and, consequently, the value of a degree Centigrade is the one-hundredth part of this fraction, that is $\frac{\pi}{100(P-\pi)}$. Suppose now that the instrument, containing again a weight P of mercury at zero is placed in a bath whose temperature we wish to determine, and that a weight p of mercury flows over. The whole apparent expansion is $\frac{p}{P-p}$, and dividing this by the value of a degree, we obtain the temperature required, which is thus expressed by the formula

$$T=\frac{p}{P-p}\div\frac{\pi}{100(P-\pi)}=100\frac{p}{\pi}\cdot\frac{P-\pi}{P-p}.$$

188. Alcohol Thermometer.—In the construction of thermometers other liquids may be introduced instead of mercury, and alcohol is very frequently employed for this purpose. But if an alcohol thermometer were constructed so as to agree with a mercurial thermometer at two fixed temperatures, and were graduated by dividing the intervening space into equal parts, and continuing the equal graduations both ways, it would be found to give different readings from a mercurial thermometer, except at or very near the two fixed temperatures. This fact may be expressed by saying, that alcohol does not expand equally for equal increments of temperature as indicated by a mercurial thermometer; or, more symmetrically, by saying that intervals of temperature which are equal as measured by the expansion of mercury, are not equal as measured by the expansion of alcohol.

In practice, alcohol thermometers are graduated by comparison with mercurial thermometers, and the degrees of an alcohol thermometer have consequently unequal volumes in different parts of the scale. The degrees, in fact, increase in length as we ascend on the scale.

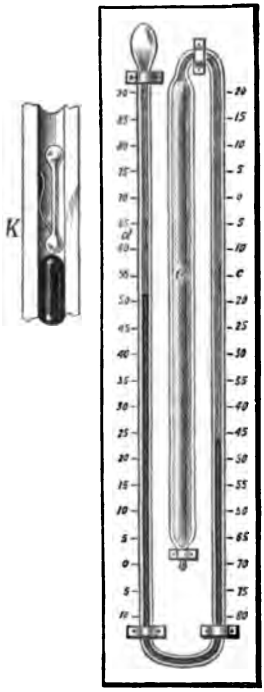

Fig. 194A.—Six's Self-registering Thermometer.

Alcohol has the disadvantage of being slower in its action than mercury, on account of its inferior conductivity; but it can be employed for lower temperatures than mercury, as the latter congeals at — 39° Cent. (—38° Fahr.), whereas the former has never congealed at any temperature yet attained.

189. Self-registering Thermometers.—It is often important for meteorological purposes to have the means of knowing the highest or the lowest temperature that occurs during a given interval. Instruments intended for this purpose are called maximum and minimum thermometers.

The oldest instrument of this class is *Six's* (Fig. 194 A), which is at once a maximum and a minimum thermometer. It has a large cylindrical bulb C filled with alcohol, which also occupies a portion of the tube. The remainder of the tube is partly filled with mercury, which occupies a portion of the tube shaped like the letter U, one extremity of the mercurial column being in contact with the alcohol already mentioned, while the other extremity is in contact with a second column of alcohol; and beyond this there is a small space occupied only with air, so as to leave room for the expansion of the liquids. When the alcohol in the bulb expands it pushes the mercurial column before it, and when it contracts the mercurial column follows it. The extreme points reached by the two ends of the mercurial column are registered by a pair of light steel indices *c*, *d* (shown on an enlarged scale at K), which are pushed before the ends of the column, and then are held in their places by springs, which are just strong enough to prevent slipping, so that the indices do not follow the mercury in its retreat. One of the indices *d* registers the maximum and the other *c* the minimum temperature which has occurred since the instrument was last set. The setting consists in bringing the indices into contact with the ends of the mercurial column, and is usually effected by means of a magnet. This instrument is now, on account of its complexity, little used. It possesses, however, the advantages of being equally quick (or slow) in its action for maximum and minimum temperatures, which is an important property when these temperatures are made the foundation for the computation of the mean temperature of the interval, and of being better able than most of the self-registering thermometers to bear slight jolts without disturbance of the indices.

Rutherford's self-registering thermometers are frequently mounted together on one frame, as in Fig. 195, but are nevertheless distinct instruments. His *minimum* thermometer, which is the only minimum thermometer in general use, has alcohol for its fluid, and is always placed with its tube horizontal, or nearly so. In the fluid column there is a small index *n* of glass or enamel, shaped like a dumb-bell.

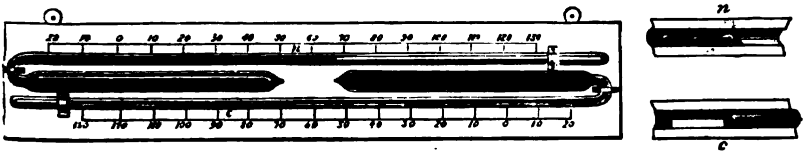

Fig. 195.—Rutherford's Maximum and Minimum Thermometers.

When contraction occurs, the index, being wetted by the liquid, is forced backwards by the contractile force of the superficial film which forms the extremity of the liquid column (§ 97); but when expansion takes place the index remains stationary in the interior of the liquid. Hence the minimum temperature is indicated by the position of the forward end of the index. The instrument is set by inclining it so as to let the index slide down to the end of the liquid column.

The only way in which this instrument is liable to derangement, is by a portion of the spirit evaporating from the column and becoming condensed in the end of the tube, which usually terminates in a small bulb. When the portion thus detached is large, or when the column of spirit becomes broken into detached portions by rough usage in travelling, "let the thermometer be taken in the hand by the end farthest from the bulb, raised above the head, and then forcibly swung down towards the feet; the object being, on the principle of centrifugal force, to send down the detached portion of spirit till it unites with the column. A few throws or swinging strokes will generally be sufficient; after which the thermometer should be placed in a slanting position, to allow the rest of the spirit still adhering to the sides of the tube to drain down to the column. But another method must be adopted if the portion of spirit in the top of the tube be small. Heat should then be applied slowly and cautiously to the end of the tube where the detached portion of spirit is lodged; this being turned into vapour by the heat will condense on the surface of the unbroken column of spirit. Care should be taken that the heat is not too quickly applied. . . . The best and safest way to apply the requisite amount of heat, is to bring the end of the tube slowly down towards a minute flame from a gas-burner; or if gas is not to be had, a piece of heated metal will serve instead."[1]

Rutherford's *maximum* thermometer is a mercurial thermometer, with the stem placed horizontally, and with a steel index *c* in the tube outside the mercurial column. When expansion occurs, the index, not being wetted by the liquid, is forced forwards by the contractile force of the superficial film which forms the extremity of the liquid column (§ 97); but when contraction takes place, the index remains stationary outside the liquid. Hence the maximum temperature is indicated by the position of the backward end of the index. The instrument is set by bringing the index into contact with the end of the liquid column, an operation which is usually effected by means of a magnet.

This thermometer is liable to get out of order after a few years' use, by chemical action upon the surface of the index, which causes it to become wetted by the mercury, and thus renders the instrument useless.

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[1] Buchan's *Handy Book of Meteorology*, p. 62.

Phillips' maximum thermometer (invented by Professor Phillips, the eminent geologist, and made by Casella) is recommended for use in the official *Instructions for Taking Meteorological Observations*, drawn up by Sir Henry James for the use of the Royal Engineers. It is a mercurial thermometer not deprived of air. It has an exceedingly fine bore, and the mercurial column is broken by the insertion of a small portion of air. The instrument is set by reducing this portion of air to the smallest dimensions which it can be made to assume, and is placed in a horizontal position. When the mercury expands it pushes forwards this intervening air and the detached column of mercury beyond it; but when contraction takes place the intervening air expands, and the detached column remains unmoved.

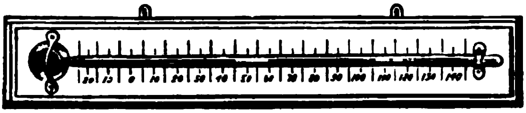

Fig. 194 B.—Phillips' Maximum Thermometer.

"The thread of mercury in these thermometers is easily broken at any point required, by simply raising the bulb end, and allowing the mercury to run into the open cell at the end; and, as it descends, detaching, with a slight jerk, as much of it as may be thought necessary, which should be an inch, or an inch and a half."[1]

The detached column is not easily shaken out of its place, and when the bore of the tube is made sufficiently narrow the instrument may even be used in a vertical position, a property which is often of great service.

In Negretti and Zambra's maximum thermometer (Fig. 194), which is employed at the Royal Observatory, Greenwich, there is an obstruction in the bent part of the tube, near the bulb, which barely leaves room for the mercury to pass when forced up by

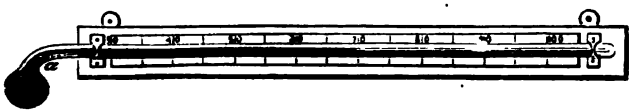

Fig. 194.—Negretti's Maximum Thermometer.

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[1] *Instructions for Taking Meteorological Observations.* By Colonel Sir Henry James, R.E., Director of the Ordnance Survey.

expansion, and is sufficient to prevent it from returning when the bulb cools.

The objection chiefly urged against this thermometer is the extreme mobility of the detached column, which renders it very liable to accidental displacement; but in the hands of a skilful observer this is of no moment. Dr. Balfour Stewart (*Elementary Treatise on Heat*, p. 20, 21), says:—“When used, the stem of this instrument ought to be inclined downwards. . . . It does not matter if the column past the obstruction go down to the bottom of the tube; for when the instrument is read, it is gently tilted up until this detached column flows back to the obstruction, where it is arrested, and the end of the column will then denote the maximum temperature. In resetting the instrument, it is necessary to shake the detached column past the obstruction in order to fill up the vacancy left by the contraction of the fluid after the maximum had been reached.”

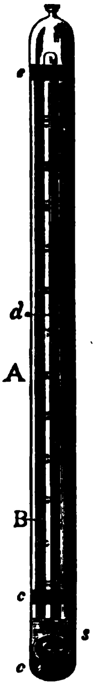

Fig. 194 c. Thomson's Protected Thermometer.

MERCURIAL MINIMUM THERMOMETERS.—As it is obviously undesirable that the minimum thermometer employed should be slower in its action than the maximum thermometer used in conjunction with it, several attempts have been made to construct a minimum thermometer in which mercury instead of alcohol shall be the expanding fluid (see a description of Casella's mercurial minimum thermometer, Stewart on *Heat*, p. 22); but no one has yet succeeded in producing such an instrument fit for general use.

DEEP-SEA AND WELL THERMOMETERS.—The instruments which have been most successfully employed for the observation of the temperature of water at great depths are self-registering thermometers either of Six's or Phillips' construction, inclosed in a glass-case, hermetically sealed, containing air and a little alcohol. The glass-case and inclosed air protect the bulb of the thermometer from the immense pressure of the superincumbent water, which by compressing the bulb would force mercury into the tube, and make the reading too high. The instrument represented in Fig. 194 c was designed by Sir Wm. Thomson, and is used by the Committee on Underground Temperature appointed by the British Association. A is the protecting case, B the Phillips' thermometer inclosed in it, and supported by three pieces of cork *ccc*. A small quantity of spirit *s* occupies the lower part of the case; *d* is the air-bubble characteristic of Phillips' thermometer, and serving to separate one portion of the mercurial column from the rest. In the figure this air-bubble is represented as expanded by the descent of the lower portion of mercury, while the upper portion remains suspended by adhesion. This instrument has been found to register correctly even under a pressure of 2½ tons to the square inch.

The use of the spirit *s* is to bring the bulb more quickly to the temperature of the surrounding medium.

Another instrument, designed, like the foregoing, for observations in wells and borings, is *Walferdin's maximum thermometer* (Fig. 196). Its tube terminates above in a fine point opening into a cavity of considerable size, which contains a sufficient quantity of mercury to cover the point when the instrument is inverted. The instrument is set by placing it in this inverted position and warming the bulb *until* the mercury in the stem reaches the point and becomes connected with the mercury in the cavity. The bulb is then *cooled* to a temperature lower than that which is to be observed, and during the operation of cooling mercury enters the tube so as always to keep it full. The instrument is then lowered in the erect position into the bore where observations are to be made, and when the temperature of the mercury rises a portion of it overflows from the tube. To ascertain the maximum temperature which has been experienced, the instrument may be immersed in a bath of known temperature, less than that of the boring, and the amount of void space in the upper part of the tube will indicate the excess of the maximum temperature experienced above that of the bath.

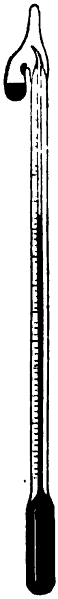

Fig. 196.

If the tube is not graduated, the maximum temperature can be ascertained by gradually raising the temperature of the bath till the tube is just full.

If the tube is graduated, the graduations can in strictness only indicate true degrees for some one standard temperature of setting, since the length of a true degree is proportional to the quantity of mercury in the bulb and tube; but a difference of a few degrees in the temperature of setting is immaterial, since 10° Cent. would only alter the length of a degree by about one six-hundredth part.

190. Thermograph.—A much more complete method of obtaining a register of the indications of a thermometer in the absence of an observer is now adopted in the principal British observatories. It consists in the photographic registration of the height of the mercurial column at every instant during the entire day. A sheet of sensitized paper is mounted on a vertical cylinder just behind the mercurial column, which is also vertical, and is protected from the action of light by a cover of blackened zinc, with the exception of a narrow vertical strip just behind the mercurial column. A strong beam of light from a lamp or gas flame is concentrated by a cylindric lens, so that if the thermometer were empty of mercury a bright vertical line of light would be thrown on the paper. As this beam of light is intercepted by the mercury in the tube, which for this purpose is made broad and flat, only the portion of the paper above the top of the mercurial column receives the light, and is photographically affected. The cylinder is made to revolve slowly by clock-work, and if the mercury stood always at the same height, the boundary between the discoloured and the unaffected parts of the paper would be straight and horizontal, in consequence of the horizontal motion of the paper itself. In reality, the rising and falling of the mercury, combined with the horizontal motion of the paper, causes the line of separation to be curved or wavy, and the height of the curve above a certain datum-line is a measure of the temperature at each instant of the day.[1] The whole apparatus is called a *thermograph*, and apparatus of a similar character is employed for obtaining a continuous photographic record of the indications of the barometer[2] and magnetic instruments.

191. Metallic Thermometers.—Thermometers have sometimes been constructed of solid metals. Breguet's thermometer, for example (Fig. 197), consists of a helix carrying at its lower end a horizontal needle which traverses a dial. The helix is composed of three metallic strips, of silver, gold, and platinum, soldered together so as to form a single ribbon. The silver, which is the most expansible, is placed in the interior of the helix; the platinum, which is the least expansible, on the exterior; and the gold serves to connect them. When the temperature rises, the helix unwinds and produces a deflection of the needle; when the temperature falls, the helix winds up and deflects the needle in the opposite direction.

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[1] Strictly speaking, the temperatures corresponding to the various points of the curve are not read off by reference to a single datum-line, but to a number of datum-lines which represent the shadows of a set of horizontal wires stretched across the tube of the thermometer at each degree, a broader wire being placed at the decades, and also at 32°, 52°, and 72°.

In order to give long degrees, the bulb of the thermometer is made very large—eight inches long, and ·4 of an inch in internal diameter.—(*Greenwich Observations*, 1847.)

[2] See § 110A.

Fig. 198 represents another dial-thermometer, in which the thermometric portion is a double strip composed of steel and brass, bent into a circular form. One extremity is fixed, the other is jointed to the shorter arm of a lever, whose longer arm carries a toothed sector. This latter works into a pinion, to which the needle is attached.

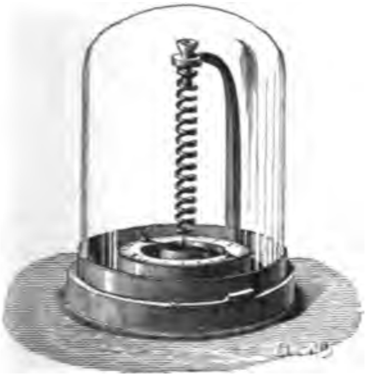

Fig. 197.—Breguet's Thermometer.

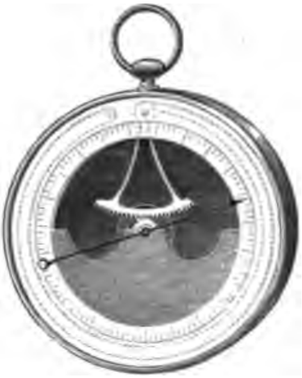

Fig. 198.—Metallic Thermometer.

It may be remarked that dial-thermometers are very well adapted for indicating maximum and minimum temperatures, it being only necessary to place on opposite sides of the needle a pair of movable indices, which could be pushed in either direction according to the variations of temperature.

Generally speaking, metallic thermometers offer great facilities for automatic registration.

In Secchi's meteorograph, for example, the temperature is indicated and registered by the expansion of a long strip of brass (about 17 metres long) kept constantly stretched by a suitable weight; this expansion is rendered sensible by a system of levers connected with the tracing point. The thermograph of Hasler and Escher consists of a steel and a brass band connected together and rolled into the form of a spiral. The movable extremity of the spiral, by acting upon a projecting arm, produces rotation of a steel axis which carries the tracer.

192. Pyrometers.—Metallic thermometers can generally be employed for measuring higher temperatures than a mercurial thermometer could bear; but there is great difficulty in constructing any instrument to measure temperatures as high as those of furnaces. Instruments intended for this purpose are called pyrometers.

Wedgwood, the famous potter, invented an apparatus of this kind, consisting of a gauge for measuring the contraction experienced by a piece of baked clay when placed in a furnace; and Brongniart introduced into the porcelain manufactory at Sèvres the instrument represented in Fig. 199, consisting of an iron bar lying in a groove in a porcelain slab, with one end abutting against the bottom of the groove, and the other projecting through the side of the furnace, where it gave motion to an indicator.

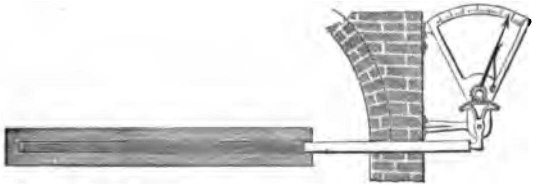

Fig. 199.—Brongniart's Pyrometer.

Neither of these instruments has, however, been found to furnish consistent indications, and the only instrument that is now relied on for the measurement of very high temperatures is the air-thermometer.

193. Differential Thermometer.—Leslie of Edinburgh invented, in the beginning of the present century, an ingenious instrument which enables us to measure small variations of temperature. A column of sulphuric acid, coloured red, stands in the two branches of a bent tube, the extremities of which terminate in two globes of equal volume (Fig. 200).

When the air contained in the two globes is at the same temperature, whatever that temperature may be, the liquid, if the instrument is in order, stands at the same height in the two branches. This point is marked zero. One of the globes being then maintained at a constant temperature, the other is raised through, for instance, 5 degrees, when the column rises on the side of the colder globe up to a point *a*, and descends on the other side to a point *b*. Suppose the space traversed by the liquid in each branch to be divided into 10 equal parts, each part will be equivalent to a quarter of a degree.

This division is continued upon each branch on both sides of zero.

The differential thermometer is an instrument of great sensibility, and enabled Leslie to make some very delicate investigations on the subject of the radiation of heat. It is now, however, superseded by the thermo-pile invented by Melloni. This latter instrument will be described in another portion of this work. Rumford's thermoscope (Fig. 201) is analogous to Leslie's differential thermometer. It differs from it in having the horizontal part much longer, and the vertical branches shorter. In the horizontal tube is an alcohol index, which, when the two globes are at the same temperature, occupies exactly the middle.

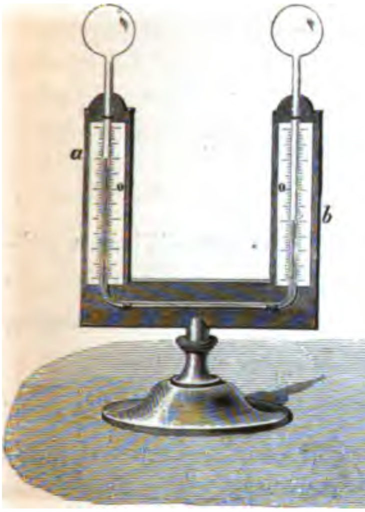

Fig. 200.—Leslie's Differential Thermometer.

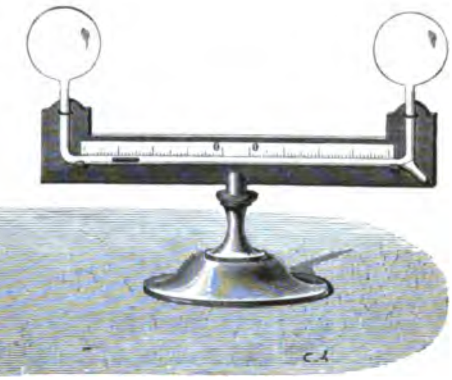

Fig. 201.—Rumford's Thermoscope

The tube is divided into equal parts, and the number of divisions traversed by the index is proportional very nearly to the difference of temperature between the two bulbs.

CHAPTER XX.

FORMULÆ RELATING TO EXPANSION.

194. Measure of Expansion, Factor of Expansion.—When a substance expands, so that its volume changes from V to $V' = V + v$, the ratio $\frac{v}{V}$ is the numerical measure of the expansion per unit volume, and is usually called simply the expansion of the substance.

Another ratio which it is frequently necessary to consider, is $\frac{V'}{V}$, or $1 + \frac{v}{V}$, that is to say, the ratio of the final to the initial volume. This may conveniently be called the *factor of expansion*, or the *expansion factor*. If m denote the expansion, $1 + m$ will be the factor of expansion, and we have

$$V' = V\,(1 + m). \tag{1}$$

195. Coefficient of Expansion.—It often happens that when the temperature of a body is increased by successive equal amounts, the successive increments of volume are also equal. In this case the body is said to expand uniformly between the extreme temperatures, and if V denote the volume of the body at 0° Cent., and V′ its volume at $t°$, then we have

$$V' = V\,(1 + K\,t); \tag{2}$$

where K is a constant number, called the coefficient of expansion per degree Centigrade.

The coefficient of expansion per degree Fahrenheit will be $\frac{5}{9}$ of this value of K, and the coefficient of expansion for the interval from 0° to 100° C. (if the expansion be uniform through this range) will be 100 K.

196. It is usual to employ the name *coefficient of expansion* to denote the coefficient of expansion per degree, the thermometric scale referred to being stated in the context.

Example. The volume of a glass vessel is 450 cubic inches at 0° C.; find its volume at 80° C., the coefficient of expansion for glass being ·00002. By (1) we have

$$V' = 450\ (1 + 80 \times \cdot 00002) = 450{\cdot}72 \text{ cubic inches.}$$

197. Cubic, Linear, and Superficial Expansion.—Thus far we have considered only expansion of volume. When a solid body expands, we may consider separately the increase of one of the linear dimensions of the body; this is called the *linear expansion*. Or we may consider the increase in area of any portion of its surface, which is called the *superficial expansion;* or finally, the increase of volume, which is called expansion of volume, or *cubical expansion*. By substituting the words "length" and "surface" for "volume" in § 194 we obtain definitions of linear and superficial expansion as numerically expressed, and we can demonstrate the two following propositions:—

(1.) The cubical expansion is three times the linear expansion.

(2) The superficial expansion is twice the linear expansion.

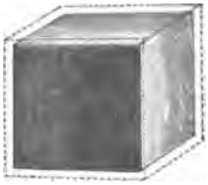

Fig. 202.

Suppose Fig. 202 to represent a cube formed of any substance, and let the length of each edge of the cube be unity at zero; the volume is consequently equal to 1, and the area of any one of the faces is also represented by 1. If the body be heated to any temperature t, each of the edges will increase by a certain quantity l, and the area of each face will become $(1+l)^2 = 1+2l+l^2$, while the volume of the cube will become

$$(1+l)^3 = 1+3l+3l^2+l^3.$$

But as the quantity l, which represents the linear expansion, is always very small, we may, without sensible error, neglect its second and third powers in comparison with its first power. We thus see that the increase in area of one of the faces of the cube is sensibly equal to $2l$, and that the increase in volume is sensibly equal to $3l$, which proves the propositions. These propositions evidently hold for the coefficients of expansion, so that we may say that *the coefficient of linear expansion is equal to one-third of the coefficient of cubical expansion, and to one-half of the coefficient of superficial expansion.*

The above demonstration supposes the body to remain similar to itself during expansion, which is not the case with bodies of a fibrous or laminated structure, nor with crystals, except those belonging to the cubic system.

198. Various Formulæ.—From equations (1) and (2) we may find the value of V in terms of V′, thus,

$$V=\frac{V'}{1+m}; \qquad (3)$$

$$V=\frac{V'}{1+Kt'}, \qquad (4)$$

that is to say, given the volume of a body at a certain temperature, the volume at zero is found by *dividing the given volume by the factor of expansion.*

Formulæ (1), (2), (3), and (4) are particular cases of a more general formula. Let V and V′ be the volumes of the same body at temperatures t and t' respectively, U the volume at zero, K the coefficient of expansion, and m and m' the respective expansions between 0 and t, and 0 and t'. We then have, by formulæ (1) and (2),

$$V=U\ (1+m)=U\ (1+Kt)$$
$$V'=U\ (1+m')=U\ (1+Kt');$$

whence by division

$$\frac{V}{V'}=\frac{1+m}{1+m'}=\frac{1+Kt}{1+Kt'}, \qquad (5)$$

or *the volumes of the same body at different temperatures are proportional to the factors of expansion.*

The above formulæ are evidently applicable also to linear and superficial expansions.

199. Influence of Temperature upon Density.—As the density of a substance is inversely as the volume occupied by unit mass, it follows from last section that *the densities of the same substance at different temperatures are inversely as the factors of expansion,* so that if D, D′, D_0 denote the densities at the temperatures t, t', 0, then

$$D=\frac{D_0}{1+m}=\frac{D_0}{1+Kt}$$

$$D'=\frac{D_0}{1+m'}=\frac{D_0}{1+Kt'}$$

$$\frac{D}{D'}=\frac{1+m'}{1+m}=\frac{1+Kt'}{1+Kt}.$$

200. Correction of Specific Gravity for Temperature.—Let d be the density of a substance at temperature t° Cent., and d_0 its density at 0° C. Also, let D be the density of water at t° C., and D_4 its density at 4° C. (the temperature of maximum density). Then if the specific gravity of the substance be computed by comparing its density with that of water at the same temperature, as in the ordinary methods of experimental determination, the specific gravity thus obtained is $\frac{d}{D}$, and has different values according to the temperature at which the determination is made.

The specific gravity as commonly given in tables is the value of $\frac{d_0}{D_4}$; that is to say, is the ratio of the density of the substance at 0° C. *to* that of water at the temperature of maximum density. The tabular specific gravity is easily derivable from the observed specific gravity $\frac{d}{D}$, if the law of expansion of the substance be known; for if $1+m$ be the factor of expansion of the substance from 0° C. to t° C., and $1+e$ the factor of expansion of water from 4° C. to t° C., then

$$d_0 = d\,(1+m)$$
$$D_4 = D\,(1+e).$$

Hence,

$$\frac{d_0}{D_4} = \frac{d}{D}\,\frac{1+m}{1+e}, \text{ or the correcting factor required is } \frac{1+m}{1+e}.$$

In the case of solid bodies this correction is generally of little importance; but in determining the specific gravities of liquids, especially those which are very expansible by heat, it cannot be neglected.

201. Formulæ for the Expansion of Gases.—The volume of a gas depends both on the temperature and on the pressure to which it is subjected; hence the formulæ of expansion for this class of bodies are somewhat more complicated. Suppose we wish to find the relation between the volumes V and V′ of the same mass of gas at temperatures t and t', and under pressures P and P′ respectively. Let U be the volume of the given mass of gas at pressure P and temperature t', and let a be the coefficient of expansion of the gas.

The two volumes V and U, being under the same pressure, are proportional to the expansion factors corresponding to their temperatures (§ 198), which gives

$$\frac{V}{U} = \frac{1+at}{1+at'}.$$

The volumes U and V′, at the same temperature t', are, by Boyle's law, inversely proportional to the pressures; whence we have

$$\frac{U}{V'} = \frac{P'}{P}.$$

From these two equations we obtain by multiplication

$$\frac{V}{V'}=\frac{P'}{P}.\frac{1+at}{1+at'}, \tag{9}$$

which means that *the volumes assumed by the same mass of gas are inversely proportional to the pressures, and directly proportional to the expansion factors corresponding to the temperatures.*

From equation (9) we may easily deduce another equation, by remarking that the densities of the same quantity of gas must be inversely as the volumes occupied. Thus we have

$$\frac{D}{D'}=\frac{P}{P'}.\frac{1+at'}{1+at}, \tag{10}$$

which signifies that *the density of a gas varies directly as the pressure, and inversely as the expansion factor corresponding to the temperature.*

CHAPTER XXI.

EXPANSION OF SOLIDS.

202. Laplace and Lavoisier's Experiments.—Laplace and Lavoisier determined the linear expansion of a great number of solids by the following method.

The bar AB (Fig. 203) whose expansion is to be determined, has one end fixed at A, while the other can move freely, pushing before it the lever OB, which is movable about the point O, and carries a telescope whose line of sight is directed to a scale at some distance. It is evident that a displacement BB′ will correspond to a considerably greater length CC′ on the scale, which will increase with the distance of the scale from the telescope. The ratio of CC′ to BB′ is equal to the ratio of OC to OB, and is the same throughout the whole course of experiments. In order to determine this ratio once for all, we have only to measure the distance CC′ on the scale corresponding to a *known* displacement BB′.

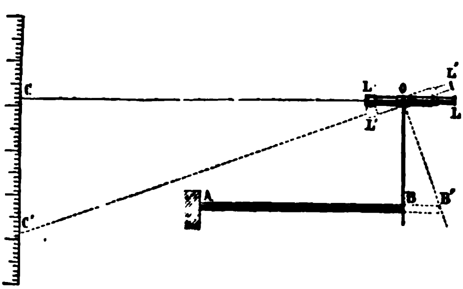

Fig. 203.
Principle of the Method of Laplace and Lavoisier.

The apparatus employed by Laplace and Lavoisier is shown in Fig. 204. The trough C, in which is laid the bar whose expansion is to be determined, is placed between four massive uprights of hewn stone N. One of the extremities of the bar rests against a fixed bar B′, firmly joined to two of the uprights; the other extremity pushes the bar B, which produces the rotation of the axis *aa′*. This axis carries with it in its rotation the telescope LL′, which is directed to the scale. The first step is to surround the bar with melting ice, and then observe the number on the scale marked by the line of sight of the

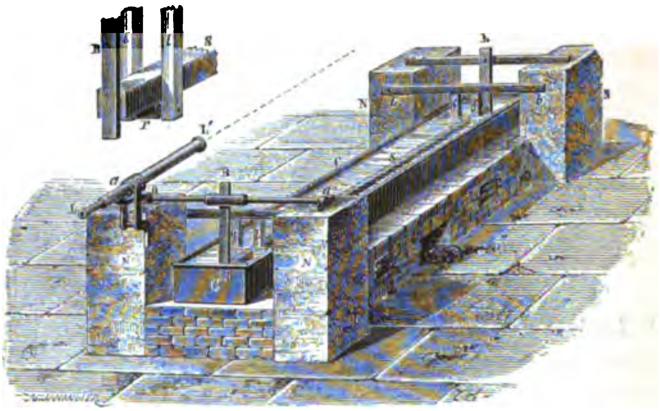

Fig. 204.—Apparatus of Laplace and Lavoisier.

telescope. The temperature of the trough is then raised, and the corresponding increase of length is measured.

Laplace and Lavoisier have discovered by this means that the expansion of solids is sensibly uniform between 0° and 100° C.; above this latter point it varies with the temperature.

The following table contains the most important results obtained by them:—

COEFFICIENTS OF LINEAR EXPANSION.

Gold, Paris standard, annealed,	0·000015153	Soft wrought iron, . . .	0·000012204
„ „ unannealed,	0·000015515	Round iron, wire drawn, .	0·000012350
Steel not tempered, . . .	0·000010792	English flint-glass, . . .	0·000008116
Tempered steel reheated to 65°,	0·000012395	Gold, procured by parting, .	0·000014660
Silver obtained by cupellation,	0·000019075	Platina, ...	0·000009918
Silver, Paris standard, . .	0·000019086	Lead, ...	0·000088483
Copper, ...	0·000017173	French glass with lead, . .	0·000008715
Brass, ...	0·000018782	Sheet zinc, ...	0·000029416
Malacca tin, ...	0·000019376	Forged zinc, ...	0·000031083
Falmouth tin, ...	0·000021729		

A simpler and probably more accurate method of observing expansions was employed by Ramsden and Roy. It consists in the direct observation of the distance moved by either end of the bar, by means of two microscopes furnished with micrometers, the microscopes themselves being attached to an apparatus which is kept at a constant temperature by means of ice.

203. Compensating Pendulum.—The pendulum, as we know, regulates the motion of a clock. Suppose the clock to keep exact time at the temperature of zero; then if the temperature rises the length of the pendulum will increase, and with it the duration of each oscillation, so that the clock will lose. The opposite effect would be produced by a fall of the temperature below zero. We thus see that clocks are liable to go too fast in winter, and too slow in summer, and that we must move the bob of the pendulum from time to time in order to insure their regularity.

The effect of temperature may be notably diminished by means of compensating pendulums, of which there are several different kinds.

1. *Harrison's Gridiron Pendulum.*—This consists of four oblong frames, the uprights of which are alternately of steel F and of brass C (Fig. 205); the brass uprights rest upon the bottom of the steel frames, and to the top of the second brass frame is attached the steel rod carrying the bob. From this arrangement it follows that the effect of the lengthening of the steel rods will be to lower the bob, while that of the lengthening of the brass rods will be to raise it. It will be seen that these two effects may be made completely to neutralize each other; we have only to insure that the whole expansion of the steel shall be equal to that of the brass.[1] If L and L′ be the

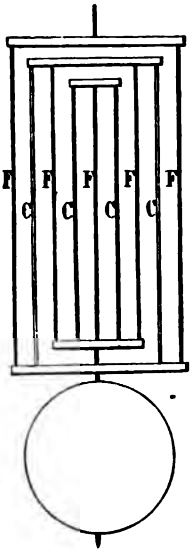

Fig. 205.
Plan of Gridiron Pendulum.

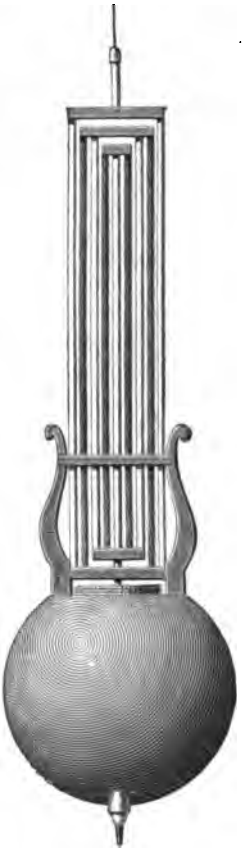

Fig. 206.
Gridiron Pendulum.

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[1] As the weight of the frame cannot be altogether neglected, and the change of dimensions produced by expansion affects the moment of inertia, the condition of compensation stated in the text can only be taken as approximate.

sum of the lengths of the steel and brass rods respectively; then this neutralization will be effected if $LKt = L'K't$, or if $LK = L'K'$, where K and K′ are the respective coefficients of expansion of steel and brass.

From the table on page 270 we learn that K is about $\frac{2}{3}$ K′; thus the sum of the lengths of the brass rods must be about $\frac{2}{3}$ of that of the steel rods. This result shows that we must have at least *two* brass frames, for with only one the compensating effect could not be produced, as the length of the steel rods would in that case be about double that of the brass. If we wish to have only a single frame of each different metal, we must choose two whose difference of expansion is much more marked; such, for instance, as iron and zinc, which are employed in Jurgensen's pendulum.

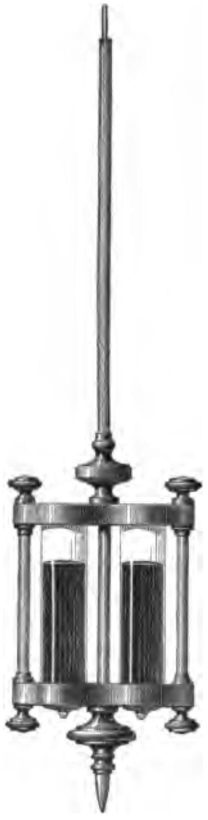

Fig. 207.—Graham's Mercurial Pendulum.

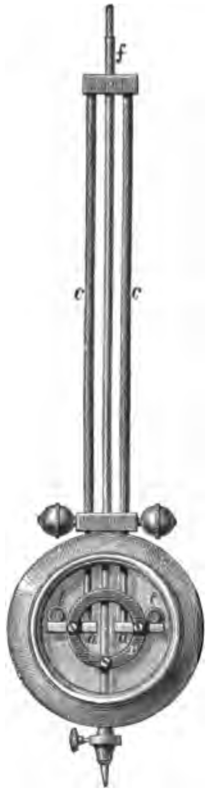

Fig. 208. Ellicott's Pendulum.

In order that the compensation may be complete, the centre of oscillation (§ 46) must remain at the same distance from the centre of suspension. The screw above the bob, shown in Fig. 206, enables us to attain this end by slightly raising or depressing the bob, and is intended to be used once for all to adjust the pendulum to the proper rate.

2. *Graham's Pendulum.*—This consists of an iron rod carrying at its lower end a frame, in which are fixed one or two glass cylinders containing mercury. When the temperature rises the lengthening of the rod lowers the centre of gravity and centre of oscillation of the whole; but the expansion of the mercury produces the contrary effect; and it will readily be understood that the quantity

of mercury in the cylinders may be such as to produce an approximately perfect compensation.

3. *Ellicott's Pendulum.*—This pendulum, which was invented in England in the last century, is known in France as Brocot's pendulum, and is frequently used in small French clocks. The main rod *f* is of iron. Attached to a cross-bar at the upper part of this rod are two brass rods *cc*, which, by means of the levers *aa* and the pins *tt*, attached to the bob, raise this latter when the temperature rises. The arms of the levers may evidently be so chosen as to maintain the centre of oscillation at an invariable distance from the axis of suspension, by means of the different expansive powers of iron and brass.

204. Force of Expansion of Solids.—The *force* of expansion is very considerable, being equal to the force necessary to compress the body to its original dimensions. Thus, for instance, iron when heated from 0° to 100° increases by ·0012 of its original length. In order to produce a corresponding change of length in a rod an inch square, a force of about 15 tons would be required. It would be useless to attempt to offer any mechanical resistance to a force so enormous; the only thing that can be done, in the case of structures in which metals are employed, is to arrange the parts in such a manner that the expansion shall not be attended with any evil effects. Thus, in a railway, the rails do not touch each other, a small interval being left to allow room for the variations of length. Iron beams employed in buildings must have the end free to move forward without encountering any obstacles, which they would inevitably overthrow. Sheets of zinc and lead employed in roofing, are so arranged as to be able in a certain extent to overlap each other on expansion.

We may further remark that the expansion of metals, though relatively very small, may practically become very considerable, if the length of metal which expands is sufficiently great. Suppose we take as an instance the length of railway from London to Edinburgh, which is about 400 miles. The extreme variations of temperature from winter to summer are about 50° C., which would produce a variation of length amounting to $400 \times 5280 \times {}^{\cdot}00061 = 1288$ feet. The actual variation is very considerable, and if the rails formed a continuous line at a certain temperature, this line would be interrupted or broken in pieces upon a change of temperature occurring in either direction.

205. Conversion of Heat into Work.—In conclusion, we may remark that heat when applied to a bar of metal produces two distinct and separate effects; one shown in the rise of temperature, and the other in the increase of volume. We may reasonably suppose that if the solid body were heated under such conditions as to preclude its expansion, the same quantity of heat would produce a much greater thermometric effect than in the former case. A similar remark applies to liquids and gases, and can be easily verified by experiment in the case of these latter. Here, then, is the first instance of a physical phenomenon of very frequent occurrence, namely, the conversion of heat into work, or reciprocally, of work into heat. Whenever a quantity of heat appears to be lost, the reason is that a corresponding amount of work is produced. If, on the other hand, work is done in compressing a body, so as to reduce it to the volume which it would occupy at a lower temperature, a rise of temperature is necessarily produced.

CHAPTER XXII.

EXPANSION OF LIQUIDS.

206. Relation between Real and Apparent Expansion.—If a vessel containing a liquid be heated, the level of the liquid rises, in consequence of the excess of the expansion of the liquid over that of the vessel. The observed increase of volume, not corrected for the expansion of the vessel, is called the apparent expansion. It is evidently less than the real expansion, for if the volume of the vessel had remained the same, the level would have risen higher.

The coefficients of real and apparent expansion are connected with the coefficient of expansion of the vessel by a very simple relation.

Let us take the case of a liquid contained in a vessel similar to a thermometer in shape, that is, suppose the tube to be divided into parts of equal capacity, and that we know by previous gauging how many divisions are equivalent to the volume of the reservoir.

Let n_0 denote the number of divisions occupied by the liquid at zero, and n_t the number of divisions occupied at temperature $t°$.

Then $\frac{n_t}{n_0}$ is the factor of apparent expansion, and $\frac{n_t}{n_0}-1$ is the apparent expansion.

Let us, for simplicity, take for our unit of volume the volume of a division at zero. Then if K be the expansion of the glass, the volume of a division at $t°$ will be $1+\mathrm{K}$.

The volume of the liquid at $t°$ is n_t of these divisions, and is therefore $n_t\,(1+\mathrm{K})$.

But if m be the real expansion of the liquid, the volume at $t°$ is $(1+m)\,n_0$, since n_0 is the volume at zero.

Hence we have

$$n_t\,(1+\mathrm{K})=(1+m)\,n_0;$$

and

$$\frac{n_t}{n_0}=\frac{1+m}{1+\mathrm{K}}.$$

That is, *the factor of apparent expansion is equal to the factor of real expansion of the liquid divided by that of the vessel.* Denote the factor of apparent expansion by $1+\Delta$.

Then since from above

$$1+\Delta=\frac{1+m}{1+K},$$

we have

$$1+m=(1+\Delta)(1+K)=1+\Delta+K+\Delta K;$$

whence

$$m=\Delta+K+\Delta K;$$

or since ΔK is much smaller than either Δ or K, and may in general be neglected,

$$m=\Delta+K.$$

That is, the real expansion of the liquid is equal to the apparent expansion plus the expansion of the vessel; and consequently, *the coefficient of real expansion is equal to the coefficient of apparent expansion plus the coefficient of expansion of the vessel.*

207. Expansion of Glass.—By means of this relation we can find the coefficient of expansion of any kind of glass; we have only to measure the coefficient of apparent expansion of the mercury in a thermometer made of this glass, and to subtract it from the coefficient of absolute expansion of the metal, which, as we shall see afterwards, is equal to $\frac{1}{5550}$. The coefficient of apparent expansion varies a little according to the quality of the glass employed; if we take it as $\frac{1}{6480}$, which is Dulong and Petit's determination of its mean value, we shall have for the coefficient of expansion of glass

$$K=\frac{1}{5550}-\frac{1}{6480}=\frac{1}{38700}.$$

208. Expansion of any Liquid.—The coefficient of expansion of the glass of which a thermometer is composed being known, we may use the instrument to measure the expansion of any liquid. For this purpose, the liquid whose coefficient of expansion is to be determined is introduced into the thermometer, and the number of divisions n_0 and n_t occupied by the liquid at the temperatures 0° and $t°$ respectively, are observed. Then, if D, Δ, K, be the coefficients of real expansion, of apparent expansion, and of expansion of the glass, each reckoned per degree Centigrade, we have

$$\frac{n_t}{n_0}-1=\Delta t, \text{ whence } \Delta \text{ is known; and}$$

$1+D=(1+\Delta)(1+K)$, or $D=\Delta+K$ nearly, whence D is known.

M. Pierre has performed an extensive series of experiments by this method upon a great number of liquids.

The apparatus employed by him is shown in Fig. 209. The thermometer containing the given liquid is fixed beside a mercurial thermometer, which marks the temperature. The reservoir and a small part of the tube are immersed in the bath contained in the cylinder below. The upper parts of the stems are inclosed in a second and smaller cylinder, the water in which is maintained at a sensibly constant temperature indicated by a very delicate thermometer.

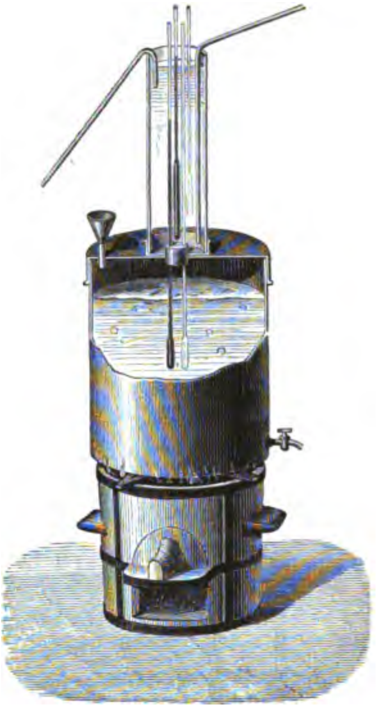

Fig. 209.—Pierre's Apparatus.

From these experiments it appears that the expansions of liquids are in general much greater than those of solids. Further, expansion does not proceed uniformly, as compared with the indications of a mercurial thermometer, but increases very perceptibly as the temperature rises. This is shown by the following table:—

	Volume at 0°.	Volume at 10°.	Volume at 40°.
Water......................	1	1·000146	1·007492
Alcohol....................	1	1·010661	1·044882
Ether	1	1·015408	1·066863
Bisulphide of carbon...	1	1·011554	1·049006
Wood-spirit..............	1	1·012020	1·050509

209. **Maximum Density of Water.**—By applying the experimental method just described to the case of water, we may easily observe the volume occupied by the same weight of this liquid at different temperatures, and it has thus been found that this volume is least at 4° Centigrade. At this temperature, accordingly, the density of water is a maximum, so that if a quantity of water at this temperature be

either heated or cooled it undergoes an increase of volume. This is a curious and unique exception to the general law of expansion by heat.

This anomaly may be exhibited by means of the apparatus represented in Fig. 210, which consists of two thermometers, one of alcohol, and the other of water. The reservoir of this latter, on account of the smaller expansive power of the liquid, is a long spiral, enveloping that of the alcohol thermometer. Both the reservoirs are contained in a metal box, which is at first filled with melting ice. The two instruments are so placed, that at zero the extremities of the two liquid columns are on the same horizontal line. This being the case, if the ice be now removed, and the apparatus left to itself, or if the process be accelerated by placing a spirit-lamp below the box, the alcohol will immediately be seen to rise, while the water will descend; and the two liquids will thus continue to move in opposite directions until a temperature of about 4° is attained. From this moment the water ceases to descend, and begins to move in the same direction as the alcohol. This experiment, although very well adapted for exhibiting the phenomenon, does not enable us to measure exactly the temperature of maximum density, since it is the apparent, and not the real, expansion of water which is thus observed. The following experiment, which is due to Hope, is more rigorous.

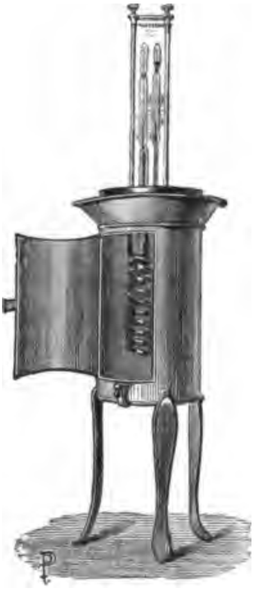

Fig. 210.—Maximum Density of Water.

A glass jar is employed, having two lateral openings, one near the top and the other near the bottom, which admit two thermometers placed horizontally. The tube is filled with water, and its middle is surrounded with a frigorific mixture. The following phenomena will then be observed.

The lower thermometer descends steadily to 4°, and there remains stationary. The upper thermometer at first undergoes very little change, but when the lower one has reached the fixed temperature, the upper one begins to fall, reaches the temperature of zero, and, finally, the water at the surface freezes, if the action of the frigorific mixture continues for a sufficiently long time. These facts admit of a very simple explanation.

As the water in the middle portion of the tube grows colder its density increases, and it sinks to the bottom. This process goes on till all the water in the lower part of the vessel has attained the temperature of 4°. But when all the water from the centre to the bottom has attained this temperature, any further cooling of the water in the centre fails to produce motion, until needles of ice are formed. These being specifically lighter than water, rise to the surface, and thus produce a circulation which causes the water near the surface to freeze, while that near the bottom remains at the temperature of 4°.

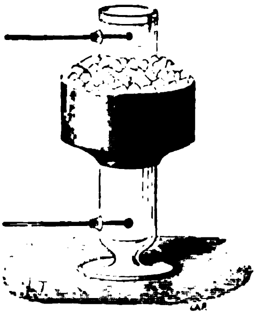

Fig. 211.
Hope's Experiment.

This experiment represents on a small scale what actually takes place during winter in pools of fresh water. The fall of temperature at the surface does not extend to the bottom of the pool, where the water, whatever be the external temperature, seldom falls below 4°. This is a fact of great interest, as exemplifying the close connection of natural phenomena, and the manner in which they contribute to a common end. It is in virtue of this anomaly exhibited by water in its expansion, taken in conjunction with the specific lightness of ice and the low conducting power of water, that the temperature at the bottom of deep pools remains moderate even during the severest cold, and that the lives of aquatic animals are preserved.

210. **Saline Solutions.**—In the case of saline solutions of different densities, the temperature of maximum density falls along with the freezing-point, and in fact falls more rapidly than this latter, so that for solutions containing a certain proportion of salt the temperature of maximum density is below the freezing-point. In order to show this experimentally, the solution must be placed in such circumstances as to remain liquid at a temperature below its freezing-point. This is a curious example of the continuity of physical laws, and of the restriction which must be applied in physics to the generally correct and logical principle of final causes. For salt water has a point of maximum density, although before that point is reached congelation takes place. In this instance the maximum density plays no part in the designs of nature, and leads to no practical benefit, being simply a proof of the permanence of a physical law, even when the circumstances on which its utility depended have disappeared.

211. Law of the Expansion of Liquids.—As we have shown above (§ 208), the expansion of liquids does not advance uniformly with the temperature; whence it follows that the mean coefficient of expansion will vary according to the limiting temperatures between which it is taken.

From a general review of the researches which have been made on this subject, it appears that for a great number of liquids the mean coefficient of expansion increases uniformly with the temperature. If, therefore, Δ be the expansion from 0 to t, we have

$$\frac{\Delta}{t} = a + bt, \text{ whence } \Delta = at + bt^2,$$

a and b being two constants specifying the expansibility of the given liquid.

For some very expansible liquids two constants are not sufficient, and the expansion is represented by the formula

$$\Delta = at + bt^2 + ct^3.$$

We subjoin a few instances of this class taken from the work of M. Pierre:—

Alcohol.................... $\Delta = 0{\cdot}0010486\ t + 0{\cdot}0000017510\ t^2 + 0{\cdot}0000000134518\ t^3$
Ether......................... $\Delta = 0{\cdot}0015132\ t + 0{\cdot}0000023592\ t^2 + 0{\cdot}000000040051\ t^3$
Bisulphide of carbon.... $\Delta = 0{\cdot}0011398\ t + 0{\cdot}0000013707\ t^2 + 0{\cdot}0000019123\ t^3$
Bromine... .... $\Delta = 0{\cdot}0010382\ t + 0{\cdot}0000017114\ t^2 + 0{\cdot}0000000054471\ t^3$

An examination of the formulæ for the different liquids which have been tested, shows that none of them have a point of maximum density; this property remains peculiar to water.

212. Absolute Expansion of Mercury.—The great importance of mercury in physical experiments, especially in connection with the barometer and thermometer, render it necessary to determine very accurately the coefficient of expansion of this liquid. This determination has been effected by Dulong and Petit by a very ingenious method, in which the observation of the heights of liquid columns is substituted for the measurement of volumes, which is always open to some uncertainty.

A and B are two tubes containing mercury, and communicating with each other by a very narrow tube CD. If the temperature of the liquid be uniform, the mercury should stand at the same height in both branches, according to the fundamental law of liquids in communicating vessels. But if the tube AC, for instance, be kept at 0° Cent., and the temperature of BD be raised, the density of the heated liquid will become less, and a greater height will consequently be required to equilibrate the pressure of the other column.

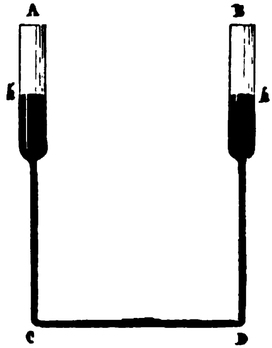

Fig. 212.—Principle of Dulong's Method.

Suppose the horizontal tube divided by a vertical section, and let h and h' denote the respective heights of the liquid in the cold and in the heated branches above the centre of gravity of this section. As the pressures on both sides must be equal, we must have the equation $hd = h'd'$, where d and d' are the densities of the mercury at zero and at the other given temperature.

But from what has been said above (§ 199), we have $d' = \frac{d}{1+m}$, where m is the expansion of the liquid from 0° to t°; and consequently, $hd = \frac{h'd}{1+m}$, whence

$$m = \frac{h' - h}{h}.$$

We thus see that it is sufficient to measure exactly the heights h' and h in order to find m.

We should remark, that the centre of gravity of the liquid section which we have been considering, is the only point of it which is in equilibrium; the upper part is subject to a greater pressure on the side of the heated liquid, and the lower part on the side of the cold; there is thus a tendency to the production of two opposite currents, which, however, is almost completely destroyed by the smallness of the bore of the connecting tube. Even supposing these currents actually to exist, the inverse effects produced by them would sensibly compensate each other, compared with the heights of the liquid in the two branches.

The following is the method by which Dulong and Petit have carried out the above principle.

The connecting tube, between the two branches A and B of the apparatus (Fig. 213), rests upon a T-shaped iron support, carrying two spirit-levels at right angles to each other for insuring horizontality. One of the branches B is inclosed in a cylinder containing melting ice; the other A is surrounded by a copper cylinder filled with oil, and heated by a furnace connected with the apparatus. In making an observation the first step is to arrange the apparatus so that, when the oil is heated to the temperature required, the mercury in the tube A may just be seen above the top of the cylinder, so as to be sighted with the telescope of a cathetometer; this may be effected by adding or taking away a small quantity of oil. The extremity of the column B is next sighted, which gives the difference of the heights h' and h. It is further necessary to determine the absolute height h.

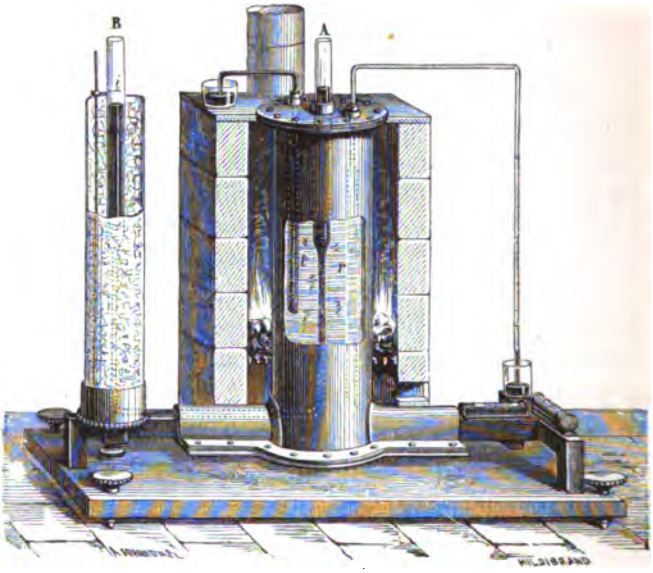

Fig. 213.—Apparatus of Dulong and Petit.

For this purpose the height of a reference mark i above the surface of the T-shaped support has been directly measured. From this height half the external diameter of the horizontal tube is to be subtracted, and it only remains to observe the distance of the end of the column B from the mark at the time of the experiment.

The temperature of the oil is given by the weight-thermometer t, and by the air-thermometer r, which latter we shall explain hereafter.

By means of this method Dulong and Petit ascertained that the expansion of mercury is sensibly uniform, as compared with the indications of an air-thermometer, between 0° and 100° C. Above this point it goes on increasing like other liquids, but not in any marked degree. Thus, the mean coefficient between 0° and 100° is $\frac{1}{5550}$. Between 0° and 200° it becomes $\frac{1}{5425}$, and $\frac{1}{5300}$ between 0° and 300°.

Regnault, without altering the principle of the experiments of Dulong and Petit, introduced several improvements into their apparatus, and added greatly to the length of the tubes A and B, thereby rendering the apparatus more sensitive. The results obtained by him do not differ very materially from those of Dulong and Petit; thus, for instance, he makes the coefficient between 0° and 100° equal to $\frac{1}{5509}$.

His experiments show that the mean coefficient between 0° and 50° is $\frac{1}{5547}$, a number almost identical with $\frac{1}{5550}$.

213. **Expansion of Iron and Platinum.**—The coefficient of absolute expansion of mercury being known, that of glass is deduced from it in the manner already indicated (§ 207). Dulong and Petit have deduced from it also the coefficients of expansion of iron and platinum, neither of which metals are attacked by mercury. The method employed is the following.

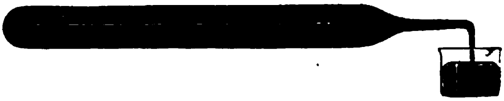

Fig. 214.—Expansion of Iron and Platinum.

The metal in question was introduced, in the shape of a cylindrical bar, into the reservoir of a weight-thermometer. Let W be the weight of the metal introduced, and D its density at zero. The process is the same as in using the weight-thermometer; that is, after having filled the reservoir with mercury at 0° C., we observe the weight w of the metal which issues at a given temperature t. The volume at 0° C. of the mercury which has issued, is $\frac{w}{d}$, d being the density of mercury at zero; the volume at $t°$ is therefore $\frac{w}{d}(1+mt)$, m being the coefficient of expansion of mercury. This volume evidently represents the expansion of the metal, *plus* that of the mercury, *minus* that of the glass. If then M denote the weight of mercury that fills the apparatus at 0° C., and if K be the coefficient of cubical expansion of glass, and x the expansion of unit volume of the given metal, we have the equation

$$\frac{w}{d}(1+mt)=\frac{W}{D}x+\frac{M}{d}mt-\left(\frac{W}{D}+\frac{M}{d}\right)Kt,$$

whence we can find x.

214. Convection of Heat in Liquids.—When different parts of a liquid are heated to different temperatures, corresponding differences of density arise, leading usually to the formation of currents which tend to produce equality of temperature as far as their presence extends. To this phenomenon, which is altogether distinct from conduction, the name of *convection* is given.

Thus, for instance, if we apply heat to the bottom of a vessel containing water, the parts immediately subjected to the action of the heat expand and rise to the surface; they are replaced by colder layers, which in their turn are heated and ascend; and thus the process continues indefinitely. The two currents can be very well shown by throwing some oak saw-dust into the water. By the movements of this substance, which has nearly the same density as water, it will be seen that the ascending current occupies the centre of the vessel, while the descending current passes along the sides.

215. Heating of Buildings by Hot Water.—This is a simple application of the principle just stated. One of the most common arrangements for this purpose is shown in Fig. 215; it is called the high-pressure system, because the water in the boiler can acquire a temperature considerably above 100° C. The boiler C is heated by a fire below it, and the products of combustion escape through the chimney A B. At the top of the house is a reservoir D, communicating with the boiler by a tube. From this reservoir the liquid flows into another reservoir E in the story immediately below, thence into another reservoir F, and so on. Finally, the last of these reservoirs communicates with the bottom of the boiler. The boiler, tubes, and reservoirs are all completely filled with water, with the exception of a small space left above in order to give room for the expansion of the liquid. An ascending current flows through the left-hand tube, and the circulation continues with remarkable regularity, so long as the temperature of the water in the boiler remains constant.

216. Currents in the Sea.—In the production of these currents convection plays an important, though perhaps not the principal part. In fact, the sea is an enormous mass of liquid whose temperature varies from point to point. Equilibrium is consequently impossible,

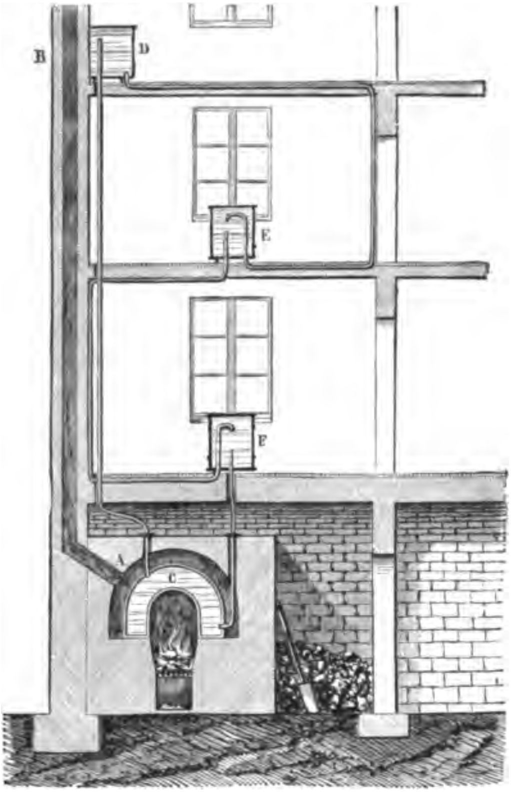

Fig. 215.—Heating by Hot Water.

and the different parts must therefore be in a state of continual motion with regard to each other. The waters of the tropical seas should, by reason of their excess of temperature, have a higher level than those of the polar seas; and the result is, a continual kind of overflowing of the waters about the equator, and consequently a vast current setting towards the poles. But to this current evidently corresponds a lower current of cold water flowing towards the equator, there to become heated, to overflow again, and so on. One of the most remarkable of oceanic currents is that which is known as the *Gulf Stream*. This current of warm water forms a kind of immense river in the midst of the sea, differing in the temperature, saltness, and colour of its waters from the medium in which it flows. Its origin is in the Gulf of Mexico, whence it issues through the straits between the Bahamas and Florida, turns to the north-west, and splits into two branches, one of which goes to warm the coasts of Ireland and Norway, the other gradually turns southwards, traverses the Atlantic from north to south, and finally loses itself in the regions of the equator.

"The Gulf Stream is a river in the ocean; in the severest droughts it never fails, and in the mightiest floods it never overflows; its banks and its bottom are of cold water, while its current is of warm; it takes its rise in the Gulf of Mexico, and empties into Arctic seas. There is on earth no other such majestic flow of waters. Its current is more rapid than the Mississippi or the Amazon, and its volume more than a thousand times greater. Its waters, as far out from the Gulf as the Carolina coasts, are of indigo blue. They are so distinctly marked that their line of junction with the common sea-water may be traced by the eye. Often one-half of the vessel may be perceived floating in Gulf Stream water, while the other half is in common water of the sea, so sharp is the line."—(Maury, *Physical Geography of the Sea.*)

Another cause of oceanic currents is to be found in the winds, which again are themselves examples of convective currents in the atmosphere. In the case of the Gulf Stream, it would appear that an accumulation of water is produced in the Gulf of Mexico by the trade-wind which blows steadily towards it over the South Atlantic. The elevation of level occasioned by this accumulation is probably to be regarded as the principal cause of the Gulf Stream. We shall discuss the origin of winds in a later chapter (Chap. xxxiv.)

CHAPTER XXIII.

EXPANSION OF GASES.

217. Experiments of Gay-Lussac.—Gay-Lussac conducted a series of researches on the expansion of gases, the results of which were long regarded as classical. He employed a thermometer with a large reservoir A, containing the gas to be operated on; an index of mercury *mn* separated the gas from the external air, while leaving it full liberty to expand. The gas had previously been dried by passing it through a tube containing chloride of calcium, or some other desiccating substance. The thermometer was fixed in a vessel which was first filled with melting ice, and when the gas had thus been brought to 0° C., the tube was so adjusted that the index coincided with the opening through which the thermometer passed.

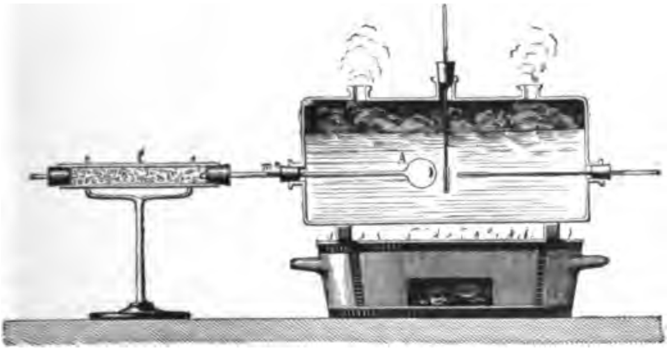

Fig. 216.—Gay-Lussac's Apparatus.

The tube being divided into parts of equal capacity, and the reservoir having been previously gauged, the volume V is known which is occupied by the gas at an external pressure H indicated by a barometer; the apparatus is then raised to a given temperature T by means of the furnace below the vessel, and the stem of the ther meter is moved until the index reaches the edge of the opening; this new temperature the gas occupies a volume V′ expressed divisions of the tube: at the same time the pressure may have vari suppose it to have become H′. From these data it is easy to ded the expansion of unit volume of the gas from 0° to T at const pressure. If D denote this expansion, the volume of the gas at the original pressure would be V (1+D). But the gas occupi volume V′ at the temperature T and pressure H′. At the pres H the volume would therefore be $V'\frac{H'}{H}$. But the divisions of thermometer have expanded in the ratio 1+KT, K being the efficient of expansion of glass: the true expression therefore for new volume of the gas at the pressure H is $\frac{V'(1+KT)H'}{H}$; whence have the equation

$$V(1+D) = V'(1+KT)\frac{H'}{H},$$

from which we can find the value of D, and consequently that o mean coefficient of expansion $\frac{D}{T}$. By means of this method Lussac arrived at the following results:—

1st.—All gases expand by the same amount between the limits of temperature.

2d.—The coefficient of expansion is independent of the pres He also found that the coefficient of expansion of air between 0° 100° was ·00375.

These laws, which, together with Boyle's law, were long rega as defining the fundamental properties of the gaseous state, are rigorously exact, but are subject to restrictions similar to which apply to Boyle's law. The absolute value of the co-effi of expansion of air as laid down by Gay-Lussac is very sen erroneous. The true value has been determined, by subsequen periments conducted with greater precision, to be ·003665, or

218. Regnault's Experiments.—The apparatus employed by Lussac had one serious imperfection. The mercurial index di constitute a sufficient barrier between the gas under investigatio the external air; so that a portion of the gas was able to escape, at the same time some of the external air became mixed with th either of which circumstances would impair the accuracy of th periment. It also appears that the means employed by Gay-L for desiccating the gas were insufficient. However this may be, the subject of the expansion of gases has been taken up by several physicists, as Pouillet, Rudberg, Magnus, and Regnault, who have performed a number of experiments on this subject, of undoubted accuracy, the result of which has been slightly to modify the conclusions arrived at by Gay-Lussac.

We shall confine ourselves to describing one of the methods employed by Regnault.

The apparatus consists of a glass ball with a narrow neck, containing the gas. This is placed in a boiler (Fig. 217), containing water,

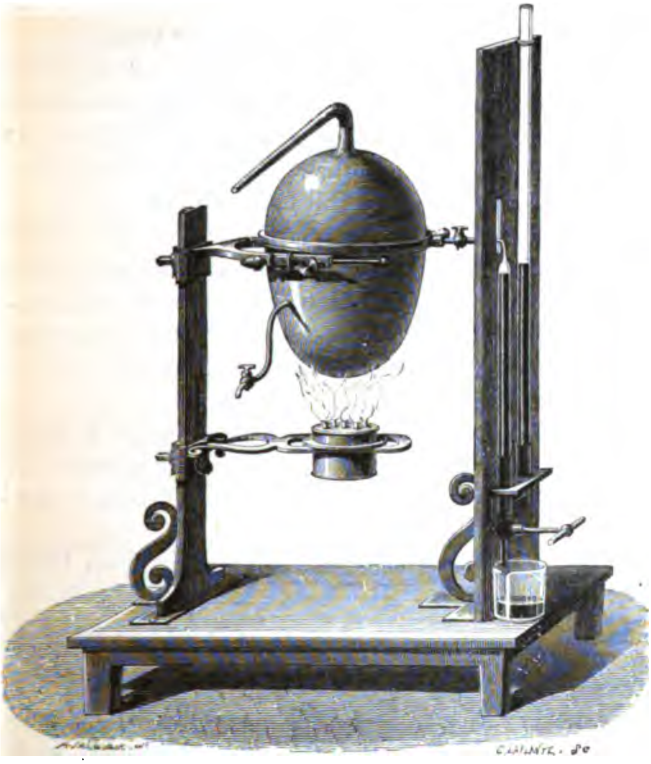

Fig. 217.—Regnault's Apparatus.

which can afterwards be raised to ebullition. A T-shaped tube, with three branches, establishes communication between the neck of the globe and a system, consisting of two tubes, containing mercury, and forming in fact a mercurial manometer, and also between the globe and a series of desiccating tubes, not shown in the figure, which are themselves in communication with a small air-pump. the first branch of the manometer, near the capillary portion of tube, is a reference mark.

The following is the mode of determining by means of this paratus the coefficient of expansion between 0° and 100° C. first step is to exhaust the globe a certain number of times, each refilling it with air, or with the gas under investigation, which been dried by passing it through the desiccating tubes. The dr may be rendered more complete and rapid by raising the tempera of the globe. This series of operations is, as Regnault has sh absolutely necessary in order to remove from the surface of the the last traces of moisture, which are exceedingly tenacious.

The gas having been admitted for the last time, the globe is rounded with melting ice, and is left to itself. The gas contracts, a fresh portion enters the globe, having first been perfectly drie passing through the desiccating tubes. The apparatus is thus with gas, and communication is established for a few seconds wit external air, so that the gas is at atmospheric pressure. Mercu then poured into the manometer so as to bring the level of the which is the same in both branches, up to the reference mark. branch of the T which established communication between the and the desiccating tubes is then hermetically sealed by dire the flame of a blowpipe upon it.

The effect of this operation has thus been to isolate a quanti gas at the atmospheric pressure H. This quantity consists of—

(1.) A volume V at the temperature of 0° C., V being the k volume of the globe.

(2.) A volume v, which is very small, extending from the ne the globe to the mark on the manometric tube. This volume v the surrounding temperature t; if brought to zero, it would be $\frac{v}{1+at}$, a being the coefficient of expansion of the gas. In the part of the experiment, therefore, we have a quantity of gas w when reduced to the temperature of zero and pressure H, would the volume

$$V+\frac{v}{1+at}.$$

The ice surrounding the globe is now removed, the boiler is with water, which is heated to ebullition; the volume and pr of the gas increase, the mercury consequently falls in the first b of the manometer, and rises in the other. When equilibrium of perature has been established, mercury is poured into the open branch, so as to raise that in the other branch to the mark. There is then found to be a difference of level h: the external pressure may have changed at the same time, and become H'; the gas is consequently subjected to a pressure $H'+h$. Its volume consists of two parts:—

(1.) The volume $V(1+KT)$ of the globe, K denoting the coefficient of expansion of the glass, and T the temperature of the boiling water *at the* moment of the experiment; this volume when reduced to zero becomes

$$\frac{V(1+KT)}{1+aT}.$$

(2.) The volume v of the tube as far as the mark, which volume at zero becomes $\frac{v}{1+at'}$, where t' is the surrounding temperature. Thus, in the second part of the experiment, the given quantity of gas under the pressure $H'+h$, would, at the temperature of zero, occupy the volume

$$\frac{V(1+KT)}{1+aT}+\frac{v}{1+at'}.$$

We have thus, by expressing that the volumes are inversely as the pressures,

$$(H'+h)\left\{\frac{V(1+KT)}{1+aT}+\frac{v}{1+at'}\right\}=\left(V+\frac{v}{1+at}\right)H,$$

whence

$$1+aT=\frac{V(1+KT)}{\left(V+\frac{v}{1+at}\right)\frac{H}{H'+h}-\frac{v}{1+at'}}.$$

To solve this equation we have recourse to a method frequently employed in physics, which is called the *method of successive approximations; v* being a very small quantity, is at first supposed to be zero; on this supposition the value of a is easily obtained. This value is then substituted in the correcting terms $\frac{v}{1+at}$, $\frac{v}{1+at'}$, whence the real value of aT is deduced. Now T is the temperature at which the water boils, and is always known, as we shall see hereafter, if the external pressure is known.

In the experiment just described, the volume of the gas remains sensibly the same, and the effect of heat is shown by an increase of pressure. We might have proceeded differently, and caused the gas to expand under a constant pressure.

We shall not stop to describe the modified form of the apparatus which is adapted to this other mode of experiment; we shall remark that the results obtained by the two processes do not exa agree, as will be seen from the following table:—

COEFFICIENTS OF EXPANSION PER DEGREE CENTIGRADE.

	From experiments at constant volume.		From experiments at constant pressure.
Air	0·003665	……	0·003670
Nitrogen	0·003668	…..	
Hydrogen	0·003667	…..	0·003661
Carbonic oxide	0·003667	……	0·003669
Carbonic acid	0·003688	……	0·003710
Nitrous oxide	0·003676	……	0·003720
Cyanogen	0·003829	……	0·003877
Sulphurous acid	0·003845	…..	0·003903

This table shows that each gas has its own coefficient of expan as we have already seen that each has its own coefficient of pressibility. Non-liquefiable gases, however, have nearly the coefficient of expansion, a result which accounts for the concl arrived at by Gay-Lussac.

As regards the differences between the two sets of numbers, it be noted that the second set alone represent expansion as dir observed. The first set directly measure the increase of pre which occurs when expansion is prevented. If Boyle's law were ously true, the two sets of results ought to be identical. In poi fact, it will be remarked that, except in the case of hydroger numbers in the second column are larger than those in the indicating that the product of volume and pressure dimin slightly as the pressure increases.

We may add that the coefficient of expansion increases very bly with the pressure; thus, between the pressures of one a three atmospheres the coefficient of expansion of air varies 0·00367 to 0·00369. This increase is still more marked in the liquefiable gases.

The coefficient of expansion per degree Fahrenheit is $\frac{5}{9}$ coefficient per degree Centigrade. For air or any non-liquefiab this may be taken as ·002036 or $\frac{1}{491}$.

219. Air-thermometer.—We have already stated, in conn with the mercurial thermometer, that the name *degree* (Centi is given to the hundredth part of the apparent expansion mercury in the glass. As the different kinds of glass employ the construction of these instruments have not the same expansion, it follows, as we have remarked, that mercurial thermometers are not rigorously comparable with each other, particularly above 100°.

The expansion of air being much more considerable than that of mercury, the variations caused by differences in the glass will be relatively less; for this reason (as well as for others which will be stated hereafter) the air-thermometer is employed to measure temperature in experiments of precision.

Any apparatus that will measure the expansion of air may serve as an air-thermometer; we have only to consider T as the unknown quantity in the expression $1+\alpha T$, writing for α the value of the coefficient of expansion given in last section.

M. Pouillet proposed to employ air in pyrometric investigations, and constructed an instrument to which he gave the name of air-pyrometer, and with which he performed some interesting experiments. Pouillet's pyrometer was very similar to Regnault's apparatus described above (§ 218); but the reservoir and part of the tube were of platinum, so as to be able to resist high temperatures. The indications of this instrument, however, could not be entirely relied on, since platinum has the property of condensing air upon its surface, and partially giving it out at high temperatures. At such temperatures, also, platinum becomes quite permeable to some of the gases of the furnace. For pyrometric purposes it is better to inclose the air in a porcelain vessel, as has been done by Deville and Troost.[1]

219A. Absolute Temperature. Absolute Zero.—If the air-thermometer be made the standard of temperature, equal differences of temperature will correspond to equal differences in the volume occupied by a given mass of air at constant pressure, the difference amounting to $\frac{1}{273}$ of the volume at 0° C. for each degree Centigrade, or to $\frac{1}{491}$ of the volume at 32° F. for each degree Fahrenheit.

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[1] The following account of a pyrometer constructed by Regnault, on the principle of the air-thermometer, is given in Dr. B. Stewart's treatise on heat. "There is a kind of flask, either cylindrical or spherical, which may be either of cast or wrought iron, of platinum or of porcelain: the mouth is closed by a plate containing a small aperture. From 15 to 20 grammes of mercury are added to this flask, which is then placed in that part of the furnace the temperature of which we desire to know. The mercury soon boils, its vapour expels the air by the orifice, and the excess of mercurial vapour goes off by the same means. When the apparatus has acquired the temperature of the furnace the flask is withdrawn and made to cool rapidly, and the mercury which remains in the flask is weighed. It may be weighed directly, or if it contains impurity, it is dissolved in acid, and estimated as a precipitate. This weight is that of the vapour of mercury which filled the flask at the temperature of the furnace, and the volume of the flask, as well as the density of mercurial vapour being known, this temperature may thus be determined."

The lowest temperature that could thus be expressed is evident
−273° C. or −459° F., since at this temperature the given mass of
would be reduced to a mathematical point. This is often called t
absolute zero of temperature, and temperatures reckoned from it
called *absolute temperatures.*

If C and F denote temperatures Cent. and Fahr. respective
273 + C and 459 + F will be the corresponding expressions for absol
temperature.

The statement of the relations between the volume, pressure
temperature of a given mass of air or other gas can be somew
simplified by the employment of this term. These relations (ass
ing the correctness of Boyle's and Gay-Lussac's laws) are as follows

1. The volume varies directly as the absolute temperature, w
the pressure is constant.

2. The pressure varies directly as the absolute temperature, w
the volume is constant.

3. For all variations, the expression $\frac{VP}{T}$ remains constant in va
V denoting volume, P pressure, and T absolute temperature.

The subject of this section will be further discussed in C
xxxii.

220. Density of Gases.—The volume of gases, owing to their g
expansibility and compressibility, is subject to enormous variat
Hence, in stating the *absolute density* of a gas, it is very impor
to specify the temperature and pressure at which we suppose it t
taken.

In stating the *specific gravity* or *relative density* of a gas as c
pared with dry atmospheric air, which is always adopted as the s
dard substance, it is to be understood that the gas is at the s
temperature and pressure as the air with which it is compared;
in consequence of the inexactness of the laws of Boyle and
Lussac, it is further necessary, for purposes of accuracy, to sp
the temperature and pressure at which the comparison is m
These are usually the temperature 0° C., and the pressure of 760
metres. The specific gravity or relative density of a gas is ther
defined as *the ratio of the weight of any volume of the gas a*
temperature 0° *C., and the pressure of* 760 *millimetres, to the w*
of the same volume of dry air at the same temperature and pres

This ratio being known, we can deduce from it the weight of
volume of the gas in question, by employing as a factor the we

of unit volume of air (§ 100). Thus, for instance, the ratio of the density of oxygen to that of air being 1·1056, and the weight of a litre of air at 0° C. and 760 millimetres being 1·293 gramme, we conclude that the weight of a litre of oxygen at this temperature and pressure is $1{\cdot}293 \times 1{\cdot}1056 = 1{\cdot}429$ gramme.

221. Measurement of the Density of a Gas.—The densities of gases have been the subject of numerous investigations; we shall here mention only the ingenious and exact method employed by Regnault. The gas is inclosed in a globe, of about 12 litres' capacity, and furnished with a stop-cock. This communicates with a three-way tube, furnished with stop-cocks *a* and *b* (Fig. 218), and through it with an air-pump on one side, and a manometer on the other. The globe is exhausted several times, and each time the gas is dried on its way to the globe by passing through a number of tubes containing pieces of pumice-stone moistened with sulphuric acid. When all moisture is supposed to be removed, the globe is surrounded with melting ice, and is allowed to fill with gas at the pressure of the atmosphere. When equilibrium of temperature has been established, the globe is taken out, carefully dried, and suspended from one of the scales of a balance. From the other scale is suspended a globe *of the same glass*, and of the same external volume (Fig. 219). The equality of the volumes is tested by weighing each in water, and noting the upward pressure of the liquid in each case. Weights are now added until equilibrium is established; and it can be proved that this equilibrium will be rigorously maintained, whatever be the variations of

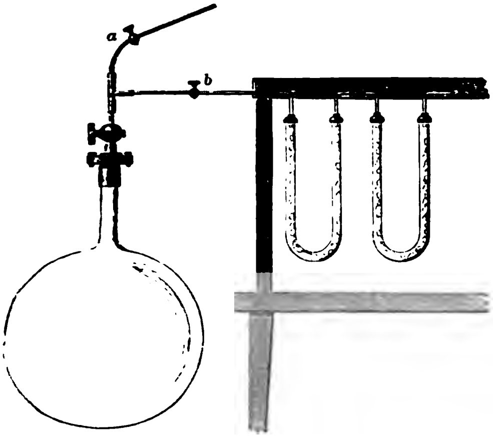

Fig. 218.—Measurement of Density of Gases.

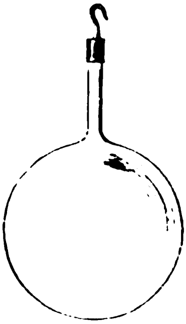

Fig. 219.—Compensating Globe.

external pressure and temperature, because these variations produce the same effect upon both globes. It is this introductio a *compensating globe* which gives Regnault's method its great cision;[1] for since all external causes that could disturb equilibri are balanced, the observed differences in weight can result only f variations in the gas inside.

The globe is then again surrounded with ice, communication the air-pump and the manometer is established, and a partial vacu is produced as far as a certain limit h. On the globe being ag suspended from the scale, the equilibrium of the balance is distur and the weight w, necessary to re-establish it, is evidently equa the weight of a volume of dry gas at 0°, and at the pressure H H being the external pressure. Hence it follows that the weigh dry gas which would completely fill the globe at the temperatur 0°, and at the pressure of 760 millimetres, is

$$w\frac{760}{H-h}.$$

The same experiment, when performed with air, would give as weight of the same volume of this gas under the same condition

$$w'\frac{760}{H'-h'}.$$

The relative density of the given gas is therefore

$$w\frac{760}{H-h}\div w'\frac{760}{H'-h'}=\frac{w}{w'}\cdot\frac{H'-h'}{H-h}.$$

222. Weight of a Litre of Air.—The preceding experiments the weight of dry air which fills a given globe at 0°, and at a pres of 760 millimetres. In order to know the weight of a litre of we have only to observe the weight of water which fills the globe at a given temperature. Let m be the difference of weig the globe when filled with water and with dry air; then the we of the water contained in the globe is evidently m *plus* the we of the dry air, which is previously known, and which we shall de by a. Let x be the volume of the globe, and e the expansion of water from 4° C. to the temperature of weighing. Then, if the gra and cubic centimetre be the units of weight and volume, we hav equation

$$x\frac{1}{1+e}=m+a,$$

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[1] The same device had previously been employed by Dr. Prout in determining the of air.

which gives the volume of the globe at a known temperature, whence the volume at zero may be deduced.

Various minute precautions are necessary in order to fill the globe with water completely free from air, and in order to insure that the temperature shall be the same throughout the whole of the considerable volume employed in the experiment. The first condition is especially difficult to fulfil; in order to attain it, Regnault first expelled the air from the globe by introducing a small quantity of water, and then exhausting the globe, the process being aided by a slight elevation of temperature; on the other hand, water, from which the air had been expelled by boiling, was forced by the pressure of steam into a tube leading to the stop-cock of the exhausted globe, so as to be nowhere exposed to the atmosphere. The difficulties of this process were skilfully overcome by Regnault, and he finally arrived at the following result.

In Paris, at a height of 60 metres above the level of the sea, a litre of dry air, at the temperature of 0° C., and a pressure of 760 millimetres, weighs 1·2932 gramme. A pressure of 760 millimetres of mercury has not the same effective value at different parts of the globe, on account of the variations in the intensity of gravity, whence it follows that the weight of the litre of air, defined by the preceding conditions, varies proportionally to the value of g. (See note to § 100.)

The following table gives the densities of several gases at 0° C., and under the pressure of 760 millimetres at Paris:—

Name of Gas.	Relative Density.	Absolute Density in grammes per litre.
Air	1	1·2932
Oxygen	1·10563	1·4298
Hydrogen	·06926	·08957
Nitrogen	·97137	1·25615
Chlorine	2·4216	3·1328
Carbonic oxide	·9569	1·2344
Carbonic acid	1·52901	1·9774
Protoxide of nitrogen	1·5269	1·9697
Binoxide of nitrogen	1·0388	1·3434
Sulphurous acid	2·1930	2·7289
Cyanogen	1·8064	2·3302
Marsh-gas	·559	·727
Olefiant gas	·985	1·274
Ammonia	·5967	·7697

223. Draught of Chimneys.—The expansion of air by heat produces the upward current in chimneys, and an approximate expression for the velocity of this current may be obtained by the application of Torricelli's theorem on the efflux of fluids from orifices (Ch xviii.).

Suppose the chimney to be cylindrical and of height h. Let air within it be at the uniform temperature t' Centigrade, and external air at the uniform temperature t. According to Torricel theorem, the square of the linear velocity of efflux is equal to product of $2g$ into the head of fluid, the term *head of fluid* be employed to denote the *pressure* producing efflux, *expressed in te of depth of the fluid.*

In the present case this head is the difference between h, whic the height of air within the chimney, and the height which a colu of the external air of original height h would have if expan upwards, by raising its temperature from t to t'. This latter hei is $h\frac{1+at'}{1+at}$; a denoting the coefficient of expansion ·00366; and head is

$$h\frac{1+at'}{1+at}-h=\frac{ha\,(t'-t)}{1+at}.$$

Hence, denoting by v the velocity of the current up the chimney, have

$$v^2=\frac{2gha\,(t'-t)}{1+at}$$

This investigation, though it gives a result in excess of the tr from neglecting to take account of friction and eddies, is sufficien explain the principal circumstances on which the strength of drau depends. It shows that the draught increases with the height the chimney, and also with the difference $t'-t$ between the int and external temperatures.

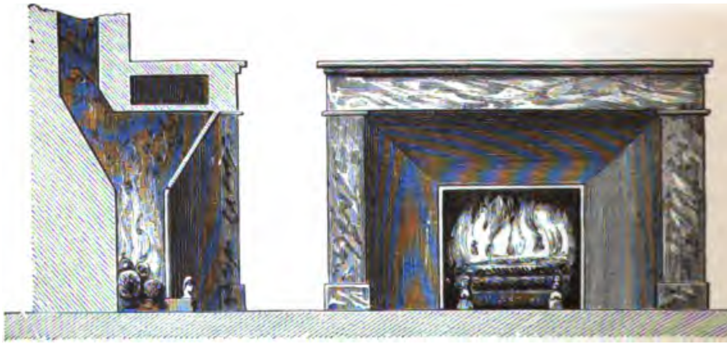

Fig. 220.—Rumford's Fire-place.

The draught is not so good when a fire is first lighted as after it has been burning for some time, because a cold chimney chills the air within it. On the other hand, if the fire is so regulated as to keep the room at the same temperature in all weathers, the draught will be strongest when the weather is coldest.

The opening at the lower end of the chimney should not be too wide nor too high above the fire, as the air from the room would then enter it in large quantity, without being first warmed by passing through the fire. These defects prevailed to a great extent in old chimneys. Rumford was the first to attempt rational improvements. He reduced the opening of the chimney and the depth of the fire-place, and added polished plates inclined at an angle, which serve both to guide the air to the fire and to reflect heat into the room (Fig. 220).

The blower (Fig. 221) produces its well-known effects by compelling all air to pass through the fire before entering the chimney. This at once improves the draught of the chimney by raising the temperature of the air within it, and quickens combustion by increasing the supply of oxygen to the fuel.

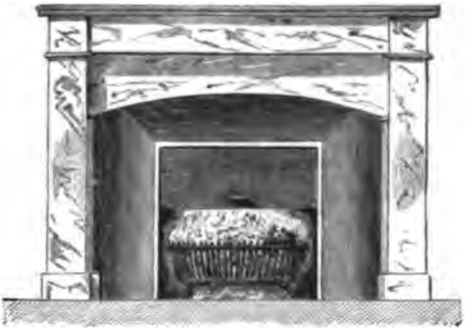
Fig. 221.—Fire-place with Blower.

224. Stoves.—The heating of rooms by open fire-places is effected almost entirely by radiation, and much even of the radiant heat is wasted. This mode of heating then, though agreeable and healthful, is far from economical. Stoves have a great advantage in point of economy, for the heat absorbed by their sides is in great measure given out to the room, whereas in an ordinary fire-place the greater part of this heat is lost. Open fire-places have, however, the advantage as regards ventilation; the large opening at the foot of the chimney, to which the air of the room has free access, causes a large body of air from the room to ascend the chimney, its place being supplied by fresh air entering through the chinks of the doors and windows, or any other openings which may exist.

Stoves are also liable to the objection of making the air of the room oo dry, not, of course, by removing water, but by raising the temperature of the air too much above the dew-point (Chap. xxviii.). ' same thing occurs with open fire-places in frosty weather, at wh time the dew-point is unusually low. This evil can be remedied placing a vessel of water on the stove. The reason why it is m liable to occur with stoves than with open fire-places, is mainly t the former raise the air in the room to a higher temperature t the latter, the defect of air-temperature being in the latter case c pensated by the intensity of the direct radiation from the glow fuel.

Fire-clay, from its low conducting power, is very serviceable b for the backs of fire-places and for the lining of stoves. In the for situation it prevents the wast escape of heat backwards into chimney, and keeps the back of fire nearly as hot as the centre. a lining to stoves, it impedes lateral escape of heat, thus ans ing the double purpose of prev ing the sides of the stove from o heating, and at the same tim keeping up the temperature of fire, and thereby promoting c plete combustion. Its use m however, be confined to that tion of the stove which serve the fire-box, as it would other prevent the heat from being g out to the apartment.

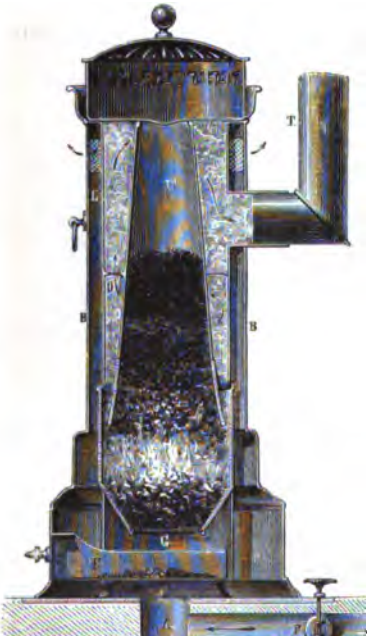

Fig. 222.—Ventilating Stove.

The stove represented in Fig. belongs to the class of what called in France *calorifères*, ar England *ventilating stoves*, b constructed with a view to pro ing the circulation and renew the air of the apartment. G is the fire-box, over which is the fe U, containing unburned fuel, and tightly closed at top by a lid, w is removed only when fresh fuel is to be introduced. The ash F has a door pierced with holes for admitting air to support

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[1] With the exception of the ventilating arrangement, this stove is identical with w known in this country as Walker's self-feeding stove.

bustion. The flame and smoke issue at the edge of the fire-box, and after circulating round the chamber O which surrounds the feeder, enter the pipe T which leads to the chimney. The chamber O is surrounded by another inclosure L, through which fresh air passes, entering below at A, and escaping into the room through perforations in the upper part of the stove as indicated by the arrows. The amount of fresh air thus admitted can be regulated by the throttle-valve P.

CHAPTER XXIV.

FUSION AND SOLIDIFICATION.

225. Fusion.—Many solid bodies, when raised to a sufficiently high temperature, become liquid. This change of state is called *melting* or *fusion*, and the temperature at which it occurs (called the melting-point, or temperature of fusion) is constant for each substance, with the exception of the variations—which in ordinary circumstances are insignificant—due to differences of pressure (§ 237). The melting-points of several substances are given in the following table:—

TABLE OF MELTING-POINTS, IN DEGREES CENTIGRADE.

Mercury,	−39	Tin,	230
Ice,	0	Bismuth,	562
Butter,	33	Lead,	320
Lard,	33	Zinc,	360
Spermaceti,	49	Antimony,	432
Stearine,	55	Bronze,	900
Yellow Wax,	62	Pure Silver,	1000
White Wax,	68	Copper,	1150
Stearic Acid,	70	Coined Gold,	1180
Phosphorus,	44	Pure Gold,	1250
Potassium,	63	Cast Iron,	1050 to 1250
Sodium,	95	Steel,	1300 to 1400
Iodine,	107	Wrought Iron,	1500 to 1600
Sulphur,	110	Platinum,	2000

Some bodies, such as charcoal, have hitherto resisted all attempts to reduce them to the liquid state; but this is to be attributed only to the insufficiency of the means which we are able to employ.

It is probable that, by proper variations of temperature and pressure, all simple substances, and all compound substances which would not be decomposed, could be compelled to assume the three forms, solid, liquid, and gaseous.

The passage from the solid to the liquid state is generally abrupt;

but this is not always the case. Glass, for instance, before reaching a state of perfect liquefaction, passes through a series of intermediate stages in which it is of a viscous consistency, and can be easily drawn out into exceedingly fine threads, or moulded into different shapes.

226. Constant Temperature during Fusion.—During the entire time of fusion the temperature remains constant. Thus if a vessel containing ice be placed on the fire, the ice will melt more quickly as the fire is hotter; but if the mixture of ice and water be constantly *stirred*, a thermometer placed in it will indicate the temperature zero without variation so long as any ice remains unmelted; it is only after all the ice has become liquid that a rise of temperature will be observed.

In the same way, if sulphur be heated in a glass vessel, the temperature indicated by a thermometer placed in the vessel will rise gradually until it reaches about 110°, when a portion of the sulphur will be seen to become liquid, and if the vessel be shaken during the time of fusion, until the whole of the sulphur is liquefied, the temperature will be observed to remain steadily at this point.

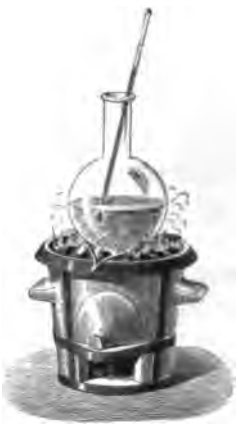

Fig. 223.—Fusion of Sulphur.

227. Latent Heat of Fusion.—This constancy of temperature is very remarkable, and leads to some important conclusions. In fact, as the action of the fire continues the same throughout the entire time of fusion, while the thermometer remains stationary, all the heat supplied after liquefaction has begun, *appears* to be lost. Hence we conclude, that in order that a body may pass from the solid to the liquid state, it must absorb a certain quantity of heat which produces no thermometric effect. Black, who was the first to investigate this subject, gave to the heat thus absorbed the name of *latent heat*, by which it is still usually designated. A similar absorption of heat without thermometric effect occurs when a boiling liquid is converted into vapour. Hence it is necessary to distinguish between the *latent heat of fusion* and the *latent heat of vaporization.*[1] Latent heat then may be defined as the heat absorbed in virtue of change of state from solid to liquid, or from liquid to gaseous. Modern philosophy teaches that the processes of fusion and vaporization involve the performance of work by heat in opposition to molecular force, and that an amount of heat disappears (or becomes latent) which is the exact equivalent of the work performed.

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[1] The former is often called the latent heat of the liquid, and the latter of the vapour. Thus we speak of the latent heat of water (which becomes latent in the melting of ice), and of the latent heat of steam (which becomes latent in the vaporization of water).

228. Heat of Fusion of Ice.—The latent heat of fusion is different for different substances. Its amount for ice (commonly called the latent heat of water) can be approximately determined by the following experiment, which is due to Black, who was the first to make accurate observations on this subject.

Take a pound of ice at 0° C. and a pound of water at 79° C.; let the water be poured over the ice, and the mixture rapidly stirred. The ice will melt, and two pounds of water at 0° will be obtained. This interesting experiment shows that all the heat necessary to raise a pound of water from 0° to 79° has been absorbed in melting a pound of ice; and it is thus directly proved that the heat required to melt a pound of ice is exactly the same as that required to raise the temperature of a pound of water from 0° to 79°.

The name *unit of heat* is given to the amount of heat necessary to raise unit mass of water one degree. It will be different according to the unit of mass and scale of temperature adopted. The pound-centigrade unit is the heat required to raise a pound of water through one degree Centigrade, and we see from above that 79 of these units are required for the melting of a pound of ice.[1]

On comparing the heat of fusion of ice with that of some other bodies as given in the table § 348, it will be seen that its amount is notably greater for ice than for any of the other substances. Ice is in this sense the most difficult to melt, and water the most difficult to freeze of all substances, a fact which is of immense importance in the economy of nature, as tending to retard the processes both of freezing and thawing. Even as it is, the effects of a sudden thaw are often very disastrous, and yet, for every pound of ice melted, as much heat is required as would raise the water produced through 79° C. or 142° F.

229. Solution.—The reduction of a body from the solid to the liquid state may be effected by other means than by the direct action of heat; it may be produced by the action of a liquid. This is what occurs when, for instance, a grain of salt or of sugar is placed in water; the body is said to *melt* or *dissolve* in the water. Solution, like fusion, is accompanied by the disappearance of heat consequent on the change from the solid to the liquid state. For example, when nitrate of ammonia is rapidly dissolved in water, a fall of from 20° to 25° Cent. is observed.

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[1] The statements in this paragraph, including the definition of a unit of heat, are only approximate. The subject will be resumed in Chap. xxxi.

Unlike fusion, it is attached to no definite temperature, but occurs with more or less freedom over a wide range. Rise of temperature usually favours it; but there are some strongly marked exceptions.

230. Freezing Mixtures.—The absorption of heat which accompanies the liquefaction of solids is the basis of the action of freezing mixtures. In all such mixtures there is at least one solid ingredient which, by the action of the rest, is reduced to the liquid state, thus occasioning a fall of temperature proportional to the latent heat of its liquefaction.

The mixture most commonly employed in the laboratory is one of snow and salt, in the proportion of two parts of the former to one of the latter. This mixture assumes a temperature of about −18° C. (0° F.), and furnished Fahrenheit with the zero of his scale. In this instance there is a double absorption of heat caused by the simultaneous melting of the snow and dissolving of the salt.

We may obtain a freezing mixture without the use of snow or ice. Such mixtures are often employed for the artificial freezing of water. Various kinds of apparatus have been invented for this purpose, one of which is shown in Fig. 224. It consists of a metal cylinder, con-

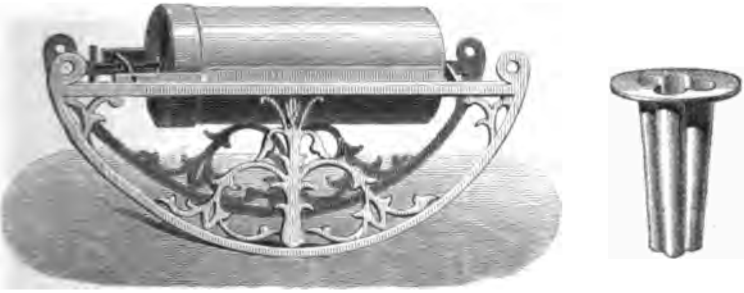

Fig. 224.—Freezing Rocker.

aining the freezing mixture (hydrochloric acid and sulphate of soda). n the cylinder is placed a mould formed of two concentric vessels ith the water between them, an arrangement which has the advange of increasing the surface of contact. The whole is set upon a adle, the rocking of which greatly assists the operation. We subjoin a table of the most important freezing mixtures, with the proportions corresponding to the maximum effect. These proportions are evidently of importance, since the amount of heat absorbed in a given time depends upon the quantity of solid matter that is reduced to the liquid state during that time. There may also be chemical action between the materials of which the mixture is composed. This is always attended with generation of heat, so that in this case the actual result depends upon the difference of two opposite effects. Thus the mixture of four parts of sulphuric acid with one of ice causes a rise of temperature of about 50° or 60°, while four parts of ice and one of sulphuric acid produce a cold of from —15° to —20° C.

TABLE.

	Proportions by Weight.	Fall of Temperature in Centigrade degrees.
Snow or Powdered Ice, ...	2	from 0° to − 21°.
Bay-salt, ...	1	
Snow, ...	3	from 0° to − 48°.
Crystallized Chloride of Calcium, . . .	4	
Nitrate of Ammonia, ...	1	from + 10° to − 15°.
Water, ...	1	
Sal-ammoniac, ...	5	from + 10° to − 15°.
Nitrate of Potash, ...	5	
Sulphate of Soda, ...	8	
Water, ...	16	
Sulphate of Soda, ...	8	from + 10° to − 17°.
Hydrochloric Acid, ...	5	

231. Solidification or Congelation.—Congelation is the inverse of fusion; that is to say, it is the passage of a substance from the liquid to the solid state. The faculty of undergoing this transformation may be regarded as common to all liquids, although some, for example, alcohol and bisulphide of carbon, have never yet been solidified.

The temperature of fusion is the highest temperature at which congelation can occur, and is frequently called the *temperature of congelation* (or the *freezing-point*); but it is possible to preserve substances in the liquid state at lower temperatures. Liquids thus cooled below their so-called freezing-points have, however, if we may so say, a *tendency to freeze, which is only kept in check by the difficulty of making a commencement.* If freezing once begins, or if ever so small a piece of the same substance in the frozen state be allowed to come in contact with the liquid, congelation will quickly extend until there is none of the liquid left at a temperature below that of fusion. The condition of a liquid cooled below its freezing-point has been aptly compared to that of a row of bricks set on end in such a manner that if the first be overturned, it will cause all the rest to fall, each one overturning its successor.

The contact of its own solid infallibly produces congelation in a liquid in this condition, and the same effect may often be produced by the contact of some other solid, especially of a crystal, or by giving a slight jar to the containing vessel.

Despretz has cooled water to −20° C. in fine capillary tubes, without freezing, and Dufour has obtained a similar result by suspending globules of water in a liquid of the same specific gravity with which it would not mix.

232. Heat set free in Congelation.—At the moment when congelation takes place, the thermometer immediately rises to the temperature of the melting-point. This may be easily shown by experiment. A small glass vessel is taken, containing water, in which a mercurial thermometer is plunged. By means of a frigorific mixture the temperature is easily lowered to −10° or −12°, without the water freezing; a slight shock is then given to the glass, congelation takes place, and the mercury rises to 0°.

The heat thus produced is the equivalent of the work done by the molecular forces of the body in the passage from the liquid to the solid state. The quantity of heat thus arising is evidently the same as that which disappears in fusion, since they are the equivalents of the same amount of work performed in opposite directions. The production of this heat may be experimentally shown in another way. If we heat a quantity of lead to its melting-point (320°), and when the metal is just beginning to melt, plunge it into water, a certain rise of temperature will be observed. If we repeat the same experiment, allowing the lead time to melt completely, the temperature being still 320°, a much more considerable increase in the temperature f the water will be produced, the reason being that the lead in olidifying in contact with the water gives out its latent heat.

233. Crystallization.—When the passage from the liquid to the olid state is a gradual one, it frequently happens that the molecules roup themselves in such a manner as to present regular geometric rms. This process is called crystallization, and the regular bodies us formed are called crystals. The particular crystalline form sumed depends upon the substance, and often affords a means of cognizing it. The forms, therefore, in which bodies crystallize e among their most important characteristics, and are to some extent analogous to the shapes of animals and plants in the organic world.

In order to make a body crystallize in solidifying, the following method is employed. Suppose the given body to be bismuth; the first step is to melt it, and then leave it to itself for a time. The metal naturally begins to solidify first at the surface and at the sides, where it is most directly exposed to cooling influences from without; accordingly, when the outer layer of the metal is solidified, the interior is still in the liquid state. If the upper crust be now removed, and the liquid bismuth poured off, the sides of the vessel will be seen to be covered with a number of beautiful crystals.

If the metal were allowed to stand too long, the entire mass would become solid, the different crystals would unite, and no regularity of structure would be observable. This is the case with a great number of solids; ice is one very remarkable instance.

234. Flowers of Ice.—The tendency of ice to assume a crystalline form is seen in the fern-leaf patterns which appear on the windows in winter, caused by the congealing of moisture on them, and still more distinctly in the symmetrical forms of snow-flakes (see Chap. xxviii.) In a block of ice, however, this crystalline structure does not show itself, owing to the closeness with which the crystals fit into each other, so that a mass of this substance appears almost completely *amorphous*. Tyndall, however, in a very interesting experiment, has succeeded in gradually *decrystallizing* ice, if we may use the expression, and thus exhibiting the crystalline elements of which it is composed. The experiment consists in causing a pencil of solar rays to fall perpendicular to the surfaces of congelation on a sheet of ice, such as is naturally formed upon the surface of water in winter. A lens placed behind the ice (Fig. 225) serves to project upon a screen

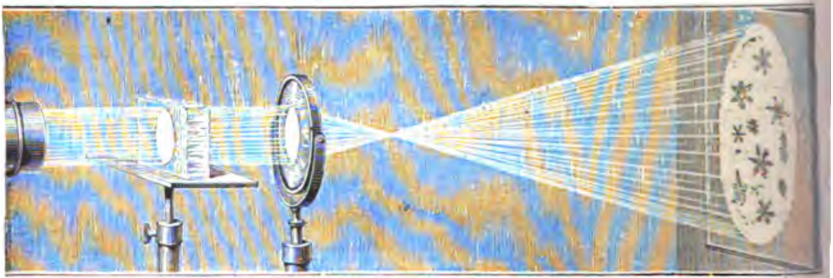

Fig. 225.—Flowers of Ice projected on a Screen.

the image of what is found in the interior of the block. The successive appearances observed upon the screen are shown in Fig. 226. A small luminous circle is first seen, from which branch out rays, resembling the petals of a flower whose pistil is the circle. Frequent changes also occur in the shape of the branches themselves, which are often cut so as to resemble fern-leaves, like those seen upon the windows during frost. In this experiment the solar heat, instead of uniformly melting the mass of ice, which it would certainly do if the mass were amorphous, acts successively upon the different crystals of which it is built up, affecting them in the reverse order of their formation. There are thus produced a number of spaces of regular shape, containing water, nd producing omparatively lark images upon he screen. In the centre of each there is generally a bright spot, hich corresponds to an empty space, depending on the fact that he water occupies a smaller volume than the ice from which it has een produced.

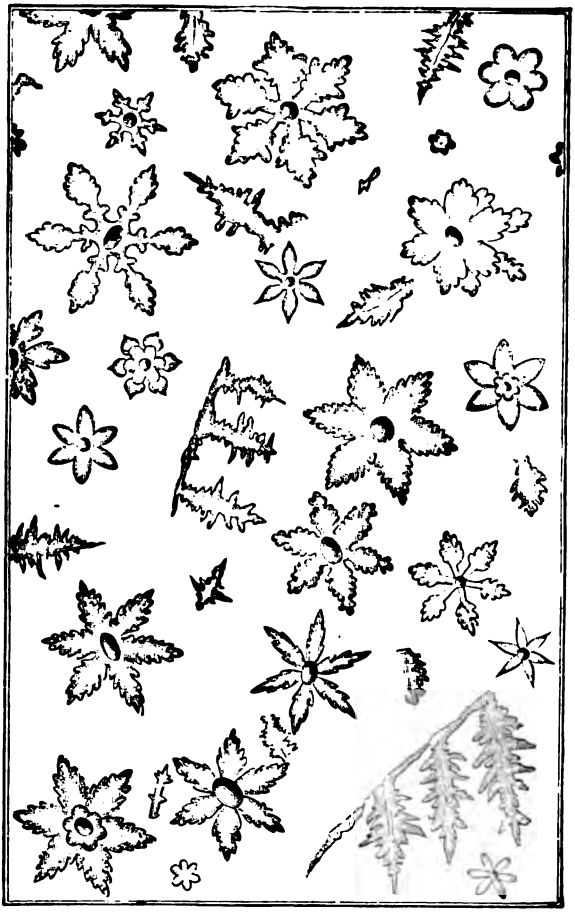

Fig. 226.—Flowers of Ice.

235. Supersaturation.—The proportion of solid matter which a liquid can hold in solution varies according to the temperature; as a general rule, though not by any means in all cases, it increases as the temperature rises. Hence it follows, that if a saturated solution be left to itself, the effect of evaporation or cooling will be gradually to diminish the quantity of matter which can be held in solution. A portion of the dissolved substance will accordingly pass into the solid state, assuming generally a crystalline form. This is an exceedingly common method of obtaining crystals, and is known as the *humid way.*

In connection with this process a phenomenon occurs which is precisely analogous to the cooling of a liquid below its freezing-point. It may be exemplified by the following experiment.

A tube drawn out at one end (Fig. 227) is filled with a warm concentrated solution of sulphate of soda. The solution is boiled, and while ebullition is proceeding freely, the tube is hermetically sealed; by this means the tube is exhausted of air. The solution when left to itself cools without the solid being precipitated, although the liquid is *supersaturated.* But if the end of the tube be broken off, and the air allowed to enter, crystallization immediately commences at the surface, and is quickly propagated through the whole length of the tube; at the same time, as we should expect, a considerable rise of temperature is observed. If the phenomenon does not at once occur on the admission of the air, it can be produced with certainty by throwing a small piece of the solid sulphate into the solution.

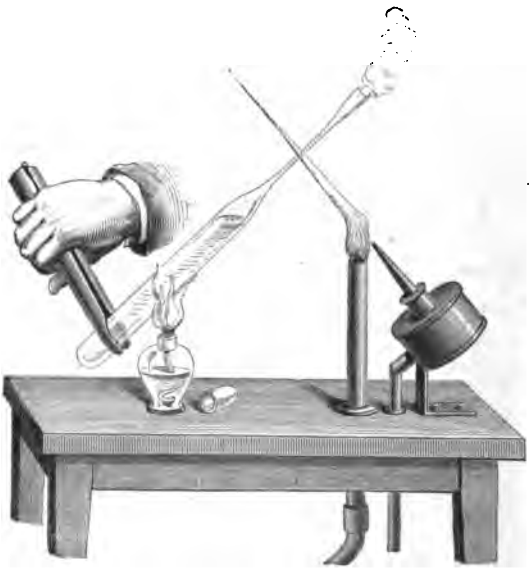

Fig. 227.—Preparation of Supersaturated Solution of Sulphate of Soda.

236. Change of Volume at the Moment of Congelation. Expansive Force of Ice.—In passing from the liquid to the solid state, bodies generally undergo a diminution of volume; there are, however, exceptions, such as ice, bismuth, silver, and cast-iron. It is this property which renders this latter substance so well adapted for the purposes of moulding, as it enables the metal to penetrate completely into every part of the mould. The expansion of ice is considerable, amounting to about $\frac{1}{12}$; its production is attended by enormous mechanical force, just as in the analogous case of expansion by heat.

Its effect in bursting water-pipes is well known. The following experiment illustrates this expansive force. A tube of forged iron (*Fig.* 228) is filled with water, and tightly closed by a screw-stopper.

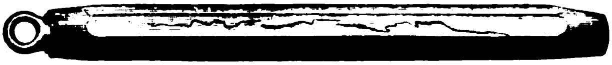

Fig. 228.—Bursting of Iron Tube by Expansion of Water in Freezing.

The tube is then surrounded with a freezing mixture of snow and salt. After some time the water congeals, a loud report is often heard, and the tube is found to be rent.

The following experiment, performed by Major Williams at Quebec, is still more striking. He filled a 12-inch shell with water and closed it with a wooden stopper, driven in with a mallet. The shell was then exposed to the air, the temperature being −28° C. (−18° F.) The water froze, and the bung was projected to a distance of more than 100 yards, while a cylinder of ice of about 8 inches in length was protruded from the hole. In another experiment the shell split in halves, and a sheet of ice issued from the rent (Fig. 229).

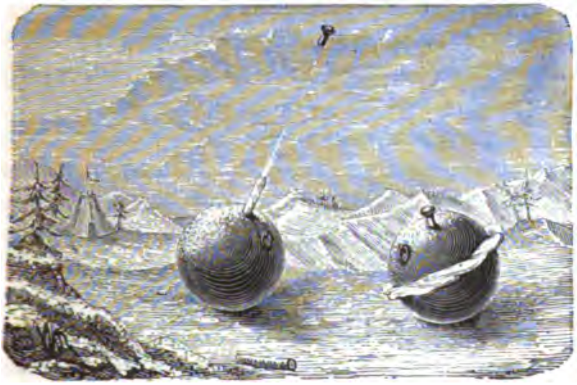

Fig. 229.—Experiment of Major Williams.

It is the expansion and consequent lightness of ice which enables it to float upon the surface of water, and thus afford a protection to animal life below.

237. Effect of Pressure on the Melting-point.—Professor James Thomson was led by theoretical considerations to the conclusion that, in the case of a substance which, like water, expands in solidifying, the freezing (or melting) point must of necessity be lowered by pressure, and that a mixture of ice and ice-cold water would fall in temperature on the application of pressure. His reasoning[1] consisted in showing that it would otherwise be possible (theoretically at least) to construct a machine which should be a perpetual source of work without supply; that is, what is commonly called a perpetual motion.

The matter was shortly afterwards put to the test of experiment by Professor (now Sir) W. Thomson, who compressed, in an Œrsted's piezometer, a mixture of ice and water, in which was inserted a very delicate thermometer protected from pressure in the same manner as the instrument represented in Fig. 194c (§189). The thermometer showed a regular fall of temperature as pressure was applied, followed by a return to 0° C. on removing the pressure. Pressures of 8·1 and 16·8 atmospheres (in excess of atmospheric pressure) lowered the freezing-point by ·106 and ·232 of a degree Fahr. respectively as indicated by the thermometer, results which agree almost exactly with Prof. J. Thomson's prediction of ·0075 of a degree Cent., or ·0135 of a degree Fahr. per atmosphere.

Mousson has since succeeded in reducing the melting-point several degrees by means of enormous pressure. He employed two forms of apparatus, by the first of which he melted ice at the temperature of −5° C., and kept the water thus produced for a considerable time at this temperature. This apparatus had windows (consisting of blocks of glass) in its sides, through which the melting of the ice was seen. His second form of apparatus, which bore a general resemblance to the first, is represented in the annexed figure. It consisted of a steel prism with a cylindrical bore, having one of its extremities closed by a conical stopper strongly screwed in, the rest of the bore being traversed by a screw-piston of steel. The apparatus was inverted, and nearly filled with water recently boiled, into which a piece of copper was dropped, to serve as an index. The apparatus,

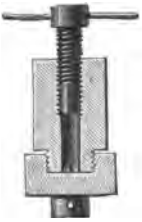
Fig. 230.
Mousson's Apparatus.

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[1] *Transactions Royal Society, Edinburgh.* January, 1849.—*Cambridge and Dublin Math. Journal.* November, 1850.

still remaining in the inverted position, was surrounded by a freezing mixture, by means of which the water was reduced to ice at the temperature of −18° C. The stopper was then screwed into its place, and the apparatus placed in the erect position. The piston was then screwed down upon the ice with great force, the pressure exerted being estimated in some of the experiments at several thousand atmospheres. The pressure was then relaxed, and, on removing the stopper, the copper index was found to have fallen to the bottom of the bore, showing that the ice had been liquefied.

Experiments conducted by Bunsen and Hopkins have shown that wax, spermaceti, sulphur, stearin, and paraffin—substances which, unlike ice, expand in melting—have their melting points *raised* by pressure, a result which had been predicted by Professor W. Thomson.

237A. Effect of Stress in general upon Melting and Solution.— In the experiments above described, the pressure applied was hydrostatical, and was therefore equal in all directions. But a solid may be exposed to pressure in one direction only, or to pull in one or more directions, or it may be subjected to shearing, twisting, or bending *forces*, all these being included under the general name of *stress*.

Reasoning, based on the general laws of energy, leads to the conclusion that stress of any kind other than hydrostatic, applied to a solid, must lower its melting-point. To quote Professor J. Thomson (*Proc. Roy. Soc.* Dec. 1861), "Any stresses whatever, tending to change he form of a piece of ice in ice-cold water, must impart to the ice a endency to melt away, and to give out its cold, which will tend to enerate, from the surrounding water, an equivalent quantity of ice ee from the applied stresses," and "stresses tending to change the rm of any crystals in the saturated solutions from which they have en crystallized must give them a tendency to dissolve away, and to nerate, in substitution for themselves, other crystals free from the plied stresses or any equivalent stresses."[1] This conclusion he ver- d by experiments on crystals of common salt. He at the same e suggested, as an important subject for investigation, the effect

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Professor Thomson draws these inferences from the following principle, which he mes (we think justly) as a physical axiom:—If any substance or system of substances n a condition in which it is free to change its state [as ice, for example, in contact water at 0° C., is free to melt], and if mechanical work be applied to it as potential gy in such a way that the occurrence of the change of state will make it lose that anical work from the condition of potential energy, without receiving other potential y as an equivalent; *then the substance or system will pass into the changed state.*

of hydrostatic pressure on the crystallization of solutions, a subject which was afterwards taken up experimentally by Sorby, who obtained effects analogous to those above indicated as occurring in connection with the melting of ice and wax.

238. Regelation of Ice.—Faraday in 1850 called attention to the fact that pieces of moist ice placed in contact with one another will freeze together even in a warm atmosphere. This phenomenon, to which Tyndall has given the name of *regelation*, admits of ready explanation by the principles just enunciated. Capillary action at the boundaries of the film of water which connects the pieces placed in contact, produces an effect equivalent to attraction between them, just as two plates of clean glass with a film of water between them seem to adhere. Ice being wetted by water, the boundary of the connecting film is concave, and this concavity implies a diminution of pressure in the interior. The film, therefore, exerts upon the ice a pressure less than atmospheric; and as the remote sides of the blocks are exposed to atmospheric pressure, there is a resultant force urging them together and producing stress at the small surface of contact. Melting of the ice therefore occurs at the places of contact, and the cold thus evolved freezes the adjacent portions of the water film, which, being at less than atmospheric pressure, will begin to freeze at a temperature a little above the ordinary freezing-point.

As regards the amount of the force urging the pieces together, if two flat pieces of ice be supported with their faces vertical, and if they be united by a film from whose lower edge water trickles away, the hydrostatic pressure at any point within this film is less than atmospheric by an amount represented, in weight of water, by the height of this point above the part from which water trickles. If, for simplicity, we suppose the film circular, the plates will be pressed together with a force equal to the weight of a cylinder of water whose base is the film and whose height is the radius.

239. Apparent Plasticity of Ice. Motion of Glaciers.—A glacier may be described in general terms as a mass of ice deriving its origin from mountain snows, and extending from the snow-fields along channels in the mountain sides to the valleys beneath.

The first accurate observations on the movements of glaciers were made in 1842, by the late Professor (afterwards Principal) J. D. Forbes, who established the fact that glaciers descend along their beds with a motion resembling that of a pailful of mortar poured into a sloping trough; the surface moving faster than the bottom and the centre faster than the sides. He summed up his view by saying, "A glacier is an imperfect fluid, or a viscous body which is urged down slopes of a certain inclination by the mutual pressure of its parts."

This apparent viscosity is explained by the principles of § 237A. According to these principles the ice should melt away at the places where stress is most severe, an equivalent quantity of ice being formed elsewhere. The ice would thus gradually yield to the applied forces, and might be moulded into new forms, without undergoing rupture. Breaches of continuity might be produced in places where the stress consisted mainly of a pull, for the pull would lower the freezing-point, and thus indirectly as well as directly tend to produce ruptures, in the form of fissures transverse to the direction of most intense pull. The effect of violent compression in any direction would, on the other hand, be, not to crack the ice, but to melt a portion of its interior sufficient to relieve the pressure in the particular part affected, and to transfer the excess of material to neighbouring parts, which must in their turn give way in the same gradual manner.

In connection with this explanation it is to be observed that the temperature of a glacier is always about 0° C., and that its structure is eminently porous and permeated with ice-cold water. These are conditions eminently favourable (the former, but not the latter, being essential) to the production of changes of form depending on the lowering of the melting-point by stresses.

This explanation is due to Professor J. Thomson[1] (*British Association Report*, 1857). Professor Tyndall had previously attempted to account for the phenomena of glacier motion by supposing that the ice is fractured by the forces to which it is subjected, and that the broken pieces, after being pushed into their new positions, are united by regelation. In support of this view he performed several very interesting and novel experiments on the moulding of ice by pressure, such as striking medals of ice with a die, and producing a clear transparent cake of ice by powerfully compressing broken pieces in a boxwood mould (Fig. 231).

Interesting experiments on the plasticity of ice may be performed by filling an iron shell with water and placing it in a freezing mixture, leaving the aperture open. As the water freezes, a cylinder of ice will be gradually protruded. This experiment is due to Mr. Christie. Professor Forbes obtained a similar result by using a very strong glass jar; and by smearing the interior, just below the neck, with colouring matter, he demonstrated that the external layer of ice which was first formed, slid along the glass as the freezing proceeded, until it was at length protruded beyond the mouth.

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[1] If it should be objected that the lowering of the melting-point by stress is too insignificant to produce the vast effects here attributed to it, the answer is that, when ice and water are present together, the slightest difference is sufficient to determine which portion of the water shall freeze, or which portion of the ice shall melt. In default of a more powerful cause, those portions of ice which are most stressed will melt first.

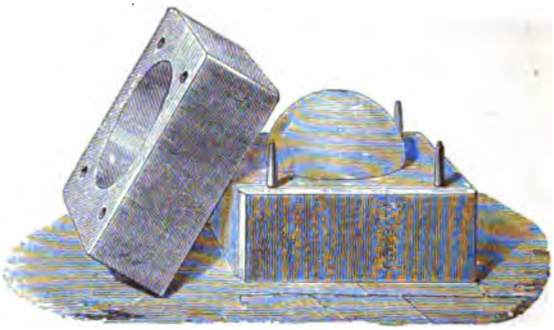

Fig. 231.—Ice Moulded by Pressure.

In the experiments of Major Williams, described in § 236, it is probable that much of the water remained unfrozen until its pressure was relieved by the bursting of the shells.

CHAPTER XXV.

EVAPORATION.

240. Transformation into the State of Vapour.—The majority of liquids, when left to themselves in contact with the atmosphere, gradually pass into the state of vapour and disappear. This phenomenon occurs much more rapidly with some liquids than with others, and those which evaporate most readily are said to be the most volatile. Thus, if a drop of ether be let fall upon any substance, it disappears almost instantaneously; alcohol also evaporates very quickly, but water requires a much longer time for a similar transformation. The change is in all cases accelerated by an increase of temperature; in fact, when we *dry* a body before the fire, we are simply availing ourselves of this property of heat to hasten the evaporation of the moisture of the body. Evaporation may also take place from solids. Thus camphor, iodine, and several other substances pass directly from the solid to the gaseous state, and we shall see hereafter that the vapour of ice can be detected at temperatures far below the freezing-point.

Evaporation, unlike fusion, occurs over a very wide range of temperature. There appears, however, to be a temperature for each ubstance, below which evaporation, if it exist at all, cannot be etected. This is the case with mercury at 0° C., and with sulphuric cid at ordinary atmospheric temperatures.

241. Vapour, Gas.—The words *gas* and *vapour* have no essential ifference of meaning. A vapour is the gas into which a liquid is anged by evaporation. Every gas is probably the vapour of a rtain liquid. The word vapour is especially applied to the gaseous ndition of bodies which are usually met with in the liquid or solid ate, as water, sulphur, &c.; while the word gas generally denotes a dy which, under ordinary conditions, is never found in any state but the gaseous. There are a few gases which experimenters have hitherto been unable to obtain under any other form. These are oxygen, hydrogen, nitrogen, nitric oxide, carbonic oxide, and marsh gas; they are sometimes called *permanent gases.*

242. Elastic Force of Vapours. Maximum Tension.—The characteristic property of gases is their expansibility or elastic force.[1] This may be exemplified in the case of vapours by the following experiment.

A glass globe A (Fig. 232) is fitted with a metal cap provided with two openings, one of which can be made to communicate with a mercurial manometer, while the other is furnished with a stop-cock R. The globe is first exhausted of air by establishing communication through R with an air-pump. The mercury rises in the left-hand and falls in the right-hand branch of the manometer; the final difference of level in the two branches differing from the height of the barometer only by the very small quantity representing the tension of the air left behind by the machine. The stop-cock R is then closed, and a second stop-cock R′ surmounted by a funnel is fixed above it. The hole in this second stop-cock, instead of going quite through the metal, extends only half-way, so as merely to form a cavity. This cavity serves to introduce a liquid into the globe, without any communication taking place between the globe and the external air. For this purpose we have only to fill the funnel with a liquid, to open the cock R, and to turn that at R′ backwards and forwards several times. It will be found, that after the introduction of a small quantity of liquid into the globe, the mercurial column begins to descend in the left branch of the manometer, thus indicating an increase of elastic force. This elastic force goes on increasing as a greater quantity of liquid is introduced into the globe; and as no liquid is visible in the globe, we must infer that it evaporates as fast as it is introduced, and that the fall of the mercurial column is caused by the elastic force of the vapour thus formed.

This increase of pressure, however, does not go on indefinitely. After a time the difference of level in the two branches of the manometer ceases to increase, and a little of the unevaporated liquid may be seen in the globe, which increases in quantity as more liquid is introduced. From this important experiment we conclude that there is a limit to the quantity of vapour which can be formed at a given temperature in an empty space. When this limit is reached, the space is said to be *saturated*, and the vapour then contained in it is at *maximum tension*, and at *maximum density*. It evidently follows from this that if a quantity of vapour at less than its maximum tension be inclosed in a given space, and then compressed at constant

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[1] The names *pressure, tension,* and *elastic force,* are used interchangeably.

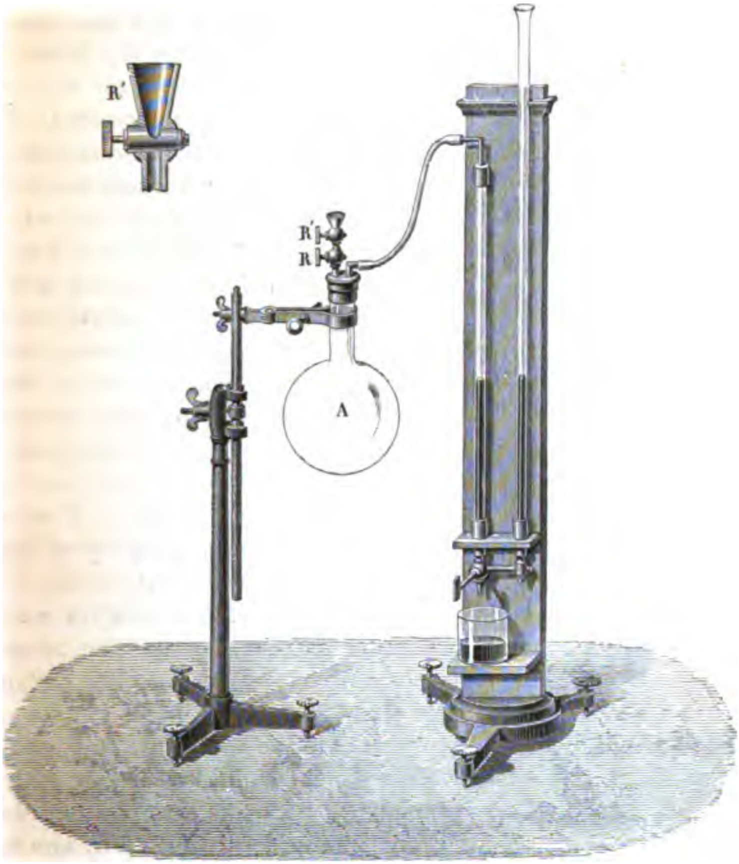

Fig. 232.—Apparatus for studying the Formation of Vapours.

temperature, its tension and density will increase at first, but that after a time a point will be reached when further compression, instead of increasing the density of the vapour, will only cause some of it to pass into the liquid state. This last result may be directly verified by the following experiment. A barometric tube *ab* (Fig. 233) is filled with mercury, with the exception of a small space, into which a few drops of ether are introduced, care having first been but the gaseous. There are a few gases which experimenters have hitherto been unable to obtain under any other form. These are oxygen, hydrogen, nitrogen, nitric oxide, carbonic oxide, and marsh gas; they are sometimes called *permanent gases.*

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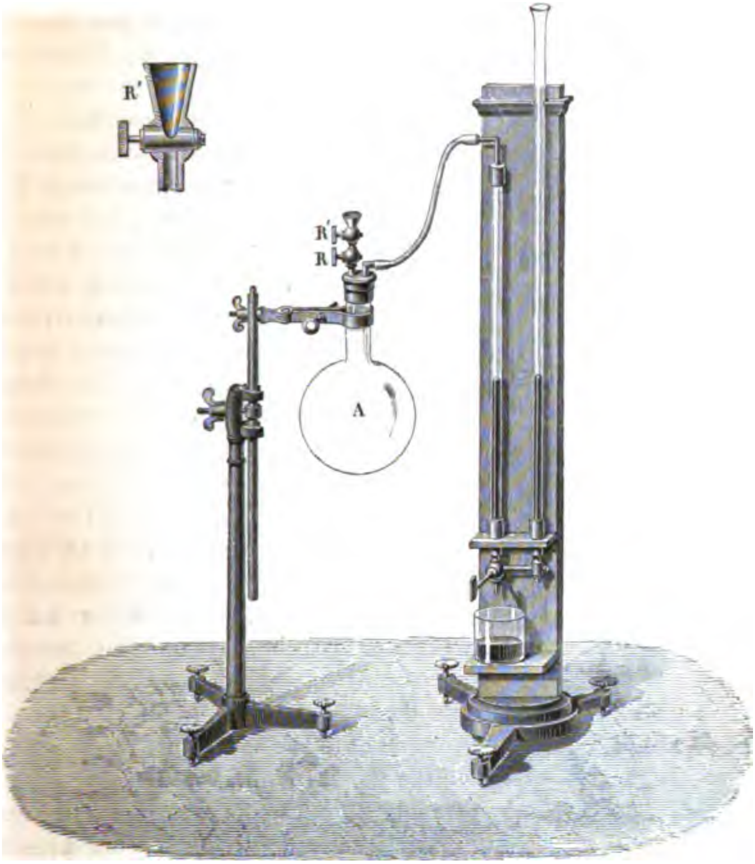

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3) is filled with mercury, with the exception of a small space, into
ich a few drops of ether are introduced, care having first been

taken to expel any bubbles of air which may have remained hering to the mercury. The tube is then inverted in the deep MN, when the ether ascends to the surface of the mercury, is converted into vapour, and produces a sensible depression of

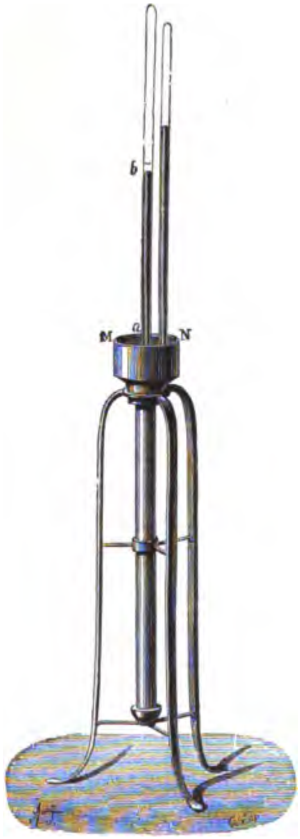

Fig. 233.—Maximum Tension of Vapours.

mercurial column. If the quantity of be sufficiently small, and if the tub kept sufficiently high, no liquid wi perceived in the space above the merc this space, in fact, is not saturated. tension of the vapour which occupies given by the difference between the h of the column in the tube and of a meter placed beside it. If the tub gradually lowered, this difference w first be seen to increase, that is, the te of the vapour of ether increases; we continue the process, a portion of ether will be observed to collect abov mercury, and after this, if we lowe tube any further, the height of the cury in it remains invariable. The effect is to increase the quantity of deposited from the vapour.[1]

243. Influence of Temperature o Maximum Tension.—Returning no the apparatus represented in Fig. 23 pose that some of the liquid remai evaporated in the bottom of the glob let the globe be subjected to an in of temperature. An increase of force will at once be indicated by the ometer, while the quantity of liqui be diminished. The maximum tens a vapour, therefore, and also its maximum density, increase wi temperature; and consequently, in order to saturate a given a quantity of vapour is required which increases with the ten ture. In a subsequent chapter we shall give the results of e ments on the maximum tension of aqueous vapour at different temperatures, and it will be seen that the increase is exceedingly rapid.

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[1] Strictly speaking, there will be a slight additional depression of the mercurial due to the weight of the liquid thus deposited on its summit; but this effect will g be very small, as the vapour occupies much more space than the liquid which it yi

Fig. 234 is a graphical representation of the rate at which the maximum density of a vapour increases with the temperature. Lengths are laid off on the base-line AB, to represent temperatures from −20° to +35° C., and ordinates are erected at every fifth degree, proportional to the weights of vapour required to saturate the same space at different temperatures. The curve CD, drawn through the extremities of these ordinates, is the curve of vapour-density as function of temperature. The figures on the right hand indicate the number of grammes of vapour required to saturate a cubic metre.

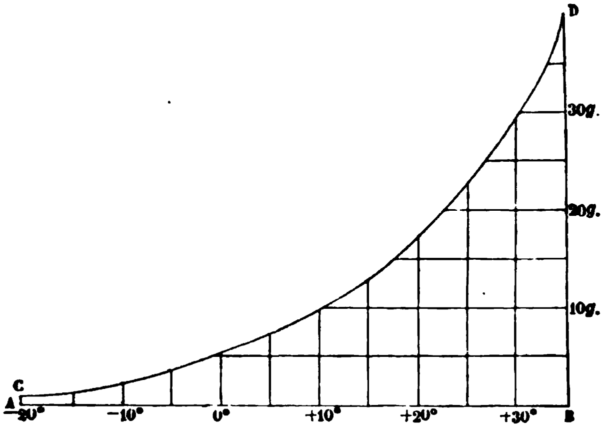

Fig. 234.—Saturation at different Temperatures.

244. Mixture of Gas and Vapour.—The experiments with the pparatus of Fig. 232 may be repeated after filling the globe with ry air, or any other dry gas, and the results finally obtained will e the same as with the exhausted globe. If, as before, we introduce ccessive small quantities of a liquid, it will be converted into vapour, d the pressure will go on increasing till saturation is attained; the astic force of vapour will then be found to be exactly the same as the case of the vacuous globe, and the quantity of liquid evarated will also be the same.

There is, however, one important difference. In the vacuum the mplete evaporation of the liquid is almost instantaneous; in a gas, the other hand, the evaporation and consequent increase of prese proceed with comparative slowness; and the difference between

the two cases is more marked in proportion as the pressure o
gas is greater.

We may lay down, then, the two following laws for the mi
of a vapour with a gas:—

1. *The weight of vapour which will enter a given space i*
same whether this space be empty or filled with gas, provided p
of time be allowed.

2. *When a gas is saturated with vapour, the actual tension*
mixture is the sum of the tensions due to the gas and vapour
rately; that is to say, it is equal to the pressure which the gas w
exert if it alone occupied the whole space, plus the maximum
sion of vapour for the temperature of the mixture.

This second law evidently comes under the general rule for
mining the pressure of a mixture of gases (§ 127); and the same
applies to a mixture of gas and vapour when the quantity o
latter *falls short of* saturation. Each element in a mixture of
and vapours exerts the same pressure on the walls of the conta
vessel as it would exert if the other elements were removed.

It is doubtful, however, whether these laws are rigorously
It would rather appear from some of Regnault's experiments
the quantity of vapour taken up in a given space is slightly, th
almost insensibly, diminished, as the density of the gas whic
cupies the space is increased.

245. Liquefaction of Gases.—When vapour exists in the st
saturation, any diminution in the volume must, if the temper
is preserved constant, involve the liquefaction of as much
vapour as would occupy the difference of volumes; and the v
which remains will still be at the original density and tensio
vapour existing by itself may therefore be completely liquefi
subjecting it to a pressure exceeding, by ever so slight an am
the maximum tension corresponding to the temperature, pro
that the containing vessel is prevented from rising in temperat

Again, if a vapour at saturation be subjected to a fall of
perature, while its volume remains unchanged, a portion of it
be liquefied corresponding to the difference between the dens
saturation at the higher and at the lower temperature. This
tion will obviously diminish the tension, since this will now
maximum tension corresponding to the lower instead of
higher temperature.

There are therefore two distinct means of liquefying a vap increase of pressure, and lowering of temperature. They are employed sometimes separately, and sometimes in conjunction.

Fig. 235 represents the apparatus usually employed for obtaining sulphurous acid in the liquid state. The gas, which is generated in a glass globe, passes first into a washing-bottle, then through a drying-tube, and finally into a *tube* surrounded with a freezing mixture of snow and salt.

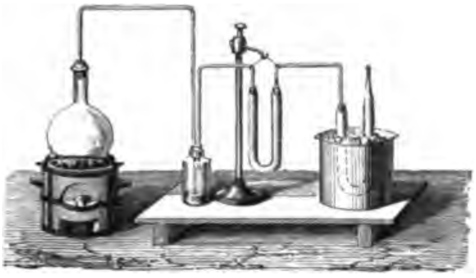

Fig. 235.—Liquefaction of Sulphurous Acid.

Pouillet's apparatus, described in § 120, serves to liquefy most gases by means of compression

In order to ascertain the pressures at which liquefaction takes place, or, in other words, the maximum tensions of gases, one of the tubes in that apparatus is replaced by a shorter tube, containing atmospheric air, and serving as a manometer.

By this means Pouillet has found that, at the temperature of 10°C., sulphurous acid is *liquefied* by a pressure of 2½ atmospheres, *nitrous* oxide by a pressure of 43, and carbonic acid by a pressure of 45 atmospheres.

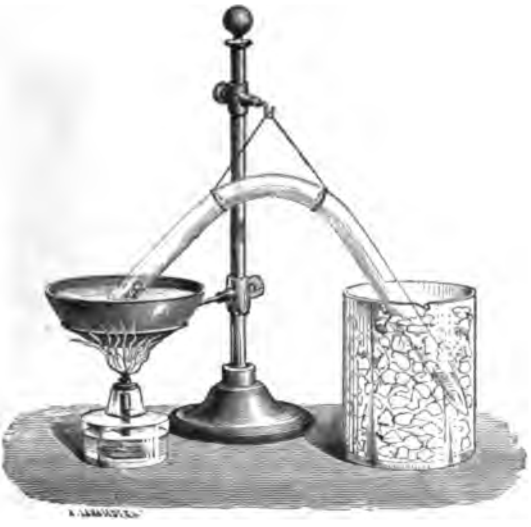

Fig. 236.—Faraday's Apparatus.

246. Faraday's Me- hod.—Faraday, who was the first to con- luct methodical ex- eriments on the quefaction of gases, mployed, in the first istance, the simple paratus represent- l in Fig. 236. It nsists of a very strong bent glass tube, one end of which contains gredients which evolve the gas on the application of heat, while the other is immersed in a freezing mixture. The pressure pro by the evolution of the gas in large quantity in a confined combines with the cold of the freezing mixture to produce liq tion of the gas, and the liquid accordingly collects in the cold the tube.

Thilorier, about the year 1834, invented the apparatus repres in Fig. 237, which is based on this method of Faraday, and tended for liquefying carbonic acid gas. This operation requir enormous pressure of about fifty atmospheres at ordinary ten tures. If a slight rise of temperature occur from the chemical attending the production of the gas, a pressure of 75 or 80 spheres may not improbably be required. Hence great care is sary in testing the strength of the metal employed in the con tion of the apparatus. It was formerly made of cast iro

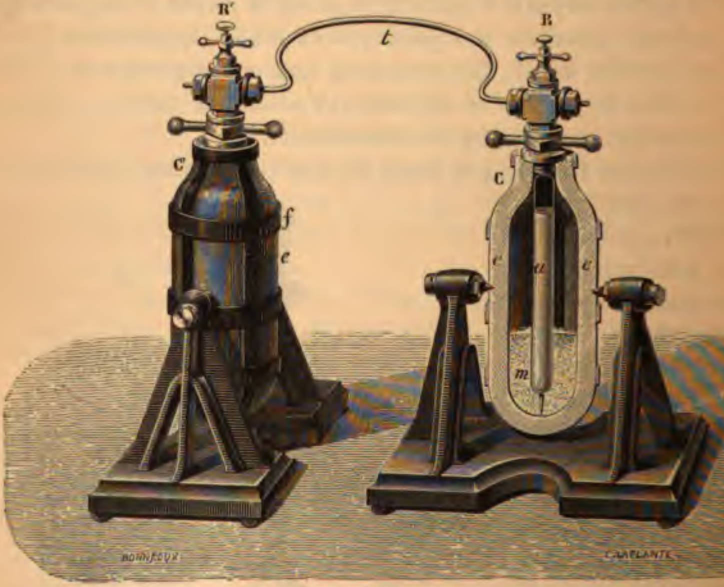

Fig. 237.—Thilorier's Apparatus.

strengthened by wrought-iron hoops; but the construction h been changed on account of a terrible explosion, which cost of one of the operators. At present the vessels are formed o parts; the inner one of lead, the next *e*, which completely e this, of copper, and finally, the hoops *ff* of wrought iron (F which bind the whole together. The apparatus consists of t tinct reservoirs. In the generator C is placed bicarbonate of soda, and a vertical tube *a*, open at top, containing sulphuric acid. By imparting an oscillatory movement to the vessel about the two pivots which support it near the middle, the sulphuric acid is gradually spilt, and the carbonic acid is evolved, and becomes liquid in the interior. The generator is then connected with the condenser C′ by the tube *t*, and the stop-cocks R and R′ are opened. As soon as the two vessels are in communication, the liquid carbonic acid passes into the condenser, which is at a lower temperature than the generator, and represents the cold branch of Faraday's apparatus. The generator can then be disconnected and recharged, and thus several pints of liquid carbonic acid may be obtained.

In the foregoing methods, the pressure which produces liquefaction is furnished by the evolution of the gas itself.

In some other forms of apparatus the pressure is obtained by the use of one or more compression-pumps, which force the gas from the vessel in which it is generated into a second vessel, which is kept cool either by ice or a freezing mixture. The apparatus of this kind which is most extensively used is that devised by Bianchi. It consists of a compression-pump driven by a crank furnished with a fly-wheel, and turned by hand.

Faraday, in his later experiments, employed two pumps, the first having a piston of an inch, and the second of only half an inch diameter. The first pump in the earlier stage of the operation forced the gas through the second into the receiver. In the later stage the second pump was also worked, so as to force the gas already condensed to 10, 15, or 20 atmospheres into the receiver at a much higher pressure. The receiver was a tube of green bottle-glass, and was immersed in a very intense freezing mixture, consisting of solid arbonic acid and ether, the cooling effect being sometimes increased y exhausting the air and vapour from the vessel containing the eezing mixture, so as to promote more rapid evaporation.

246 A. Continuity of the Liquid and Gaseous States.—Remarkable sults were obtained by Cagniard de la Tour[1] by heating volatile quids (alcohol, petroleum, and sulphuric ether) in closed tubes of eat strength, and of capacity about double the volume of the closed liquid. At certain temperatures (36° C. for alcohol, and 42° ether) the liquid suddenly disappeared, becoming apparently con-ted into vapour.

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[1] *Ann. de Chim.* II. xxi.

Drion,[1] by similar experiments upon hydrochloric ether, hyponi acid, and sulphurous acid, showed—

1. That the coefficients of apparent expansion of these liq increase rapidly with the temperature.

2. That they become equal to the coefficient of expansion of at temperatures much lower than those at which total conver into vapour occurs.

3. That they may even become double and more than double coefficient of expansion of air; for example, at 130° C. the coeffic of expansion of sulphurous acid was ·009571.

Thilorier had previously shown that the expansion of liquid bonic acid between the temperatures 0° and 30° C. is four time great as that of air.

Drion further observed, that when the temperature was ra very gradually to the point of total vaporization, the free surface its definition, and was replaced by a nebulous zone without defi edges and destitute of reflecting power. This zone increased in both upwards and downwards, but at the same time became visible, until the tube appeared completely empty. The s appearances were reproduced in inverse order on gradually coo the tube.

When the liquid was contained in a capillary tube, or wh capillary tube was partly immersed in it, the curvature of the meni and the capillary elevation decreased as the temperature rose, at length, just before the occurrence of total vaporization, the su became plane, and the level was the same within as without the

Dr. Andrews, by a series of elaborate experiments on carb acid, with the aid of an apparatus which permitted the pressure temperature to be altered independently of each other, has sh that at temperatures above 31° C. this gas cannot be liquefied, when subjected to intense pressure, becomes reduced to a cond in which, though homogeneous, it is neither a liquid nor a When in this condition, lowering of temperature under constant sure will reduce it to a liquid, and diminution of pressure at cons temperature will reduce it to a gas; but in neither case can any br of continuity be detected in the transition.

On the other hand, at temperatures below 31°, the subst remains completely gaseous until the pressure reaches a certain depending on the temperature, and any pressure exceeding this ses liquefaction to commence and to continue till the whole of the is liquefied, the boundary between the liquefied and unliquefied tions being always sharply defined.

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[1] *Ann. de Chim.* III. lvi.

he temperature 31°C., or more exactly 30·92°C. (87·7°F.), may efore be called the critical temperature for carbonic acid; and it robable that every other substance, whether usually occurring in gaseous or in the liquid form, has in like manner its own critical perature. Dr. Andrews found that nitrous oxide, hydrochloric , ammonia, sulphuric ether, and sulphuret of carbon, all exhibited cal temperatures, which, in the case of some of these substances, e above 100°C.

is probable that, in the experiments of Cagniard de la Tour and n, the so-called total conversion into vapour was really conver- into the intermediate condition.

he continuous conversion of a gas into a liquid may be effected irst compressing it at a temperature above its critical tempera- , until it is reduced to the volume which it will occupy when efied, and then cooling it below the critical point.

he continuous conversion of a liquid into a gas may be obtained irst raising it above the critical temperature while kept under sure sufficient to prevent ebullition, and afterwards allowing it xpand.

hen a substance is a little above its critical temperature, and pies a volume which would, at a lower temperature, be com- ble with partial liquefaction, very great changes of volume are uced by very slight changes of pressure.

the other hand, when a substance is at a temperature a little v its critical point, and is partially liquefied, a slight increase of erature leads to a gradual obliteration of the surface of demarca- between the liquid and the gas; and when the whole has thus reduced to a homogeneous fluid, it can be made to exhibit an arance of moving or flickering striæ throughout its entire mass ightly lowering the temperature, or suddenly diminishing the ure.

e apparatus employed in these remarkable experiments, which escribed in the Bakerian Lecture (*Phil. Trans.* 1869), is shown g. 237A, where *cc* are two capillary glass tubes of great strength, f them containing the carbonic acid or other gas to be experi- ed on, the other containing air to serve as a manometer. These onnected with strong copper tubes *dd*, of larger diameter, containing water, and communicating with each other through *ab*, water being separated from the gases by a column of mercury cupying the lower portion of capillary tube. The steel screw are the instruments for appl pressure. By screwing eithe them forward into the water, contents of both tubes are pressed, and the only use of ha two is to give a wider rang compression. A rectangular case (not shown in the figure), cl before and behind with plate-g surrounds each capillary tube, allows it to be maintained at required temperature by the of a stream of water.

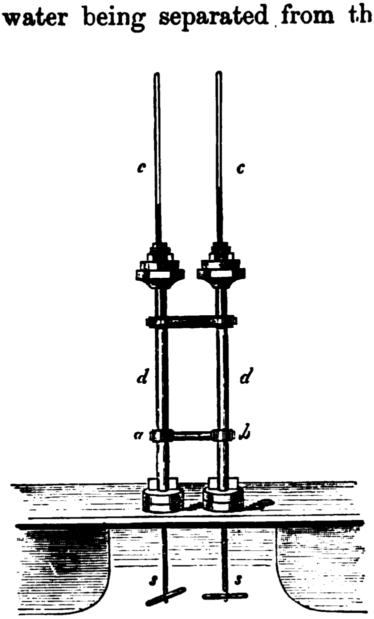

Fig. 237 A.—Andrews' Apparatus.

247. Latent Heat of Vaporiza Cold produced by Evaporation.— passage from the liquid to the eous state is accompanied by disappearance of a large quanti heat. Whenever a liquid evaporates without the application of a depression of temperature occurs. Thus, for instance, if any portion of the skin be kept moist with alcohol or ether, a decided sensation of cold is felt. Water produces the same effect in a smaller degree, because it evaporates less rapidly.

Fig. 238.—Leslie's Experi

The heat which thus disappears in virtue of the passage of a liquid into the gaseous condition, is called the *latent heat of vaporization*. Its amount varies according to the temperature at which the change is effected, and it is exactly restored when the vapour returns to the liquid form, provided that both changes have been effected at the same temperature. Its amount for vapour of water at the temperature 10 is 536°; that is to say, the quantity of heat which disappears i evaporation of a pound of water at this temperature, and which reappears in the condensation of a pound of steam at the same temperature, would be sufficient to raise the temperature of 536 pounds of water from 0° to 1°.

The latent heat of vaporization plays an important part in the heating of buildings by steam. A pound of steam at 100°, in becoming reduced to water at 30°, gives out as much heat as about $8\frac{2}{3}$ lbs. of water at 100° in cooling down to the same temperature.

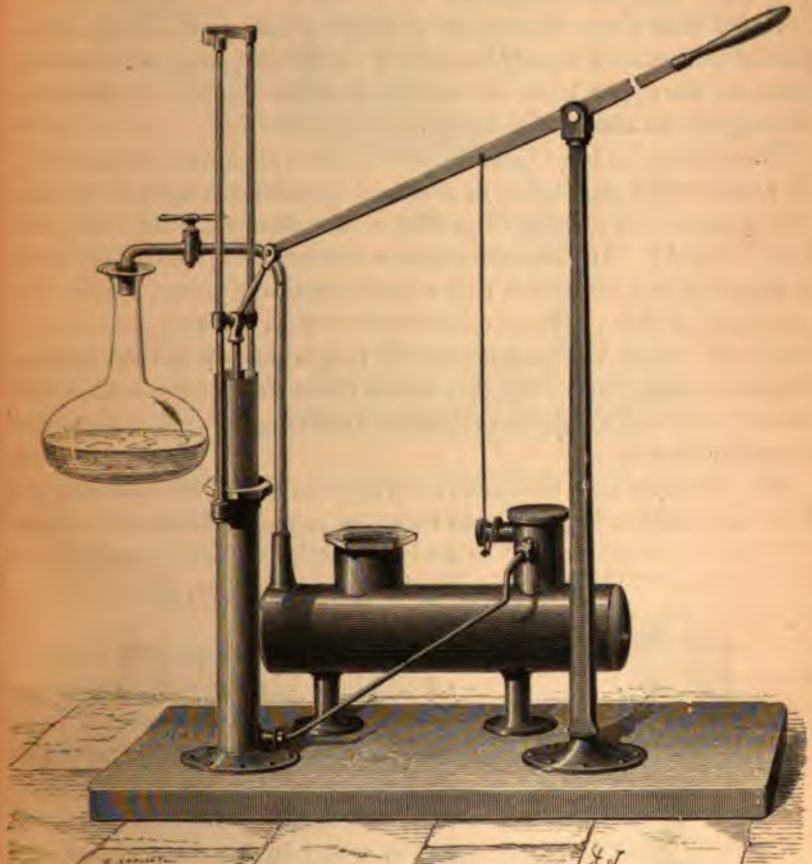

Fig. 239.—Carré's Apparatus.

248. Leslie's Experiment.—Water can be easily frozen by the cold resulting from its own evaporation, as was first shown by Leslie in a celebrated experiment. A small capsule (Fig. 238) of copper is taken, containing a little water, and is placed above a vessel taining strong sulphuric acid. The whole is placed under the rec of an air-pump, which is then exhausted. The water evaporates great rapidity, the vapour being absorbed by the sulphuric ac fast as it is formed, and ice soon begins to appear on the su The experiment is, however, rather difficult to perform success This arises from various causes.

In the first place, the vapour of water which occupies the part of the receiver is only imperfectly absorbed; and, in the se place, as the upper layer of the acid becomes diluted by abso the vapour, its affinity for water rapidly diminishes.

These obstacles have been removed by an apparatus invente M. Carré, which enables us to obtain a considerable mass of ic few minutes. It consists (Fig. 239) of a leaden reservoir conta sulphuric acid. At one extremity is a vertical tube, the end of is bent over and connected with a flask containing water. The extremity of the reservoir communicates with an air-pump, t handle of which is fitted a metallic rod, which drives an ag immersed in the acid. By this means the surface of the acid i tinually renewed, absorption takes place with regularity, and the is rapidly frozen.

249. Cryophorus.—Wollaston's cryophorus (Fig. 240) consist bent tube with a bulb at each end. It is partly filled with and hermetically sealed whil liquid is in ebullition, thus exp the air.

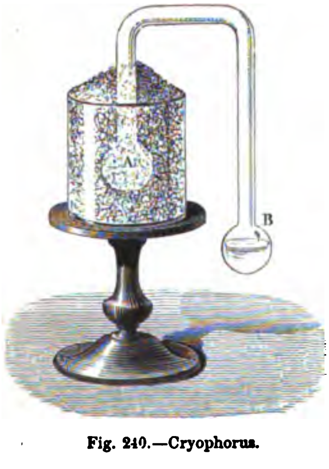

Fig. 240.—Cryophorus.

When an experiment is to be all the liquid is passed into th B, and the bulb A is plunged freezing mixture, or into pound The cold condenses the vapour and thus produces rapid evapo of the water in B. In a short needles of ice appear on the sur the liquid.

250. Freezing of Water by th poration of Ether.—Water is p into a glass tube dipped into which is contained in a glass vessel for the purpose (Fig. 241) means of a pair of bellows a current of air is made to pass th ether; evaporation is quickly produced, and at the end of a few utes the water in the tube is frozen.

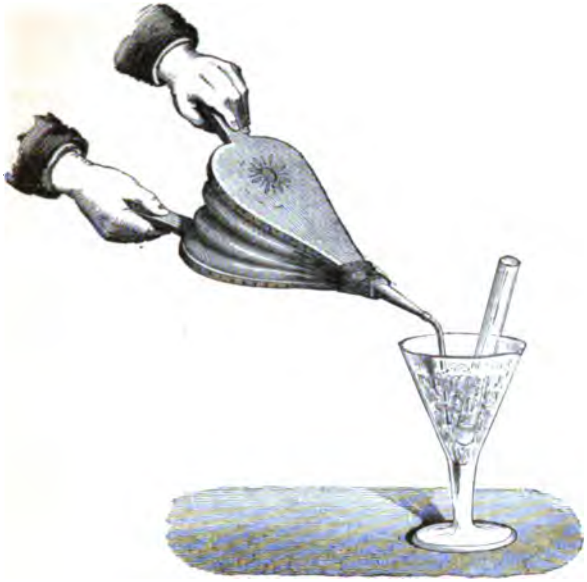

Fig. 241.—Freezing of Water by Evaporation of Ether.

instead of promoting evaporation of the ether by means of a nt of air, the vessel were placed under the exhausted receiver air-pump, a much greater fall of temperature would be obd, and even mercury might easily be frozen. This experiment, ver, is injurious to the pump, owing to the sol- action of the ether on the oil with which the s and other moving parts are lubricated.

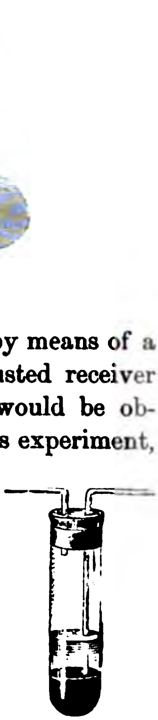

Fig. 242. Freezing of Mercury by Evaporation of Sulphurous Acid.

1. **Freezing of Mercury by means of Sulphurous** —Mercury may be frozen by means of liquid urous acid, which is much more volatile than In order to escape the suffocating action of the the experiment is performed in the following er:—

a glass vessel (Fig. 242) are poured successively ry and liquid sulphurous acid. The vessel is by an india-rubber stopper, in which two glass are fitted. One of these dips to the bottom of the sulphurcid, and is connected at its outer end with a bladder full of

air. Air is passed through the liquid by compressing the blad
and escapes, charged with vapour, through the second open
which is fitted with an india-rubber tube leading to the open
Evaporation proceeds with great rapidity, and the mercury s
freezes.

252. Carré's Apparatus.—The apparatus invented some years
by M. Carré for making ice is another instance of the applicatio
cold produced by evaporation. It consists (Figs. 243 and 244) of
parts, a boiler and a cooler. The boiler is of wrought iron, and i
constructed as to give a very large heating surface. It is th

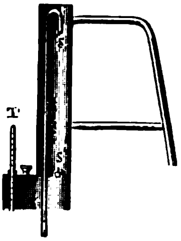

Fig. 243.

Carré's Apparatus.

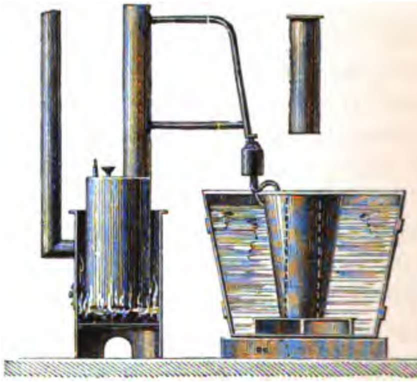

Fig. 244.

quarters filled with a saturated solution of ammonia, which cont
from six to seven hundred times its volume of gas. The cooler
an annular form, and in the central space is placed a vessel cont
ing the water to be frozen. In the sides of the cooler are a nur
of small cells, the object of which is to increase the surface of
tact of the metal with the liquid.

In the first part of the experiment, which is represented ir
figure, the boiler is placed upon a fire, and the temperature raise
130°, while the cooler is surrounded with cold water. Ammon
gas is given off, passes into the cooler by the valve s' opening
wards, and is condensed in the numerous cells above mentio
This first part of the operation, in the small machines for dom
use, occupies about three-quarters of an hour. In the second pa
the operation, the cylindrical vessel containing the water to be fr
is placed in the central space; the cooler is surrounded with an enve
of felt, which is a very bad conductor of heat, and the boiler is

rsed in cold water. The water in the boiler, as it cools, is able
ain to receive and dissolve the gas, which enters by the valve *s* of
bent siphon-shaped tube. The liquid ammonia in the cooler
ordingly evaporates with great rapidity, producing a fall of tem-
ature which freezes the water in the inclosed vessel.

253. Solidification of Carbonic Acid.—When a small orifice is opened
a vessel containing liquid carbonic acid, evaporation proceeds so
idly that the cold resulting from it freezes a portion of the vapour,
ich takes the form of fine snow, and may be collected in consider-
e quantity.

This carbonic acid snow, which was first obtained by Thilorier, is
dily dissolved by ether, and forms with it one of the most intense
zing mixtures known. By immersing tubes containing liquefied
es in this mixture, Faraday succeeded in reducing several of them,
uding carbonic acid, cyanogen, and nitrous oxide, to the form of
r transparent ice, the fall of temperature being aided, in some
is experiments, by employing an air-pump to promote more rapid
poration of carbonic acid from the mixture. By the latter pro-
he was enabled to obtain a temperature of −166° F. (−110° C.)
ndicated by an alcohol thermometer, the alcohol itself being re-
ed to the consistence of oil. Despretz, by means of the cold
luced by a mixture of solid carbonic acid, liquid nitrous oxide,
ether, rendered alcohol so viscid that it did not run out when
vessel which contained it was inverted.

CHAPTER XXVI.

EBULLITION.

254. Ebullition.—When a liquid contained in an open vessel is
jected to a continual increase of temperature, it is gradually cha
into vapour, which is dissipated in the surrounding atmosp
This action is at first confined to the surface; but after a certain
bubbles of vapour are formed in the i
rior of the liquid, which rise to the
and set the entire mass in motion
more or less vehemence, accompanied
characteristic noise; this is what is m
by ***ebullition*** or ***boiling***.

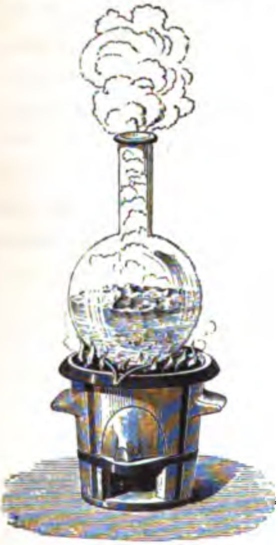

Fig. 245.—Ebullition.

If we observe the gradual progre
this phenomenon,—for example, in a
vessel containing water, we shall per
that, after a certain time, very mi
bubbles are given off; these are bu
of dissolved air. Soon after, at the bo
of the vessel, and at those parts of the
which are most immediately expose
the action of the fire, larger bubbl
vapour are formed, which decrease in
ume as they ascend, and disappear b
reaching the surface. This stage is ac
panied by a peculiar sound, indicati
approaching ebullition, and the liquid is said to be *singing*.
sound is probably caused by the collapsing of the bubbles as the
condensed by the colder water through which they pass. Fin
the bubbles increase in number, growing larger as they ascend,
they burst at the surface, which is thus kept in a state of agita
the liquid is then said to boil.

255. **Laws of Ebullition.**—1. *At the ordinary pressure, ebullition nmences at a temperature which (roughly speaking) is definite for h liquid.*

This law is analogous to that of fusion (§ 225). It follows from s that the boiling-point of any liquid is a *specific* element, serving letermine its nature.

The following table gives the boiling-points of several liquids at pressure of 760 millimetres:—

Sulphurous acid,	−10° C.	**Spirits** of turpentine,	+130° C.
Hydrochloric ether,	+11°	**Phosphorus**,	290°
Common ether,	37°	**Concentrated** sulphuric acid,	325°
Alcohol,	79°	**Mercury**,	353°
Distilled water,	100°	**Sulphur**,	440°

. *The temperature, in ordinary circumstances, remains constant ing ebullition.* If a thermometer be introduced into the glass el of Fig. 245, the temperature will be observed to rise gradually ing the different stages preceding ebullition; but, when active llition has once commenced, no further variation of temperature be observed. This phenomenon points to the same conclusion he cold produced by evaporation.

nce, notwithstanding the continuous action of the fire, the perature remains constant, the conclusion is inevitable, that all heat produced is employed in doing the work necessary to change liquid into vapour. The constancy of temperature during ebulli- explains the fact that vessels of pewter, tin, or any other easily le metal, may be safely exposed to the action of even a very hot provided that they contain water, since the liquid remains at a erature of about 100°, and its contact prevents the vessel from -heating.

The elastic force of the vapour given off during ebullition is l to the pressure of the external air.

is important proposition may be experimentally demonstrated e following manner:—

e take a bent tube A, open at the longer extremity, and closed ie shorter. The short branch is filled with mercury, all but a l space containing water; in the long branch the mercury stands le higher than the bend. Water is now boiled in a glass vessel, during ebullition, the bent tube is plunged into the steam. vater occupying the upper part of the short branch is partially erted into steam, the mercury falls, and it *assumes the same* *level in both branches.* Thus the pressure exerted by the atmosp at the open extremity of the tube is exactly equal to that exerte the vapour formed by wate ebullition.

256. Theory of Ebullitio This latter circumstance sup the true physical definitio ebullition. *A liquid is in eb tion when it gives off vapo the same tension as the a sphere above it.*

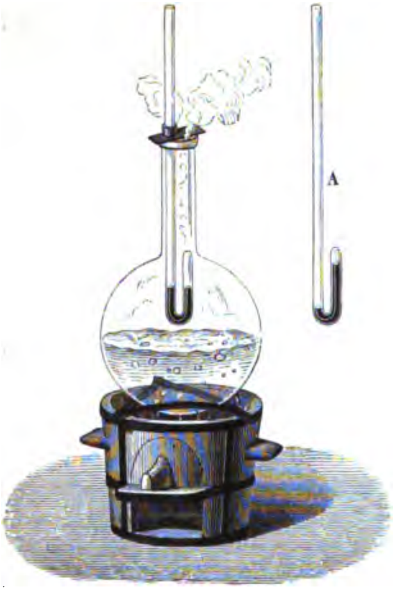

Fig. 246.—Tension of Vapour during Ebullition.

The necessity of this equ of tension is easily explained a bubble of vapour exists i interior of a liquid as at *m* 247), it is subject to a pre exceeding atmospheric by weight of the liquid above it. the bubble rises, the latter ment of pressure becomes and the tension of the v composing the bubble accord diminishes, until it is reduc atmospheric pressure on reaching the surface.

The boiling-point of a liquid is therefore necessarily fixed, si is the temperature at which the tension of the vapour at satur is equal to that of the atmosphere. It must be remarked, how that this temperature varies in the different layers of the liquid that it increases with the depth below the surface. According determining the second fixed point of the thermometer, we stated that the instrument should be plunged int steam, and not into the water.

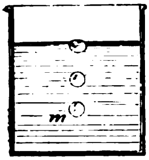

Fig. 247.

257. Effect of Pressure upon the Boiling-poin evidently follows from the foregoing consideration the boiling-point of a liquid must vary with the sure on the surface; and experiment shows tha is the case. Water, for instance, boils at 100° the external pressure of 760 millimetres; but pressure decreases, ebullition occurs at a lower temperature. the receiver of an air-pump, water may be made to boil at any perature between 0° and 100°. In Carré's apparatus (Fig. 239) the water in the glass bottle is observed to enter into active ebullition a few moments before the appearance of the ice. The reason, therefore, why boiling water has come to be associated in our minds with a fixed temperature is that the variations of atmospheric pressure are comparatively small.

At Paris, for instance, the external pressure varies between 720 and 790 millimetres (28·3 and 31·1 inches), and the boiling-point, in consequence, varies from 98·5° to 101·1°.

258. Franklin's Experiment.—The boiling of water at a temperature lower than 100° may be shown by the following experiment:—

A little water is boiled in a flask for a sufficient time to expel most of the air contained in it. The flask is then removed from the source of heat, and is at the same time securely corked. To render the exclusion of air still more certain, it may be inverted with the corked end immersed in water which has been boiled. Ebullition ceases almost immediately; but if cold water be now poured over the vessel, or, better still, if ice be applied to it, the liquid again begins to boil, and continues to do so for a considerable time. This fact may easily be explained: the contact of the cold water or the ice lowers the temperature and tension of the steam which presses upon the surface of the liquid, and the decrease of tension causes the renewal of ebullition.

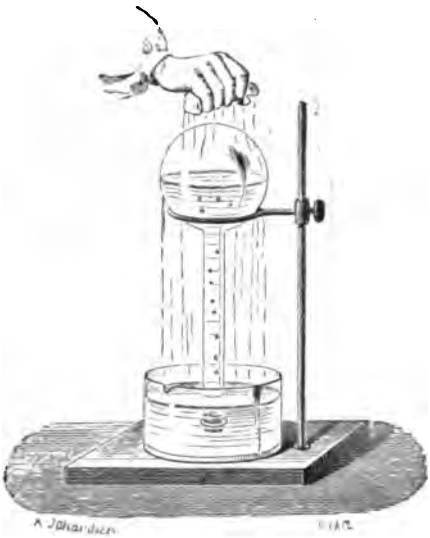
Fig. 248.—Franklin's Experiment.

260. Determination of Heights by Boiling-point.—Just as we can determine the boiling-point of water when the external pressure is given, so, if the boiling-point be known, we can determine the external pressure. In either case we have simply to refer to a table of maximum tensions of aqueous vapour at different temperatures.

As the barometer is essentially unsuitable for portability, Wollaston proposed to substitute the observation of boiling-points as a means of determining pressures. For this purpose he employed a the meter with a large bulb and with a scale extending only a fe grees above and below 100°. He called this instrument the bar tric thermometer.

Regnault has constructed a small instrument for the same pur which he calls the *hypsometer.* It consists of a little boiler he by a spirit-lamp, and terminating in a scope tube with an opening at the through which the steam escapes. A mometer dips into the steam, and pro through the top of the tube so as to the temperature of ebullition to be rea

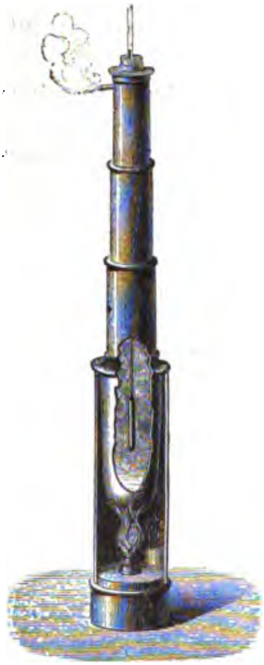

Fig. 250.—Hypsometer.

This temperature at once gives the spheric pressure by reference to a ta vapour-tensions, and the subsequent putations for determining the height a same as when the barometer is emp (§ 112).

When only an approximate result sired, it may be assumed that the h above sea-level is sensibly proportion the difference between the observed bo point and 100° C., and Soret's formula be employed, viz.:

$$h=295\,(100-t),$$

where *h* is expressed in metres an degrees Centigrade.

Thus, at Quito, where the boiling of water is about 90·1°, the height abo level would be 9·9 × 295=2920 metres, which agrees nearly wi true height 2808 metres.

At Madrid, at the mean pressure, the boiling-point is 97·8°, gives 2·2 × 295=649 metres; the actual height being 610 met

261. Papin's Digester.—While a decrease of pressure lowe boiling-point, an increase of pressure raises it. Accordingly, b ting the boiler in communication with a reservoir containing the pressure of several atmospheres, we can raise the boiling-p 110°, 115°, or 120°; a result often of great utility in the arts.

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[1] If *h* be expressed in feet, and *t* in degrees Fahrenheit, the formula become

$$h=538\,(212-t).$$

er that the liquid may actually enter into ebullition, the space
ve the liquid must be sufficiently large and cool to allow of the
lensation of the steam. In a confined vessel, water may be raised
higher temperature than would be possible in the open air, but
ill not boil. This is the case in the apparatus invented by the
orated Papin, and called after him *Papin's digester.* It is a
ize vessel of great strength, covered with a lid secured by a
erful screw. It is employed for raising water to very high tem-
tures, and thus obtaining effects which would not be possible with
er at 100°, such for example as dissolving the gelatine contained
ones.

is to be observed that the tension of the steam increases rapidly
the temperature, and may finally acquire an enormous power.
s, at 200°, the pressure is that of 16 atmospheres, that is about
ounds on the square inch.
rder to obviate the risk of
osion, Papin introduced a
e for preventing the pres-
from exceeding a definite
. This invention has since
applied to the boilers of
-engines, and is well
n as the *safety-valve.* It
sts of an opening, closed
conical valve or stopper,
is pressed down by a
loaded with a weight.
ose the area of the lower
of the stopper to be 1
e inch, and that the pres-
s not to exceed 10 atmo-
es, corresponding to a
rature of 180°. The
tude and position of the weight are so arranged that the pres-
n the hole is 10 times 15 pounds. If the tension of the steam
l 10 atmospheres, the lever will be raised, the steam will escape,
e pressure will thus be relieved.

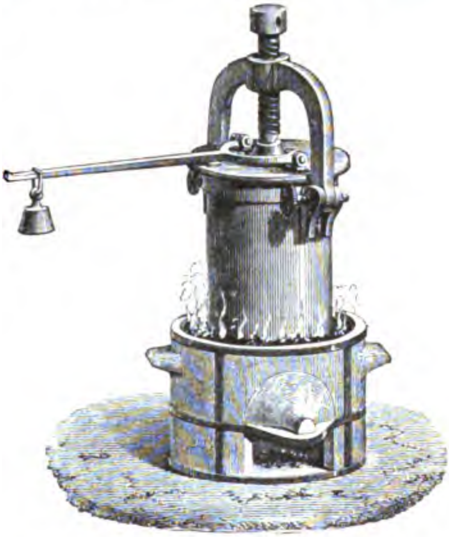

Fig. 251.—Papin's Digester.

en the tension of the steam contained in the digester has be-
considerable, if the lever be raised, so as to permit some steam
pe, it rushes out with a loud noise, and produces a cloud in

the air. On placing the hand in this cloud, scarcely any sensa of heat is experienced, whereas, on performing the same experi with steam at the ordinary pressure, the hand would certainl scalded. This apparently paradoxical result is completely in acc ance with the principles which have already been stated more once. The steam formed at 100°, being at atmospheric pressure, serves its pressure and temperature on issuing into the air. Or other hand, the steam generated in Papin's digester has a pres greatly exceeding that of the atmosphere, and accordingly exp rapidly upon its exit, and thus performs work in forcing back external air. The performance of this work is accompanied by loss of an equivalent quantity of heat, and the temperature of jet is consequently considerably lowered.

262. Boiling-point of Saline Solutions.—When water holds s matters in solution, the boiling-point rises as the proportion of s matter in the water increases. Thus with sea-salt the boiling- can be raised from 100° to 108°.

When the solution is not saturated, the boiling-point is not f but rises gradually as the mixture becomes concentrated; but certain stage the salt begins to be precipitated, and the temper then remains invariable. This is to be considered the normal bo point of the saturated solution. Supersaturation, however, some occurs, the temperature gradually rising above the normal bo point without any deposition of the salt, until all at once prec tion begins, and the thermometer falls several degrees.

The steam emitted by saline solutions consists of pure water it is frequently asserted to have the same temperature as the s of pure water boiling under the same pressure; but the experi of Magnus and others have shown that this is not the case. Ma for example,[1] found that when a solution of chloride of calcium boiling at 107°, a thermometer in the steam indicated 105¼ when by concentration the boiling-point had risen to 116°, the mometer in the steam indicated 111·2°.

These and other observations seem to indicate that the s emitted by a saline solution when boiling, is in the condition in w the steam of pure boiling water would be, if heated, under spheric pressure, to the temperature of the boiling solution. I therefore be cooled down to the boiling-point of pure water wi undergoing any liquefaction. When cooled to this point, it be urated, and precisely resembles the steam of pure water boiling ler the same pressure. When saturated steam loses heat, it does cool, but undergoes partial liquefaction, and it does not become ipletely liquefied till it has lost as much heat as would have cooled e than a thousand times its weight of superheated[1] steam one ree Centigrade.

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[1] Poggendorff's *Annalen*, cxii. p. 415.

62 A. **Boiling-point of Liquid Mixtures.**—A mixture of two liquids ch have an attraction for each other, and will dissolve each other ly in all proportions—for example, water and alcohol—has a boil- point intermediate between those of its constituents. But a hanical mixture of two liquids between which no solvent action s place—for example, water and sulphide of carbon—has a boiling- it lower than either of its constituents. If steam of water is ed into liquid sulphide of carbon, or if sulphide of carbon vapour assed into water, a mixture is obtained which boils at 42·6° C., g four degrees lower than the boiling-point of sulphide of carbon e. This apparent anomaly is a direct consequence of the laws of ours stated in § 244; for the boiling-point of such a mixture is the perature at which the sum of the vapour-tensions of the two pendent ingredients is equal to one atmosphere.

33. **Influence of Dissolved Air upon the Boiling-point.**—The ence of air in the midst of the liquid mass is a necessary ition of regularity of ebullition, and of its production at the al temperature; this is shown by several convincing experi- s.

Donny's Experiment.—We take a glass tube bent twice, and inated at one of its extremities by a series of bulbs. The first is to wash it carefully with alcohol and ether, finally leaving in ne diluted sulphuric acid. These operations are for the purpose moving the solid particles adhering to the sides, which always n portions of air. Water is then introduced and boiled long gh to expel the air dissolved in it, and while ebullition is pro- ng, the end of the apparatus is hermetically sealed. The other mity is now plunged in a strong solution of chloride of calcium, has a very high boiling-point, and the tube is so placed that e water shall lie in this extremity; it will then be found that mperature may be raised to 135° without producing ebullition.

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t is steam heated above the temperature of saturation. Philosophically speaking, ated steam is merely nonsaturated steam; but the name is never used except where perature exceeds the atmospheric boiling-point.

At about this temperature bubbles of steam are seen to be form and the entire liquid mass is thrown forward with great violen

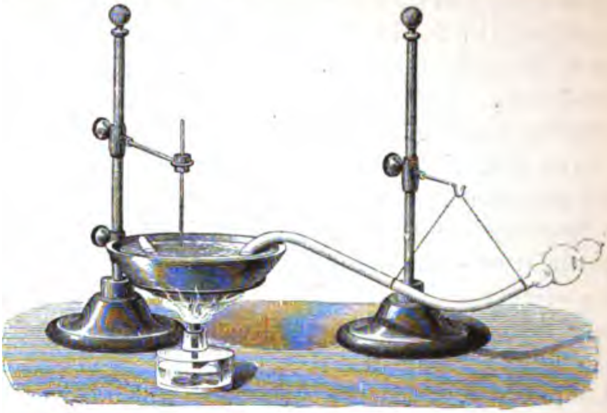

Fig. 252.—Donny's Experiment.

The bulbs at the end of the tube are intended to diminish the sh thus produced.

2. *Dufour's Experiment.*—This experiment is still more decis A mixture of linseed-oil and oil of cloves, whose respective densi are about ·93 and 1·01, is so prepared that, for temperatures near 1 the density of the whole is nearly that of water. This mixtu placed in a cubical box of sheet-iron, with two holes opposite other, which are filled with glass, so as to enable the observe perceive what is passing within. The box is placed in a met envelope, which permits of its being heated laterally. When temperature of 120° has been reached, a large drop of water is allo to fall into the mixture, which, on reaching the bottom of the is partially converted into vapour, and breaks up into a numb smaller drops, some of which take up a position between the windows, so as to be visible to the observer. The temperature now be raised to 140°, 150°, or even 180°, without producing eva tion of any of these drops. Now the maximum tension of stea 180° is equal to 10 atmospheres, and yet we have the remark phenomenon of a drop of water remaining liquid at this tempera under no other pressure than that of the external air increased b inch or two of oil. The reason is that the air necessary to eva tion is not supplied. If the drops be touched with a rod of m or, better still, of wood, they are immediately converted into va

great violence, accompanied by a peculiar noise. This is ained by the fact that the rods used always carry a certain tity of condensed air upon their surface, and by means of this he evaporation is produced. The truth of this explanation is ed by the fact, that when the rods have been used a certain er of times, they lose their power of provoking ebullition, owing, ubt, to the exhaustion of the air which was adhering to their ces.

Production of Ebullition by the formation of Bubbles of Gas e midst of a Liquid.—A retort is carefully washed with uric acid, and then charged with water slightly acidulated, from the air has been expelled by repeated boiling. The retort unicates with a manometer and with an air-pump. The air is sted until a pressure of only 150 millimetres is attained, corre-ing to 60° as boiling-point. Dufour has shown that under conditions the temperature may be gradually raised to 75° ut producing ebullition. But if, while things are in this con-, a current of electricity is sent through the liquid by means of latinum wires previously immersed in it, the bubbles of oxygen ydrogen which are evolved at the wires immediately produce t ebullition, and a portion of the liquid is projected explosively, Donny's experiment.

m these experiments we may conclude that liquid, when not tact with gas, has a difficulty in *making a beginning* of vapor-, and may hence remain in the liquid state even at tempera-t which vaporization would upon the whole involve a fall of ial energy.

t vapour (as well as air) can furnish the means of overcoming ifficulty, is established by the fact noted by Professor G. C. [1] that when a liquid has been boiling for some time in a retort, times ceases to exhibit the movements characteristic of ebulli-though the amount of vapour evolved at the surface, as mea-y the amount of liquid condensed in the receiver, continues nished. In these circumstances, it would appear that the ial layer of liquid, which is in contact with its own vapour, nly part that is free to vaporize.

preceding remarks explain the reluctance of water to boil in essels carefully washed, and the peculiar formation, in these tances, of large bubbles of steam, causing what is called *boil-*

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[1] Watts's *Dictionary of Chemistry*, art. "Heat," p. 88.

ing by bumping. In the case of sulphuric acid, the phenomen much more marked; if this liquid be boiled in a glass vessel, enor bubbles are formed at the sides, which, on account of the vi nature of the liquid, the mass of the li above them, and the it fall back with violence as sometim break the vessel. inconvenience may avoided by using a nular brazier (Fig. by means of which the upper part only of the liquid is heated.

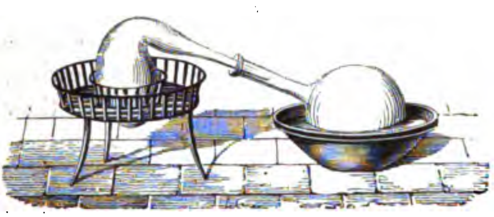

Fig. 253.—Apparatus for boiling Sulphuric Acid.

The ebullition of ether and alcohol presents some similar feat probably because these liquids dissolve the fatty particles on surface of the glass, and thus adhere to the sides very strongly.

264. Spheroidal State.—This is the name given to a peculiar dition which is assumed by liquids when exposed to the acti very hot metals.

If we take a smooth plate of iron or silver, and let fall a dr water upon it, the drop will evaporate more rapidly as the tem ture of the plate is increase to a certain point. Whe temperature of the plate ex this limit, which, for wate pears to be about 150°, the assumes a spheroidal form, about like a ball or spins axis, and frequently exhi beautiful rippling, as repres in the figure. While in thi dition, it evaporates much slowly than when the plat at a lower temperature. latter circumstance is impo and is easily verified by e ment. If the plate be allowed to cool, a moment arrives whe globule of water flattens out, and boils rapidly away with a h noise.

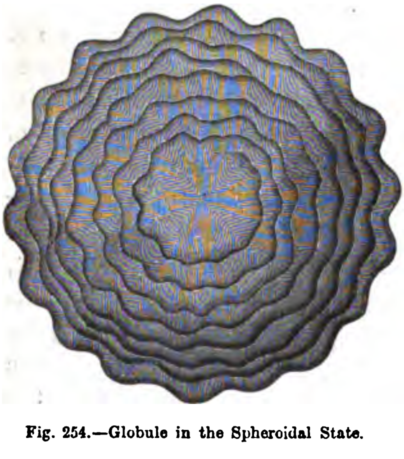

Fig. 254.—Globule in the Spheroidal State.

These phenomena have been long known, and were studi

enfrost and Klaproth; but the subject has recently been more
pletely investigated by Boutigny. All liquids are probably capable
ssuming the spheroidal state. Among those which have been
d are alcohol, ether, liquid sulphurous acid, and liquid nitrous
e. When in this state they do not boil. Sometimes bubbles of
n are seen to rise and burst at the top of the globule, but these
lways owing to some roughness of the surface, which prevents
team from escaping in any other way; when the surface is smooth,
ubbles are observed.

the temperature of the liquid be measured by means of a ther-
eter with a very small bulb, or a thermo-electric junction, it is
ys found to be below the boiling-point.

5. **Freezing of Water and Mercury by means of the Spheroidal**
.—This latter property enables us to obtain some very striking
aradoxical results. The boiling-point of liquid sulphurous acid
0°C., and that of liquid nitrous oxide is about −70° C. If a
or platinum crucible be heated to redness by a powerful lamp,
ome liquid sulphurous acid be then poured into it, this latter
es the spheroidal state; and drops of water let fall upon it are
diately frozen. Mercury can in like manner be frozen in a red-
rucible by employing liquid nitrous oxide in the spheroidal

ese experiments are due to Boutigny, who called attention to
as remarkable exceptions to the usual tendency of bodies to
brium of temperature. The exception is of the same kind as
resented by a vessel of water boiling at a constant temperature
° over a hot fire, the heat received by the liquid being in both
expended in producing evaporation.

. **The Metal not in Contact with the Liquid.**—The basis of the
theory of liquids in the spheroidal state is the fact that the
and the metal plate do not come into contact. This fact can
ved by direct observation.

plate used must be quite smooth and accurately levelled.
the plate is heated, a little water is poured upon it, and
es the spheroidal state. By means of a fine platinum wire
passes into the globule, the liquid is kept at the centre of the
plate. It is then very easy, by placing a light behind the
e, to see distinctly the space between the liquid and the plate.
pearance thus presented may be easily thrown on a screen by
of the electric light.

The interruption of contact can also be proved by conne (through a galvanometer) one pole of a battery with the hot while a wire from the other pole is dipped in the liquid. The rent refuses to circulate if the liquid is in the spheroidal state, immediately established when, on cooling the plate, the liquid to boil.

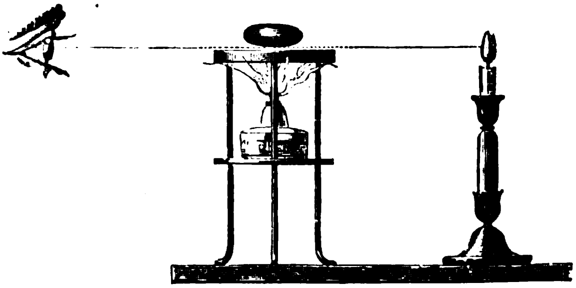
Fig. 255.—Separation between Globule and Plate.

This separation is maintained by the rush of steam from the surface of the globule, which is also the cause of the peculiar ments above described.

In consequence of the separation, heat can only pass to the g by radiation, and hence its comparatively low temperature counted for.

The absence of contact between a liquid and a metal at a temperature may be shown by several experiments. If, for ins a ball of platinum be heated to bright redness, and plunged (Fig into water, the liquid is seen to recede on all sides, leaving an en of vapour round the ball. This latter remains red for several se and contact does not take place till its temperatu fallen to about 150°. An active ebullition then place, and an abundance of steam is evolved.

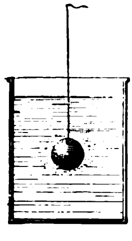
Fig. 256.—Red-hot Ball in Water.

If drops of melted sugar be let fall on water, the float for a short time, though their density is g than that of water (§ 79), contact being prevent their high temperature. A similar phenomenon served when a fragment of potassium is thrown on The water is decomposed; its hydrogen takes fi burns with a red flame; its oxygen combines wi potassium to form potash; and the globule of potash floats up surface without touching it, owing to the high temperature ı it is formed. After a few seconds the globule cools sufficiently ne into contact with the water, and bursts with a slight noise.

/. **Distillation.**—Distillation consists in boiling a liquid and nsing the vapour evolved. It enables us to separate a liquid the solid matter dissolved in it, and to effect a partial separa- of the more volatile constituent of a mixture from the less le.

: apparatus employed for this purpose is called a still. One of mpler forms, suitable for distilling water, is shown in Fig. 257. onsists of a retort *a*, the neck of which *c* communicates with a

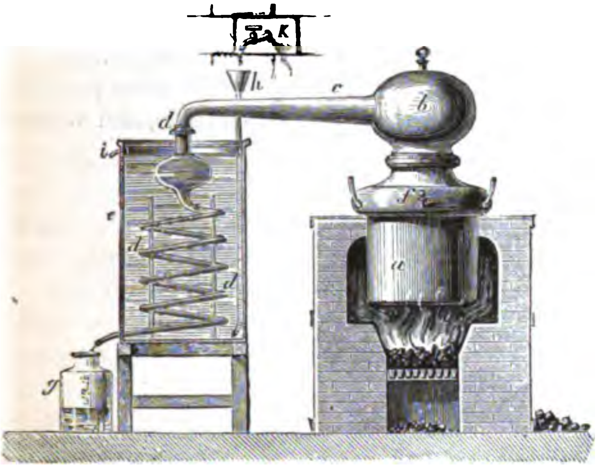

Fig. 257.—Still.

ube *dd* called the *worm*, placed in the vessel *e*, which contains ter. The water in the retort is raised to ebullition, the steam ff is condensed in the worm, and the *distilled water* is col- n the vessel *g*.

ıe condensation of the steam proceeds, the water of the cooler s heated, and must be renewed; for this purpose a tube ing to the bottom of the cooler is supplied with a continuous of cold water from above, while the superfluous water flows the tube *i* at the upper part of the cooler. In this way the vater, which rises to the top, is continually removed. The filled about three-quarters full, and the water in it can from time be renewed by the opening *f*; but it is advisable not to e process of distillation too far, and to throw away the liquid remaining in the boiler when its volume has been reduced to a fo or a fifth of what it was originally. By exceeding this limit we the risk of impairing the purity of the water by the carrying of some of the solid matter contained in the liquid in the boiler.

269. Circumstances which Influence Rapidity of Evaporation.- the case of a liquid exposed to the air, and at atmospheric tem ture, the rapidity of evaporation increases with the extent of surface, the dryness of the air, and the rapidity of renewal of air immediately above the surface.

In the case of a liquid evaporated by boiling, the quantity ev rated in a given time is proportional to the heat received. depends upon the intensity of the source of heat, the facility which heat passes through the sides of the vessel, and the ar *heating surface*, that is to say, of surface (or more properly lan which is in contact with the liquid on one side, and with the s of heat on the other.

CHAPTER XXVII.

MEASUREMENT OF THE MAXIMUM TENSIONS OF VAPOURS.

). **Tension of Aqueous Vapour.**—The knowledge of the maximum »n of the vapour of water at various temperatures is important, nly from a theoretical, but also from a practical point of view, uch as this tension is the motive force in the steam-engine. riments for the purpose of determining the values of this nt have accordingly been undertaken by several experimenters ferent countries. The researches conducted by Regnault are ally remarkable for the range of temperature which they em- as well as for the number of observations which they include, ie extreme precision of the methods employed. Next to these portance are the experiments of Magnus in Germany and of airn and Tate in England.

. **Dalton's Apparatus.**—The first investigations in this subject have any pretensions to accuracy were those of Dalton. The ıtus which he employed is represented in Fig. 259. Two baro- tubes A and B are inverted in the same cistern H; one is an ıry barometer, the other a vapour-barometer; that is, a baro- in which a few drops of water have been passed up through ercury. The two tubes, attached to the support CD, are sur- ›d by a cylindrical glass vessel containing water which can be to different temperatures by means of a fire. The first step ll the vessel with ice, and then read the difference of level of ercury in the two tubes. This can be done by separating the nts of ice. The difference thus observed is the tension of s vapour at zero Centigrade. The ice is then replaced by and the action of the fire is so regulated as to give different atures, ranging between 0° and 100° C., each of which is pre- constant for a few minutes, the water being at the same time well stirred by means of the agitator pq, so as to insure unifor of temperature throughout the whole mass. The difference of in the two barometers is read off in case; and we have thus the mea constructing, with the aid of grap or numerical interpolation, a com table of vapour-tensions from 0° to C. At or about this latter temper the mercury in the vapour-baror falls to the level of the cistern; an method is therefore inapplicabl higher temperatures. Such a tabl constructed by Dalton.

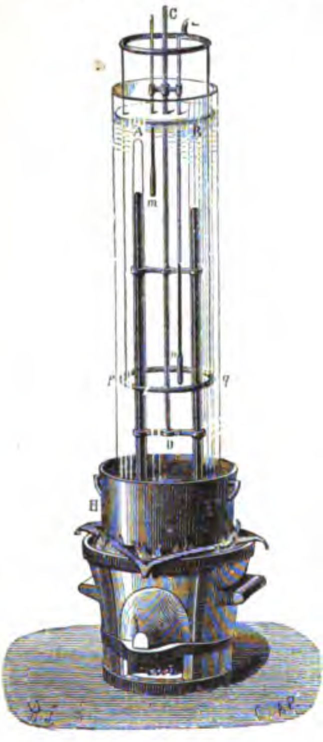

Fig. 259.—Dalton's Apparatus.

272. Regnault's Modifications.— ton's method has several defects the first place, it is impossible to i that the temperature shall be e where the same in a column as lo that which is formed by the vap 70°, 75°, and higher temperature the second place, there is always a deal of uncertainty in observin difference of level through the si the cylindrical glass vessel. Re employed this method only up t temperature of 50° C. At this ten ture the tension of the vapour i about 9 centimetres (less than 4 inches) of mercury, and it i unnecessary to heat the barometers throughout their entire l The improved apparatus is represented in Fig. 260. The two metric tubes, of an interior diameter of 14 millimetres, traver holes in the bottom of a metal box. In one of the sides of t is a large opening closed with plate-glass, through which the nec observations can be made with great accuracy. On account shortness of the liquid column it was very easy, by bringing a lamp within different distances of the box, to maintain for a cient time any temperature between 0° and 50° C.

The difference of level between the two mercurial columns be reduced to 0° C. by the ordinary correction. We should als into consideration the short column of water which is abo ury in the vapour barometer, and which, by its weight, produces ression that may evidently be expressed in mercury by dividing eight of the column by 13·59.

adapt this apparatus to low eratures, it is modified in the ving way. The upper ex- ity of the vapour barometer is drawn out and connected a small copper tube of three hes, one of which communi- with an air-pump, and an- with a glass globe of the ity of about 500 cubic centi- s. In the interior of this is a small bulb of thin glass ining water, from which all r has been expelled by boil- The globe is several times sted of air, and after each ex- on is refilled with air which een passed over desiccating nces. After the last exhaus- the tube which establishes unication with the air-pump metically sealed, the box is vith ice, and the tension at the dry air left behind in be by the air-pump is mea- it is of course exceedingly Heat is then applied to the globe, the little bulb bursts, and be, together with the space above the mercury, is filled with . This form of apparatus can also be employed to measure s at temperatures up to 50°, the only difference being that the eplaced by water at different temperatures, allowance being in each case, for the elastic force of the unexhausted air.

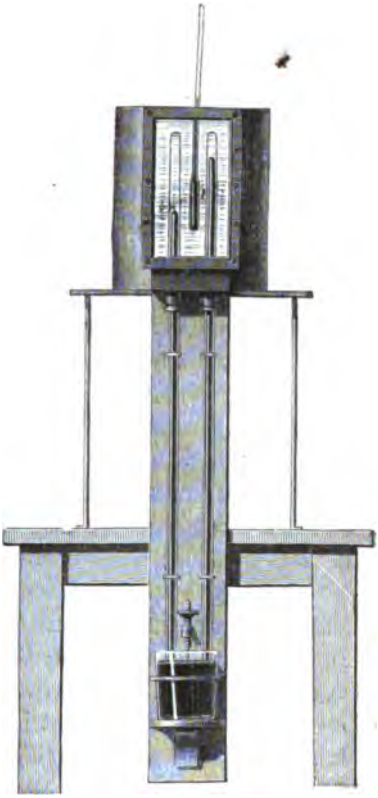
Fig. 260.—Modified Form of Dalton's Apparatus.

he case of temperatures below zero, the box is no longer d, and the globe alone is placed in a vessel containing a freez- xture. The barometric tubes are surrounded by the air of the ent.

is case the space occupied by the vapour is at two different temperatures in different parts, but it is evident that equilib can exist only when the tension is the same throughout. But tension of the vapour in the globe can never exceed the maxi tension for the actual temperature; this must therefore be the te throughout the entire space, and is consequently that which c sponds to the difference of level observed.

In reality what happens is as follows:—The low temperatu the globe causes some of the vapour to condense; equilibriu consequently destroyed, a fresh quantity of vapour is produced, e the globe, and is there condensed, and so on, until the tensi everywhere the same as the maximum tension due to the tem ture of the globe. This condensation of vapour in the cold p the space was utilized by Watt in the steam-engine; it is the *ciple of the condenser.*

Before Regnault, Gay-Lussac had already turned this princi account in a similar manner for the measurement of low tem tures.

By using chloride of calcium mixed with successively incre quantities of snow or ice, the temperature can be brought as l —32°C. (—25·6° F.), and it can be shown that the tension of the v of water is quite appreciable even at this point.

273. Measurement of Maximum Tensions for Temperatures abo —In investigating the tension of the vapour of water at temper above 50°, Regnault made use of the fact that the maximum t of steam at the boiling-point is equal to the external pressure.

His apparatus consists (Fig. 261) of a copper boiler cont water which can be raised to different temperatures indicat very delicate thermometers. The vapour produced passes thro tube inclined upwards, which is kept cool by a constant curr water; in this way the experiment can be continued for any of time, as the vapour formed by ebullition is condensed in the and flows back into the boiler. The tube leads to the lower a large reservoir, in which the air can be either rarefied or pressed at will. This reservoir is in communication with a meter. The apparatus shown in the figure is that employ pressures not exceeding 5 atmospheres. Much greater pre extending to 28 atmospheres, can be attained by simply alteri dimensions of the apparatus without any change in its pri The manometer employed in this case was the same as that testing Boyle's law, consisting of a long column of mercury (S using this apparatus, the air in the reservoir is first rarefied the water boils at about 50° C.; the occurrence of ebullition recognized by its characteristic sound, and by the temperature

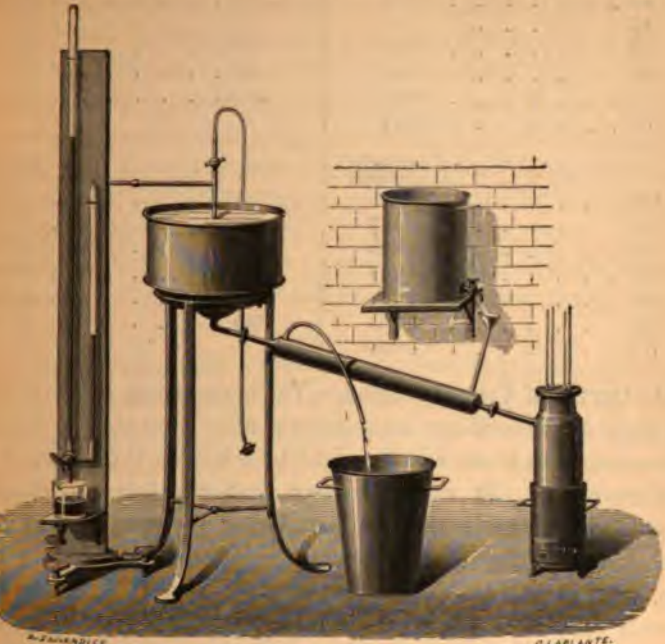

Fig. 261.—Regnault's Apparatus for High Temperatures.

ing invariable. This steadiness of temperature is of great age in making the observations, inasmuch as it enables the meters to come into perfect equilibrium of temperature with ter. The tension indicated by the manometer during ebulli- exactly that of the vapour produced. By admitting air into ervoir, the boiling-point is raised by successive steps until it 100°. After this, air must be forced into the reservoir by a ssion-pump.

following is an abstract of the results thus obtained:—

tures ade.	Tensions in Millimetres of Mercury.	Temperatures Centigrade.	Tensions in Millimetres of Mercury.
2°	0·32	5°	6·53
)	0·93	10	9·17
)	2·09	15	12·70
5	3·11	20	17·39
)	4·60	25	23·55

Temperatures Centigrade.	Tensions in Millimetres of Mercury.	Temperatures Centigrade.	Tensions i Millimetres Mercury.
30°	31·55	70°	233·09
35	41·82	75	288·51
40	54·91	80	354·64
45	71·39	85	433·04
50	91·98	90	525·45
55	117·47	95	633·77
60	148·70	100	760·00
65	186·94		

	Tensions in Atmospheres.		Tension Atmospl
100°	1	180°	9·92
121	2·025	189	12·12
134	3·008	199	15·06
144	4·000	213	19·99
152	4·971	225	25·12
159	5·966	239	27·53
171	8·036		

274. Curves of Vapour-tension.—The comparison of these te
with their corresponding temperatures affords no clue to any s
relation between them which might be taken as the physical
the phenomena. It would appear that the law of variat
maximum tensions is incapable of being thrown into any
expression—judging at least from the failure of all efforts hi
made. An attentive examination of the above table will ena
to assert only that the maximum tension varies very rapidly
the temperature. Thu
tween 0° and 100° the va
is only 1 atmosphere, b
tween 100° and 200° it is
15, and between 200° an
about 13 atmospheres.

The clearest way of
senting to the mind the l
cording to which vapo
sion varies with tempera
by means of a curve wh
dinates represent vapo
sions, while the abscissæ
sent the corresponding
peratures. Such a curve is exhibited in Fig. 262. Lengths p
tional to the temperatures, reckoned from 0° C., are laid off
base-line (called the line of abscissæ), and perpendiculars (calle

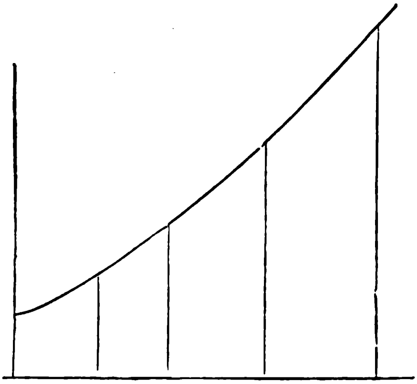

Fig. 262.

) are erected at their extremities, the lengths of these perpendi-
s being made proportional to the vapour-tensions. The scales
oyed for the two sets of lengths are of course quite independent
e another, their selection being merely a question of convenience.
curve itself is obtained by joining the extremities of the per-
iculars, taking care to avoid sudden changes of direction; and
only serves to convey to the mind an idea of the amounts of
ır-tensions and their rates of variation at different tempera-
but also furnishes the readiest means of determining the
ır-tensions at temperatures intermediate between those of
vation (see § 88 B).

will be noticed that the curve becomes steeper as the tempera-
ncreases, indicating that the tension increases faster than the
rature.

. **Empirical Formulæ.**—Though all attempts at finding a rational
la for vapour-tension in terms of temperature have hitherto
it is easy to devise empirical formulæ which yield tolerably
te results within a limited range of temperature; and by
g the values of the constants in such a formula by successive
it may be adapted to represent in succession the different por-
f the curve above described.

simplest of these approximate formulæ[1] is that of Dulong and
which may be written—

$$\left(\frac{40+C}{140}\right)^5 \quad \text{or} \quad \left(\frac{38+F}{250}\right)^5,$$

ves the maximum tension *in atmospheres*, corresponding to
nperature C° Centigrade, or F° Fahrenheit. This formula is
sly correct at 100° C., and gives increasing errors as the tem-
e departs further from this centre, the errors amounting to
½ per cent. at the temperatures 80° C. and 225° C. Hence it
that between these limits the maximum tension of aqueous
is nearly proportional to the fifth power of the excess of the
ature above –40° C.

Tensions of the Vapours of Different Liquids.—Dalton held that
*ours of different liquids have equal tensions at temperatures
removed from their boiling-points.* Thus the boiling-point
ol being 78°, the tension of alcohol vapour at 70° should be
that of the vapour of water at 92°. If this law were correct,

---

[1] For a more accurate formula, see *Rankine on Steam-engine*, p. 237.

it would only be necessary to know the boiling-point of any liq order to estimate the tension of its vapour at any given temper but subsequent experiment has shown that the law is far from rigorously exact, though it is approximately correct for temper differing by only a few degrees from the boiling-points.

Regnault has performed numerous experiments on the v tensions of some of the more volatile liquids, employing fo purpose the same form of apparatus which had served for mining the tensions of aqueous vapour. The following are s his results:—

VAPOUR OF ALCOHOL.

Temperatures Centigrade.	Tensions in Millimetres.	Temperatures Centigrade.	Tension Millime
−20°	3·24	+ 30°	78·5
0	12·70	100	1697·5
+10	24·23	155	6259·1

VAPOUR OF ETHER.

−20°	68·90	+ 30°	634·
0	184·39	100	4953·
+10	286·83	120	7719·

VAPOUR OF SULPHIDE OF CARBON.

−20°	47·30	+ 30°	434·
0	127·91	100	3325·
+10	198·46	150	9095·

277. Expression of Vapour-tension in Absolute Measure maximum tension of a given vapour at a given temperature i its very nature, independent of geographical position, and therefore, properly speaking, be denoted by one and the same at all places. This numerical uniformity will not exist if the be expressed, as in the preceding sections, in terms of the le a column of mercury which balances it. For example, in o adapt Regnault's determinations to London, we must multipl by the fraction $\frac{3456}{3457}$, inasmuch as 3456 millimetres of mercur the same pressure at London as 3457 at Paris. In general, t determinations of pressure made at a place A, to another plac must multiply them by the fraction

$$\frac{\text{gravity at A}}{\text{gravity at B}}.$$

If the length of mercurial column at 0° C. which balances a tension at a given place be multiplied by the value of g (d intensity of gravity) at that place, the product may be cal *absolute value* of the vapour-tension; for the products thus obtained will be the same at all localities. This is the proper mode of treating determinations of vapour-tension made at various localities when it is desired to establish a rigorous comparison between them.

278. Laws of Combination by Volume.—It was discovered by Gay-Lussac, that when two or more gaseous elements at the same temperature and pressure enter into chemical combination with each other, the two following laws apply:—

1. The volumes of the components bear a very simple ratio to each other, such as 2 to 3, 1 to 2, or 1 to 1.

2. The volume of the compound has a simple ratio to the sum of the volumes of the components.

Ammonia, for example, is formed by nitrogen and hydrogen uniting in the proportion of one volume of the former to three of the latter, and the volume of the ammonia, if reduced to the same pressure as each of its constituents, is just half the sum of their volumes. Further investigation has led to the conclusion (which is now generally received, though hampered by some apparent exceptions), that these laws apply to all cases of chemical combination, the volumes compared being those which would be occupied respectively by the combining elements and the compound which they form, *when reduced to the state of vapour*, at such a temperature and pressure as to be very far removed from liquefaction, and consequently to possess the properties of what we are accustomed to call permanent gases.

It is obvious that if all gases and vapours were equally expansible by heat, the volume-ratios referred to in this law would be the same at all temperatures; and that, in like manner, if they were all equally compressible (whether obeying Boyle's law, or departing equally from it at equal pressures), the volume-ratios would be independent of the pressure at which the comparison was made.

In reality great differences exist between different vapours in both respects, and these inequalities are greater as the vapours are nearer to saturation. It is accordingly found that the above laws of volume-ratio often fail to apply to vapours when under atmospheric pressure and within a few degrees of their boiling-points, and that, in such cases, a much nearer fulfilment of the law is obtained by employing very high temperatures, or operating in inclosures at very low pressures.

278 A. Relation of Vapour-densities to Chemical Equivalents.—Chem-

ists have determined with great accuracy the combining propo
by weight of most of the elements. Hence the preceding la
be readily tested for bodies which usually exist in the solid or
form, if we are able to compare the densities of their vapour
fact, if two such elements combine in the ratio, by weight, of w_1
we have

$$v_1 = \frac{w_1}{d_1}, \quad v_2 = \frac{w_2}{d_2},$$

v_1 v_2 d_1 d_2 denoting the volumes and densities of the vapo
weights w_1 w_2 of the two substances.

Hence we have the equation—

$$\frac{v_1}{v_2} = \frac{w_1}{w_2} \cdot \frac{d_2}{d_1},$$

which gives the required volume-ratio of the vapours, if the r
their densities be known.

The densities themselves will differ enormously according
pressure and temperature at which they are taken, but thei
will only vary by comparatively small amounts, and would no
at all if they were equally expansible by heat, and equall
pressible. Hence comparison will be facilitated by tabulati
ratios of the densities to that of some standard gas, namely air,
the same conditions of pressure and temperature, rather th
absolute densities. This is accordingly the course which is ge
pursued, so generally indeed, that by the *vapour-density of*
stance is commonly understood the relative density as measu
this ratio.

The process most frequently employed for the determina
this element is that invented by Dumas.

279. Dumas' Method.—The apparatus consists of a glass g
containing the substance which is to be converted into vapou

The globe is placed in a vessel C, containing some liquid
can be raised to a suitable temperature. If the substance
operated on is one which can be vaporized at 100°C., the ba
sists simply of boiling water. When higher temperatur
required, a saline solution, oil, or a fusible alloy is employed.
cases, the liquid should be agitated, that its temperature may
same in all parts. This temperature is indicated by the t
meter *t*.

When the substance in the globe has attained its boiling
evaporation proçeeds rapidly, and the vapour escapes, carryi

r along with it. When the vapour ceases to issue, we may
ie, if the quantity of matter originally taken has been suffi-
y large, that all the
s been expelled, and
the globe is full of
r at the temperature
by the thermometer,
: the external pres-
l. The globe is then
tically sealed at the
nity *p* of the neck,
has been previously
out into a fine tube.

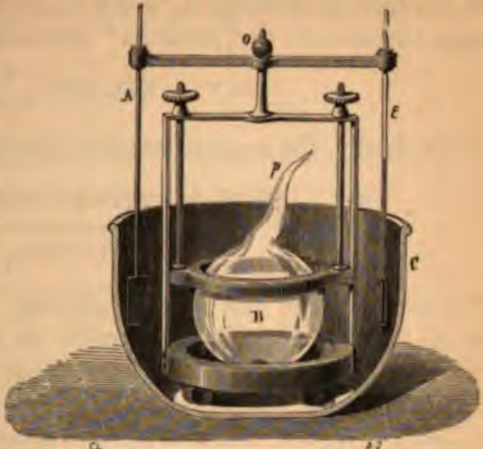

Fig. 263.—Dumas' Apparatus.

**. Calculation of the
ment.**—As already
ced, the densities of
s given in treatises
mistry express *the
f the weight of a given volume of the vapour to that of the
olume of air at the same temperature and pressure.* In order
ıce this ratio from the preceding experiment, we must first find
ight of the vapour. This is done by weighing the globe with
ents, after allowing it to cool. Suppose the weight thus found
V. Before the experiment the globe had been weighed full of
at a known temperature t and pressure h. Suppose this
to be W′; the difference W − W′ evidently represents the
of the weight of the vapour above that of the air. If, then,
W − W′ to the weight of the air, we shall evidently have the
of the vapour. Now the weight of the air is easily deduced
e known volume of the globe. If V denote this volume at
pressed in litres, the weight in grammes of the air con-
n the globe at the time of weighing is

$$V(1+Kt)\ 1{\cdot}293\times\frac{1}{1+at}\cdot\frac{h}{760},$$

ting the coefficient of cubical expansion of glass, and a the
nt of expansion of air. The weight of the vapour contained
lobe is consequently

$$A=W-W'+V(1+Kt)\times1{\cdot}293\times\frac{1}{1+at}\cdot\frac{h}{760}.$$

e the pressure, and T the temperature at the time of sealing the globe. The volume occupied by the vapour under these c stances was V (1+KT). The density of the vapour will th be obtained by dividing A by the weight of this volume of air same temperature and pressure. But this weight is

$$A' = V\,(1+KT)\,.\,1{\cdot}293\,.\,\frac{1}{1+aT}\,.\,\frac{H}{760};$$

hence, finally, the required relative density is

$$D = \frac{A}{A'} = \frac{W - W' + V\,(1+Kt)\,.\,1{\cdot}293\,.\,\frac{1}{1+at}\,.\,\frac{h}{760}}{V\,(1+KT)\,.\,1{\cdot}293\,.\,\frac{1}{1+aT}\,.\,\frac{H}{760}}.$$

The correctness of this formula depends upon the assumptic no air is left in the globe. In order to make sure that this tion is fulfilled, the point *p* of the neck of the globe is brc under mercury; the liquid then rushes in, and, together w condensed vapour, fills the globe completely, if no air has b behind.

This last operation also affords a means of calculating the vol for we have only to weigh the mercury contained in the glob measure it in a graduated tube, in order to ascertain its vo the actual temperature, whence the volume at zero can ea deduced.

281. Example.—In order better to illustrate the method, v take the following numerical results obtained in an investiga the vapour-density of sulphide of carbon:—

Excess of weight of vapour above weight of air, W—W′=·3 g temperature of the vapour T=59°; external pressure H=752 metres; volume of the globe at a temperature of 12°, 190 cubi metres; temperature of the dry air which filled the globe at t of weighing, $t=15°$; pressure $h=765$; $K = \frac{1}{38700}$.

The volume V of the globe at zero is

$$\frac{190}{1+\frac{12}{38700}} = 189{\cdot}94 \text{ cubic centimetres} = {\cdot}18994 \text{ litre.}$$

The weight of the air contained in the globe is

$$\cdot18994 \times 1{\cdot}293\,.\left(1 + \frac{12}{38700}\right).\,\frac{1}{1+12\times\,.\,00366}\,.\,\frac{765}{760} = {\cdot}23658 \text{ gramm}$$

Weight of the vapour,

$$\cdot23658 + W - W' = {\cdot}53658 \text{ gramme.}$$

The weight of the same volume of air at the same temperature and pressure is

$$\cdot 18994 \times 1\cdot 293 \left(1 + \frac{59}{38700}\right) \cdot \frac{1}{1 + \cdot 00366 \times 59} \cdot \frac{752\cdot 5}{760} = \cdot 20019 \text{ gramme.}$$

The density is therefore

$$\frac{\cdot 53658}{\cdot 20019} = 2\cdot 67.$$

Deville and Troost have effected several improvements in the application of Dumas' method to vapours at high temperatures. These temperatures are obtained by boiling various substances, such as chloride of zinc, cadmium, which boils at 860° C., or zinc, which boils at 1040° C. For temperatures above 800°, the glass globe is replaced by a globe of porcelain, which is hermetically sealed with the oxyhydrogen blowpipe. The globe itself serves as a pyrometer to determine the temperature; and since the weight of air becomes very inconsiderable at high temperatures, some heavier vapour, such as that of iodine, is substituted in its place. If we suppose, as we may fairly do, that at these high temperatures the coefficient of expansion of the vapour of iodine is the same as that of air, the temperature may easily be deduced from the weight of iodine contained in the globe. We subjoin a table of some relative densities of vapours obtained by this method:—

Water,	0·622	**Phosphorus,**	4·5
Alcohol,	1·6138	**Cadmium,**	3·94
Ether,	2·586	**Chloride of aluminium,**	9·347
Spirit of turpentine,	5·0130	**Bromide of aluminium,**	18·62
Iodine,	8·716	**Chloride of zirconium,**	8·1
Sulphur,	2·23	**Sesquichloride of iron,**	11·39

282. Limiting Values of Relative Densities.—In investigating the relative density of acetic acid vapour, Cahours found that it went on decreasing as the temperature increased, up to a certain point, after which no further change was observable. A similar circumstance is observed in the case of all substances, only in different degrees. The vapour of sulphur, for instance, has a relative density of 6·65 at 500° C., while at about 1000° C. it is only 2·23. This indicates that the vapours in question are more expansible by heat than air until the limiting temperatures are attained. Indeed it may be laid down as a general principle, that the nearer a vapour is to saturation the greater is the change produced in its absolute density by a given change whether of temperature or pressure. The limiting density-ratio is always that which it is most important to determine, and we should consequently take care that the temperature vapour is sufficiently high to enable us to obtain it.

283. Gay-Lussac's Method.—Gay-Lussac determined the den the vapour of water and of some other liquids by a method a more complicated than that described above, and which fo reason has not been generally adopted in the laboratory. We p to describe it, however, on account both of its historical intere of the importance of the question which it has assisted in solv

A graduated tube divided into cubic centimetres, suppose, is with mercury, and inverted in a cast-iron vessel containing th liquid. The inverted tube is surrounded by a glass envelop taining water, as in Dalton's apparatus. A small glass bulb co ing a given weight *w* (expressed in grammes) of distilled w passed into the tube, and rises to the surface of the mercury. temperature of the apparatus is raised by means of a fire below, th bursts, and the water which it con is converted into vapour. If the tity of water be not too great, i converted into vapour; this is to be the case when, at the temp of about 100°, the mercury stands in the tube than in the vessel, there were any liquid-water p the space would be saturated, a tension of the vapour would be e the external pressure. This a ment accordingly gives the weig known volume of the vapour of This volume, which may at once be read upon the graduated equal to V (1 + KT), V being the number of divisions occup the vapour. The temperature T is marked by a thermomet mersed in the water contained in the envelope. The tension vapour is evidently equal to the external pressure *minus* the of the mercury in the tube.

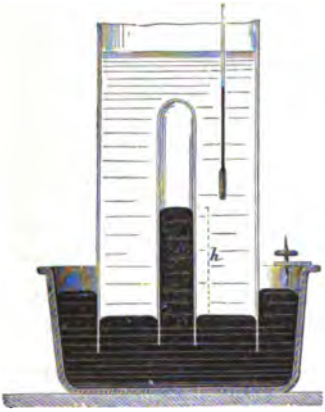

Fig. 264.—Gay-Lussac's Apparatus.

In order to find the relative density, we must divide *w* weight of a volume V (1 + KT) of air at the temperature T an sure H − *h*, giving

$$\frac{w}{V\,(1+KT)\times\cdot001293\times\frac{1}{1+aT}\cdot\frac{H-h}{760}}.$$

The relative density of the vapour of water, as thus determined by Gay-Lussac, is about $\frac{5}{8}$, or ·625. Several recent investigations have given as mean result ·622, which agrees with the theoretical density deduced from the composition of water.[1]

284. Volume of Vapour formed by a given Weight of Water.—When the density of the vapour of water is known, the increase of volume which occurs when a given quantity of water passes into the state of vapour may easily be calculated. Suppose, for instance, that we wish to find the volume which a cubic centimetre of water at 4° will occupy in the state of vapour at 100°. At this temperature the tension of the vapour is equal to one atmosphere, and its weight is equal to ·622 times the weight of the same volume of air at the same temperature and pressure. If then V be the volume in litres, we have (in grammes)

$$V \times 1{\cdot}293 \times \frac{1}{1+100a} \times {\cdot}622 = 1,$$

whence

$$V = \frac{1+100a}{1{\cdot}293 \times {\cdot}622} = \frac{1{\cdot}366}{{\cdot}804246} = 1{\cdot}698 \text{ lit.} = 1698 \text{ cubic centimetres.}$$

Hence we see that water at 4° gives about 1700 times its volume of vapour at 100° C.

The latent heat of evaporation is doubtless connected with this increase of volume; and it may be remarked that both these elements appear to be greater for water than for any other substance.

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[1] Water is composed of 2 volumes of hydrogen, and 1 volume of oxygen, forming 2 volumes of vapour of water. The sum of the density of oxygen and twice the density of hydrogen is 1·244, and the half of this is exactly ·622.—*D.*

CHAPTER XXVIII.

HYGROMETRY.

285. Humidity.—The condition of the air as regards mo
involves two distinct elements: (1) the amount of vapour p
in the air, and (2) the ratio of this to the amount which
saturate the air at the actual temperature. It is upon the sec
these elements that our sensations of dryness and moisture c
depend, and it is this element which meteorologists have agre
denote by the term *humidity;* or, as it is sometimes called, *re*
humidity. It is usually expressed as a percentage; for exam
the weight of vapour present is seven-tenths of that requir
saturation, the humidity is said to be 70.

The words *humid* and *moist,* as applied to air in ordinar
guage, nearly correspond to this technical use of the word *hum*
and air is usually said to be dry when its *humidity* is consid
below the average. In treatises on physics, "dry air" u
denotes air whose humidity is zero.

The air in a room heated by a hot stove contains as much
weight for weight as the open air outside; but it is drier, beca
capacity for vapour is greater. In like manner the air is d
noon than at midnight, though the amount of vapour present is
the same; and it is for the most part drier in summer than in
though the amount of vapour present is much greater.

Bearing in mind that a cubic foot of air is able to take up th
amount of vapour as a cubic foot of empty space, we may defi
humidity of the air as *the weight of aqueous vapour in a*
volume of air, expressed as a percentage of the weight of
at saturation which would occupy the same volume at the
temperature.

Also, since aqueous vapour nearly fulfils Boyle's law, the hu of the air may be obtained by comparing the *tension* of the vapour present in the air with the *maximum tension* for the actual temperature.

286. Dew-point.—When air containing aqueous vapour is gradually cooled at constant pressure, its density increases, and the rate of increase is sensibly the same for the vapour as for the dry air with which it is mixed (inasmuch as vapours not in contact with their liquids nearly fulfil Gay-Lussac's law), until a point is reached at which the density of the vapour becomes equal to the maximum density corresponding to the temperature. This temperature is called the *dew-point* of the given mass, and any further reduction of temperature will be accompanied by the condensation of a portion of the vapour, which will take the form of dew, rain, snow, or hoar-frost, according to circumstances. If the cooling is produced by the low temperature of the sides of the containing vessel, the deposit will be dew or hoar-frost, according as the temperature of the sides is above or below the freezing-point. If the cooling takes place in the interior of the mass of air, the deposit will be rain or snow, according as the temperature of deposition is above or below the freezing-point.

In the operation of cooling down to the dew-point, the density of the vapour, as we have seen, increases. Let t denote the initial temperature, and T the dew-point, and let d and D be the densities of the vapour at these temperatures. Then we have, by Gay-Lussac's law,

$$D = d \cdot \frac{1 + at}{1 + aT}$$

But the *tension* of the vapour *is not sensibly changed* by the operation, since the whole pressure is by hypothesis preserved constant, and the changes of temperature and volume affect the dry and the vaporous constituent nearly alike.

If the reduction of temperature from t to T took place *at constant volume* (in a closed receiver, for example), we should then have

$$P = p \cdot \frac{1 + aT}{1 + at},$$

p and P denoting the vapour-tensions at the temperatures t and T; and the density would remain constant, since no vapour enters or escapes. In this case the vapour would not begin to be condensed till a somewhat lower temperature had been attained.

287. Hygroscopes.—Anything which serves to give rough indica-

tions of the state of the air as regards moisture may be c
hygroscope (ὑγρος, moist). Many substances, especially those
are composed of organic tissue, have the property of absorbi
moisture of the surrounding air, until they attain a condit
equilibrium, such that their affinity for the moisture absor
exactly equal to the force with which the latter tends to eva
Hence it follows that, according to the dampness or dryness
air, such a substance will absorb or give up vapour, either of
processes is always attended with a variation in the dimensions
body. The nature of this variation depends upon the peculiar
ture of the substance; thus, for instance, bodies formed of fil
exhibit a greater increase in the direction of their breadth
their length. Membranous bodies, on the other hand, such as
or parchment, formed by an interlacing of fibres in all dire
expand or contract almost as if they were homogeneous.
composed of twisted fibres, as ropes and strings, swell und
action of moisture, grow shorter, and are more tightly twisted.
opposite is the case with catgut, which is often employed in
hygroscopes.

288. Hygrometers.—Instruments intended for furnishing
measurements of the state of the air as regards moisture are
hygrometers. They may be divided into four classes:—

1. Hygrometers of absorption, which should rather be
hygroscopes.

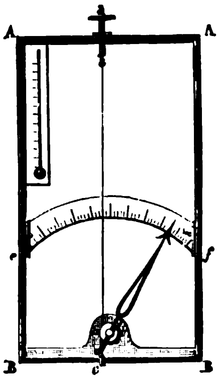

Fig. 266.
De Saussure's Hygroscope.

2. Hygrometers of condensation, or dew
instruments.

3. Hygrometers of evaporation, or w
dry bulb thermometers.

4. Chemical hygrometers, for directly m
ing the weight of vapour in a given vol
air.

289. De Saussure's Hygrometer.—Th
hygrometer of absorption is that of De Sa
consisting of a hair deprived of grease, wh
its contractions moves a needle (Fig. 266).
the hair relaxes, the needle is caused to m
the opposite direction by a weight, which
to keep the hair always equally tight. T
contracts as the humidity increases, but not in simple prop
and Regnault's investigations have shown that, unless the

minute precautions are adopted in the construction and graduation of each individual instrument, this hygrometer will not furnish definite numerical measures.

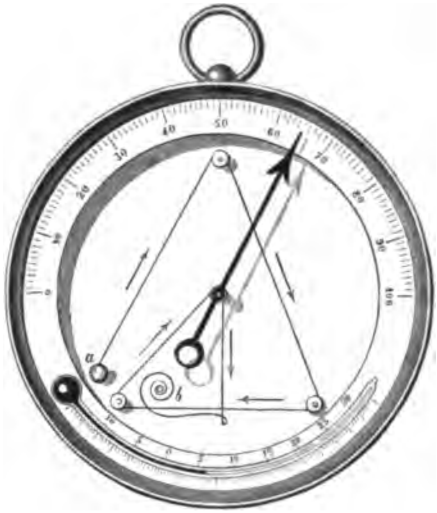

Fig. 267.—Monnier's Hygroscope.

Fig. 267 represents Monnier's modification of De Saussure's hygrometer, in which the hair, after passing over four pulleys, is attached to a light spring, which serves instead of a weight, and gives the advantage of portability.

These instruments are never employed for scientific purposes in this country.

290. Dew-point Hygrometers.—These are instruments for the direct observation of the dew-point, by causing moisture to be condensed from the air upon the surface of a body artificially cooled to a known temperature.

The dew-point, which is itself an important element, gives directly, as we have seen in § 286, the tension of vapour; and if the temperature of the air is at the same time observed, the tension requisite for saturation is known. The ratio of the former to the latter determines the humidity.

The principle of these instruments may be illustrated by a description of their simplest type, the hygrometer of Leroy, a French philosopher of the last century.

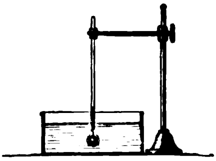

Fig. 268. Leroy's Hygrometer.

291. Leroy's Hygrometer.—The instrument consists of a tin vessel containing water, in which a thermometer is immersed. The temperature of the water and containing vessel is gradually lowered by the introduction of ice, and when it has fallen below the dew-point of the adjacent air, a portion of the vapour will be condensed as dew upon the exterior of the vessel. This is at once recognized by the metallic surface losing its brilliancy.

We may observe that the deposition of dew does not begin till the point of saturation has been passed, and that the indication thermometer is consequently somewhat too low. Leroy propo empirical correction of half a degree. There are, however, defects in the instrument; the use of ice does not afford a and regular diminution of temperature; and it is especially obj able to place an open vessel containing water in the very place the humidity of the air is to be determined.

292. Daniell's Hygrometer.—Daniell's hygrometer is an inst of much greater precision, and has been very extensively us consists of a bent tube with a globe at each end, and is partl with ether. The rest of the space is occupied with vapour of the air having been expelled. One globes A contains a thermometer *t*. Thi is generally made of black glass, which p a brilliant surface. The method of us instrument is as follows:—The whole liquid is first passed into the globe A, a the other globe B, which is covered with is moistened externally with ether. The ration of this ether from the muslin c partial condensation of vapour of ether interior of the globe, which produces evaporation from the surface of the liqui thus lowering the temperature of that part of the instrume carefully watching the surface of the globe, the exact mo the deposition of dew may be ascertained. The temperature read on the inclosed thermometer. This temperature is a littl than the dew-point.

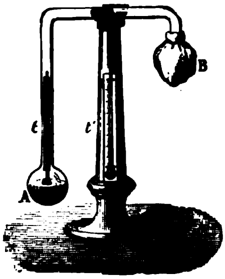

Fig. 269.
Daniell's Hygrometer.

If the instrument be now left to itself, the exact momen disappearance of the dew may be observed; this correspond indicated temperature a little above the dew-point, and t plan is to take the mean between this temperature and t observed. The temperature of the surrounding air is give thermometer *t′* attached to the stand.

Daniell's hygrometer, though capable of furnishing accu dications, has some defects, which have been removed by the i ments effected by Regnault.

293. Regnault's Hygrometer.—Regnault's hygrometer consi 270) of a glass tube closed at the bottom by a very thin silve The opening at the upper end is closed by a cork, throug passes the stem of a thermometer T, and a glass tube *t* open at both ends. The lower end of the tube and the bulb of the thermometer dip into ether contained in the silver cap. A side tube establishes communication between this part of the apparatus and a vertical tube U V, which is itself connected with an aspirator[1] A, placed at a convenient distance. By allowing the water in the aspirator to escape, a current of air is produced through the ether, which has the effect of keeping the liquid in agitation, and thus producing uniformity of temperature throughout the whole. It also tends to hasten evaporation; and the cold thus produced speedily causes a deposition of dew, which is observed from a distance with a telescope, thus obviating the risk of vitiating the observation by the too close proximity of the observer. The observation is facilitated by the contrast offered by the appearance of the second cap, which has no communication with the first, and contains a thermometer for giving the temperature of the external air. By regulating the flow of liquid from the aspirator, the temperature of the ether can be very nicely controlled, and the dew can be made to appear and disappear at temperatures nearly identical. The mean of the two will then very accurately represent the dew-point.

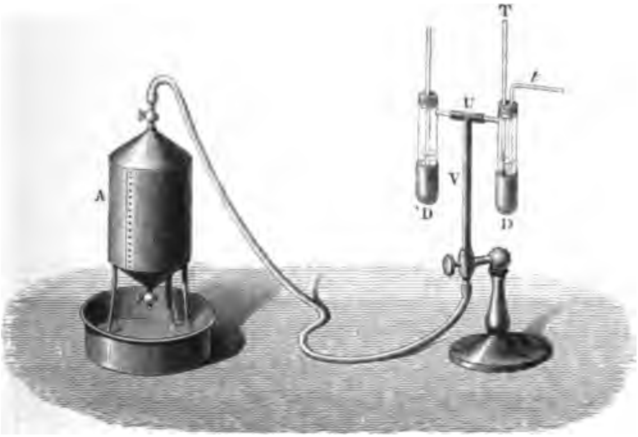

Fig. 270.—Regnault's Hygrometer.

The liquid employed in Regnault's hygrometer need not be ether.

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[1] An aspirator is a vessel into which air is sucked at the top to supply the place of water which is allowed to escape at the bottom; or, more generally, it is any apparatus for sucking in air or gas.

Alcohol, a much less volatile liquid, will suffice. This is an im advantage; for, since the boiling-point of ether is 36° C. (97° not easy to preserve it in hot climates.

294. Wet and Dry Bulb Hygrometer.—This instrument, w also called Mason's hygrometer, and is known on the Conti August's psychrometer, consists (Fig. 271) of two precisely thermometers, mounted at a short distance from each other, t of one of them being covered with which is kept moist by means of a wick leading from a vessel of wate evaporation which takes place fr moistened bulb produces a depre temperature, so that this thermomet lower than the other by an amoun increases with the dryness of the a instrument must be mounted in suc that the air can circulate very freel the wet bulb; and the vessel contai water should be small, and should b some inches to the side. The leve vessel must be high enough to f supply of water which keeps the thoroughly moist, but not high en cause a drop to form at the botto bulb. Unless these precautions are c the depression of temperature wil sufficiently great, especially in calm

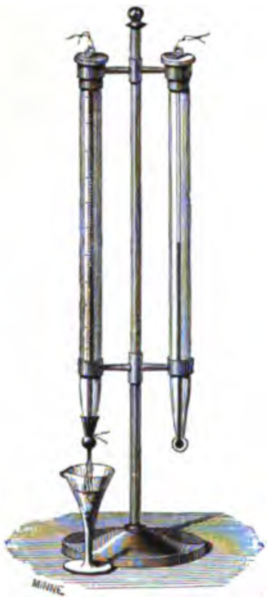

Fig. 271.
Wet and Dry Thermometers.

In frosty weather the wick cease and the bulb must be dipped in water some time before ta observation, so that all the water on the bulb may be froze little time allowed for evaporation from the ice before the r taken.

The great facility of observation afforded by this instrum brought it into general use, to the practical exclusion of oth of hygrometer. As the theoretical relation between the in of its two thermometers and the humidity as well as the d of the air is rather complex, and can scarcely be said to b with certainty, it is usual, at least in this country, to e reduction by means of tables which have been empirically co by comparison with the indications of a dew-point instrume tables universally employed by British observers were constructed by Mr. Glaisher, and are based upon a comparison of the simultaneous readings of the wet and dry bulb thermometers and of Daniell's hygrometer taken for a long series of years at Greenwich observatory, combined with some similar observations taken in India and at Toronto.[1]

According to these tables, the difference between the dew-point and the wet-bulb reading bears a constant ratio to the difference between the two thermometers, when the temperature of the dry-bulb thermometer is given. When this temperature is 53° F., the dew-point is as much below the wet-bulb as the wet-bulb is below the temperature of the air. At higher temperatures the wet-bulb reading is nearer to the dew-point than to the air-temperature, and the reverse is the case at temperatures below 53°.

In order to obtain a clue to the construction of a rational formula for deducing the dew-point from the indications of this instrument, we shall assume that the wet-bulb is so placed that its temperature is not sensibly affected by radiation from surrounding objects, and hence that the heat which becomes latent by the evaporation from its surface is all supplied by the surrounding air. When the temperature of the wet-bulb is falling, heat is being consumed by evaporation faster than it is supplied by the air; and the reverse is the case when it is rising. It will suffice to consider the case when it is *stationary*, and when, consequently, the heat consumed by evaporation in a given time is exactly equal to that supplied by the air.

Let t denote the temperature of the air, which is indicated by the dry-bulb thermometer; t' the temperature of the wet-bulb; T the temperature of the dew-point, and let f, f', F be the vapour-tensions corresponding to saturation at these three temperatures. Then, as shown in § 286, the tension of the vapour present in the air at its actual temperature t is also equal to F.

We shall suppose that wind is blowing, so that continually fresh portions of air come within the sphere of action of the wet-bulb. Then each particle of this air experiences a depression of temperature and an increase of vapour-tension as it comes near the wet-bulb, from both of which it afterwards recovers as it moves away and mixes with the general atmosphere.

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[1] The first edition of these Tables differs considerably from the rest, and is never used; but there has been no material alteration since the second edition (1856).

If now it is legitimate to assume[1] that this depression of te ture and exaltation of vapour-tension are always proportiona another, not only in comparing one particle with itself at c times, but also in comparing one particle with another, we h means of solving our problem; at all events, if we may m additional assumptions that a portion of the air close to t bulb is at the temperature of the wet-bulb, and is saturated.

On these assumptions the greatest reduction of temperatur air is $t-t'$, and the greatest increase of vapour-tension is f'- the corresponding changes in the whole mass are proporti these. The three temperatures t, t', T must therefore be so that the heat lost by a mass of air in cooling through the ran is just equal to the heat which becomes latent in the formati much vapour as would raise the vapour-tension of the mas amount $f'-$F.

Let h denote the height of the barometer, s the specific air (Chap. xxxi.), D the relative density of vapour (§ 278A latent heat of steam.

Then the mass of the air is to that of the vapour require duce the additional tension, as h to D ($f'-$F), and we are to

$$\text{LD}\,(f'-\text{F}) = s\,(t-t')\,h,$$

or

$$f'-\text{F} = (t-t')\,h\,.\,\frac{s}{\text{LD}}, \qquad (1)$$

which is the required formula, enabling us, with the aid of of vapour-tensions, to determine F, and therefore the dew- when the temperatures t, t' of the dry and wet bulb, and the of the barometer, have been observed. The expression for tive humidity will be $\frac{\text{F}}{f}\,100$.

Properly speaking, s denotes the specific heat not of dry a air containing the actual amount of vapour, and therefore to some extent upon the very element which is to be det but its variation is inconsiderable. L also varies with the quantity t', but its variations are also small within the limits which occur in practice. The factor $\frac{s}{\mathrm{LD}}$ may therefore be regarded as constant, and its value, as adopted by Dr. Apjohn for the Fahrenheit scale, is $\frac{1}{2610}$ or $\frac{1}{30} \times \frac{1}{87}$. We thus obtain what is known as *Apjohn's formula*,

---

[1] The assumption which Dr. Apjohn actually makes is as follows:—"When i bulb hygrometer the stationary temperature is attained, the caloric which v water is necessarily exactly equal to that which the air imparts in descendin temperature of the atmosphere to that of the moistened bulb; and the air which gone this reduction becomes saturated with moisture" (*Trans. R.I.A.* Nov. 1

This implies that unless air passes near enough to the wet-bulb to become saturated, it experiences no depression of temperature whatever—a very harsh but August independently makes the same assumption.

$$\mathrm{F} = f' - \frac{t - t'}{87} \cdot \frac{h}{30}. \qquad (2)$$

When the wet-bulb is frozen, L denotes the sum of the latent heats of liquefaction and vaporization, and the formula becomes

$$\mathrm{F} = f' - \frac{t - t'}{96} \cdot \frac{h}{30}. \qquad (3)$$

In calm weather, and also in very dry weather, the humidity, as deduced from observations of wet and dry thermometers, is generally too great, probably owing mainly to the radiation from surrounding objects on the wet-bulb, which makes its temperature too high.

295. Chemical Hygrometer.—The determination of the quantity of aqueous vapour in the atmosphere may be effected by ordinary chemical analysis in the following manner:—

An aspirator A, of the capacity of about 50 litres, communicates at its upper end with a system of U-tubes 1, 2, 3, 4, 5, 6, filled with pieces of pumice soaked in sulphuric acid. The aspirator being full of water, the stop-cock at the bottom is opened, and the air which

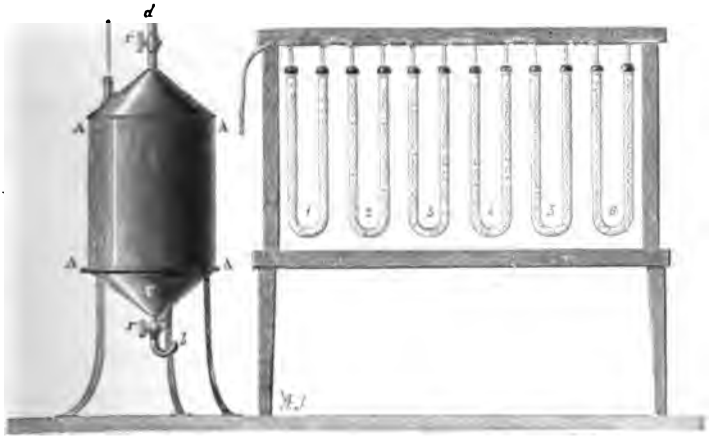

Fig. 272.—Chemical Hygrometer.

enters the aspirator to take the place of the water is obliged
through the tubes, where it leaves all its moisture behin
moisture is deposited in the first tubes only. The last
intended to absorb any moisture that may come from the as
Suppose *w* to be the increase of weight of the first tubes 4, 5
is evidently the weight of the aqueous vapour contained in
which has passed through the apparatus. The volume V of
which we will suppose to be expressed in litres, may easily b
by measuring the amount of water which has escaped.
has been again saturated by contact with the water of the a
and the aqueous vapour contained in it is consequently at th
mum tension corresponding to the temperature indicated by
mometer attached to the apparatus. Let this tension be
by *f*. The volume occupied by this air when in the atm
where the temperature is T, is known by the regular formulæ
been

$$V \cdot \frac{H-f}{H-x} \cdot \frac{1+aT}{1+at},$$

x denoting the tension of the aqueous vapour in the atmosph
H the total atmospheric pressure as indicated by the baromet
since the relative density of steam is ·622, and the weight o
of air at temperature 0° C. and pressure 760 mm. is 1·293 g
the weight of vapour which this air contained must have bee

$$V \cdot \frac{H-f}{H-x} \cdot \frac{1+aT}{1+at} \times 1{\cdot}293 \times {\cdot}622 \cdot \frac{x}{760} \cdot \frac{1}{1+aT},$$

which must be equal to the known weight *w*, and thus we
equation from which we find

$$x = \frac{w\,(1+at)\,760\,H}{V\,(H-f) \times {\cdot}622 \times 1{\cdot}293 + w\,(1+at)\,760}.$$

A good approximation will be obtained by writing

$$w = V \times 1{\cdot}293 \times {\cdot}622\,\frac{x}{760},$$

whence

$$x = 945\,\frac{w}{V}.$$

This method has all the exactness of a regular chemical
but it involves great labour, and is, besides, incapable of
the sudden variations which often occur in the humidity
atmosphere. It can only give the mean quantity of moist
given volume of air during the time occupied by the exp

Its accuracy, however, renders it peculiarly suitable for checking the results obtained by other methods.

296. Weight of a given Volume of Moist Air.—The laws of vapours and the known formulæ of expansion enable us to solve a problem of very frequent occurrence, namely, the determination of the weight of a given volume of moist air. Let V denote the volume of this air, H its pressure, f the tension of the vapour of water in it, and t its temperature. The entire gaseous mass may be divided into two parts, a volume V of dry air at the temperature t and the pressure $H-f$, whose weight is, by the known formulæ,

$$V \times 1{\cdot}293 \times \frac{1}{1+at} \cdot \frac{H-f}{760},$$

and a volume V of aqueous vapour at the temperature t and the pressure f; the weight of this latter is

$$\frac{5}{8}V \times 1{\cdot}293 \times \frac{1}{1+at} \cdot \frac{f}{760}.$$

The sum of these two weights is the weight required, viz.

$$V \times 1{\cdot}293 \times \frac{1}{1+at} \cdot \frac{H-\frac{3}{8}f}{760}.$$

297. Ratio of the Volumes occupied by the same Air when saturated at Different Temperatures and Pressures.—Suppose a mass of air to be in presence of a quantity of water which keeps it always saturated; let H be the total pressure of the saturated air, t its temperature, and V its volume.

At a different temperature and pressure t' and H′, the volume occupied V′ will in general be different. The two quantities V and V′ may be considered as the volumes occupied by a mass of dry air at temperatures t and t' and pressures $H-f$ and $H'-f'$; we have then (§ 201) the relation

$$\frac{V}{V'} = \frac{H'-f'}{H-f} \cdot \frac{1+at}{1+at'} \qquad (1)$$

In passing from one condition of temperature and pressure to another, it may be necessary, for the maintenance of saturation, that a new quantity of vapour should be formed, or that a portion of the vapour should be condensed, or again, neither the one nor the other change may take place. To investigate the conditions on which these alternatives depend, let D and D′ be the maximum densities of vapour at the temperatures t and t' respectively. Suppose we have $t' > t$, and that, without altering the pressure f, the temperature of the is raised to t', all contact with the generating liquid being pre The vapour will no longer remain saturated; but, on increasi pressure to f', keeping the temperature unchanged, saturati again be produced. This latter change does not alter the quantity of vapour, and if we suppose its coefficient of expan be the same as that of air, we shall have (§ 201)

$$\frac{D}{D'}=\frac{f}{f'}\cdot\frac{1+at'}{1+at}, \qquad (2)$$

and, by multiplying together equations (1) and (2), we have

$$\frac{VD}{V'D'}=\frac{H'f-ff'}{Hf'-ff'}. \qquad (3)$$

From this result the following particular conclusions may duced:—

1. If $H'f=Hf'$, $VD=V'D'$, that is, the mass of vapour is th in both cases; consequently, neither condensation nor evap takes place.

2. If $H'f>Hf'$, $VD>V'D'$, that is, partial condensation occ

3. If $H'f<Hf'$, $VD<V'D'$, that is, a fresh quantity of va required to maintain saturation. In this case the formula only be applied when we are sure that there is a sufficient e liquid to produce the fresh quantity of vapour which is requi

The general formulæ (1), (2), (3) furnish the solution of particular problems which may be proposed by selecting so of the variables for the unknown quantity.

298. Aqueous Meteors.—The name *meteor*, from the Greek μ *aloft*, though more especially applied to the bright objects ot called shooting-stars and their like, likewise includes all the phenomena which have their seat in the atmosphere; for e clouds, rain, and lightning. This use of the word *meteor* is somewhat rare; but the correlative term *meteorology* is inv employed to denote the science which treats of these phenom fact, the *science of matters pertaining to weather.*

By *aqueous meteors* are to be understood the phenomena result from the condensation of aqueous vapour contained in such as rain, dew, and fog. This condensation may occur i of two ways. Sometimes it is caused by the presence of a col which reduces the film of air in contact with it to a temp below the dew-point, and thus produces the liquefaction or so tion of a portion of its vapour in the form of dew or hoar-frost. The circumstances connected with the formation of dew will be treated in Chapter xxix. All that we need remark now is that it essentially consists in the deposition of moisture on the very body whose low temperature causes the condensation.

When, on the contrary, the condensation of vapour takes place in the interior of a large mass of air, the resulting liquid or solid *falls* in obedience to gravity. This is the origin of rain and snow.

299. Cloud and Mist.—When vapour is condensed in the midst of the air, the first product is usually *mist* or *cloud*, a cloud being merely a mist at a great elevation in the air.

Natural clouds are similar in constitution to the cloudy substance which passes off from the surface of hot water, or which escapes in puffs from the chimney of a locomotive. In common language this substance is often called steam or vapour, but improperly, for steam is, like air, transparent and invisible, and the appearance in question is produced by the presence of particles of liquid water, which have been formed from vapour by cooling it below its dew-point.

Naturalists are not agreed as to the nature of these particles, the difference of opinion having arisen in the attempt to explain their suspension in the atmosphere. Some have endeavoured to account for it by maintaining that they are hollow;[1] but even if we could conceive of any causes likely to lead to the formation of such bubbles, it would furnish no solution of the difficulty, for the air inclosed in a bubble is no rarer, but in fact denser, than the external air (§ 97 B); the bubble and its contents are therefore heavier than the air which it displaces.

It is more probable that the particles are solid spheres differing only in size from rain-drops. It has been urged against this view, that such drops ought to exhibit rainbows, and the objection must be allowed to have some weight. The answer to it is probably to be found in the excessive smallness of the globules. Indeed, the non-occurrence of bows may fairly be alleged as proving that the diameters of the drops are comparable with the lengths of waves of light.

This smallness of the particles is amply sufficient to explain all the observed facts of cloud suspension, without resorting to any special theory. It probably depends on the same principle as the suspension of the motes which are rendered visible when a beam of su traverses a darkened room. It is true that these motes, whi small particles of matter of the most various kinds, are neve resting stationary in the air; but neither are the particles compose clouds. All who have ever found themselves in mo mists must have observed the excessive mobility of their cons parts, which yield to the least breath of wind, and are carried by it like the finest dust. Sometimes, indeed, clouds ha *appearance* of being fixed in shape and position; but this illusion due to distance which renders small movements in In many cases, the fixity is one of form and not of materi example, the pern cloud on a mounta often consists of sive portions of air, become cloudy b densation as the through the cold at the top of the tain, and recover transparency as th away.

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[1] Those who adopt this view call them *vesicles* (*vesica*, a bladder), and call mist or cloud *apour in the vesicular state.*

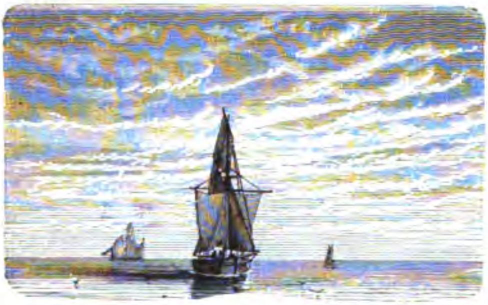

Fig. 273.—Cirrus.

300. Varieties of

—The cloud nomenclature generally adopted by meteorologis devised by Howard, and is contained in his work on the clin London. The fund tal forms, accordi him, are three— *cumulus*, and *stra*

1. *Cirrus* consi fibrous, wispy, or fe clouds, occupying highest region of mosphere. The *mare's-tails*, whi given them by describes their well. They are higher than the greatest elevations attain balloons, and are probably composed of particles of ice. It this species of cloud, and its derivatives, that haloes are u

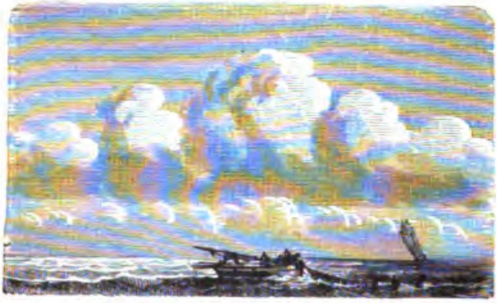

Fig. 274.—Cumulus.

seen; and their observed forms and dimensions seem to agree with the supposition that they are formed by refractions and reflections from ice-crystals.

2. *Cumulus* consists of rounded masses, convex above and comparatively flat below. Their form bears a strong resemblance to *heaps* of cotton wool, hence the names *balls of cotton* and *wool-packs* applied to these clouds by sailors. They are especially prevalent in *summer*, and are probably formed by columns of ascending vapour which become condensed at their upper extremities.

3. *Stratus* consists of horizontal sheets. Its situation is low in the atmosphere, and its formation is probably due to the cooling of the earth and the lower portion of the air by radiation. It is very frequently formed at sunset, and disappears at sunrise.

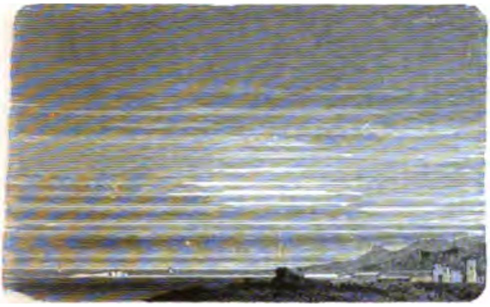

Fig. 275.—Stratus.

Of the intermediate forms it may suffice to mention *cirro-cumulus*, which floats at a higher level than cumulus, and consists usually of small roundish masses disposed with some degree of regularity. This is the cloud which forms what is known as a *mackerel sky*.

As a distinct form not included in Howard's classification, may be mentioned *scud*, the characteristic of which is that, from its low elevation, it *appears* to move with excessive rapidity.

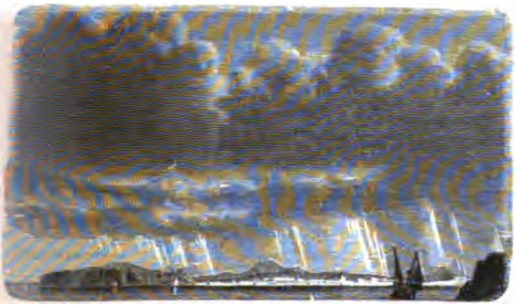

Fig. 276.—Nimbus.

Howard gives the name of *nimbus* to any cloud which is discharging rain; and, for no very obvious reason, he regards this rain-cloud as compounded of (or intermediate between) the three elementary types above defined.

The classification of clouds is a subject which scarcely admits of

precise treatment; the varieties are so endless, and they sl
gradually into one another.

301. Causes of the Formation of Cloud and Mist.—Since clo
merely condensed vapour, their formation is regulated by the
which tend to convert vapour into liquid. Such liquefaction
the presence of a quantity of vapour greater than that which
actual temperature, would be sufficient for saturation, a cond
things which may be brought about by the cooling of a mass c
air in any of the following ways:—

(1.) By radiation from the mass of air to the cold sky.

(2.) By the neighbourhood of cold ground, for example, mo
tops.

(3.) By the cooling effect of expansion, when the mass of air
into regions of diminished pressure. This cooling of the as
mass is accompanied by a corresponding warming of the ai
descends, it may be in some distant locality, to supply its pla

Causes (2) and (3) combine to produce the excessive
which generally characterizes mountainous districts.[1]

It is believed that waterspouts are produced by the rapi
of a stream of air up the axis of an aerial vortex.

(4.) By the contact and mixture of cooler air.[2] It is obvio
ever, that this cooler air must itself be warmed by the proce
as both the temperature and vapour-density of the mixture
intermediate between those of the two components, it does n
ously follow (as is too often hastily assumed) that such conta
to produce precipitation. Such is however the fact, and it
upon the principle that the density of saturation increase
than the temperature; or, what is the same thing, that th
in which temperature is the abscissa and maximum vapour-
the ordinate, is everywhere concave upwards.

It will be sufficient to consider the case of the mixing of tw
volumes of saturated air at different temperatures, which
denote by t_1 and t_2. Let the ordinates AA′, BB′ repres
densities of vapour for saturation at these temperatures,
being the intermediate portion of the curve, and Cm the

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[1] The rainiest place at present known in Great Britain is about a mile sou
thwaite in Cumberland, where the annual rainfall is about 165 inches. The raini
the world is believed to be Cherra Ponjee, in the Khasyah Mountains, about 300
of Calcutta, where the annual fall is about 610 inches.

[2] *Contact* with cooler air may be regarded as equivalent to mixing: for vapo
readily.

at the middle point of AB, representing therefore the density of saturation for the temperature $\frac{1}{2}(t_1+t_2)$. When the equal volumes are mixed, since the colder *mass* is slightly the greater, the temperature of the mixture will be something less than $\frac{1}{2}(t_1+t_2)$, and, if there were no condensation of vapour, the density of vapour in the mixture would be $\frac{1}{2}(AA'+BB') = Cn$. But the density for saturation is something less than Cm. The excess of vapour is therefore represented by something more than mn. The amount actually precipitated will, however, be less than this, since the portion which is condensed gives out its latent heat, and thus contributes to keep up the temperature of the whole.

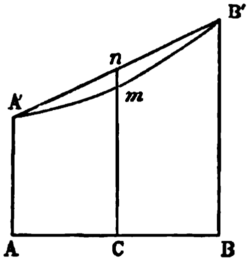

Fig. 273a.

The cause here indicated combines with (3) to produce condensation when masses of air ascend.

On the surface of the earth mists are especially frequent in the morning and evening; in the latter case extending over all the surface; in the former principally over rivers and lakes. The mists of evening are due simply to the rapid cooling of the air after the heat of the sun has been withdrawn. In the morning another cause is at work. The great specific heat of water causes it to cool much more slowly than the air, so that the vapour rising from a body of water enters into a colder medium, and is there partly condensed, forming a mist, which, however, confines itself to the vicinity of the water, and is soon dissipated by the heat of the rising sun.

302. Rain.—In what we have stated regarding the constitution of clouds, it is implied that clouds are always raining, since the drops of which they are composed always tend to obey the action of gravity. But, inasmuch as there is usually a non-saturated region intervening between the clouds and the surface of the earth, these drops, when very small, are usually evaporated before they have time to reach the ground. Ordinary rain-drops are formed by the coalescing of a number of these smaller particles.

By the amount of annual rainfall at a given place is meant the depth of water that would be obtained if all the rain which falls there in a year were collected into one horizontal sheet; and the depth of rain that falls in any given shower is similarly reckoned.

It is the depth of the pool which would be formed if the g were perfectly horizontal, and none of the water could get The instrument employed for determining it is called a *rain-*

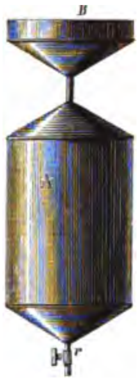

Fig. 277.
Rain-gauge.

It has various forms, one of which is represen the adjoining figure. B is a funnel into whi rain falls, and from which it trickles into the re A. It is drawn off by means of the stop-cock measured in a graduated glass, each division of corresponds to $\frac{1}{100}$ of an inch of rain.[1]

It is essential that the receiving surface—the the funnel—should be truly horizontal, otherwi gauge will catch too much or too little accord the direction of the wind.

The best place for a rain-gauge is the centr level and open plot; and the height of its re surface should be not less than 6 inches, to av splashing. The roof of a house is a bad place count of the eddies which abound there.

A circumstance which has not yet been fully explained is t higher a gauge is above the ground the less rain it catches. case of gauges on the top of poles in an open situation, the a collected is diminished by $\frac{1}{10}$th part of itself by doubling the of the receiving surface, as shown by comparing gauges in th plot of ground at heights ranging from 6 inches to 20 feet.[2]

By means of tipping-buckets and other arrangements, aut records of rainfall are obtained at the principal observatories best of these pluviometers is Osler's, which, by means of springs stretched by the weight of the water, furnishes a cont record, until a quarter of an inch has been collected, wh reservoir empties itself, and a fresh record begins.

The mean annual rainfall, according to Mr. Symons, is 20 in Lincoln and Stamford; 21 at Aylesbury, Bedford, and Witham London and Edinburgh; 30 at Dublin, Perth, and Salisbury Exeter and Clifton; 35 to 36 at Liverpool and Manchester Glasgow and Cork; 50 at Galway; 64 at Greenock and Inver at Dartmoor; and 91 on Benlomond.

[1] The best work on the subject of rain and its measurement is the little treatis *Rain* by Mr. G. J. Symons, a gentleman who devotes his life to the investi British rainfall. Mr. Symons would probably tell us that the funnel in the ab is too flat, and would cause some of the rain to splash out.

[2] This appears from the table in Symons on *Rain*, p. 19.

303. Snow and Hail.—Snow is probably formed by the direct passage of vapour into the solid state. Snow-flakes, when examined under the microscope, are always found to be made up of elements possessing hexagonal symmetry. In Fig. 278 are depicted various forms observed by Captain Scoresby during a long sojourn in the Arctic regions.

In these cold countries the air is often filled with small crystals of ice which give rise to the phenomena of haloes and parhelia.

Hail is probably due to the freezing of rain-drops in their passage through strata of air colder than those in which they were formed. Even in fine summer weather, a freezing temperature exists at the height of from 10,000 to 20,000 feet, and it is no unusual thing for a colder stratum to underlie a warmer, although, as a general rule, the temperature diminishes in ascending.

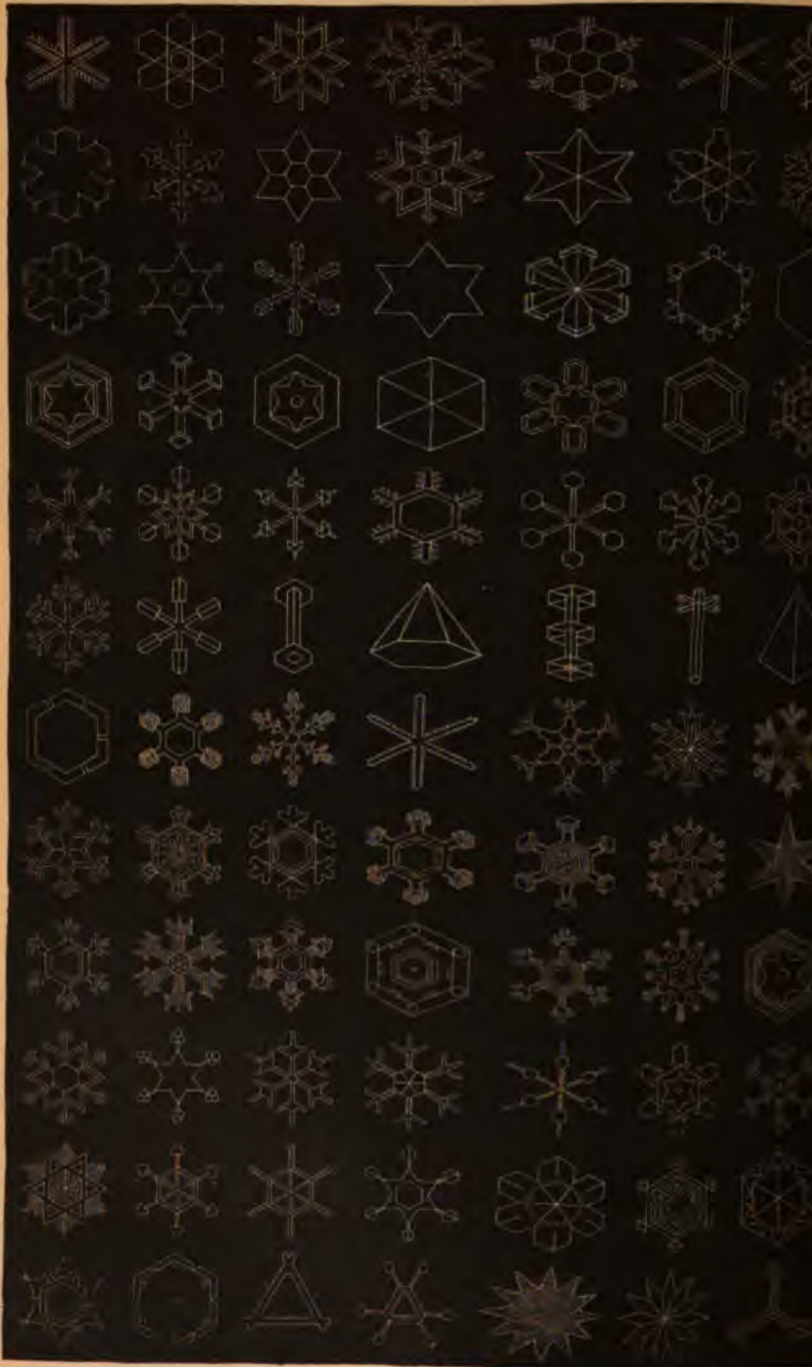

Fig. 278.—Snow-crystals.

CHAPTER XXIX.

RADIANT HEAT.

[illegible] **Radiation.**—When two bodies at different temperatures are brought opposite to each other, an unequal exchange of heat takes place through the intervening distance; the temperature of the hotter body falls, while that of the colder rises, and after some time the temperature of both becomes the same. This propagation of heat across an intervening space is what is meant by *radiation*, and the heat transmitted under these conditions is called *radiant heat*. Instances of heat communicated by radiation are the heat of a fire received by a person sitting in front of it, and the heat which the earth receives from the sun.

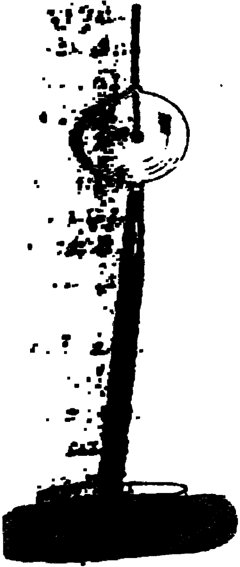

Fig. 279.—Rumford's Experiment.

This last instance shows us that radiation as a means of propagating heat is independent of any ponderable medium. But since the solar heat is accompanied by light, it might still be questioned whether dark heat could in the same way be propagated through a vacuum.

This was tested by Rumford in the following way:—He constructed a barometer (Fig. 279), the upper part of which was expanded into a globe, and contained a thermometer hermetically sealed into a hole at the top of the globe, so that the bulb of the thermometer was at the centre of the globe. The globe was thus a Torricellian vacuum-chamber. By melting the tube with a blow-pipe, the globe was separated, and was then immersed in a vessel containing hot water, when the thermometer was immediately observed to rise to a temperature evidently higher than could be

due to the conduction of heat through the stem. The heat ha
fore been communicated by direct radiation through the
between the sides of the globe and the bulb *a* of the thermo

306. **Radiant Heat travels in Straight Lines.**—In a uniform
the radiation of heat takes place in straight lines. If, for i
between a thermometer and a source of heat, there be placed
ber of screens, each pierced with a hole, and if the scree
arranged that a straight line can be drawn without interrupti
the source to the thermometer, the temperature of the latte
diately rises; if a different arrangement be adopted, the
stopped by the screens, and the thermometer indicates no ef

The heat which travels along any one straight line is call
of heat. Thus we say that rays of heat issue from all point
surface of a heated body, or that such a body emits rays of

Such language may be thought to imply the hypothesis t
is a substance (caloric) which is accumulated in bodies, and
by them in all directions. A ray of heat would thus con
series of molecules of caloric issuing forth one after the ot
straight line. But, in fact, the definition which we have ju
of a ray of heat is independent of any hypothesis, and is
experimental; it amounts merely to the expression of the in
able fact that the direction of radiation is rectilinear. W
idea we may form about the nature of heat, it must be su
imply this rectilinear propagation.

It is now generally admitted that both heat and light ar
a vibratory motion which is transmitted through space by
a fluid called ether. According to this theory the rays of l
heat are lines drawn in all directions from the origin of mot
along which the vibratory movement advances.

307. **Law of Cooling.**—It is often important to know
according to which a body cools when placed in an inclosure
temperature than its own; for we are thus enabled to take
of the heat which a body loses during the progress of an exp
This law, when stated in such terms as to be applicable to all
differences of temperature and all possible conditions of the s
ing medium, becomes exceedingly complex; but when the d
of temperature is small, amounting only to a few degrees,
known as Newton's law of cooling can be applied without
error. It is this:—*the rate at which the body loses heat is*
tional to the difference between the temperature of its sur

that of the inclosure. If the body be of sensibly uniform temperature throughout its whole mass, as in the case of a vessel with thin metallic sides containing water which is kept stirred, or of the quicksilver in the bulb of a thermometer, the fall of temperature is proportional to the loss of heat, and Newton's law as applied to such a body asserts that *the rate at which the temperature falls is proportional to the excess of the temperature of the body above that of the inclosure.*

To test this law experimentally, we observe from time to time the excess of the temperature of a thermometer above that of the air in which it is cooling. It is found, that, if the observations are made at equal intervals of time, the observed excesses form a decreasing geometric series.

To express this fact algebraically, let θ_0 denote the initial excess of temperature, and $\frac{1}{m}$ the ratio of the series. Then the excess at the end of a unit of time will be $\frac{\theta_0}{m}$, at the end of two units $\frac{\theta_0}{m^2}$, and after t units $\frac{\theta_0}{m^t}$; so that if θ denote the excess at time t, we have

$$\theta = \frac{\theta_0}{m^t} = \theta_0 m^{-t}. \qquad (1)$$

One pair of observations is sufficient to determine the value of the constant m, which is different for different thermometers.

By *rate of cooling* is meant the *fall of temperature per unit time* which is taking place at the instant considered. This is computed approximately by dividing the fall of temperature in a small interval of time by the length of the interval. Its exact value is given by the differential calculus, and is

$$-\frac{d\theta}{dt} = \theta_0 m^{-t} \log_e m = \theta \log_e m, \qquad (2)$$

which is proportional to θ, as asserted by Newton's law

A precisely similar law holds for *warming.*

307A. Cooling by Radiation in Vacuo.—The cooling of a thermometer in air is effected partly by the contact of the air, and partly by radiation. When the thermometer is placed in the centre of a vacuous space, radiation alone can operate, namely, radiation from the thermometer to the walls of the inclosure. The law of cooling under these conditions has been investigated experimentally by Dulong and Petit, and was reduced by them to a formula which was found to be accurate within the limits of experimental error for all the ranges of temperature employed, the excess of the tempe of the thermometer above that of the walls of the inclosure r from 20° to 240° C. They found that the rate of cooling d depend upon the difference of temperature alone, but was fa high than at low temperatures; also that, for a given tempera the walls, the rate of cooling was not simply proportional excess, but increased more rapidly. Both these results are ex by their formula

$$V = ca^t (a^\theta - 1), \qquad (3)$$

where V denotes the rate of cooling, c and a constants, t th perature of the walls of the inclosure, and θ the excess of tempe so that $t+\theta$ is the temperature of the thermometer. If we this by t' the formula may be thrown into the more symm form

$$V = c\,(a^{t'} - a^t), \qquad (4)$$

which suggests the idea that an unequal *exchange* of heat take between the thermometer and the walls, the thermometer gi the walls a quantity of heat represented by $a^{t'}$, and recei exchange[1] only the quantity a^t. The former of these a remains the same at all temperatures of the inclosure, and th is the same for all temperatures of the thermometer.

When the temperatures are Centigrade, the constant a is When they are Fahrenheit it is 1·0043, the form of the ex for V being unaffected by a change of the zero from which t peratures are reckoned. The value of c depends upon the siz bulb and some other circumstances, and is changed by a ch zero.

By developing a^θ in ascending powers of θ, it will be fou formula (3) agrees sensibly with Newton's law when the e temperature does not exceed a few degrees.

308. Law of Inverse Squares.—If we take a delicate therm and place it at successively increasing distances from a source the temperature indicated by the instrument will exceed tha atmosphere by decreasing amounts, showing that the inte radiant heat diminishes as the distance increases. The law o tion may be discovered by experiment. In fact, when the excess of temperature of the thermometer becomes fixed, we know that the heat received is equal to that lost by radiation; but this latter is, by Newton's law, proportional to the excess of temperature above that of the surrounding air; we may accordingly consider this excess as *the* measure of the heat received. It has been found, by experiments at different distances,[1] that the excess is inversely proportional to the *square* of the distance; we may therefore conclude that *the intensity of the heat received from any source of heat varies inversely as the square of the distance.*

---

[1] According to the *theory of exchanges*, the heat emitted by the thermometer i a constant term depending on the zero of the temperature scale employed, and absorbed by it is ca^t plus the same constant. It may be remarked that the fa depends upon the zero from which temperatures are reckoned as well as upon t of the degrees, whereas a depends on the latter only.

The following experiment, devised by Tyndall, supplies another simple proof of this fundamental law:—

The thermometer employed is a Melloni's pile, the nature of which we shall explain in § 313. This is placed at the small end of a hollow cone, blackened inside, so as to prevent any reflection of heat from its inner surface. The pile is placed at S and S′ in front of a vessel filled with boiling water, and coated with lamp-black on the side next the pile. It will now be observed that the temperature indicated by the pile remains constant for all distances. This result proves the law of inverse squares. For the arrangement adopted prevents the pile from receiving more heat than that due to the area of AB in the first case, and to the area A′B′ in the second. These are the areas of two circles, whose radii are respectively proportional to SO and S′O; and the areas are consequently proportional to the

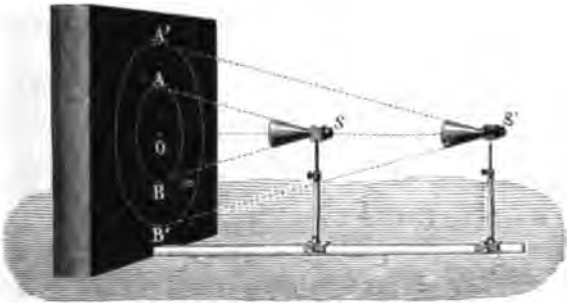

Fig. 280.—Law of Inverse Squares.

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[1] The dimensions of the source of heat must be small in comparison with the distance of the thermometer, as otherwise the distances of different parts of the source of heat from the thermometer are sensibly different. In this case, the amount of heat received varies directly as the solid angle subtended by the source of heat.

squares of SO and S′O. Since, therefore, these two areas co cate the same quantity of heat to the pile, the intensity of ra must vary inversely as the squares of the distances SO and S

The law of inverse squares may also be established *a priori* following manner:—

Suppose a sphere of given radius to be described about a ra particle as centre. The total heat emitted by the particle received by the sphere, and all points on the sphere will ex the same calorific effect. If now the radius of the sphere be d the surface will be quadrupled, but the total amount of heat the same as before, namely, that emitted by the radiating Hence we conclude that the quantity of heat absorbed by area on the surface of the large sphere is one-fourth of that a by an equal area on the small sphere; which agrees with stated above.

This demonstration is valid, whether we suppose the radi heat to consist in the emission of matter or in the emission of for energy as well as matter is indestructible, and remains u in amount during its propagation through space.

309. Law of the Reflection of Heat.—When a ray of heat a polished surface, it is reflected in a direction determined laws.

If at the point of incidence, that is, the point where the ra the surface, a line be drawn normal or perpendicular to the the plane passing through this line and the incident ray is ca *plane of incidence.* With this explanation we proceed to laws of the reflection of heat:—

1. When a ray of heat is reflected by a surface, the line tion lies in the plane of incidence.

2. The angle of reflection is equal to the angle of inciden is, the reflected and incident rays make equal angles normal to the surface at the point of incidence.

We shall see hereafter that these laws are precisely the those of the reflection of light. In the case of light, they can strictly verified. And since, in all phenomena involving heat and light, we find the same laws holding for calorifi luminous rays, we may consider the demonstration of these the case of light as sufficient to prove that they hold good of heat also.

310. Burning-mirrors.—The laws of the reflection of heat, may be directly verified by some well-known experiments, especially by means of a concave spherical mirror (Fig. 281), that is, a mirror consisting of a portion of a hollow sphere, with its concave side polished. The principal axis of this mirror is a line CF passing through the centre of the sphere, and through the middle point A (which may be called the *pole*) of the mirror itself. It may easily be deduced from the law of reflection that if a pencil of rays parallel to the axis are incident upon the mirror, they will all, after reflection, pass through or very near the point F which bisects the radius, and is called the principal focus. We may remark that the rays need not be parallel to the principal axis; it is only necessary that they be parallel to each other, in order that they should meet in a focus situated on the line drawn through the centre in a direction parallel to the pencil of rays.

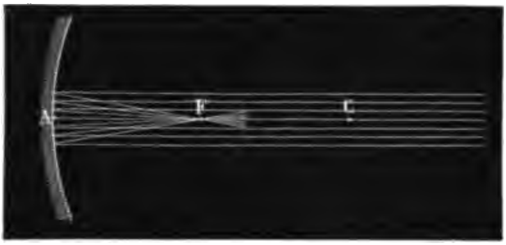

Fig. 281.—Focus of Concave Mirror.

These theoretical conclusions have been verified by experiments with burning-mirrors. These are spherical[1] concave mirrors, turning about an axis, so as to receive the rays of the sun perpendicularly upon their surface. The heat produced under these circumstances at the focus of the mirror, is sufficiently great to inflame any combustible material placed there, or even to produce more striking effects. Tschirnhausen's mirror, for instance, which was constructed in 1687, and was about 6½ feet in diameter, was able to melt copper or silver, and to vitrify brick. Instead of curved mirrors, Buffon employed a number of movable plane mirrors, which were arranged so t different pencils of heat-rays reflected by them converged t the same focus. In this obtained an extremely p effect, and was able, for i to set wood on fire at a of between 80 and 90 This is the method whic medes is said to have e for the destruction of the fleet in the siege of S and though the truth of t is considered doubtful, i altogether absurd.

---

[1] *Parabolic* mirrors are perhaps more frequently employed for experiments on the reflection of heat. All rays falling on a parabolic mirror parallel to its axis are *accurately* reflected to its focus, and all rays incident upon it from a source of heat or light at the focus are reflected parallel to the axis.

The mirrors in the experiment of § 311 are most easily adjusted by first placing a source of light (such as the flame of a candle) in one focus, and forming a luminous image in the other. We have thus a convincing proof that heat and light obey the same laws as regards direction of reflection.

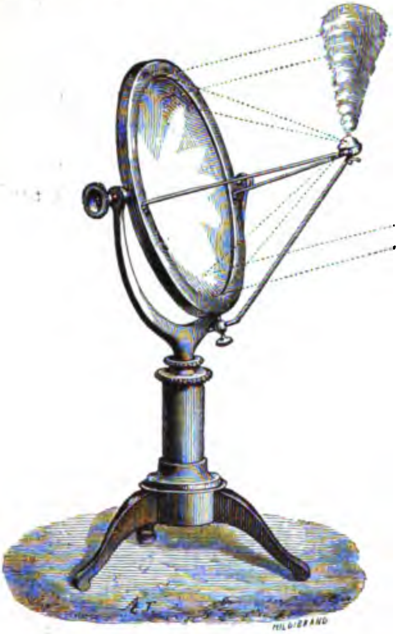
Fig. 282.—Burning Mirror.

311. Conjugate Mirr more rigorous demonstr the laws of the reflection is afforded by the celebr periment of the conjug rors, which is generally to Pictet of Geneva.

Two spherical mirrors ed at any convenient with their concave surf wards each other and their axes in the same straight line. focus of one of them is placed a small furnace, or a red-hot ball, and in the focus of the other some highly inflammab rial, such as phosphorus or gun-cotton. On exciting the with bellows, the substance in the other focus immediate fire. With two mirrors of 14 inches diameter, gun-cotton n be set on fire at a distance of more than 30 feet. The exp is very easy. The rays of heat coming from the focus of mirror are reflected in parallel lines, and, on impinging u surface of the second mirror, converge again to its focus, thus concentrated upon the inflammable material placed the

It might appear at first sight that the experiment is not v cise, owing to the comparatively large size of the source of h we must remark that only a very small portion of this actu duces any effect; the rest serving merely to maintain a su high temperature at the points which really send rays to t

Again, the image of the source of heat which is formed at the focus of the second mirror, is so small, that the slightest change in the position of either mirror causes the failure of the experiment. Great exactness is therefore required to insure success; and we may consequently regard the experiment as a tolerably severe test of the truth of the laws of reflection which have been quoted above.

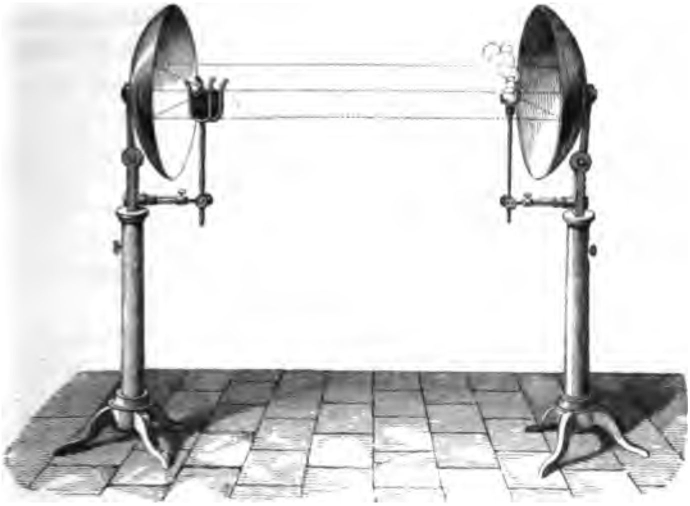

Fig. 283.—Conjugate Mirrors.

312. Different Properties of Bodies with respect to Radiant Heat.—Suppose a quantity of heat denoted by unity to be incident upon he surface of a body. This quantity will be divided into several istinct parts.

1. A portion will be regularly reflected according to the law given ove. If the fraction of heat thus reflected be denoted by $\frac{1}{r}$, then is the measure of the *reflecting power.*

2. A portion $\frac{1}{d}$ will be irregularly reflected, and will be scattered diffused through space in all directions. Thus $\frac{1}{d}$ is the measure the *diffusive power.*

3. A portion $\frac{1}{a}$ will penetrate into the body so as to be absorbed it, and to contribute to raise its temperature; $\frac{1}{a}$ is therefore the sure of the *absorbing power.*

4. Finally, we shall have, in many cases, a fourth portion
passes through the body without contributing to raise its t
ture. This fraction, which exists only in the case of diathe
bodies, is the measure of *diathermancy* or transmissive pow

The sum of these fractional parts must evidently make
original unit; that is

$$\frac{1}{r}+\frac{1}{d}+\frac{1}{a}+\frac{1}{\delta}=1.$$

312 A. Relation between Absorption and Emission.—A differe
of expressing absorbing power is sometimes required—a moc
shall express the rate at which the body gains heat when co
inclosed in an exhausted space whose walls have a higher
ture than its own.

Let the walls be covered with lamp-black, and have a sma
of temperature θ, and let the area of the body's surface (wh
have no concavities) be S; then the rate at which the bo
heat is proportional to $S\theta$, and may be denoted by $AS\theta$, A
coefficient which is independent of θ provided that θ is s
which is not necessarily the same at high as at low temp
We shall call this factor A the *coefficient of absorption;* and
gained by the body in a short interval of time τ will be

$$q=AS\theta\tau.$$

If now we suppose the inclosed body to be warmer than
by a small excess θ', the rate of losing heat will be propor
$S\theta'$, and may be denoted by $ES\theta'$, E being called the coef
emission; and the quantity of heat lost by the body in t
time τ is

$$q'=ES\theta'\tau.$$

Now experiment shows that, for any given body at a given
ture, the values of A and E are equal; for example, a ther
inclosed in a vacuous globe, as in the experiments of Du
Petit (§ 307 A), takes the same time to rise 1° as to fall 1° if t
difference of temperature between the thermometer and t
was 2° in each case.

The *coefficient of emission* then, at any given temperature
to the coefficient of absorption; and we may therefore give
metrical name, and call it the *coefficient of radiation.* A s
for which this coefficient has a large value is said to be a good

Now it is obviously impossible for a body to absorb m than falls upon it. There must therefore be a limiting value of A applicable to a body whose absorbing power $\frac{1}{a}$ is unity; and the same limiting value must exist for E which is equal to A.

Hence it appears that good radiation depends rather upon defect of resistance than upon any positive power. A perfect radiator would be a substance whose surface opposed no resistance to the communication of radiant heat in either direction; while an imperfect radiator is one whose surface allows a portion to be communicated through it, and reflects another portion regularly or irregularly.

We may conveniently employ $\frac{1}{e}$ to denote the ratio of the heat emitted by a surface to that which would be emitted by a perfect radiator at the same temperature. We can then assert that for any one kind of heat

$$\frac{1}{e}=\frac{1}{a},$$

or the *emissive and absorptive powers are equal.*

The reflecting and diffusive powers of lamp-black are so insignificant, at temperatures below 100° C., that this substance is commonly adopted as the type of a perfect radiator, and the emissive and absorptive powers of other substances are usually expressed by comparison with it.

Recent experiments by Sir W. Thomson show that the coefficient of radiation A or E between two lamp-black surfaces radiating to each other in vacuo is about $\frac{1}{8000}$, q being expressed in gramme-degrees (§ 339), θ in degrees,[1] S in square centimetres of surface of the inclosed body, and τ in seconds.

312a. Different Kinds of Heat-rays.—A beam of radiant heat or light is not, generally speaking, homogeneous, but (as we shall more fully explain in connection with optics) is made up of rays differing in wave-length, and capable of being separated by transmission through a prism, those which have the shortest wave-lengths being refracted or turned out of their original direction to the greatest extent. A beam of radiant heat or light may also possess peculiar properties comprehended under the name of polarization.

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[1] It is immaterial whether the Cent. or Fahr. scale be employed—for the degree and gramme-degree change in the same ratio.

The experiments referred to (which will shortly be published in the Proc. Roy. Soc.), were conducted by observing the cooling of a copper ball in an inclosure filled with air. The total loss of heat corresponded to a co-efficient $\frac{1}{2000}$, and it was estimated that half the loss was due to atmospheric contact, and the other half to radiation.

Almost all substances exhibit the phenomenon of *selective*
tion, that is to say, they absorb some kinds of heat more
than others; and it has been completely established by a v
experiments, that the heat which a body emits when radi
bodies colder than itself, consists chiefly of the same elementa
rays which it absorbs most copiously from bodies hotter tha

312 c. Theory of Exchanges.—Many of the phenomena of r
are most simply explained by the view now usually called th
of exchanges, but originally promulgated by Prévost of Gene
the name of the theory of *mobile equilibrium of temperatur*
theory asserts that all bodies are constantly giving out radia
at a rate depending upon their substance and temperat
independent of the substance or temperature of the bodie
surround them;[1] and that, when a body is kept at a unifo
perature, it receives back just as much heat as it gives out.

According to this view, two bodies at the same tem
exposed to mutual radiation, exchange equal amounts of hea
two bodies have unequal temperatures, that which is at th
temperature gives to the other more than it receives in exch

As a necessary accompaniment of this view, it is maintai
for each particular kind of heat, the emissive and absorbin
$\frac{1}{e}$ and $\frac{1}{a}$ (§ 312 A) are equal for any one body, and at any given t
ture; this inference being drawn from the well-known expe
fact that a body completely surrounded by an inclosure wh
are preserved at a constant temperature, will ultimately t
temperature. For the details of the reasoning we must
special treatises.[2]

According to this theory, the coefficient A or E (§ 312 A) fo
at temperature $t°$, represents the difference between the
emission at this temperature and at the temperature $t° + 1°$.

**313. Thermoscopic Apparatus employed in Researches c
with Radiant Heat.**—An indispensable requisite for the s
study of radiant heat is an exceedingly delicate thermomet
this purpose Leslie, about the beginning of the present
invented the differential thermometer, with which he conduc
very important investigations, the main results of which
acknowledged to be correct. Modern investigators, as

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[1] See § 307 A, formula (4) and foot-note.

[2] *Report on the Theory of Exchanges* by Balfour Stewart, in British Associati
1861, p. 97; and Stewart on *Heat*, book ii. chap. iii.

Laprovostaye, &c., have exclusively employed Nobili's thermo-multiplier, which is an instrument of much greater delicacy than the differential thermometer.

The thermo-pile inverted by Nobili, and improved by Melloni, consists essentially of a chain (Fig. 284) formed of alternate elements of bismuth and antimony. If the ends of the chain be connected by a wire, and the alternate joints *slightly* heated, a thermo-electric current will be produced, as will be *explained* hereafter. The amount of current increases with the number of elements, and with the difference of temperatures of the opposite junctions.

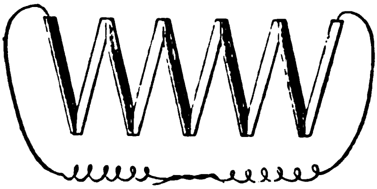

Fig. 284.—Nobili's Thermo-electric Series.

In the pile as improved by Melloni, the elements are arranged side by side so as to form a square bundle (Fig. 285), whose opposite

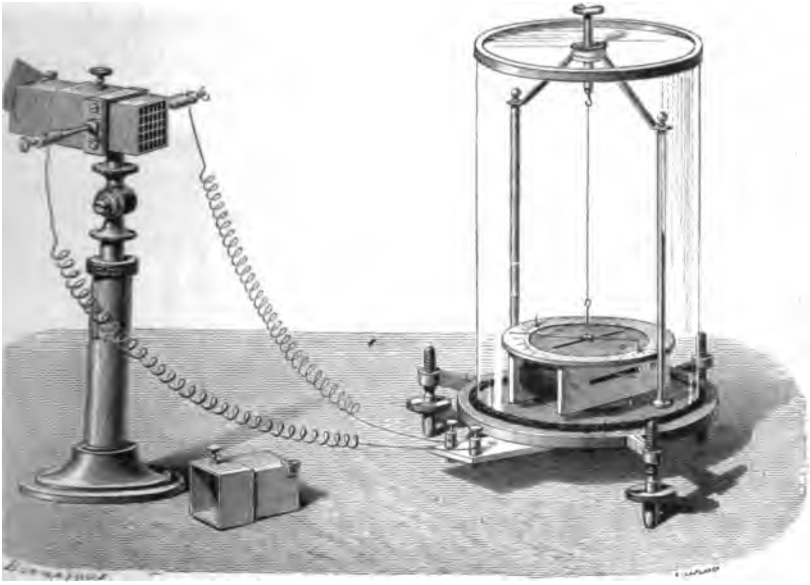

Fig. 285.—Melloni's Thermo-multiplier.

consist of the alternate junctions. The whole is contained in per case, with covers at the two ends, which can be removed it is desired to expose the faces of the pile to the action of heat.

Two metallic rods connect the terminals of the thermo-elec with wires leading to a galvanometer,[1] so that the existen current will immediately be indicated by the deflection of t The amounts of current which correspond to different defle known from a table compiled by a method which we shal hereafter. Consequently, when a beam of radiant heat st pile, an electric current is produced, and the amount of th is given by the galvanometer. We shall see hereafter, come to treat of thermo-electric currents, that within certa which are never exceeded in investigations upon radiant current is proportional to the difference of temperature bet two ends of the pile. Accordingly, as soon as all parts o have acquired their permanent temperatures, the quantit received during any interval of time from the source of he equal to that lost to the air and surrounding objects. But is, by Newton's law, proportional to the excess of temperat the surrounding air, and therefore to the difference of ter between the two ends of the pile. The current is therefor tional to the quantity of heat received by the instrument. thus in Nobili's pile a thermometer of great delicacy, and adapted to the study of radiant heat; in fact, the immens

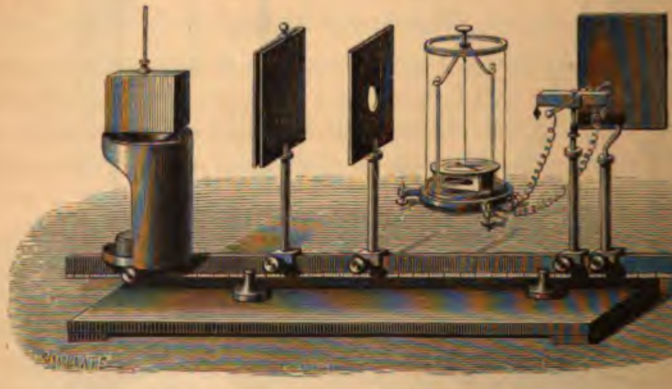

Fig. 286.—Measurement of Emissive Powers.

which has been made in this department of physics is mai to this invention of Nobili.

314. Measurement of Emissive Power.—The following arr was adopted by Melloni for the comparison of emissive p graduated horizontal bar (Fig. 286) carries a cube, the different sides of which are covered with different substances. This is filled with water, which is maintained at the boiling-point by means of a spirit-lamp placed beneath. The pile is placed at a convenient distance, and the radiation can be intercepted at pleasure by screens arranged for the purpose. The whole forms what is called Melloni's apparatus.

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[1] The pile and galvanometer together constitute the thermo-multipl

If we now subject the pile to the heat radiated from each of the faces in turn, we shall obtain currents proportional to the emissive powers of the substances with which the different faces are coated.

From a number of experiments of this kind it has been found that lamp-black has the greatest radiating power of all known substances, while the metals are the worst radiators. Some of the most important results are given in the following table, in which the emissive powers of the several substances are compared with that of lamp-black, which is denoted by 100:—

RELATIVE EMISSIVE POWERS AT 100° C.

Lamp-black,	100	Steel,	17
White-lead,	100	Platinum,	17
Paper,	98	Polished brass,	7
Glass,	90	Copper,	7
Indian ink,	85	Polished gold,	3
Shellac,	72	Polished silver,	3

316. Absorbing Power.—The method which most naturally suggests itself for comparing absorbing powers, is to apply coatings of different substances to that face of the pile which is exposed to the action of the source of heat. But this would involve great risk of injury to the pile.

The method employed by Melloni was as follows:—He placed in front of the pile a very thin copper disc (Fig. 287), coated with lamp-black on the side next the pile, and on the other side with the substance whose absorbing power was required. The disc absorbed heat by radiation from the source, of amount proportional to the absorbing power of this coating, and at the same time emitted heat from both sides in all directions. When its temperature became stationary, the amounts of heat absorbed and emitted were necessarily equal, and its two faces had sensibly equal temperatures.

Let E and E′ denote the coefficients of emission of lamp-black and of the substance with which the front was coated, and θ the excess of temperature of the disc above that of the air; then $(E+E')\theta$ is the heat emitted in unit time, if the area of each face is unity, must be equal to the heat absorbed in unit time.

But the indications of the thermo-pile are proportional to

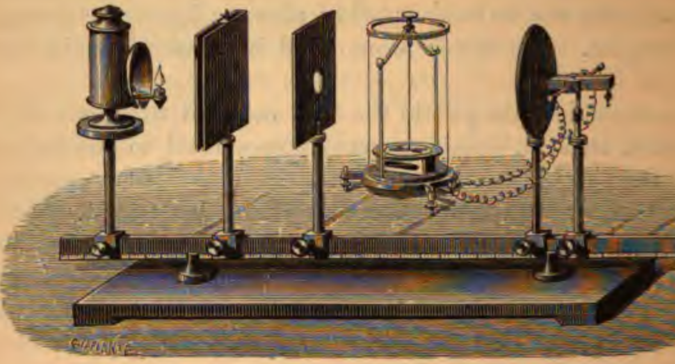

Fig. 287.—Measurement of Absorbing Powers.

radiated from the back alone, that is, to Eθ. The heat ab therefore represented by the indication of the pile multi $\frac{E+E'}{E}$.

In this way the absorbing powers given in the following been calculated from experiments of Melloni, the source of h a cube filled with water at 100° C.

Relative Absorbing Powers at 100° C.

Lamp-black,	100	Indian ink,	
White-lead,	100	Shellac,	
Isinglass,	91	Metal,	

It will be observed that these numbers are identical with tho represent the emissive powers of the same substances.

317. Variation of Absorption with the Source.—The a power varies according to the source of heat employed. lishing this important fact, Melloni employed the following s heat:—

1. Locatelli's lamp, a small kind of oil-lamp, in which th the oil remains invariable, and which has a square-cut so As a source of heat it is of tolerably constant action, and it employed in most of the experiments upon diathermancy shown in Fig. 287.

2. Incandescent platinum. This is a spiral of platinum wire (Fig. 288) suspended over a spirit-lamp so as to envelop the flame. The metal is heated to a bright white heat; and since the radiating powers of the flame are very feeble, the metal may be regarded as the sole source of radiation. The flame, in fact, is scarcely distinguishable.

3. Copper heated to about 400° C. This is effected by placing a spirit-lamp behind a curved copper plate (Fig. 289).

4. Copper covered with lamp-black at 100° C. This is a cube containing boiling water (Fig. 290) similar to that already described in

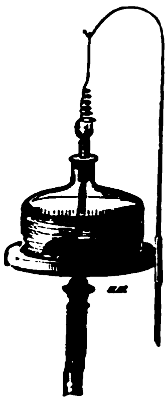

Fig. 288.
Incandescent Platinum.

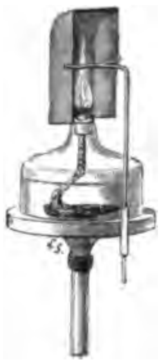

Fig. 289.
Copper heated to 400°.

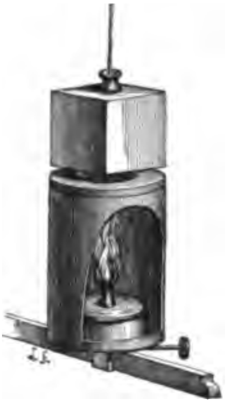

Fig. 290.
Cube at 100°.

onnection with the measurement of emissive powers. The face whose adiation is employed is covered with lamp-black.

If these different sources of heat be severally used in measuring sorbing powers, it will be found that these powers vary consider- ly according to the particular source of heat employed, and that if e *denote* the absorption of lamp-black in each case by 100, the lative absorbing powers of other substances are in general greater *the* temperature of the source is lower. In establishing this portant principle by experiment, the sources of heat are first *ced* at such distances that the direct radiation upon the pile ll be the same for each, and the pile is then replaced by the The following table contains some of the results obtained Melloni:—

Substances.	Locatelli's Lamp.	Incandescent Platinum.	Heated Copper.	Hot-wa Cube
Lamp-black, . . .	100	100	100	100
Indian ink, . . .	96	95	87	85
White-lead, . . .	53	56	89	100
Isinglass, ...	52	54	64	91
Shellac, ...	48	47	70	72
Metallic surface, .	14	13·5	13	13

319. Reflecting Power.—The reflecting power of a surface sured by the proportion of incident heat which is regularly from it. This subject has been investigated by Melloni, Laprovostaye and Desains. The arrangement used for the p shown in Fig. 291.

The substance under investigation is placed upon the circu D, which is graduated round the circumference. The pile E

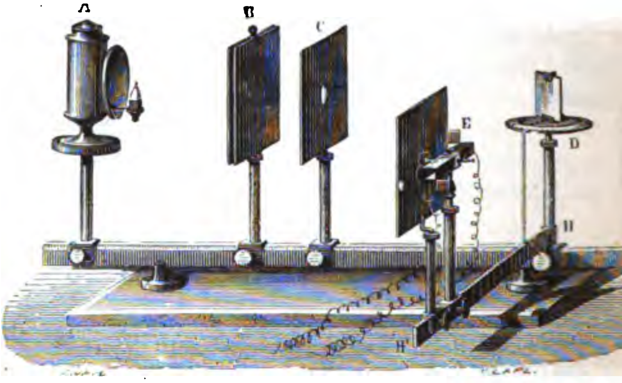

Fig. 291.—Measurement of Reflecting Power.

by the horizontal bar H H′, which turns about the pillar s the plate D. This bar is to be so adjusted as to make the refl impinge upon the pile, the adjustment being made by th the divisions marked on the circular plate.

In making an observation, the bar H H′ is first placed coincide with the prolongation of the principal bar, and the of direct radiation is thus observed. The pile is then place receive the reflected rays, and the ratio of the intensity thu to the intensity of direct radiation is the measure of the power.

The following are some of the results obtained by Laprovostaye and Desains, the source of heat employed being a Locatelli lamp:—

	Reflecting Power.		Reflecting Power.
Silver plate,	·97	Polished platinum,	·80
Gold,	·95	Steel,	·83
Brass,	·93	Zinc,	·81
Speculum metal,	·86	Iron,	·77
Tin,	·85		

Laprovostaye and Desains have also shown that, in the case of diathermanous substances, the reflecting power varies considerably with the angle of incidence, which is also the case for luminous rays.

In the case of metals, the change in the reflecting power produced by a change in the angle of incidence is not nearly so great; the reflecting power remains almost constant till about 70° or 80°, and when the angle of incidence exceeds this limit, the reflecting power decreases, whereas the opposite is the case with transparent bodies.

Finally, Laprovostaye and Desains have shown that, contrary to what was previously supposed, the reflecting power varies according to the source of heat. Thus the reflecting power of polished silver, which is ·97 for rays from a Locatelli lamp, is only ·92 for solar rays. In either case it will be seen that the reflecting powers of polished silver are very great; and since experiment has shown that luminous and calorific rays from the same source are reflected in nearly equal proportions, the advantages attending the use of silvered specula in telescopes can easily be understood.

320. Diffusive Power.—Diffusion is the irregular reflection of heat, doubtless owing to the minute inequalities of surface which are met with on even the most finely-polished bodies. The existence of this power may very easily be verified. We have only to let a beam of radiant heat fall upon any dead surface, for example on carbonate of lead. On placing the pile before the surface in any position, a deviation of the galvanometer is observed, which cannot be attributed to radiation from the surface, since in that case the effect, instead of instantly attaining its maximum, as it actually does, would increase gradually as the substance became warmed by the heat falling upon it.

Moreover the heat thus diffused, when the source of heat is a body at high temperature, such as a lamp-flame, is found to agree in its properties with the heat radiated from a body at high temperature, and to be altogether different from that which the diffusing surface is capable of radiating at its actual temperature. The diffused heat, for example, passes through a plate of alum without undergoi absorption.

The diffusive power of powders, especially if white, is siderable, as is shown by the following table taken from th published by Laprovostaye and Desains:—

DIFFUSIVE POWER.	
White-lead,	·82
Powdered silver,	·76
Chromate of lead,	·66

The knowledge of this property enables us to explain th heat which is felt in the neighbourhood of a white wall li by the sun.

Diffusion takes place in different proportions accordin direction, and is a maximum for points near the directi regularly-reflected ray.

The intensity of the diffused rays varies very considerab ing to the source of heat employed. This was shown by the following manner:—

He directed a ray of heat upon the surface of a disc of card covered with a substance capable of diffusing the r back of the disc was coated with lamp-black. When the parts had acquired their permanent temperatures, the pile in corresponding positions first before, and then behind th as to receive the heat due to both radiation and diffusion disc in the first case, and that due to radiation only in th It was found that the ratio of the two indications of the pi two positions varied very much according to the source of general rule being that the ratio of the diffused to the rad was greatest when the source of heat was luminous, and temperature.

321. Peculiar Property of Lamp-black.—If a similar expe performed with a card covered on both sides with lamp-bla be found that the difference between the indications of t the two positions is very small. This difference, such as be accounted for by a slight difference of temperature be two faces of the disc. We may therefore conclude that the the heat has been absorbed by the lamp-black. This import has been confirmed by direct experiments, which have fail cover any trace of reflecting or diffusive power in this Further, in the above experiment, the ratio of the indicati two positions of the pile remains constant for all sources of heat; whence we see that the absorption of rays of heat by lamp-black is in all cases independent of the nature of the source. We thus see the advantage of applying a coating of lamp-black to all thermoscopic apparatus intended for the absorption of radiant heat.

322. **Diathermancy.**—It has long been known that some of the heat from an intensely luminous body, like the sun, could pass through certain transparent substances, such as glass; but it was, till recently, supposed that this could not happen in the case of dark, or even feebly luminous rays.

Pictet, of Geneva, was the first to establish the fact of diathermancy for radiant heat in general. He showed that a thermometer rose in temperature when exposed to radiation from a source of heat, notwithstanding the interposition of a transparent lamina; and the idea that this could be owing to the absorption and subsequent radiation of heat from the lamina was completely exploded by Prévost, who showed that the effect occurred even when the interposed substance was a sheet of ice. It is to Melloni, however, that we are indebted for the principal results which have been obtained in connection with this subject.

323. Influence of the Nature of the Substance.—The arrangement adopted by Melloni for testing the diathermancy of a solid body is that shown in Fig. 292. The Locatelli lamp A radiates its heat upon

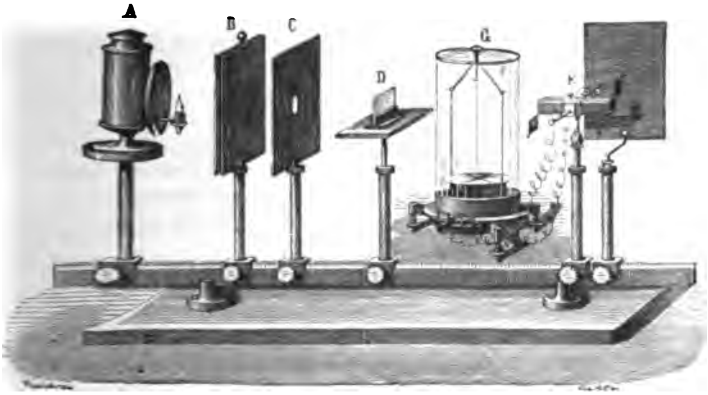

Fig. 292.—Measurement of Diathermancy.

pile E when the screen B is lowered; the hole in the screen C is the purpose of limiting the pencil of rays. Direct radiation is t allowed to take place, and the resulting current as indicated by the galvanometer G is noted. The diathermanous plate D interposed between the lamp and the pile, and the current measured; the ratio of the latter current to the former is the sion of the diathermancy of the plate.

In the case of liquids, Melloni employed narrow troughs wi of very thin glass; the rays were first transmitted through th vessel, and then through the same vessel filled with liquid; ference of the two results thus obtained being the measure of stopped by the liquid. Specimens of the results are giver following table:—

HEAT TRANSMITTED BY DIFFERENT SUBSTANCES FROM AN ARGAND L

(The direct heat is represented by 100.)

SOLIDS.

Colourless Glass.

(Thickness 1·88 mm.)

Flint-glass,	from 67 to 64
Plate-glass,	62 to 59
Crown-glass (French),	58
Crown glass (English),	49
Window-glass,	54 to 50

Coloured Glass.

(Thickness 1·85 mm.)

Deep violet,	53
Pale violet,	45
Very deep blue,	19
Deep blue,	33
Light blue,	42
Mineral green,	23
Apple green,	26
Deep yellow,	40
Orange,	44
Yellowish red,	53
Crimson,	51

LIQUIDS.

(Thickness 9·21 mm.—A plate of glas thickness gives 53.)

Colourless Liquids

Distilled water,
Absolute alcohol,
Sulphuric ether,
Sulphide of carbon,
Spirits of turpentine,
Pure sulphuric acid,
Pure nitric acid,
Solution of sea-salt,
Solution of alum,
Solution of sugar,
Solution of potash,
Solution of ammonia,

Coloured Liquids.

Nut-oil (yellow),
Colza-oil (yellow),
Olive-oil (greenish yellow),
Oil of carnations (yellowish)
Chloride of sulphur (reddish bro
Pyroligneous acid (brown),
White of egg (slightly yellow),

CRYSTALLIZED BODIES.

(Thickness 3·62 mm.—A plate of glass of the same thickness gives 62.)

Colourless.

Rock-salt,	92
Iceland-spar,	12
Rock-crystal,	57
Brazilian topaz,	54
Carbonate of lead,	52
Borate of soda,	28
Sulphate of lime,	20
Citric acid,	15
Rock alum,	12

Coloured.

Smoky quartz (brown),
Aqua-marina (light blue),
Yellow agate,
Green tourmaline,
Sulphate of copper (blue),

It will be seen from this table that though diathermancy and transparency for light usually go together, the one is far from being a measure of the other. We see, for instance, that colourless nitric acid is much less diathermanous than strongly-coloured chloride of sulphur; and perfectly colourless alum allows much less heat to pass than deeply-coloured glass of the same thickness. Tyndall has shown that a solution of iodine in sulphide of carbon, though excessively opaque to light, allows heat to pass in large quantity.

The substance possessing the greatest diathermanous power is rock-salt, which allows the passage of ·92 of the incident heat. Common sea-salt only allows ·12 to pass. No such difference, however, attaches to solutions of these substances.

The diathermancy of gases has been investigated by Tyndall. The gases were contained in a long metallic tube with rock-salt ends; and, in order to obtain greater sensitiveness, a compensating cube filled with hot water was employed. This cube was placed at such a distance from one end of the thermo-pile as exactly to counterbalance the effect of the radiation from the principal source of heat when the tube was vacuous, so that the needle of the galvanometer in these circumstances stood at zero. The tube was then filled with different gases in turn, the compensating cube remaining unmoved; and the indications of the galvanometer were found to vary according to the gas employed. Compound gases stopped more than simple ones; the vapours of aromatic substances increased the absorptive power of dry air from 30 to 300 fold, and a similar effect was produced by the vapour of water, air more or less charged with aqueous vapour being found to exercise from 30 to 70 times the absorption of pure dry air.

It is probable that the aqueous vapour which is always present in the atmosphere greatly mitigates the heat of the solar rays, and also greatly retards the cooling of the earth by radiation at night. On the other hand, vapour being a better absorber is also a better radiator than dry air, a circumstance which conduces to the cooling and condensation of the upper portions of masses of vapour in the atmosphere, and the consequent formation of cloud.

324. Influence of Thickness.—From the experiments of Jamin and Masson, it appears that, when heat of definite refrangibility passes through a plate, the amount transmitted decreases in geometrical progression as the thickness increases in arithmetical progression; a result which may also be expressed by saying, that if a plate be divided in imagination into laminæ of equal thickness, the the heat absorbed to the heat transmitted is the same for the

In the case of mixed radiation, such as is emitted by ne available sources of heat, we must suppose this law to hold separate constituent; but some of these are more easily absorb others, and as these accordingly diminish in amount more than the others, the beam as it proceeds on its way through t acquires a character which fits it for transmission rather than tion. Hence the foremost layers absorb much more than t ones, if the plate be of considerable thickness.

In the case of bodies which are opaque to heat, absorpt radiation are mere surface-actions. But in diathermanous sub as we have seen, absorption goes on in the interior, so that plate absorbs more heat than a thin one. The same thing is regards radiation:—a diathermanous substance radiates interior as well as from its surface, as proved by the fact that plate radiates more heat than a thin one at the same temper

325. **Relation between Radiant Heat and Light.**—The pro virtue of which particular substances select particular kinds for absorption and other kinds for transmission, was called by *thermochrose* (literally heat-colour), from its obvious analogy we call colour in the case of light. A piece of coloured example, selects rays of certain colours (or vibration-peri absorption, and transmits the rest, what we call the colou glass being determined by those which it transmits.

It is now believed that thermochrose and colour are not analogous but essentially identical. Prismatic analysis sh rays exist of refrangibilities much greater and much less th which compose the luminous spectrum. The spectrum of th light, for example, extends on both sides of the visible spec distances considerably exceeding the length of the visible s itself. The invisible ultra-violet rays can be detected chemical action, or by causing them to fall upon certain su (called *fluorescent*) which become luminous when exposed action, but have exceedingly small heating effect. The heat considerable in the yellow portion of the spectrum, stronge red, and goes on increasing in the invisible portion beyond up to a certain point, beyond which it gradually diminish becomes inappreciable.

It would, however, be an error to suppose that there is spectrum consisting of distinct rays from those which form the luminous spectrum, and that the two spectra are superimposed one upon the other. There is every reason for believing that the contrary is the fact, and that the radiations which constitute heat and light are essentially identical. In operating upon rays of definite refrangibility, it is never found possible to diminish their heating and illuminating powers in unequal proportions; an interposed plate of any partially transparent material, if it stops half the light, also stops half the heat.

It is true that the most intense heat is not found in the most luminous portion of the spectrum; but it is probable that the eye, like the ear, is more powerfully affected by quick than by slow vibrations when the amount of energy is the same; and as a treble note contains far less energy than a bass note which strikes the ear as equally loud, so a blue ray contains much less energy than a red ray if they strike the eye as equally bright.

The invisibility—at least to human eyes—of the ultra-red and ultra-violet rays may be due either to the absorption of these rays by the humours of the eye before they can reach the retina, or to the inability of our visual organs to take up vibrations quicker than the violet or slower than the red.

A body at a low temperature (say 100° C.) emits only dark heat. As the temperature rises, the emission of dark heat becomes more energetic, and at the same time rays of a more refrangible character are added. This strengthening of the rays formerly emitted, with the continual addition of new rays of higher refrangibility, goes on as long as the temperature of the body continues to rise. The luminosity of the body begins with the emission of the least refrangible of the visible rays, namely the red, and goes on to include rays of other colours as it passes from a red to a white heat. Tyndall, by thus gradually raising the temperature of a platinum spiral, obtained the following measures of the heat received in a definite position in the dark portion of the spectrum:—

Appearance of Spiral.	Heat Received.	Appearance of Spiral.	Heat Received.
Dark,	1	Full red,	27
Dark,	6	Orange,	60
Faint red,	10	Yellow,	93
Dull red,	13	Full white,	122
Red,	18		

Generally speaking, the rays which fall within the limits of the

visible spectrum are the most transmissible, and the extreme
both ends of the complete spectrum are the soonest absorbe
is probably the reason why the invisible portion of the sol
trum, though extending to a considerable distance in both di
is less extensive than that of the electric light. The extre
have probably been absorbed by the earth's atmosphere.

Ordinary glass is comparatively opaque to both classes
rays. Rock-salt surpasses all other substances in its trans
to the dark rays beyond the red; and quartz (rock-crystal
transparent to the dark rays beyond the violet. Alum is ren
as a substance which is exceedingly opaque to the ultra-r
though exceedingly transparent to visible rays; and Tyn
found that a solution of iodine in sulphide of carbon is, on
trary, highly transparent to the ultra-red and opaque to the l
rays.

Great interest was excited some years ago by Stokes' d
that the ultra-violet rays, when they fall upon fluorescent su
undergo a lowering of refrangibility which brings them wi
limits of human vision. Akin subsequently proposed the
whether it was possible, by a converse change, to transf
ultra-red into visible rays, and Tyndall, by taking advantag
peculiar property of the solution of iodine, succeeded in effe
transformation. He brought the rays of the electric lamp t
by means of a reflector, and, after stopping all the luminous
interposing a vessel with rock-salt sides, containing the so
iodine, he found that a piece of platinum foil, when brought
focus, was heated to incandescence, and thus emitted light a
heat. To this transformation of dark radiant heat into light
the name of *calorescence*.

326. **Selective Emission and Absorption.**—In order to
together the various phenomena which may be classed u
general title of selective radiation and absorption, it is nec
form some such hypothesis as the following. The atoms
cules of which any particular substance is composed, must
posed to be capable of vibrating freely in certain periods,
the case of gases, are sharply defined, so that a gas is like
string, which will vibrate in unison with certain definite r
with no intermediate ones. The particles of a solid or liqui
other hand, are capable of executing vibrations of any peri
between certain limits; so that they may perhaps be compar

body of a violin, or to the sounding-board of a piano; and these limits (or at all events the upper limit) alter with the temperature, so as to include shorter periods of vibration as the temperature rises.

These vibrations of the particles of a body are capable of being excited by vibrations of like period in the external ether, in which case the body absorbs radiant heat. But they may also be excited by the internal heat of the body; for whenever the molecules experience violent shocks, which excite tremors in them, these are the vibrations which they tend to assume. In this case the particles of the body excite vibrations of like period in the surrounding ether, and the body is said to emit radiant heat.

One consequence of these principles is that a diathermanous body is particularly opaque to its own radiation. Rock-salt transmits 92 per cent. of the radiation from most sources of heat; but if the source of heat be another piece of rock-salt, especially if it be a thin plate, the amount transmitted is much less, a considerable proportion being absorbed. The heat emitted and absorbed by rock-salt is of exceedingly low refrangibility.

Glass largely absorbs heat of long period, such as is emitted by bodies whose temperatures are not sufficiently high to render them luminous, but allows rays of shorter period, such as compose the luminous portion of the radiation from a lamp-flame, to pass almost entire. Accordingly glass when heated emits a copious radiation of non-luminous heat, but comparatively little light.

Experiment shows that if various bodies, whether opaque or transparent, colourless or coloured, are heated to incandescence in the interior of a furnace, or of an ordinary coal-fire, they will all, while in the furnace, exhibit the same tint, namely the tint of the glowing coals. In the case of coloured transparent bodies, this implies that the rays which their colour prevents them from transmitting from the coals behind them are radiated by the bodies themselves most copiously; for example, a glass coloured red by oxide of copper permits only red rays to pass through it, absorbing all the rest, but it does not show its colour in the furnace, because its own heat causes it to radiate just those rays which it has the power of absorbing, so that the total radiation which it sends to the eye of a spectator, consisting partly of the radiation due to its own heat, and partly of rays which it transmits from the glowing fuel behind it, is exactly the same in kind and amount as that which comes direct from the other parts of the fire. This explanation is verified by the fact that such glass, if heated to a high temperature in a dark room, glows green light.

A plate of tourmaline cut parallel to the axis has the prop breaking up the rays of heat and light which fall upon it i equal parts, which exhibit opposite properties as regards polar One of these portions is very largely absorbed, while the transmitted almost entire. When such a plate is heated t descence, it is found to radiate just that description of heat a which it previously absorbed; and if it is heated in a furn traces of polarization can be detected in the light which com it, because the transmitted and emitted light exactly com each other, and thus compose ordinary or unpolarized light.

Spectrum analysis as applied to gases furnishes perhaps st striking illustrations of the equality of selective radiation and tion. The radiation from a flame coloured by vapour of sodi example, the flame of a spirit-lamp with common salt sprin the wick—consists mainly of vibrations of a definite perio sponding to a particular shade of yellow. When vapour of so interposed between the eye and a bright light yielding a co spectrum, it stops that portion of the light which correspond particular period, and thus produces a dark line in the yellow of the spectrum.

An immense number of dark lines exist in the spectrum sun's light, and no doubt is now entertained that they indi presence, in the outer and less luminous portion of the sun sphere, of gaseous substances which vibrate in periods corres to the positions of these lines in the spectrum.

327. **Dew.**—By this name we denote those drops of wat are seen in the morning on the leaves of plants, and are e noticeable in spring and autumn. We have already seen (§ 2 dew does not *fall*, as it is not formed in the atmosphere, but tact with the bodies on which it appears, being in fact due cooling after the sun has sunk below the horizon, when t heat by radiation to the sky. The lowering of temperatu thus occurs, is much more marked for grass, stones, or bare ea for the air, whose radiating power is considerably less. T sequence is a considerable difference of temperature betw surface of the ground and the air at the height of a few fe ference which is found by observation to amount sometimes 10° C., and it is this which causes the deposition of dew. Th

of the earth, as it gradually cools, lowers the temperature of the adjacent air, which thus becomes saturated, and, on further cooling, yields up a portion of its vapour in the liquid form. If the temperature of the soil falls below 0° C., the dew is frozen, and takes the form of *hoar-frost.*

According to this theory, it would appear that the quantity of dew deposited upon a body should increase with the radiating power of its surface, and with its insulation from the earth or other bodies from which it might receive heat by conduction, both which conclusions are verified by observation.

The amount of deposition depends also in a great measure on the degree of exposure to the sky. If the body is partially screened, its radiation and consequent cooling are checked. This explains the practice which is common with gardeners of employing light coverings to protect plants from frost—coverings which would be utterly powerless as a protection against the cold of the surrounding air. The lightness of the dew on cloudy nights is owing to a similar cause; clouds, especially when overhead, acting as screens.

The deposition of dew is favoured by a slight motion of the atmosphere, which causes the lower strata of air to cool down more rapidly; but if the wind is very high, the different strata are so intermingled that very little of the air is cooled down to its dew-point, and the deposit is accordingly light. When these two obstacles are combined, namely a high wind and a cloudy sky, there is no dew at all.

CHAPTER XXX.

CONDUCTION OF HEAT.

328. **Conduction.**—When heat is applied to one end of a
metal, it is propagated through the substance of the bar, pr
a rise of temperature, which is first perceptible at near and aft
at remote portions. This transmission of heat is called *cond*
and it differs notably from radiation (1), in being gradual in
instantaneous; and (2), in exhibiting no preference for rec
transmission, the propagation of heat being as rapid thr
crooked as through a straight bar.

328A. **Definition of Conductivity or Specific Conducting Po**
the application of heat to one end of the bar be continued for
ciently long time, and with great steadiness, the different por
the bar will at length cease to rise in temperature, and wil
steadily the temperatures which they have acquired. We m
distinguish two stages in the experiment: 1st, the variabl
during which all portions of the bar are rising in temperatur
2d, the permanent state, which may subsist for any length
without alteration. In the former stage the bar is gainin
that is, it is receiving more heat from the source than it g
to surrounding bodies. In the latter stage the receipts and e
ture of heat are equal, and are equal not only for the bar as a
but for every small portion of which it is composed.

In the permanent state no further accumulation of hea
place. All the heat which reaches an internal particle is tran
by conduction, and the heat which reaches a superficial pa
given off partly by radiation and air-contact, and partly by
tion to colder neighbouring particles. In the earlier stage,
contrary, only a portion of the heat received by a particle
disposed of, the remainder being accumulated in the parti
serving to raise its temperature.

In order to obtain results depending on conduction free from complications arising from differences of specific heat (§ 340), we must, in all cases, wait for the permanent state. In the earlier stage great specific heat acts as an obstacle to rapid transmission, and a body of great specific heat would be liable to be mistaken for a body of small conductivity.

The measurement of the conductivity of a substance is still further simplified by making the flow of heat through it take place entirely in one definite direction (that is to say in parallel lines), avoiding all cross-currents. To this end it is necessary that all points in the same cross section should have the same temperature, a condition which is not strictly fulfilled in the bar above described, as the surface will be cooler than the interior. It is nearly fulfilled in the axial portions of the bar, and it is very nearly fulfilled in the central portions of a uniform plate whose breadth in all directions is a very large multiple of its thickness, when the whole of one face is maintained as nearly as practicable at one uniform temperature, and the other face at another uniform temperature. In the central portions of such a plate, the flow of heat will be perpendicularly through the plate; and when the permanent state has arrived, the amount of heat that passes in a unit of time through a cross section of area A, will be expressed by the formula

$$Q = k\,A\,\frac{t_2 - t_1}{x}, \qquad (1)$$

where x is the thickness of the plate, t_1 and t_2 the temperatures of its two faces, and k a coefficient depending on the material of the plate. This coefficient k is the *conductivity* of the material. It may be defined as *the quantity of heat which flows in unit time through a cross section of unit area, when the thickness of the plate is unity, and one face is warmer by* 1° *than the other.*[1]

---

[1] The method of taking account of conductivity during the variable stage may be illustrated by considering the simplest case,—that in which the flow of heat is in parallel lines.

Let x denote distance measured in the direction in which heat is flowing, v the temperature at the time t at a point specified by x, k the conductivity, and c the thermal capacity per unit volume (both at the temperature v). Then the flow of heat per unit time past a cross section of area A is $-k\,A\,\frac{dv}{dx}$, and the flow past an equal and parallel section further on by the small distance δx is greater by the amount

$$A\,\frac{d}{dx}\left(-k\,\frac{dv}{dx}\right)\delta x.$$

This latter expression therefore represents the loss of heat from the intervening prism $A\,\delta x$,

Forbes' experiments have shown that the conductivity o
stance is not the same at all temperatures. In view of this f
the above formula denotes the average conductivity betw
temperatures t_1 and t_2. The variation of conductivity with t
ture is, however, comparatively small.

329. Differences of Conductivity.—The following experim
often adduced in illustration of the different conducting p
different solids.

Two bars of the same size but of different materials (Fig.
placed end to end, and small wooden balls are attached by
their under-surfaces at equal distances. The bars are the
at their contiguous ends, and, as the heat extends along t

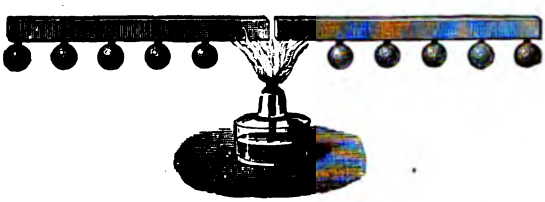

Fig. 293.—Balls Melted off.

wax melts, and the balls successively drop off. If the he
continued till the permanent state arrives, it may generally
cluded that the bar which has lost most balls is the best co
especially if both bars have been coated with the same varn
to make their radiating powers equal.

The well-known experiment of Ingenhousz is of the sa
The apparatus consists of a copper box having a row of hol
of its faces, in which rods of different materials can be fix

and the resulting fall of temperature is the quotient of the loss by the therm
$cA\delta x$, which quotient is

$$\frac{1}{c}\frac{d}{dx}\left(-k\frac{dv}{dx}\right).$$

This, then, is the fall of temperature per unit time, or is $-\frac{dv}{dt}$. If the ra
perature is small enough to admit of our regarding k and c as constant, t
becomes

$$\frac{dv}{dt}=\frac{k}{c}\frac{d^2v}{dx^2},$$

which applies approximately to the variations of temperature in the soil near th
the earth, x being in this case measured vertically. For the integral of this e
Trans. Roy. Soc. Edin. vol. xxii. part ii. p. 438.

rods having previously been coated with wax, the box is filled with boiling water, which comes into contact with the inner ends of the rods. The wax gradually melts as the heat travels along the rods; and, if the experiment is continued till the melting reaches its limit, those rods on which it has extended furthest are, generally speaking, the best conductors.

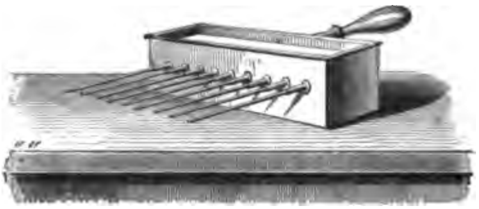

Fig. 294.—Ingenhousz's Apparatus.

It is thus found that metals are unequally good conductors of heat, and that they may be arranged in the following order, beginning with the best conductors:—*Silver, copper, gold, brass, tin, iron, lead, platinum, bismuth.*

In both these experiments we must beware of attempting to measure conductivity by the *quickness* with which the melting advances. This quickness may be simply an indication of small specific heat.[1]

330. Conducting Power of Metals.—Metals, though differing con- iderably one from another, are as a class greatly superior in con- luctivity to other substances, such as wood, marble, brick. This xplains several familiar phenomena. If the hand be placed upon a etal plate at the temperature of 10° C., or plunged into mercury at his temperature, a very marked sensation of cold is experienced. his sensation is less intense with a plate of marble at the same mperature, and still less with a piece of wood. The reason is that e hand, which is at a higher temperature than the substance to hich it is applied, gives up a portion of its heat, which is conducted ay by the substance, and consequently a larger portion of heat is rted with, and a more marked sensation of cold experienced, in the e of the body of greater conducting power.

331. Davy Lamp.—The conducting power of metals explains the ious property possessed by wire-gauze of cutting off a flame. If, example, a piece of wire-gauze be placed above a jet of gas, the ne is prevented from rising above the gauze. If the gas be first wed to pass through the gauze, and then lighted above, the flame t off from the burner, and is unable to extend itself to the under- ace of the gauze. These facts depend upon the conducting power

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[1] Strictly speaking, *small specific heat per unit volume*, not, as usual, *per unit mass.*

of metallic gauze, in virtue of which the heat of the flame is
dissipated at the points of contact, the result being a diminu
temperature su
to prevent ign

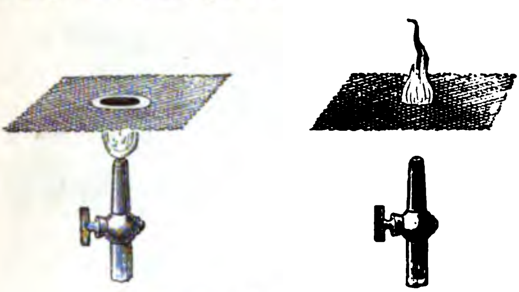
Fig. 295.—Action of Wire-gauze on Flame.

This prop
metallic gau
been turned
count for vari
poses, but it
useful applic
in the safety-
Sir Humphre

It is well
that a gas called *fire-damp* is often given off in coal-mines.
compound of carbon and hydrogen, and is a large ingredient
nary coal-gas.

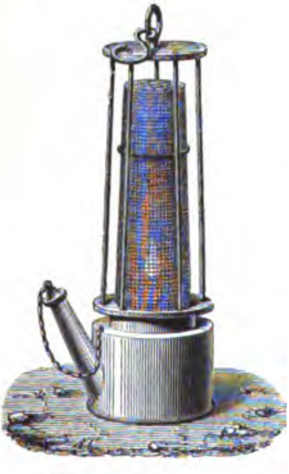
Fig. 296.—Davy Lamp.

This fire-damp, when mixed with eight or ten times its vo
air, explodes with great violence on coming in contact with a
body. To obviate this danger, D
vented the safety-lamp, which is
nary lamp with the flame incl
wire-gauze. The explosive ga
through the gauze, and burn in
lamp, in such a manner as to w
miner of their presence; but the
unable to pass through the gauze.

332. Various Applications.—Th
ledge of the relative conducting
of different bodies has several in
practical applications.

In cold countries, where the h
duced in the interior of a house s
as far as possible prevented from
the walls should be of brick o
which have feeble conducting pow
they are of stone, which is a better conductor, a greater t
is required. Thick walls are also useful in hot countries in
the power of the solar rays during the heat of the day.

We have already alluded (§ 224) to the advantage of en
fire-brick, which is a bad conductor, as a lining for stoves.

The feeble conducting power of brick has led to its employment in the construction of ice-houses. These are round pits, generally from 6 to 8 yards in diameter at top, and somewhat narrower at the bottom, where there is a grating to allow the escape of water. The inside is lined with brick, and the top is covered with straw, which, as we shall shortly see, is a bad conductor. In order to diminish as much as possible the extent of surface exposed to the action of the air, the separate pieces are dipped in water before depositing them in the ice-house, and, by their subsequent freezing together, a solid mass is produced, capable of remaining unmelted for a very long time.

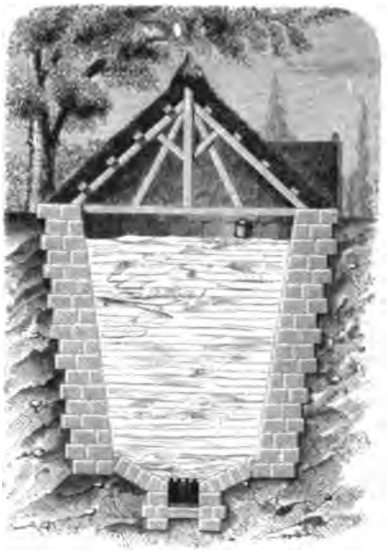

Fig. 297.—Ice-house.

333. Experimental Determination of Conductivity.—Several experimenters have investigated the conductivity of metals, by keeping one end of a metallic bar at a high temperature, and, after a sufficient lapse of time, observing the permanent temperatures assumed by lifferent points in its length.

If the bar is so long that its further end is not sensibly warmer han the surrounding air, and if, moreover, Newton's law of cooling e assumed true for all parts of the surface, and all parts of a cross ection be assumed to have the same temperature, the conductivity eing also assumed to be independent of the temperature, it is easily iown that the temperatures of the bar at equidistant points in its ngth, beginning from the heated end, must exceed the atmospheric mperature by amounts forming a decreasing geometric series. iedemann and Franz, by the aid of the formula to which these sumptions lead,[1] computed the relative conducting powers of several

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If p and s denote the perimeter and section of the bar, k the conductivity, and h the ficient of emission of the surface at the temperature v, the heat emitted in unit time the length δx is $hvp\,\delta x$, if we assume as our zero of temperature the temperaof the surrounding air. But the heat which passes a section is $sk\frac{dv}{dx}$, and that of the metals, from experiments on thin bars, which were steadily heated at one end, the temperatures at various points in the length being determined by means of a thermo-electric junction clamped to the bar. The following were the results thus obtained:—

RELATIVE CONDUCTING POWERS.

Silver,	100
Copper,	77·6
Gold,	53·2
Brass,	33
Zinc,	19·9
Tin,	14·5
Steel,	12
Iron,	11·9
Lead,	8·5
Platinum,	8·2
Palladium,	6·3
Bismuth,	1·9

The *absolute* conductivity of wrought iron was investigated with great care by Professor J. D. Forbes, by a method which avoided some of the questionable assumptions above enumerated. The end of the bar was heated by a bath of melted lead kept at a uniform temperature, screens being interposed to protect the rest of the bar from the heat radiated by the bath. The temperatures at other points were observed by means of thermometers inserted in small holes drilled in the bar, and kept in metallic contact by fluid metal. In order to determine the loss of heat by radiation at different temperatures, a precisely similar bar, with a thermometer inserted in it, was raised to about the temperature of the bath, and the times of cooling down through different ranges were noted.

The conductivity of one of the two bars experimented on, varied from ·01337 at 0° C. to ·00801 at 275° C., and the corresponding numbers for the other bar were ·00992 and ·00724, the units being the foot, the minute, the degree (of any scale), and the foot-degree[1] (of the same scale). In both instances, the conductivity decreased regularly with increase of temperature.

Absolute determinations have also been made of the conductivity of the soil or rock at three localities in or near Edinburgh, by Pro-

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which passes a section further on by the amount δx is less by the amount $sk\frac{d^2v}{dx^2}\delta x$; and this difference must equal the amount emitted from the intervening portion of the surface. Hence we have the equation $\frac{d^2v}{dx^2}=\frac{hp}{ks}v$, the integral of which for the case supposed is

$$v=Ve^{-x\sqrt{\frac{hp}{ks}}},$$

V denoting the temperature at the heated end.

[1] See §§ 339, 340.

fessor Forbes and Sir W. Thomson. When expressed in terms of the above units, they are—

Trap-rock of Calton Hill, ...	·000268
Sand of Experimental Garden, ...	·000169
Sandstone of Craigleith Quarry, ...	·000689

These determinations were derived from observations on the temperature of the soil as indicated by thermometers having their bulbs buried at depths of from 3 to 24 feet. The annual range of temperature diminished rapidly as the depth increased; and this diminution of range was accompanied by a retardation of the times of maximum and minimum. To deduce the conductivity,[1] it was necessary first to reduce the annual variation of each thermometer to the sum of a number of terms each of which would express a *simple harmonic variation* or *simple vibration* (§ 53A), the most important of these being the annual term, which represents a simple vibration whose period is a year. By comparing the amplitudes of this term at two different depths, we obtain the value of $\frac{k}{c}$, k denoting conductivity, and c capacity per unit volume; and another independent determination of the same element is obtained by comparing the *epochs,* in other words by noting the *retardation of phase* which this term undergoes. The value of c was determined by direct experiments conducted by Regnault, and lastly, this value multiplied by $\frac{k}{c}$ gave the conductivity k.

334. Conducting Powers of Liquids.—With the exception of mercury and other melted metals, liquids are exceedingly bad conductors of heat. This can be shown by heating the upper part of a column of liquid, and observing the variations of temperature below. These will be found to be scarcely perceptible, and to be very slowly produced. If the heat were applied below (Fig. 298), we should have the process called *convection of heat;* the lower layers of liquid would rise to the surface, and be replaced by others which would rise in their turn, thus producing a circulation and a general heating of the liquid. On the other hand, when heat is applied above, the expanded layers remain in their place, and the rest of the liquid can be heated by conduction and radiation only.

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[1] The process of reduction is fully explained, both theoretically and practically, in two papers (by Sir W. Thomson and the Editor of this work) in the *Trans. Roy. Soc. Edin.* 1860.

The following experiment is one instance of the very feeble conducting power of water. A piece of ice is placed at the bottom of a glass tube (Fig. 299), which is then partly filled with water; heat is applied to the middle of the tube, and the upper portion of the water is readily raised to ebullition, without melting the ice below.

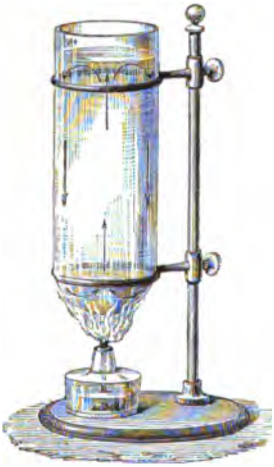

Fig. 298.—Liquid heated from below.

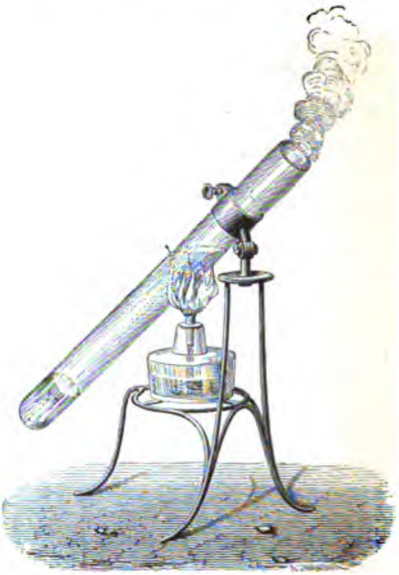

Fig. 299.—Boiling of Water over Ice.

335. Measure of the Conducting Power of Water.—The power of conducting heat possessed by water, though very small, is yet capable of measurement. This was established by Despretz by the following experiment. He took a cylinder of wood (Fig. 300) about a yard in height and eight inches in diameter, which was filled with water. In the side of this cylinder were arranged twelve thermometers one above another, their bulbs being all in the same vertical through the middle of the liquid column. On the top of the liquid rested a metal box, which was filled with water at 100°, frequently renewed during the course of the experiment. Under these circumstances Despretz observed that the temperature of the thermometers rose gradually, and that a long time—about 30 hours—was required before the permanent state was assumed. Their permanent differences, which formed a decreasing geometric series, were very small, and were inappreciable after the sixth thermometer.

The increase of temperature indicated by the thermometers might be attributed to the heat received from the sides of the cylinder, though the feeble conducting power of wood renders this idea some-

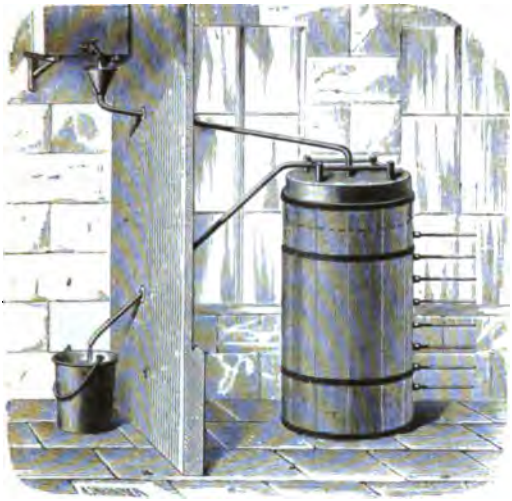

Fig. 300.—Despretz's Experiment.

what improbable. But Despretz observed that the temperature was higher in the axis of the cylinder than near the sides, which proves that the elevation of temperature was due to the passage of heat lownwards through the liquid.

From recent experiments by Professor Guthrie,[1] it appears that vater conducts better than any other liquid except mercury.

336. Conducting Power of Gases.—Of the conducting powers of ases it is almost impossible to obtain any direct proofs, since it is xceedingly difficult to prevent the interference of convection and irect radiation. However, we know at least that they are exceed-gly bad conductors. In fact, in all cases where gases are inclosed small cavities where their movement is difficult, the system thus rmed is a very bad conductor of heat. This is the cause of the ble conducting powers of many kinds of cloth, of fur, eider-down, t, straw, saw-dust, &c. Materials of this kind, when used as icles of clothing, are commonly said to be *warm*, because they der the heat of the body from escaping. If a garment of eider-down or fur were compressed so as to expel the greater part of the air, and to reduce the substance to a thin sheet, it would be found to be a much less warm covering than before, having become a better conductor. We thus see that it is the presence of air which gives these substances their feeble conducting power, and we are accordingly justified in assuming that air is a bad conductor of heat.

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[1] *B. A. Report*, 1868, and *Trans. R. S.* 1869.

337. Norwegian Stove.—A curious application of the bad conducting power of felt is occasionally to be seen in the north of Europe, in a kind of self-acting cooking-box. This is a box lined inside with a thick layer of felt, into which fits a metallic dish with a cover. The dish is then covered with a cushion of felt, so as to be completely surrounded by a substance of very feeble conducting power. The method of employing the apparatus is as follows:—The meat which it is desired to cook is placed along with some water in the dish, the whole is boiled for a short time, and then transferred from the fire to the box, where the cooking is completed *without any*

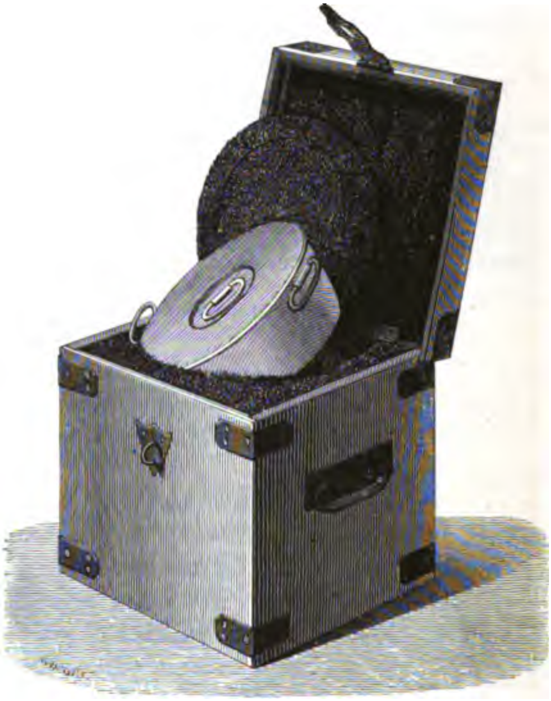

Fig. 301.—Norwegian Cooking-box.

farther application of heat. The isolating power of the stuffing of the box, as far as regards heat, is exceedingly great; in fact, it may be shown that at the end of three hours the temperature of the water has fallen by only about 10° or 15° C., and has accordingly remained during all that time sufficiently high to conduct the operation of cooking.

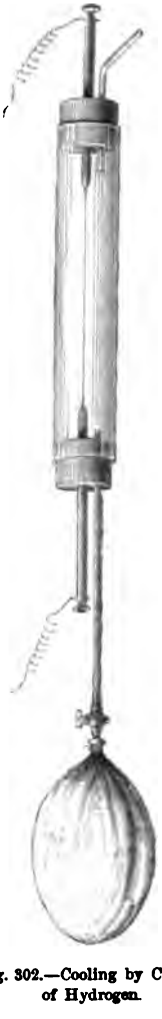

Fig. 302.—Cooling by Contact of Hydrogen.

338. Conductivity of Hydrogen.—The *conducting* power of hydrogen is much superior to that of the other gases—a fact which agrees with the view entertained by chemists, that this gas is the vapour of a metal. The good conductivity of hydrogen is shown by the following experiments:—

1. Within a glass tube (Fig. 302) is stretched a thin platinum wire, which is raised to incandescence by the passage of an electric current. When air, or any gas other than hydrogen, is passed through he tube, the incandescence continues, hough with less vividness than in vacuo; ut it disappears as soon as hydrogen is mployed.

2. A thermometer is placed at the bot-m of a vertical tube, and heated by a essel containing boiling water which is aced at the top of the tube. The tube exhausted of air, and different gases e successively admitted. In each case e indication of the thermometer is found be lower than for vacuum, except when gas is hydrogen. With this gas, the ference is in the opposite direction, showing that the diminution radiation has been more than compensated by the conducting ver of the hydrogen.

CHAPTER XXXI.

CALORIMETRY.

339. Quantities of Heat.—Calorimetry consists in the measurement of quantities of heat. This can be effected without making any assumption as to what heat is. It merely presupposes the power of identifying equal quantities.

If two different thermic actions, of which one may be friction, and the other combustion, when separately applied to the heating of a pound of water, raise its temperature in each case from 0° C. to 1° C., we say that the water receives equal quantities of heat in both cases; and the quantity of heat required to raise m pounds of water from 0° C. to 1° C. would be m times as great.

In order to test whether the quantity of heat required to raise the temperature of a pound of water by 1° C. is the same at all initial temperatures of the water, we may employ the method of mixtures. Let us, for example, mix 3 lbs. of water at 15° with 5 lbs. at 35°, and observe the temperature of the mixture. If this temperature be denoted by x, the 3 lbs. have risen through the range $x-15$, and the 5 lbs. have fallen through the range $35-x$. If a rise or fall of 1° involves the same gain or loss of heat at all temperatures, the quantity of heat gained by the 3 lbs. may be represented by $3\,(x-15)$, and the heat lost by the 5 lbs. will be represented by $5\,(35-x)$. But what the one has gained the other has lost; we have therefore

$$3\,(x-15)=5\,(35-x),$$

whence

$$x=27\cdot5.$$

Calculation, based on this principle, is found to agree very accurately with experiment up to about 40° C. We may therefore define the *unit of heat* as the *quantity of heat required to raise the temperature of unit mass of water* 1°, between the limits 0° C. and 40° C.

Several different units of heat are employed, all having reference to pure water between these limits of temperature.

The gramme-degree (Centigrade) is the quantity of heat required to raise a gramme of water 1° (Centigrade).

The kilogramme-degree (Centigrade) is the heat required to raise a kilogramme of water 1° (Centigrade). It is sometimes called the *calorie.*

The pound-degree (Fahrenheit or Centigrade) is the heat required to raise a pound avoirdupois of water 1°.

The foot-degree (Fahrenheit or Centigrade) is the heat required to raise a cubic foot of water 1°.

We shall briefly describe in this chapter three of the most important applications of calorimetry:—

1. In connection with changes of temperature (specific heat);
2. In connection with change of state (latent heat);
3. In connection with chemical actions (heat of combination).

340. Thermal Capacity—Specific Heat.—The thermal capacity of a body is the quantity of heat required to raise the temperature of the body one degree. It is numerically equal to the mass or volume of water (according to the unit of heat employed) which would be raised one degree by the same quantity of heat.

The *specific heat* of a substance is the thermal capacity of unit mass of the substance, or, more simply, is the *ratio of the thermal capacity of the substance to that of an equal weight of water.* It is obviously independent of the unit of mass and scale of temperature adopted.

It is sometimes necessary to consider the thermal capacity of unit *volume* of a substance.[1] This will be the same as the ratio of the thermal capacity of the substance to that of an equal volume of water, if we employ as unit of heat the heat required to raise unit volume of water one degree. In discussions relating to conduction, if the foot be made the linear unit, the unit of heat employed should be the heat required to raise a cubic foot of water one degree. Thermal capacity per unit volume is, like thermal capacity per unit mass, independent of the units employed, provided they are employed consistently.

Experiment shows that nearly equal quantities of heat are required to raise a body through 1° at all temperatures between 0° C. and

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[1] This is evidently equal to *specific heat multiplied by density;* that is, to the thermal capacity of unit mass multiplied by the number of units of mass which are contained in a unit of volume.

100° C.; in other words, specific heat varies but very slightly with temperature between these limits. The quantity of heat which must be added to or taken from a body to raise or lower its temperature by T° will therefore be proportional to T, and will, in fact, be given by the formula

$$Q = W S T,$$

where W denotes the weight of the body, and S its specific heat. The product W S is the thermal capacity of the whole body.

The specific heats of different substances differ very widely from one another. This is easily tested by the following experiment.[1]

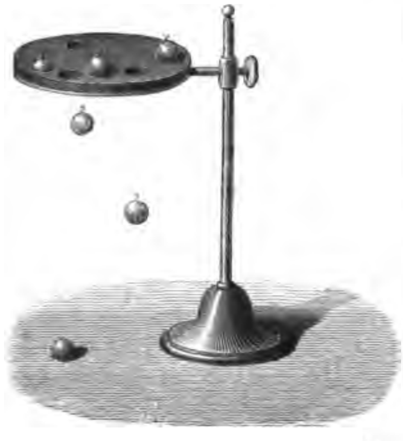

Fig. 303.—Balls Melting their way through Disc.

A number of balls, of different materials and of equal weight, are heated to the same temperature—suppose 200°—and placed upon a wax disc. Each of the balls gives up part of its heat to the wax, causing it to melt, and thus making a hole through which the ball finally drops. The balls of the greatest specific heat drop through first; thus the iron ball falls before the copper, the copper ball before the tin, &c., while the balls of lead and bismuth are much slower in their action, and, if the disc be moderately thick, never get through at all.

As another example we may take the following experiment, which serves to show the great difference between the specific heat of mercury and that of water.

A kilogramme of water at 10° and a kilogramme of mercury at 100° are poured into the same vessel and well shaken together. The temperature of the whole is then found to be about 13°, the water having gained 3 units of heat, which have been furnished by the mercury, and this loss has caused the temperature of the latter to fall through

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[1] Critically considered, this experiment gives undue advantage to the balls of heavy material, because they have not to make such large holes for themselves as the lighter ones, in order to get through. If the balls are all made of the same *size*, we have a pretty fair test of their *thermal capacities per unit volume.*

87°. Thus we see that the specific heat of mercury is about $\frac{3}{87}$ or $\frac{1}{29}$, that of water being unity.[1]

In order to determine this element exactly, a number of minute precautions are required; in fact, the experiment just described is to be regarded merely as a means of readily exhibiting the difference between the specific heats of the two bodies. The methods of accurately determining specific heats are various; we shall describe only two of them.

341. Method by Fusion of Ice.—A hole is scooped in a solid block of ice, and a lid of the same substance is fitted over it. A body of weight W is heated to a temperature T and placed in the hole, which is immediately covered with the lid. The body, in cooling down to 0°, gives off a quantity of heat which melts some of the ice. The water thus obtained is wiped up with cotton wool, which is then weighed, having previously been weighed dry. If the weight of water thus found be denoted by m, the quantity of heat necessary to produce it is, by § 228, 79 m. But this heat is supplied by the body, and has produced in it a fall of temperature amounting to T°, which correspond to WST units of heat, S being the specific heat of the body. Hence we have $\mathrm{WST}=79\,m$, whence $\mathrm{S}=\frac{79\,m}{\mathrm{WT}}$.

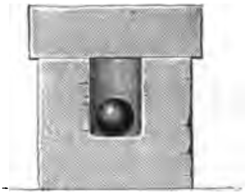

Fig. 304.—Ice-block Calorimeter.

This method is due to the Swedish philosopher Wilke, and is difficult of application in our climate. An apparatus, based on the same principle, and called the ice-calorimeter, was employed by Lavoisier and Laplace, and has recently been improved by Bunsen (*Phil. Mag.* March, 1871).

342. Method of Mixtures—Its Principle.—The method of mixtures resembles the experiment which we have already described as illustrating the difference between the specific heats of mercury and water.[2]

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[1] The specific heat of mercury is almost exactly $\frac{1}{30}$; see table § 344.

[2] If weights w, w' of two substances whose specific heats are s, s' and temperatures t, t', are mixed, the substances being supposed not to act chemically on one another, and no heat being gained or lost externally, it is easily shown that the temperature of the mixture will be

$$\frac{w\,s\,t+w'\,s'\,t'}{w\,s+w'\,s'},$$

and by appending additional terms to numerator and denominator, the formula becomes applicable to a mixture of any number of substances.

A body of a known weight W is raised to a fixed temperature T, and then plunged into a quantity of water of weight W′ and temperature t, which is contained in a copper vessel called a *calorimeter*. As T is supposed to exceed t, the water gains in temperature by the immersion of the body, and finally attains a maximum temperature θ, which is noted. In the change from t to θ, the water has gained a quantity of heat equal to $W'(\theta-t)$, and the body immersed has lost a quantity equal to $Wx(T-\theta)$, x being the specific heat required. Equating these two quantities, we have

$$W'(\theta-t)=Wx(T-\theta), \qquad (a)$$

whence

$$x=\frac{W'(\theta-t)}{W(T-\theta)}.$$

343. Sources of Error.—Such is the principle of the method of mixtures; but, upon closer examination, it will easily be seen that equation (*a*), for various reasons, must be regarded as only approximate.

I. The equation assumes that the only exchange of heat is between the body and the water, which is not actually the case.

1. The body is often contained in an envelope which cools along with it, and thus furnishes part of the heat given up.

2. The heat of the body is not given up exclusively to the water in the calorimeter, but partly to the calorimeter itself, to the thermometer, and to such other instruments as may be employed in the experiment, as, for instance, a rod to stir the liquid for the purpose of establishing uniformity of temperature throughout it.

The equation for the most general case can easily be written down. We have only to express that the quantity of heat given up by the body and its envelope is equal to that gained by the water, the calorimeter, the thermometer, and the rod.

Let W denote the weight of the body, T its initial temperature, x its specific heat, m the weight of the envelope, a its specific heat, W′ the weight of the water in the calorimeter, w the weight of the calorimeter, c its specific heat, w' the weight of the glass of the thermometer, c' its specific heat, w'' the weight of the mercury, c'' its specific heat, w''' the weight of the rod for stirring, c''' its specific heat, θ the final temperature.

Under these circumstances we evidently have the equation

$$Wx(T-\theta)+ma(T-\theta)=(W'+wc+w'c'+w''c''+w'''c''')(\theta-t),$$

whence we find

$$x=\frac{(\mathrm{W}'+wc+w'c'+w''c''+w'''c''')\,(\theta-t)-ma\,(\mathrm{T}-\theta)}{\mathrm{W}\,(\mathrm{T}-\theta)}.$$

The above equation is the general type of all equations which occur in questions of this kind; the only difference that can arise is in the number of the terms, since each term represents a quantity of heat gained or lost by one of the bodies employed in the experiment.

In the above expression for x, the coefficient of $(\theta-t)$ is what is called the *water-equivalent of the calorimeter*,[1] representing, in fact, a mass of water such that, supposing it to receive exclusively all the heat given up in the experiment, a thermometer placed in it would indicate the variation of temperature actually observed. Among the elements which enter this coefficient are the specific heat of the material of the calorimeter and of the rod for stirring; these are generally made of brass, and their specific heat may be considered as known with sufficient approximation from previous experiments. The two terms connected with the thermometer may be directly determined by a previous experiment with a body whose specific heat is known.

II. The calorimeter, when heated by the body immersed in the liquid, loses a certain quantity of heat by radiation, which must be taken into account if we wish to obtain a rigorous result. For this purpose a very simple method of compensation was proposed by Rumford. It consists in lowering the initial temperature of the calorimeter below that of the surrounding air by a number of degrees equal to the excess of the final temperature above that of the surrounding air, which may easily be effected by means of a previous trial. In this case the experiment may be divided into two parts, during the first of which the calorimeter gains heat by radiation, while during the second it loses heat by the same process. As the difference of temperature between the calorimeter and the air is the same in both cases, we may consider the quantities of heat as equal. The compensation, however, is not exact, since the two periods are of unequal length, and accordingly the method adopted by most investigators has been different. This consists in determining the constant in the expression for the rate of cooling (§ 307), and employing it in the direct calculation of the number of degrees lost by radiation. In order to effect this calculation, instead of supposing the variation of temperature to be continuous, they divide the entire length of the experiment into a number of parts, during each of which they suppose the excess constant, and this method of approximation is always found to be sufficient.

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[1] This is only another name for the *thermal capacity* of the calorimeter and its contents.

III. The calorimeter loses heat also by the supports on which it rests. This source of error can never be entirely removed, but it may easily be rendered so small as to be quite insensible. This is effected by making the supports of some substance which is a very bad conductor, and by diminishing the extent of surface of contact between them and the calorimeter.

Finally, we should ascertain carefully the initial temperature of the body, and provide against any fall of temperature in transferring it from the place of heating to the calorimeter.

344. Regnault's Apparatus.—The subject of specific heat, both for solids and liquids, has been investigated with great care by Regnault, who employed for that purpose an apparatus in which the advantages of convenience and precision are combined. The body whose specific heat is required is divided into small fragments, which are placed in a cylindrical basket G of very fine brass wire. This basket is suspended inside the steamer A, the suspending thread being fixed by

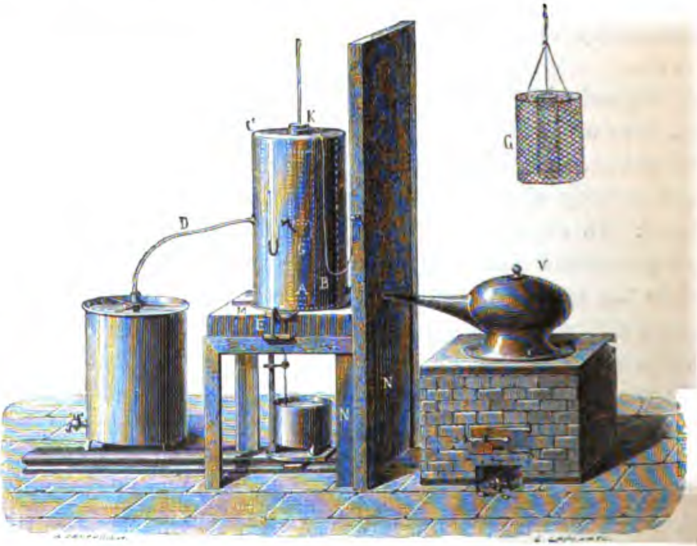

Fig. 305.—Regnault's Apparatus.

the cork K, through which passes the stem of a thermometer, the reservoir of which rests in a small tube in the centre of the basket, and made of the same material. The steamer is a double cylinder, the inside compartment B of which is filled with steam supplied by the boiler V, and afterwards conducted by the tube D into a condenser. In the bottom of the steamer is a sliding-door E, which can be drawn out when required. The outside compartment C is filled with air, to prevent the temperature of the inside from cooling by contact with the air outside.

This entire apparatus rests, by means of a sheet of cork, upon a hollow metal vessel MN, filled with water, the vertical face of which N serves as a screen to protect the calorimeter against the heat of the fire. The calorimeter itself is a vessel of very thin polished brass, resting upon silk threads stretched across the bottom of a larger vessel. This latter rests by three points upon a small wooden sled, which runs smoothly along a rail. The thermometer for measuring the temperature of the water in the calorimeter is carried by a support attached to the sled. After this explanation of the details, we proceed to indicate the course of an experiment.

The body is placed in the basket, introduced into the steamer, and there exposed to the action of the steam which is admitted from the boiler. During this part of the experiment the calorimeter is kept as far as possible from the steamer. After the lapse of an hour or two, the thermometer of the steamer remains stationary.

The calorimeter is now pushed below the sliding-door E, the door is drawn, and the basket is rapidly lowered into the calorimeter, which is immediately slid back to its former place. The basket is moved about in it for some time, and the final temperature of the water is observed. Thus we have all the elements required for the equation given above.

To determine the specific heats of liquids, a thin glass vessel is employed, in which the liquid is contained, and the effect produced by this envelope is taken into consideration in the general equation.

The same method is adopted for bodies soluble in water, or upon which water has a chemical action, some other liquid being in this case substituted, as, for instance, oil of turpentine. The specific heats of several substances are given in the following table:—

Water, 1·00000

SOLIDS.

Antimony,	0·05077	Brass,	0·09391
Silver,	0·05601	Nickel,	0·10860
Arsenic,	0·08140	Gold,	0·03244
Bismuth,	0·03084	Phosphorus,	0·18870
Cadmium,	0·05669	Platinum,	0·03243
Charcoal,	0·24150	Lead,	0·03140
Copper,	0·09215	Plumbago,	0·21800
Diamond,	0·14680	Sulphur,	0·20259
Tin,	0·05623	Glass,	0·19768
Iron,	0·11379	Zinc,	0·09555
Iodine,	0·05412	Ice,	0·5040

LIQUIDS.

Mercury,	0·03332	Benzine,	0·3952
Acetic acid,	0·6589	Ether,	0·5157
Alcohol at 36°	0·6735	Oil of turpentine,	0·4629

345. Great Specific Heat of Water.—This table illustrates the important fact, that, of all substances, water has the greatest specific heat; that is to say, it absorbs more heat in warming, and gives out more heat in cooling, through a given range of temperature, than an equal weight of any other substance. The quantity of heat which raises a pound of water from 0° to 100° C. would suffice to raise a pound of iron from 0° to about 900° C., that is to a bright red heat; and conversely, a pound of water in cooling from 100° to 0°, gives out as much heat as a pound of iron in cooling from 900° to 0°. This property of water is utilized in the heating of buildings by hot water, and in other familiar instances, such as the bottles of hot water used for warming beds, and railway foot-warmers.

It is of especial importance from the influence which it exerts on terrestrial temperatures. In fact, when we consider this property in conjunction with those which have been indicated in § § 228 and 247, we perceive that all the thermic modifications which water undergoes are accompanied by the evolution or absorption of remarkably large quantities of heat. If, for instance, the external temperature rises, much of the additional heat is consumed in warming the water, or in converting it into vapour, or in melting ice. If, on the other hand, the temperature falls, a large amount of heat is given up to the air by the cooling of the water, the condensation of vapour, or the formation of ice. In both cases, the change of temperature is greatly mitigated. If the water of the globe were removed, the difference of temperature between day and night, and between summer and winter, would very far exceed what are observed at present.

346. Dulong and Petit's Law.—Dulong and Petit were the first to observe that the product of the specific heat of any body, and what is called in chemistry its atomic weight, is constant. This law is of considerable importance, for it shows that atoms require each the same amount of heat to raise them through the same number of degrees. In fact, if w be the atomic weight of a body, and c its specific heat, the quantity of heat necessary for a variation of temperature of 1° is cw, and it was this product which Dulong and Petit found to be constant.

347. Specific Heats of Gases.—The limits of this treatise do not permit us to enter into the details of the complicated processes by which the specific heats of gases have been determined. It is necessary, however, to remark that, in the case of gases, two kinds of specific heat must be distinguished.

1. *Specific Heat at Constant Pressure.*—This is the quantity of heat required to raise the temperature of unit weight of the gas by one degree, when the gas is allowed to expand to such an extent that its pressure remains unchanged during the whole operation of heating.

2. *Specific Heat at Constant Volume.*—This is the amount of heat required to raise the temperature of unit weight of a gas by one degree, when the gas is compelled to retain its original volume.

The former of these exceeds the latter by the amount of heat consumed in producing the expansion.

A similar distinction exists in the case of liquids and solids, but it is not often attended to. What is always understood by specific heat in the case of these bodies is in fact their specific heat at constant pressure. The resistance of solids and liquids to compression is so enormous that a pressure of one or two atmospheres may be neglected as regards its effect upon their temperature and thermal capacity. But, in dealing with gases, the case is far otherwise, and one of the most important data for the solution of questions in gaseous mechanics is the ratio of their two specific heats.

The best experiments on the specific heats of gases are those of Regnault. They have established the two following conclusions for specific heat at constant pressure:—

(1) The specific heat (or thermal capacity per unit mass) of one and the same gas, whether simple or compound, is *the same at all pressures and temperatures.*

(2) The specific heats of different simple gases are approximately in the inverse ratio of their relative densities (§ 220), or of their atomic

weights, according to Dulong and Petit's law. This may be otherwise expressed by saying that *all simple gases have nearly the same thermal capacity per unit volume,* when at the same pressure and temperature.

The specific heat of dry air (at constant pressure), according to Regnault, is ·2375; that is to say, the thermal capacity of a given weight of air is ·2375 of that of an equal weight of water.

The above conclusions (1) and (2) are also true of specific heat at constant volume, as far as regards simple gases and air (which, being merely a mechanical mixture, obeys the same laws as a simple gas). The two following consequences have been experimentally confirmed.

(3) The *difference* between the two *thermal capacities per unit volume,* at a given pressure and temperature, is the same for all permanent gases.

(4) The *ratio* of the two specific heats is constant for all simple gases, its value being about 1·41.

It is important to remark, that the temperatures of the gases in Regnault's experiments were given by an air-thermometer, so that his result as regards the constancy of specific heat at all temperatures may be thus stated:—*If equal quantities of heat be successively added to a gas at constant pressure, the volume of the gas will increase in arithmetical progression.*

347 A. Relation of Pressure to Volume when no Heat enters or escapes.—Boyle's law asserts that the density of a gas varies directly as the pressure, when the temperature is the same in the cases compared. If no heat is allowed to enter or escape, the temperature of the gas will rise when the pressure is increased, and the volume will not be so much diminished as it would be if the temperature remained constant.

Suppose that a quantity of gas at volume V, pressure P, and temperature T, receives a small addition of heat q, the gas being allowed to expand at constant pressure, so that its volume becomes $V+v$, and its temperature $T+t$, its pressure being still P.

Now let the gas be compressed without allowing any heat to enter or escape, till its volume is V as at first; and let its temperature now be $T+t+\beta t$, and its pressure $P+p$.

It is now sensibly in the same condition as if the quantity q of heat had been imparted to it without allowing any change of volume to take place; so that the same quantity q of heat which produces an elevation of temperature t at constant pressure, would produce an elevation $t+\beta t$ at constant volume. The specific heats are inversely as these elevations of temperature; hence we have

$$\frac{\text{Specific heat at constant pressure}}{\text{Specific heat at constant volume}}=1+\beta. \qquad (2)$$

To determine the change of pressure, we have

$$\frac{P+p}{P}=\frac{V+v}{V}\cdot\frac{1+\alpha\,(T+t+\beta t)}{1+\alpha\,(T+t)};\ \text{and}\ \frac{V+v}{V}=\frac{1+\alpha\,(T+t)}{1+\alpha T},$$

hence

$$\frac{P+p}{P}=\frac{1+\alpha\,(T+t+\beta t)}{1+\alpha T}=1+\frac{\alpha\,(t+\beta t)}{1+\alpha T};$$

therefore $\frac{p}{P}=\frac{\alpha\,(t+\beta t)}{1+\alpha T}$; but we have $\frac{v}{V}=\frac{\alpha t}{1+\alpha T}$;

therefore

$$\frac{p}{P}=(1+\beta)\,\frac{v}{V}, \qquad (3)$$

a result which is of great importance in connection with the velocity of sound. In fact, experiments on the velocity of sound have furnished the most precise determinations hitherto made of the value of $1+\beta$, which, as above indicated, is the ratio of specific heat at constant pressure to specific heat at constant volume, and is about 1·41.[1]

348. Heat of Fusion.—The method of mixtures may be employed to determine the quantity of heat absorbed by a body in its passage from the solid to the liquid state. For instance, a piece of ice at zero is carefully weighed and plunged into water contained in a calorimeter. The temperature of the water falls until a certain limit is attained, which is then observed. Suppose now that we have the following data:—

m the thermal capacity of the calorimeter, or the equivalent of the calorimeter in weight of water; t its initial temperature; θ its final temperature; w the weight of the ice; x the latent heat of fusion.

The heat required to melt the ice is wx, and the heat required to

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[1] It is easily shown, by integrating equation (3), that the general relation between volume and pressure, when no heat enters or escapes, is

$$\frac{P_2}{P_1}=\left(\frac{V_1}{V_2}\right)^{1+\beta}$$

V_1, V_2 denoting the volumes at pressures P_1, P_2.

raise the water which it yields to the temperature θ is $w\,\theta$. The sum of these two quantities must be equal to $m\ (t-\theta)$, the heat given up by the calorimeter; that is,

$$w\ (x+\theta)=m\ (t-\theta),$$

from which equation x is easily found. Since this experiment necessarily occupies a considerable time, the radiation from the calorimeter must be taken into account. For this purpose a continual series of observations is taken of the temperatures indicated by the thermometer immersed in the liquid, and the quantities of heat gained or lost are estimated by the method described above (§ 343). In this way the heat of fusion of ice was fixed by Laprovostaye and Desains at 79·25 Centigrade, which is equivalent to 142·65 Fahrenheit.

When the body melts at a high temperature, the method is reversed; the body is first melted, and then immersed in the calorimeter. In doing this, precautions must be taken to prevent the vaporization of a portion of the water; for instance, the body may be inclosed in a small thin box which is not completely opened until towards the close of the experiment. Suppose we have W, the equivalent of the calorimeter in water; t its initial temperature; θ its final temperature; w the weight of the body; T its initial temperature; T′ its melting-point; c its specific heat in the solid state; c' its specific heat in the liquid state; from these data the equation will evidently be

$$W\ (\theta-t)=wx+wc'\ (T-T')+wc\ (T'-\theta),$$

neglecting the correction for radiation, which can be determined by the ordinary methods.

One of the quantities which enter into this equation is the specific heat of the body in the solid state, which may be considered as known. The specific heat of the body in the liquid state may be deduced by combining this equation with another of the same kind, in which the initial temperature of the melted body is different. In the case of bodies which, like mercury and bromine, are liquid at ordinary temperatures, the specific heat in the solid state can be found by a similar but inverse process.

The following table gives the heats of fusion of several substances, together with their specific heats in both states.

Substances.	Melting-point.	Specific Heats		Latent Heat of Fusion.
		In the Solid State.	In the Liquid State.	
Water, ...	0°	·5040	1·0000	79·250
Phosphorus, . .	44·20	·2000	·2000	5·400
Sulphur, . . .	111	·2020	·2340	9·368
Bromine, . . .	−7·32	·0840	·1670	16·185
Tin, ...	232	·0560	·0640	14·252
Bismuth, . . .	266	·0308	·0363	12·640
Lead, ...	326	·0314	·0402	5·369
Mercury, ...	−39	·0319	·0333	2·820

349. Heat of Evaporation.—The latent heat of evaporation of water, and of some other liquids, can be determined by means of Despretz's apparatus, which is shown in Fig. 306.

The liquid is boiled in a retort C, which is connected with a worm S

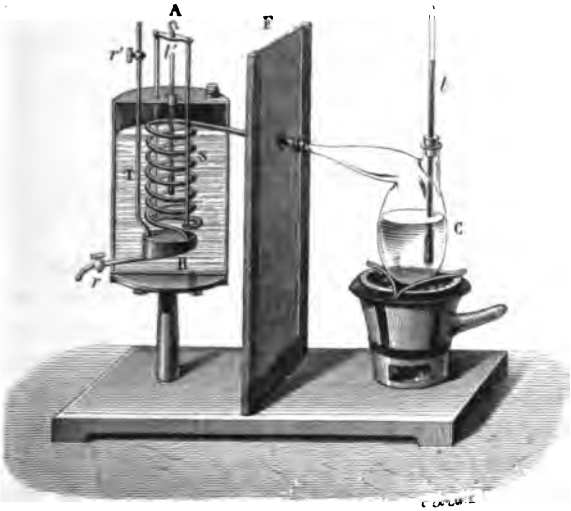

Fig. 306.—Despretz's Apparatus.

rounded by cold water, and terminating in the reservoir R. The our is condensed in the worm, and collects in the reservoir, whence an be drawn by means of the stop-cock *r*. The tube T, which is d with a stop-cock *r′*, serves to establish communication between reservoir and the atmosphere, or between the reservoir and a e where a fixed pressure is maintained, so as to produce ebulli- at any temperature required, which is indicated by the thermometer t. A is an agitator for keeping the water at a uniform temperature, which is indicated by the thermometer t'.

In using the apparatus, the first step is to boil the liquid in the retort, and when it is in active ebullition, it is put in communication with the worm. The temperature of the calorimeter has previously been lowered a certain number of degrees below that of the surrounding air, and the experiment ceases when it has risen to the same number of degrees above. The compensation may thus be considered as complete, since the rate of heating is nearly uniform.

If W be the equivalent of the calorimeter in water, t its initial temperature, θ its final temperature; then the quantity of heat gained by it is $W(\theta - t)$. This heat comes partly from the latent heat disengaged at the moment of condensation of the vapour, partly from the loss of temperature of the condensed water, which sinks from T, the boiling-point of the liquid, to the temperature of the calorimeter. If, then, x denote the latent heat of evaporation, w the weight of the liquid collected in the box R, and c its specific heat, we have the equation

$$W(\theta - t) = wx + wc(T - \theta).$$

This experiment is exposed to some serious causes of error. The calorimeter may be heated by radiation from the screen F which protects it from the direct radiation of the furnace. Heat may also be propagated by means of the neck of the retort. Again, the vapour is not *dry* when it passes into the worm, but carries with it small drops of liquid. Finally, some of the vapour may be condensed at the top of the retort, and so pass into the worm in a liquid state. This last objection is partly removed by sloping the neck of the retort upwards from the fire, but it sometimes happens that this precaution is not sufficient.

350. Regnault's Experiments.—The labours of Regnault in connection with the subject of latent heat are of the greatest importance, and have resulted in the elaboration of a method in which all these sources of error are entirely removed. The results obtained by him are the following:—

The quantity of heat required to convert a kilogramme of water at 100° into vapour, without change of temperature, is 537 kilogramme-degrees or calories.

If the water were originally at zero, the total amount of heat required to convert it into vapour at 100° would be 637 calories—

100 to raise it to 100°, and 537 more to convert it into vapour. It is this total amount which is most important to know in the applications of heat in the arts.

In general, if Q denote the total quantity of heat[1] required to transform water at zero into vapour at the temperature T, the value of Q may be deduced with great exactness from the following equation:—

$$Q=606{\cdot}5+{\cdot}305\,T. \tag{a}$$

From what we have said above, it will be seen that if λ denote the latent heat of evaporation at temperature T, we must have

$$Q=\lambda+T,$$

whence, by substituting for Q in (*a*), we have

$$\lambda=606{\cdot}5-{\cdot}695\,T. \tag{b}$$

From (*a*) we can find the total heat for any given temperature, and from (*b*) the latent heat of evaporation at any given temperature. The results for every tenth degree between 0° and 230° are given in the following table:—

Temperatures Centigrade.	Latent Heat.	Total Heat.	Temperatures Centigrade.	Latent Heat.	Total Heat.
0° . . .	606	606	120°. . .	522	642
10 . . .	600	610	130 . . .	515	645
20 . . .	593	613	140 . . .	508	648
30 . . .	586	616	150 . . .	501	651
40 . . .	579	619	160 . . .	494	654
50 . . .	572	622	170 . . .	486	656
60 . . .	565	625	180 . . .	479	659
70 . . .	558	628	190 . . .	472	662
80 . . .	551	631	200 . . .	464	664
90 . . .	544	634	210 . . .	457	667
100 . . .	537	637	220 . . .	449	669
110 . . .	529	639	230 . . .	442	672

To reduce latent heat and total heat from the Centigrade to the Fahrenheit scale, we must multiply by $\frac{9}{5}$. Thus the latent and total heat of steam at 212° F. are 966·6 and 1146·6. We subjoin a table, taken from the researches of Favre and Silbermann, giving the latent heat of evaporation of a number of liquids at the temperature of their boiling-point, referred to the Centigrade scale:—

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[1] Called by Regnault the total heat of saturated vapour at T°, or the total heat of vaporization at T°.

	Boiling-point.	Latent Heat.		Boiling-point.	Latent Heat.
Wood-spirit, . . .	66·5	264	Acetic acid, . . .	120°	102
Absolute alcohol, . .	78	208	Butyric acid, . . .	164	115
Valeric alcohol, . .	78	121	Valeric acid, . . .	175	104
Ether, ...	38	91	Acetic ether, . . .	74	100
Ethal, ...	38	58	Oil of turpentine, . .	156	69
Valeric ether, . . .	113·5	113·5	Essence of citron, .	165	70
Formic acid, . . .	100	169			

351. Heat Disengaged in Chemical Combinations.—A very convenient apparatus has been invented by Favre and Silbermann for measuring the heat given off in chemical reactions.[1] It is a kind of large mercurial thermometer (Fig. 307), the reservoir R of which is made of iron, and contains one or more cylindrical openings similar to that shown at *m*. Into these are fitted tubes of glass or platinum, in which the chemical reaction takes place. One of the substances is introduced first, and the other, which is liquid, is then added by means of a pipette bent at B, and containing the liquid in a globe, as shown in the figure. This is effected by raising the pipette into the position indicated by the dotted lines in the figure.

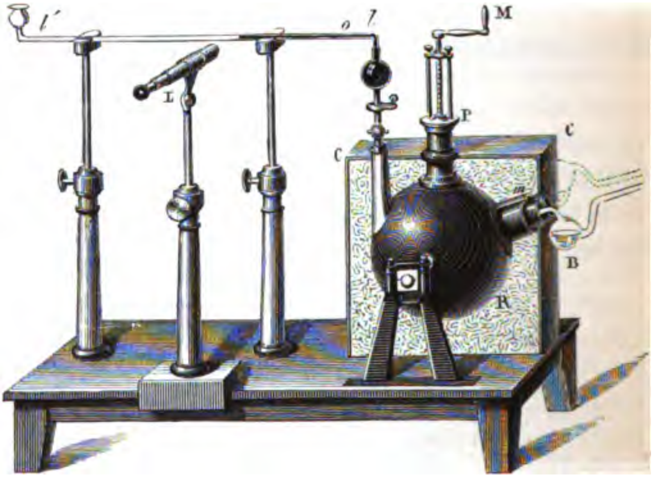

Fig. 307.—Calorimeter of Favre and Silbermann.

In the upper part of the reservoir is an opening fitted with a tube containing a steel plunger P, which descends into the mass of mercury, and can be screwed down or up by turning the handle M. To prepare the apparatus for use, the plunger is so adjusted that the mercury stands at the zero-point of the graduated tube tt', the reaction is then allowed to take place, and the movement of the mercurial column is observed with the telescope L. In order to measure the quantity of heat corresponding to this displacement, a known weight of hot water is introduced into the reservoir, and allowed to give up its heat to the mercury; the displacement of the mercurial column is then observed, and since the quantity of heat corresponding to this displacement is known, that corresponding to any other displacement can easily be calculated. The iron reservoir is inclosed in a box filled with wadding or some other non-conducting material.[1]

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[1] Chemical actions are called *reactions*, and a substance which acts chemically is called a *reagent*. The names are awkward, but are in general use.

When the chemical reaction takes the orm of combustion, a different arrangenent is necessary. An apparatus of great erfection has been devised by Favre and ilbermann for this purpose, but it is of o complex a construction to be deribed here. A sufficiently accurate idea the general method adopted in these ses will be obtained by a study of the uch simpler apparatus employed for e same purpose by Dulong.

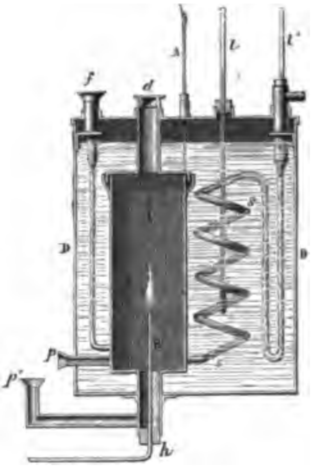

Fig. 308.—Dulong's Calorimeter for Combustion.

It consists of a combustion-chamber C rounded by the water contained in a orimeter D, in which moves an agitator ose stem is shown at A. The comtible substance, if it be a gas, is conted into the chamber through the e *h*, and the oxygen necessary for combustion enters by one of the tubes *f* or *p'*. The products ombustion pass through the worm *s*, and finally escape, but

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the mode of experimentation adopted by Dr. Andrews, the combination takes place hin copper vessel inclosed in a calorimeter of water to which it gives up its heat; and e of temperature in the water is observed with a very delicate thermometer, the being agitated either by stirring with a glass rod or by making the whole apparatus about a horizontal axis.

xperimenting on the heat of combustion, the oxygen and the substance to be burned roduced into the thin copper vessel, which is inclosed in the calorimeter as above, ition or explosion is produced by means of electricity.

not until they have fallen to the temperature of the water in the calorimeter. This condition is necessary to the exactness of the result, and its precise fulfilment is verified by observing the temperatures indicated by the thermometers *t* and *t′*, the former of which gives the temperature of the water, and the latter that of the products of combustion at their exit. These two temperatures should always agree. The progress of the combustion is observed through the opening *p*, which is closed by a piece of glass. Some of the results obtained by this method are given in the following table, the combustion being supposed to take place in oxygen, with the exception of the second example on the list

HEATS OF COMBUSTION.[1]

(Referred to the Centigrade Scale.)

Hydrogen,	34,462
Hydrogen with chlorine,	23,783
Carbonic oxide,	3,403
Marsh-gas,	13,063
Charcoal,	8,080
Graphite,	7,797
Diamond,	7,770
Native sulphur,	2,261
Soft sulphur,	2,258
Sulphide of carbon,	3,400
Olefiant gas,	11,857
Ether,	9,028
Alcohol,	7,184
Stearic acid,	9,616
Oil of turpentine,	10,852
Olive-oil,	9,862

Of all substances hydrogen possesses by far the greatest heat of combustion. This fact accounts for the intense heating effects which can be obtained with the oxy-hydrogen blowpipe, in which an annular jet of hydrogen is completely burned by means of a central jet of oxygen. It is to be observed that the heat of combination observed by the above methods includes the heating or cooling effect of the changes of volume which usually accompany chemical combination.

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[1] These numbers denote the number of times its own weight of water which would be raised one degree Centigrade by the heat evolved in the combustion of the substance. For example, the combustion of a pound of charcoal gives out enough heat to raise 8080 pounds of water one degree.

CHAPTER XXXII.

THERMO-DYNAMICS.

354. Connection between Heat and Work.—That heat can be made to produce work is evident when we consider that the work done by steam-engines and other heat-engines is due to this source.

Conversely, by means of work we can produce heat. Fig. 310 represents an apparatus sometimes called the pneumatic tinder-box, consisting of a piston working tightly in a glass barrel. If a piece of gun-cotton be fixed in the cavity of the piston, and the air be then suddenly compressed, so much heat will be developed as to inflame the gun-cotton.

A singular explanation of this effect was at one time put forward. It was maintained that heat or *caloric* was a kind of imponderable fluid, which, when introduced into a body, produced at once an increase of volume and an elevation of temperature. If, then, the body was compressed, the caloric which had served to dilate it was, so to speak, *squeezed out*,[1] and hence the development of heat. An immediate consequence of this theory is that heat cannot be increased or diminished in quantity, but that any addition to the quantity of heat in one part of a system must be compensated by a corresponding loss in another part. But we know that there are cases in which heat is produced by two bodies in contact, without our being able to observe any traces of this compensating process. An instance of this is the production of heat by friction.

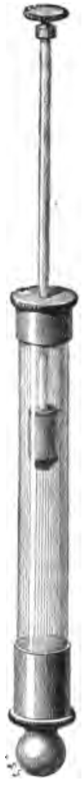

Fig. 310. Pneumatic Tinder-box.

355. Heat produced by Friction.—Friction is a well-known source of heat. Savages are said to obtain fire by rubbing two pieces of dry wood together. The friction between the wheel and axle in railway-carriages frequently produces the same effect, when they have been insufficiently greased; and the stoppage of a train by applying a brake to the wheels usually produces a shower of sparks.

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[1] In other words, the thermal capacity of the body was supposed to be diminished, so that the amount of heat contained in it, without undergoing any increase, was able to raise it to a higher temperature.

The production of heat by friction may be readily exemplified by the following experiment, due to Tyndall. A glass tube containing water (Fig. 311), and closed by a cork, can be rotated rapidly about its axis. While thus rotating, it is pressed by two pieces of wood, covered with leather. The water is gradually warmed, and finally enters into ebullition, when the cork is driven out, followed by a jet of steam. Friction, then, may produce an intense heating of the bodies rubbed together, without any corresponding loss of heat elsewhere.

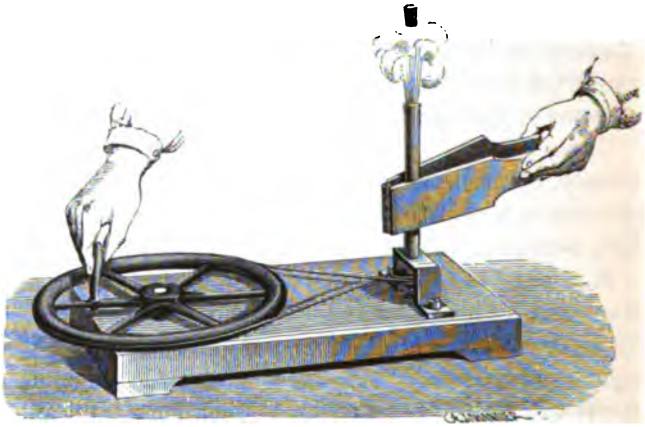

Fig. 311.—Heat produced by Friction.

At the close of last century, Count Rumford (an American in the service of the Bavarian government) called attention to the enormous amount of heat generated in the boring of cannon, and found, in a special experiment, that a cylinder of gun-metal was raised from the temperature of 60° F. to that of 130° F. by the friction of a blunt steel borer, during the abrasion of a weight of metal equal to about $\frac{1}{950}$ of the whole mass of the cylinder. In another experiment, he surrounded the gun by water (which was prevented from entering the bore), and, by continuing the operation of boring for $2\frac{1}{2}$ hours, he made this water boil. In reasoning from these experiments, he strenuously maintained that heat cannot be a material substance, but must consist in motion.

The advocates of the caloric theory endeavoured to account for these effects by asserting that caloric, which was latent in the metal when united in one solid mass, had been forced out and rendered sensible by the process of disintegration under heavy pressure. This supposition was entirely gratuitous, no difference having ever been detected between the properties of entire and of comminuted metal as regards thermal capacity; and, to account for the observed effect, the latent heat thus supposed to be rendered sensible in the abrasion of a given weight of metal, must be sufficient to raise 950 × 70, that is 66,500 times its own weight of metal through 1°.

Yet, strange to say, the caloric theory survived this exposure of its weakness, and the, if possible, still more conclusive experiment of Sir Humphrey Davy, who showed that two pieces of ice, when rubbed together, were converted into water, a change which involves not the evolution but the absorption of latent heat, and which cannot be explained by diminution of thermal capacity, since the specific heat of water is much greater than that of ice.

Davy, like Rumford, maintained that heat consisted in motion, and the same view was maintained by Dr. Thos. Young; but the doctrine of caloric nevertheless continued to be generally adopted until about the year 1840, since which time, the experiments of Joule, the eloquent advocacy of Meyer, and the mathematical deductions of Thomson, Rankine, and Clausius, have completely established the mechanical theory of heat, and built up an accurate science of thermodynamics.

356. Foucault's Experiment.—The relations existing between electrical and thermal phenomena had considerable influence in leading to correct views regarding the nature of heat. An experiment devised by Foucault illustrates these relations, and at the same time furnishes a fresh example of the production of heat by the performance of mechanical work.

The apparatus consists (Fig. 312) of a copper disc which can be made to rotate with great rapidity by means of a system of toothed wheels. The motion is so free that a very slight force is sufficient to maintain it. The disc rotates between two pieces of iron, constituting the armatures of one of those temporary magnets which are obtained by the passage of an electric current (called electro-magnets). If, while the disc is turning, the current is made to pass, the armatures become strongly magnetized, and a peculiar action takes place between them and the disc, consisting in the formation of induced currents in the latter, accompanied by a resistance to motion. As long as the magnetization is continued, a considerable effort is necessary to maintain the rotation of the disc; and if the rotation be continued for two or three minutes, the disc will be found to have risen some 50° or 60° C. in temperature, the heat thus acquired by the disc being the equivalent of the work done in maintaining the motion. It is to be understood that, in this experiment, the rotating disc does not touch the armatures; the resistance which it experiences is due entirely to invisible agencies.

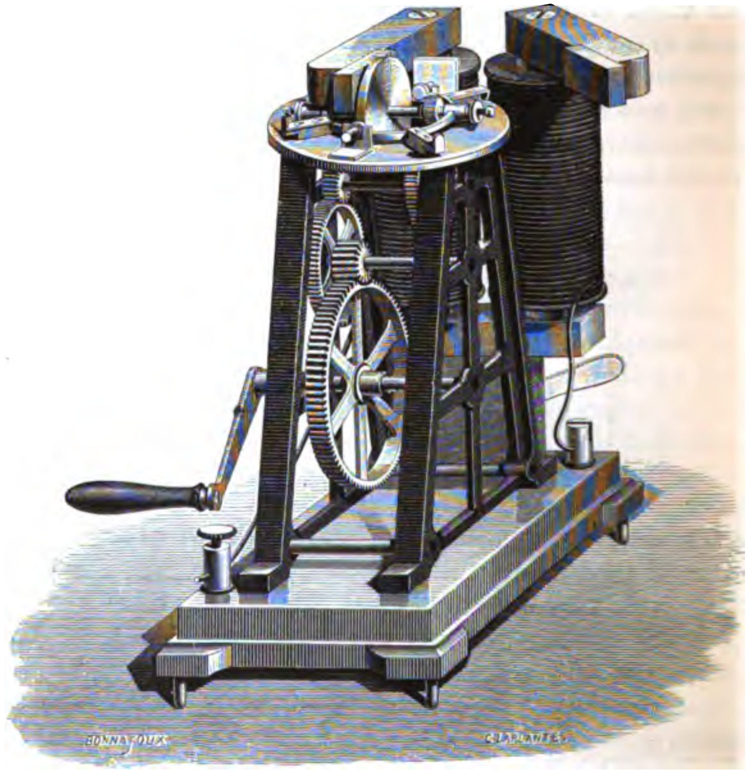

Fig. 312.—Foucault's Apparatus.

The experiment may be varied by setting the disc in very rapid rotation, while no current is passing, then leaving it to itself, and immediately afterwards causing the current to pass. The result will be, that the disc will be brought to rest almost instantaneously, and will undergo a very slight elevation of temperature, the heat gained being the equivalent of the motion which is destroyed.

357. Mechanical Equivalent of Heat.—The most precise determination yet made of the numerical relation subsisting between heat and mechanical work was obtained by the following experiment of Joule. He constructed an agitator which is somewhat imperfectly represented in Fig. 313, consisting of a vertical shaft carrying several sets of paddles revolving between stationary vanes, these latter serving to prevent the liquid in the vessel from being bodily whirled in the direction of rotation. The vessel was filled with water, and the agitator was made to revolve by means of a cord, wound round the upper part of the shaft, carried over a pulley, and attached to a weight, which by its descent drove the agitator, and furnished a measure of the work done. The pulley was mounted on friction-wheels, and the weight could be wound up without moving the paddles. When all corrections had been applied, it was found that the heat communicated to the water by the agitation amounted to one pound-degree Fahrenheit for every 772 foot-pounds of work spent in producing it. This result was verified by various other forms of experiment, and may be assumed to be correct within about one foot-pound. The experiments were made at Manchester, where

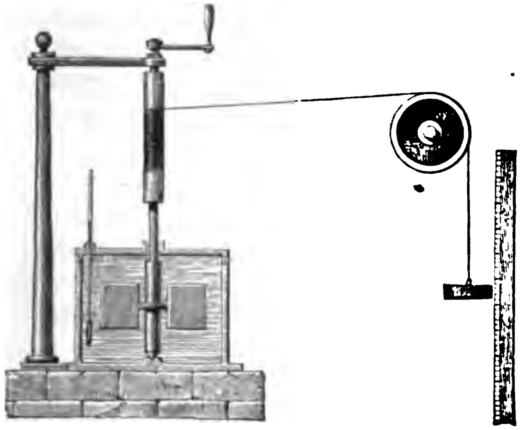

Fig. 313.—Determination of the Mechanical Equivalent of Heat.

g is 32·194, and it is to be borne in mind that a foot-pound does not denote precisely the same amount of work at all places on the earth's surface, but varies in direct proportion to the intensity of gravity. The difference in its value in passing from one place to another on the earth is, however, not greater than the probable error of the number 772. We may therefore, with about as much accuracy as is warranted by the present state of our knowledge, assert that the energy comprised in one pound-degree Fahrenheit is about 772 terrestrial foot-pounds.[1]

The mechanical equivalent of the pound-degree Centigrade is $\frac{9}{5}$ of this, or about 1390 foot-pounds.

The number 772 or 1390, according to the scale of temperature adopted, is commonly called *Joule's equivalent*, and is denoted in formulæ by the letter J. If we take the kilogramme-degree Centigrade for unit of heat, and the kilogrammetre for unit of work, the value of J will be 424.

357 A. First Law of Thermo-dynamics.—Whenever work is performed by the agency of heat, an amount of heat disappears equivalent to the work performed; and whenever mechanical work is spent in generating heat, the heat generated is equivalent to the work thus spent; that is to say, we have in both cases

$$W = JH;$$

W denoting the work, H the heat, and J Joule's equivalent. This is called the *first law of thermo-dynamics*, and it is a particular case of the great natural law (§ 53 K) which asserts that energy may be transmuted, but is never created or destroyed.

It may be well to remark here that work is not energy, but is rather the process by which energy is transmuted (§ 53 J), amount of work being measured by the amount of energy transmuted. Whenever work is done, it leaves an effect behind it in the shape of energy of some kind or other, equal in amount to the energy consumed in performing the work, or, in other words, equal to the work itself.

As regards the nature of heat, there can be little doubt that heat properly so called, that is sensible as distinguished from latent heat, consists in some kind of motion, and that quantity of heat is quantity of energy of motion, or kinetic energy (§ 53 I), whereas latent heat consists in energy of position or potential energy (§ 53 J).

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[1] In absolute units of work, of which a foot-pound contains g, the equivalent of a pound-degree Fahrenheit is $772 \times 32{\cdot}194 = 24854$, which is within less than 1 per cent. of 25,000. Hence the heat-equivalent of the kinetic energy of a mass of m pounds moving with a velocity of v feet per second is approximately $\frac{1}{2} mv^2 \div 25000$, or $2 mv^2 \div 100000$.

We have already had, in the experiments of Rumford, Davy, Foucault, and Joule, some examples of transmutation of energy; but it will be instructive to consider some additional instances.

When a steam-engine is employed in hauling up coals from a pit, an amount of heat is destroyed in the engine equivalent to the energy of position which is gained by the coal.

In the propulsion of a steam-boat with uniform velocity, or in the drawing of a railway train with uniform velocity on a level, there is no gain of potential energy, neither is there, as far as the vessel or train is concerned, any gain of kinetic energy. In the case of the steamer, the immediate effect consists chiefly in the agitation of the water, which involves the generation of kinetic energy; and the ultimate effect of this is a warming of the water, as in Joule's experiment. In the case of the train, the work done in maintaining the motion is spent in friction and concussions, both of which operations give heat as the ultimate effect. Here, then, we have two instances in which heat, after going through various transformations, reappears as heat at a lower temperature.

In starting a train on a level, the heat destroyed in the engine finds its equivalent mainly in the energy of motion gained by the train; and this energy can again be transformed into heat by turning off the steam and applying brakes to the wheels.

When a cannon-ball is fired against an armour plate, it is heated red-hot if it fails to penetrate the plate, the energy of the moving ball being in this case obviously converted into heat. If the plate is penetrated, and the ball lodges in the wooden backing, or in a bank of earth, the ball will not be so much heated, although the total amount of heat generated must still be equivalent to the energy of motion destroyed. The ruptured materials, in fact, receive a large portion of the heat. The heat produced in the rupture of iron is well illustrated by punching and planing machines, the pieces of iron punched out of a plate, or the shavings planed off it, being so hot that they can scarcely be touched, although the movements of the punch and plane are exceedingly slow. The heat gained by the iron is, in fact, the equivalent of the work performed, and this work is considerable on account of the great force required.

357 B. Heat Lost in Expansion.—The difference between the specific heat of a gas at constant pressure and at constant volume, is almost exactly the equivalent of the work which the gas at constant pressure performs in pushing back the surrounding atmosphere. Joule immersed two equal vessels in water, one of them containing highly-compressed air, and the other being exhausted; and when they were both at the temperature of the water he opened a stop-cock which placed the vessels in communication. The compressed air thus expanded to double its volume, but the temperature of the surrounding water was unaltered, the heat converted into energy of motion by the expansion being, in fact, compensated by the heat generated in the destruction of this motion in the previously vacuous vessel. This experiment shows that, when air expands without having to overcome external resistances, its temperature is not sensibly changed by the expansion.

Suppose we have a cylinder whose internal section is a square decimetre, and that a piston travelling in this cylinder is pushed outwards by the expansion of air in the cylinder. Let the air be initially at 0° C., and occupy 1 decimetre in length of the cylinder, so that its volume at this temperature is a cubic decimetre, and let this air be expanded at the constant pressure of 760 mm. by the addition of heat, until it occupies double its initial volume, and has therefore pushed the piston a distance of 1 decimetre. Since air expands by $\frac{1}{273}$ of its volume at 0° for each degree, the final temperature will, in this case, be 273° C., and since the specific heat of air at constant pressure is ·237, and the weight of the air is 1·293 gramme, the heat taken up by the air is

$$273 \times \cdot 237 \times 1\cdot 293 = 83\cdot 66 \text{ gramme-degrees.}$$

Now the pressure of 760 mm. is equivalent to 1·033 kilogrammes per square centimetre; hence the total pressure resisting the advance of the piston is 103·3 kilogrammes, which is overcome through a distance of 1 decimetre, so that the work done against atmospheric resistance is 10·33 kilogrammetres.

Joule's equivalent for a kilogramme-degree is 424 kilogrammetres. The heat-equivalent of the work done in the present case is therefore $\frac{10\cdot 33}{424} = \cdot 02436$ kilogramme-degree = 24·36 gramme-degrees.

Hence, of the whole heat 83·66 received by the air in the cylinder, 24·36 has been spent in doing external work, leaving 59 3 as the amount which would have sufficed to raise the air to the same temperature if no external work had been performed. Now the ratio of 83·66 to 59·3 is 1·41, which, as we have already seen (§ 347), is the ratio of specific heat at constant pressure to specific heat at constant volume.

In the case of air and perfect gases, the heat gained by compression or lost in expansion is almost the exact equivalent of the external work performed.

In view of these facts, the specific heat of a gas at constant volume is often called the *true specific heat* of the gas. The heat required to produce a given change of temperature in a gas, when its volume changes in any specified way, may be computed to a very close approximation by calculating the work done by the gas against external resistances during its change of volume, and adding the heat-equivalent of this work to the heat which would have produced the same change of temperature at constant volume. The true specific heat of air in this sense is ·168.

358. Thermic Engines.—In every form of thermic engine, work is obtained by means of expansion produced by heat, the force of expansion being usually applied by admitting a hot elastic fluid to press alternately on opposite sides of a piston travelling in a cylinder. Of the heat received by the elastic fluid from the furnace, a part leaks out by conduction through the sides of the containing vessels, another part is carried out by the fluid when it escapes into the air or into the condenser, the fluid thus escaping being always at a temperature lower than that at which it entered the cylinder, but higher than that of the air or condenser into which it escapes; but a third part has disappeared altogether, and ceased to exist as heat, having been spent in the performance of work. This third part is the exact equivalent of the work performed by the elastic fluid in driving the piston,[1] and may therefore be called the *heat utilized*, or the *heat converted*.

The *efficiency of an engine* may be measured by *the ratio of the heat thus converted to the whole amount of heat which enters the engine;* and we shall use the word *efficiency* in this sense.

358A. Carnot's Investigations.—The first approach to an exact science of thermo-dynamics was made by Carnot in 1824. By reasoning based on the theory which regards heat as a substance, but which can be modified so as to remain conclusive when heat is regarded as a form of energy, he established the following principles:—

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[1] If negative work is done by the fluid in any part of the stroke (that is, if the piston presses back the fluid), the algebraic sum of work is to be taken.

I. *The thermal agency by which mechanical effect may be obtained is the transference of heat from one body to another at a lower temperature.* These two bodies he calls the *source* and the *refrigerator.* Adopting the view generally received at that time regarding the nature of heat, he supposed that all the heat received by an engine was given out by it again as heat; so that, if all lateral escape was prevented, all the heat drawn by the engine from the source was given by the engine to the refrigerator, just as the water which by its descent turns a mill-wheel, runs off in undiminished quantity at a lower level. We now know that, when heat is let down through an engine from a higher to a lower temperature, it is diminished in amount by the equivalent of the work done by the engine against external resistances.

He further shows that the amount of work which can be obtained by letting down a given quantity of heat—or, as we should say with our present knowledge, by partly letting it down and partly consuming it in work, is increased by raising the temperature of the source, or by lowering the temperature of the refrigerator; and establishes the following important principle:—

II. *A perfect thermo-dynamic engine is such that, whatever amount of mechanical effect it can derive from a certain thermal agency; if an equal amount be spent in working it backwards, an equal reverse thermal effect will be produced.* This is often expressed by saying that a *completely reversible engine* is a *perfect engine.*

By a *perfect engine* is here meant an engine which possesses the maximum of efficiency compatible with the given temperatures of its source and refrigerator; and Carnot here asserts that all completely reversible engines attain this maximum of efficiency. The proof of this important principle, when adapted to the present state of our knowledge, is as follows:—

Let there be two thermo-dynamic engines, A and B, working between the same source and refrigerator; and let A be completely reversible. Let the efficiency of A be m, so that, of the quantity Q of heat which it draws from the source, it converts m Q into mechanical effect, and gives Q $-$ m Q to the refrigerator, when worked forwards. Accordingly, when worked backwards, with the help of work m Q applied to it from without, it takes Q $-$ m Q from the refrigerator, and gives Q to the source.

In like manner, let the efficiency of B be m', so that, of heat Q' which it draws from the source, it converts $m'Q'$ into mechanical effect, and gives $Q'-m'Q'$ to the refrigerator.

Let this engine be worked forwards, and A backwards. Then, upon the whole, heat to the amount $Q'-Q$ is drawn from the source, heat $m'Q'-mQ$ is converted into mechanical effect, and heat $Q'-Q-(m'Q'-mQ)$ is given to the refrigerator.

If we make $m'Q'=mQ$, that is, if we suppose the external effect to be nothing, heat to the amount $Q'-Q$ or $\left(\frac{m}{m'}-1\right)Q$ is carried from the source to the refrigerator, if m be greater than m', that is, if the reversible engine be the more efficient of the two. If the other engine be the more efficient, heat to the amount $\left(1-\frac{m}{m'}\right)Q$ is transferred from the refrigerator to the source, or heat pumps itself up from a colder to a warmer body, and *that* by means of a machine which is self-acting, for B does work which is just sufficient to drive A. Such a result we are entitled to assume impossible, therefore B cannot be more efficient than A.

Another proof is obtained by making $Q'=Q$. The source then neither gains nor loses heat, and the refrigerator gains $(m-m')Q$, which is derived from work performed upon the combined engine from without, if A be more efficient than B. If B were the more efficient of the two, the refrigerator would lose heat to the amount $(m'-m)Q$, which would yield its full equivalent of external work, and thus a machine would be kept going and doing external work by means of heat drawn from the coldest body in its neighbourhood, a result which cannot be admitted to be possible.

358B. Second Law of Thermo-dynamics.—It follows, from the principle thus established, that all *reversible engines* with the same temperatures of source and refrigerator have the same efficiency, whether the working substance employed in them be steam, air, or any other material, gaseous, liquid, or solid. Hence we can lay down the following law, which is called the second law of thermo-dynamics: *the efficiency of a completely reversible engine is independent of the nature of the working substance, and depends only on the temperatures at which the engine takes in and gives out heat; and the efficiency of such an engine is the limit of possible efficiency for any engine.*

As appendices to this law it has been further established:

1. That when one of the two temperatures is fixed, the efficiency is simply proportional to the difference between the two, provided

this difference is very small. This holds good for all scales of temperature.

2. From calculations relating to an imaginary engine of special simplicity, in which a permanent gas is the working substance, it has been determined that the efficiency of a reversible engine is approximately $\frac{T-T'}{T}$, T denoting the upper, and T′ the lower temperature between which the engine works, reckoned from absolute zero (§ 219 A), on the air-thermometer. This is more easily remembered when stated in the following more symmetrical form. Let Q denote the quantity of heat taken in at the absolute temperature T, Q′ the quantity given out at the absolute temperature T′, and consequently Q—Q′ the heat converted into mechanical effect, then we shall have approximately

$$\frac{Q}{T} = \frac{Q'}{T'} = \frac{Q-Q'}{T-T'}.$$

358c. Absolute Scale of Temperature.—In ordinary thermometers, temperatures are measured by the apparent expansion of a liquid in a glass envelope. If two thermometers are constructed, one with mercury and the other with alcohol for its liquid, it is obviously possible to make their indications agree at two fixed temperatures. If, however, the volume of the tube intervening between the two fixed points thus determined be divided into the same number of equal parts in the two instruments, and the divisions be numbered as degrees of temperature, the two instruments will give different indications if plunged in the same bath at an intermediate temperature, and they will also differ at temperatures lying beyond the two fixed points. It is a simple matter to test equality of temperature, but it is far from simple to decide upon a test of equal differences of temperature. Different liquids expand not only by different amounts but by amounts which are not proportional, no two liquids being in this respect in agreement.

In the case of permanent gases expanding under constant pressure, the discordances are much less, and may, in ordinary circumstances, be neglected. Hence gases would seem to be indicated by nature as the proper substances by which to measure temperature, if differences of temperature are to be measured by differences of volume.

It is also possible to establish a scale of temperature by assuming that some one substance rises by equal increments of temperature on receiving successive equal additions of heat; in other words, by making some one substance the standard of reference for specific heat, and assuming the specific heat of this substance to be the same at all temperatures. Here, again, the scale would be different according to the liquid chosen. A mixture of equal weights of water at 0° C. and 100° C. will not have precisely the same temperature as a mixture of equal weights of mercury at these temperatures. If, however, we resort to permanent gases, we find again a very close agreement, so that, if one gas be assumed to have the same specific heat at all temperatures (whether at constant volume or at constant pressure), the specific heat of any other permanent gas will also be sensibly independent of temperature. More than this;—the measurement of temperature by assuming the specific heats of permanent gases to be constant, agrees almost exactly with the measurement of temperature by the expansion of permanent gases. For, as we have seen (§ 347), a permanent gas under constant pressure has its volume increased by equal amounts on receiving successive equal additions of heat.

The air-thermometer, or gas-thermometer, then, has a greatly superior claim to the mercury thermometer to be considered as furnishing a natural standard of temperature.

But a scale which is not only sensibly but absolutely independent of the peculiarities of particular substances, is obtained by *defining temperature in such a sense as to make appendix* (2) *to the second law of thermo-dynamics rigorously exact.* According to this system, the ratio of any two temperatures is the ratio of the two quantities of heat which would be drawn from the source and supplied to the refrigerator by a completely reversible thermo-dynamic engine working between these temperatures. This ratio will be rigorously the same, whatever the working substance in the engine may be, and whether it be solid, liquid, or gaseous.

358 D. Heat required for Change of Volume and Temperature.—The amount of heat which must be imparted to a body to enable it to pass from one condition, as regards volume and temperature, to another, is not a definite quantity, but depends upon the course by which the transition is effected. It is, in fact, the sum of two quantities, one of them being *the heat which would be required if the transition were made without external work*—as in Joule's experiment of the expansion of compressed air into a vacuous vessel—and the other being *the heat equivalent to the external work which the body performs in making the transition.* As regards the first of these quantities, its amount, in the case of permanent gases, depends almost entirely upon the difference between the initial and final temperatures, being sensibly independent of the change of volume, as Joule's experiment shows. In the case of liquids and solids, its amount depends, to a very large extent, upon the change of volume, so that, if the expansion which heat tends to produce is forcibly prevented, the quantity of heat required to produce a given rise of temperature is greatly diminished. This contrast is sometimes expressed by saying that expansion by heat involves a large amount of internal work in the case of liquids and solids, and an exceedingly small amount in the case of gases; but the phrase *internal work* has not as yet acquired any very precise meaning.

External work performed against uniform hydrostatic or pneumatic pressure, may be computed by *multiplying the increase of volume by the pressure per unit area.* For, if we suppose the expanding body to be immersed in an incompressible fluid without weight, confined in a cylinder by means of a movable piston under constant pressure, the work done by the expanding body will be spent in driving back the piston. Let A be the area of the piston, x the distance it is pushed back, and p the pressure per unit area. Then the increment of volume is Ax, and the work done is the product of the force pA by the distance x, which is the same as the product of p by Ax.

As an illustration of the different courses by which a transition may be effected, suppose a quantity of gas initially at 0° C. and a pressure of one atmosphere, and finally at 100° C. and the same pressure, the final volume being therefore 1·366 times the initial volume. Of the innumerable courses by which the transition may be made, we will specify two:—

1st. The gas may be raised, at its initial volume, to such a temperature that, when afterwards allowed to expand against pressure gradually diminishing to one atmosphere, it falls to the temperature 100° C. Or,

2d. It may be first allowed to expand, under pressure diminishing from one atmosphere downwards, until its final volume is attained, and may then, at this constant volume, be heated up to 100°.

In both cases it is to be understood that no heat is allowed to enter or escape during expansion.

Obviously, the first course implies the performance of a greater amount of external work than the second, and it will require the communication to the gas of a greater quantity of heat,—greater by the heat-equivalent of the difference of works.

When a body passes through changes which end by leaving it in precisely the same condition in which it was at first, we are not entitled to assume that the amounts of heat which have entered and quitted it are equal. They are not equal unless the algebraic sum of external work done by the body during the changes amounts to zero. If the body has upon the whole done positive work, it must have taken in more heat than it has given out, otherwise there would be a creation of energy; and if it has upon the whole done negative work, it must have given out more heat than it has taken in, otherwise there would be a destruction of energy. In either case, *the difference between the heat taken in and given out must be the equivalent of the algebraic sum of external work.*

These principles are illustrated in the two following sections.

358 E. Lowering of Freezing-point by Pressure.—When a litre (or cubic decimetre) of water is frozen under atmospheric pressure, it forms 1·087 of a litre of ice, thus performing external work amounting to ·087 × 103·3 = 9 kilogramme-decimetres = ·9 of a kilogrammetre, since the pressure of one atmosphere or 760 mm. of mercury is 103·3 kilogrammes per square decimetre. Under a pressure of n atmospheres, the work done would be ·9 n kilogrammetres, neglecting the very slight compression due to the increase of pressure. If the ice is allowed to melt in vacuo, no external work is done upon it in the melting, and therefore, in the whole process, at the end of which the water is in the same state as at the beginning, heat to the amount of $\frac{\cdot 9\,n}{424} = \cdot 00212\,n$ of a calorie is made to disappear. This process is *reversible*, for the water might be frozen in vacuo and melted under pressure; and hence, by appendix (2) to the second law of thermo-dynamics, we have

$$\cdot 00212\,n \; : \; Q \; :: \; T - T' \; : \; T;$$

where Q denotes the heat taken in in melting, which is 79·25 calories, T the absolute temperature at which the melting occurs, about 273°, and T′ the absolute temperature of freezing under the pressure of n atmospheres. Hence we have

$$\cdot 00212\,n \; : \; 79\cdot 25 \; :: \; T - T' \; : \; 273;$$

whence

$$T - T' = \cdot 0073\,n;$$

that is, the freezing-point is lowered by ·0073 of a degree Cent. for each atmosphere of pressure.

358 F. Latent Heat at Temperatures below the Melting-point.—We have seen (§ 231) that water may be preserved in the liquid state at temperatures considerably below its normal freezing-point. When ice is formed at these low temperatures, the latent heat absorbed must be less than 79·25, which is the latent heat at 0° C. For, neglecting the trifling amount of external work performed, we can assert that the heat lost from a mass of water at 0°, in its passage to ice at $-t°$, is independent of the order of operations. Now the specific heat of ice is ·504, that of water being 1; hence the heat lost in making the transition in the ordinary way is $79·25 + ·504\,t$, while that lost by first cooling down, and then freezing at $-t°$ is $l+t$, l denoting the latent heat at $-t°$. Equating these two expressions, we have $l=79·25 - ·496\,t$.

We have here assumed that the specific heat of ice is ·504 at all temperatures. Person's experiments seem to show that within a degree or two of the melting-point it has a much larger value.

If the specific heats both of ice and water were constant at all temperatures, we might determine the position of the absolute zero of temperature by putting $l=0$, which gives $-t° = -160°$ nearly, a result which differs widely from that deduced from the laws of gaseous expansion.

Allowing for possible variations of specific heat with temperature, the expressions for the heat evolved in passing from water at 0° to ice at $-t°$ are $79·25 + ts'$ and $l+ts$, in which s denotes the mean specific heat of water between 0° and $-t°$, and s' the mean specific heat of ice between the same limits. Equating these two expressions, we find

$$l = 79·25 - t(s - s').$$

When freezing once begins at a temperature below 0°, it proceeds very rapidly, accompanied by a rise of temperature, and ceases as soon as the temperature of the whole mass has risen to 0°. To compute the proportions of ice at 0° and water at 0° which a given mass of water at $-t°$ will yield, we may reason as follows:—Calling the whole mass unity, to raise it from its initial condition to 0°, and then convert a fraction m of it into ice at 0°, would require first the addition of t heat-units, and then the subtraction of $79·25\,m$ heat-units, that is upon the whole an addition of $t - 79·25\,m$. Now the external work being inconsiderable, the amount of heat required from without for the conversion of a mass 1 of water at $-t^\circ$ into m of ice at 0° and $1-m$ of water at 0°, will be independent of the order of procedure; and in the given case no heat is added from without; we have therefore

$$t-79{\cdot}25\,m = 0, \quad m = \frac{t}{79{\cdot}25}.$$

359. Animal Heat and Work.—We have every reason to believe that animal heat and motions are derived from the energy of chemical combinations, which take place chiefly in the act of respiration, the most important being the combination of the oxygen of the air with carbon which is furnished to the blood by the animal's food. The first enunciation of this view has been ascribed to Lavoisier. Rumford certainly entertained very clear and correct ideas on the subject, for he says, in describing his experiments on the boring of cannon:—

"Heat may thus be produced merely by the strength of a horse, and, in a case of necessity, this heat might be used in cooking victuals. But no circumstances could be imagined in which this method of procuring heat would be advantageous; for more heat might be obtained by using the fodder necessary for the support of a horse as fuel."

When the animal is at rest, the heat generated by chemical combination is equal to that given off from its body; but when it works, an amount of heat disappears equivalent to the mechanical effect produced. This may at first sight appear strange, in view of the fact that a man becomes warmer when he works. The reconciliation of the apparent contradiction is to be found in the circumstance that, in doing work, respiration is quickened, and a greater quantity of carbon consumed.

Elaborate experiments on this subject were conducted by Hirn. He inclosed a man in a box containing a tread-mill, the shaft of which passed through the side of the box; and the arrangements were such that the man could either drive the mill against external resistance, by continually stepping from one tread to the next above in the usual way, or could resist the motion of the mill when driven from without, by continually descending the treads, thus doing negative work. Two flexible tubes were connected, one with his nostrils, and the other with his mouth. He inhaled through the former, and exhaled through the latter, and the air exhaled was collected and analyzed. The heat given off from his body to the box was also measured with some degree of approximation. The carbon exhaled, and heat generated, were both tolerably constant in amount when the man was at rest. When he was driving the mill by ascending the treads, the heat given out was increased, but the carbon exhaled was increased in a much greater ratio. When he was doing negative work by descending the treads, the heat given out, though less in absolute amount, was greater in proportion to the carbon exhaled, than in either of the other cases.

360. Heat of Chemical Combination.—There is potential energy between the particles of two substances which would combine chemically if the opportunity were afforded. When *combination* actually takes place, this potential energy runs down and yields an equivalent of heat. We may suppose that the particles rush together in virtue of their mutual attraction, and thus acquire motions which constitute heat.

In every case of *decomposition*, an amount either of heat or some other form of energy must be consumed equivalent to the heat of combination.

360A. Vegetable Growth.—In the growth of plants, the forces of chemical affinity do negative work. Particles which were previously held together by these forces are separated, and potential energy is thus obtained. When wood is burned, this potential energy is converted into heat.

We are not, however, to suppose that plants, any more than animals, have the power of *creating* energy. The forces which are peculiar to living plants are merely *directive*. They direct the energy of the solar rays to spend itself in separating the carbon and oxygen which exist united in the carbonic acid of the air; the carbon being taken up by the plant, and the oxygen left.

Coal is the remnant of vegetation which once existed on the earth. Thus all the substances which we are in the habit of employing as fuel, are indebted to the sun for the energy which they give out as heat in their combustion.

361. Solar Heat.—The amount of heat radiated from the sun is great almost beyond belief. The best measures of it have been obtained by two instruments which are alike in principle—Sir John Herschel's *actinometer* and Pouillet's *pyrheliometer*. We shall describe the latter, which is represented in Fig. 314. At the upper end, next the sun, is a shallow cylinder composed of very thin copper or silver, filled with water in which the bulb of a thermometer is inserted, the stem being partially inclosed in the hollow tube which supports the cylinder. At the lower end of the tube is a disc equal and parallel to the base of the cylinder. This is intended to receive the shadow of the cylinder, and thus assist the operator in pointing the instrument directly towards the sun. The cylinder is blackened, in order that its absorbing power may be as great as possible.

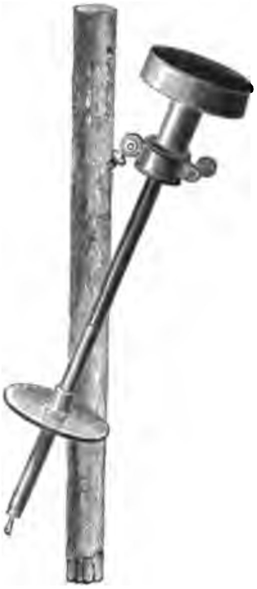

Fig. 314.—Pyrheliometer.

The instrument, initially at the temperature of the atmosphere, is first placed for five minutes in a position where it is exposed to the sky, but shaded from the sun, and the increase or diminution of its emperature is observed; suppose it to be fall of θ°. The screen which shaded it rom the sun is then withdrawn, and its ise of temperature is observed for five inutes with the sun shining upon it; call is rise T°. Finally, it is again screened om the sun, and its fall in five minutes is ted;—call this θ'°. From these observa- ns it is inferred, that the instrument, ile exposed to the sun, lost $\frac{\theta+\theta'}{2}$ to the and surrounding objects, and that the whole heat which it eived from the sun was $T+\frac{\theta+\theta'}{2}$, or rather was the product of s difference of temperature by the thermal capacity of the inder and its contents. This is the heat which actually reaches instrument from the sun, but a large additional amount has been rcepted by absorption in the atmosphere. The amount of this rption can be roughly determined by comparing observations n when the sun has different altitudes, and when the distance ersed in the air is accordingly different. Including the amount absorbed, Pouillet computes that *the heat sent yearly by the sun e earth would be sufficient to melt a layer of ice* 30 *metres thick, d over the surface of the earth;* and Sir John Herschel's estimate t very different.

e earth occupies only a very small extent in space as viewed the sun; and if we take into account the radiation in all direc- the whole amount of heat emitted by the sun will be found to out 2100 million times that received by the earth, or sufficient to melt a thickness of two-fifths of a mile of ice per hour over the whole surface of the sun.

361 A. Sources of Solar Heat.—The only causes that appear at all adequate to produce such an enormous effect, are the energy of the celestial motions, and the potential energy of solar gravitation. The motion of the earth in its orbit is at the rate of about 96,500 feet per second. The kinetic energy of a pound of matter moving with this velocity is equivalent to about 104,000 pound-degrees Centigrade, whereas a pound of carbon produces by its combustion only 8080. The inferior planets travel with greater velocity, the square of the velocity being inversely as the distance from the sun's centre; and the energy of motion is proportional to the square of velocity. It follows that a pound of matter revolving in an orbit just outside the sun would have kinetic energy about 220 times greater than if it travelled with the earth. If this motion were arrested by the body plunging into the sun, the heat generated would be about 2800 times greater than that given out by the combustion of a pound of charcoal. We know that small bodies are travelling about in the celestial spaces; for they often become visible to us as meteors, their incandescence being due to the heat generated by their friction against the earth's atmosphere; and there is reason to believe that bodies of this kind compose the immense circumsolar nebula called the zodiacal light, and also, possibly, the solar corona which becomes visible in total eclipses. It is probable that these small bodies, being retarded by the resistance of an ethereal medium, which is too rare to interfere sensibly with the motion of such large bodies as the planets, are gradually sucked into the sun, and thus furnish some contribution towards the maintenance of solar heat. But the perturbations of the inferior planets and comets furnish an approximate indication of the quantity of matter circulating within the orbit of Mercury, and this quantity is found to be such that the heat which it could produce would only be equivalent to a few centuries of solar radiation.

Helmholtz has suggested that the smallness of the sun's density—only $\frac{1}{4}$ of that of the earth—may be due to the expanded condition consequent on the possession of a very high temperature, and that this high temperature may be kept up by a gradual contraction. Contraction involves approach towards the sun's centre, and therefore the performance of work by solar gravitation. By assuming that the work thus done yields an equivalent of heat, he brings out the result that, if the sun were of uniform density throughout, the heat developed by a contraction amounting to only one ten-thousandth of the solar diameter, would be as much as is emitted by the sun in 2100 years.

361 B. Sources of Energy available to Man.—Man cannot produce energy; he can only apply to his purposes the stores of energy which he finds ready to his hand. With some unimportant exceptions, these can all be traced to three sources:—

I. The solar rays.

II. The energy of the earth's rotation.

III. The energy of the relative motions of the moon, earth, and sun, combined with the potential energy of their mutual gravitation.

The fires which drive our steam-engines owe their energy, as we have seen, to the solar rays. The animals which work for us derive their energy from the food which they eat, and thus, indirectly, from the solar rays. Our water-mills are driven by the descent of water, which has fallen as rain from the clouds, to which it was raised in the form of vapour by means of heat derived from the solar rays.

The wind which propels our sailing-vessels, and turns our windmills, is due to the joint action of heat derived from the sun, and the earth's rotation.

The tides, which are sometimes employed for driving mills, are due to sources II. and III. combined.

The work which man obtains, by his own appliances, from the winds and tides, is altogether insignificant when compared with the work done by these agents without his intervention, this work being chiefly spent in friction. It is certain that all the work which they do, involves the loss of so much energy from the original sources; a loss which is astronomically insignificant for such a period as a century, but may produce, and probably has produced, very sensible effects in long ages. In the case of tidal friction, great part of the loss must fall upon the energy of the earth's rotation; but the case is very different with winds. Neglecting the comparatively insignificant effect of aerial tides, due to the gravitation of the moon and sun, wind-friction cannot in the slightest degree affect the rate of the earth's rotation, for it is impossible for any action exerted between parts of a system to alter the angular momentum of the system (53 F.) The effect of easterly winds in checking the earth's rotation must therefore be exactly balanced by the effect of westerly winds in accelerating it. In applying this principle, it is to be remembered that the couple exerted by the wind is jointly proportional to the force of friction resolved in an easterly or westerly direction, and to the distance from the earth's axis.

361c. Dissipation of Energy.—From the principles laid down in the present chapter it appears that, although mechanical work can be entirely spent in producing its equivalent of heat, heat cannot be entirely spent in producing mechanical work. Along with the conversion of heat into mechanical effect, there is always the transference of another and usually much larger quantity of heat from a body at a higher to another at a lower temperature. In conduction and radiation heat passes by a more direct process from a warmer to a colder body, usually without yielding any work at all. In these cases, though there is no loss of energy, there is a running to waste as far as regards convertibility; for a body must be hotter than neighbouring bodies, in order that its heat may be available for yielding work. This process of running down to less available forms has been variously styled *diffusion*, *degradation*, and *dissipation* of energy, and it is not by any means confined to heat. We can assert of energy in general that it often runs down from a higher to a lower grade (that is to a form less available for yielding work), and that, if a quantity of energy is ever raised from a lower to a higher grade, it is only in virtue of the degradation of another quantity, in such sort that there is never a gain, and is generally a loss, of available energy.

This general tendency in nature was first pointed out by Sir W. Thomson. It obviously leads to the conclusion that the earth is gradually approaching a condition in which it will no longer be habitable by man as at present constituted.

CHAPTER XXXIII.

STEAM AND OTHER HEAT ENGINES.

362. **Heat-engines.**—The name of heat-engine or thermo-dynamic engine is given to all machines which yield work in virtue of heat which is supplied to them. Besides the steam-engine, it includes the air-engine and the gas-engine. We shall first describe one of the best forms of the air-engine.

363. **Stirling's Air-engine.**—Fig. 315 is a perspective view, and Fig. 316 a section of the engine invented by Dr. Robert Stirling. The particular form here represented is that which has been adopted in France by M. Laubereau. It consists of two cylinders of different diameters, which are in communication with each other. The larger cylinder is divided into two compartments by a kind of large piston made of plaster of Paris, which, however, does not touch the sides of the cylinder, and thus leaves an annular space for communication between the two compartments.

The bottom of the large cylinder, which is directly exposed to the action of the furnace, is slightly concave; the top is double, thus affording an intermediate space, through which cold water is kept circulating by means of a pump which is driven by the machine. From this arrangement it follows that, when the mass of plaster is at the bottom of the cylinder, it will intercept the heat of the fire, being a very bad conductor, and thus the air in the cylinder will be cooled by the water in the double top. On the other hand, when the piston is in contact with the refrigerator, the air will be exposed to the action of the fire, and its elastic force will, consequently, be increased.

The smaller cylinder is open above, and contains a piston which drives a crank on the axle of a heavy fly-wheel of cast-iron. The communication between the two cylinders is in the lower part of each.

Suppose now that the large piston is in contact with the refrigerator, while the small piston is in its lowest position. The air is thus exposed to the action of heat, expands, and raises the small piston. If we now suppose the large piston shifted to the bottom of the cylinder, the air will cool, and its tension will diminish, becoming equal to or even less than that of the atmosphere. The small piston will thus be carried to the bottom of the cylinder by the movement of the fly-wheel, and will again be pushed up by the expanding air, if we suppose the large piston to rise again to the top of its cylinder.

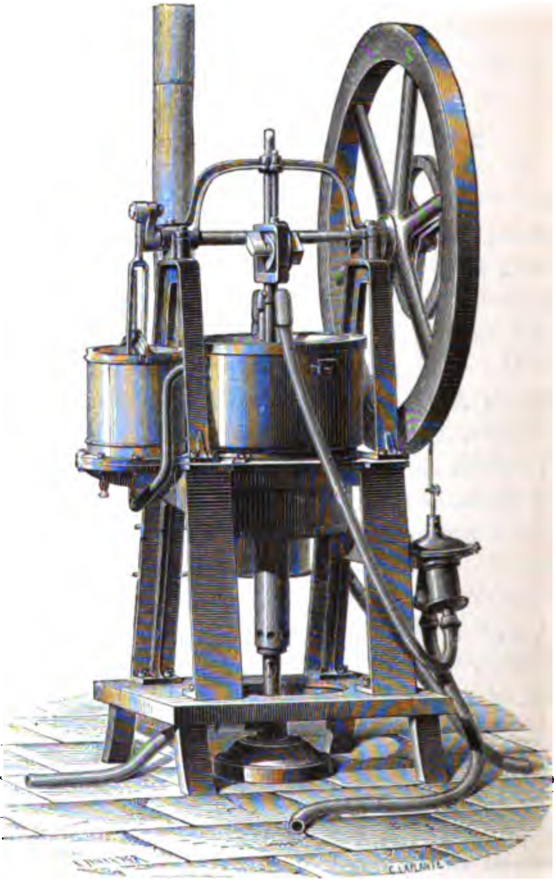

Fig. 315.—Stirling's Air-engine.

This motion of the large piston is effected, as shown in the figure, by means of an eccentric on the axle of the fly-wheel. The engine is of small size, and is intended for purposes requiring but little power. To obtain high efficiency, according to the principles of the preceding chapter, the difference of temperature between the two ends of the large cylinder should be very great. This amounts to saying that the lower end must be kept very hot, since it is practically impossible to keep the upper end much cooler than the surrounding atmosphere. The facility of maintaining a very high temperature constitutes at once the strength and the weakness of the air-engine. The bottom of the cylinder becomes rapidly oxidized, and needs frequent renewal. Partly for this reason, and partly on account of the small expansibility of air as compared with the expansion which takes place when water is converted into steam, air-engines are seldom employed for high powers.

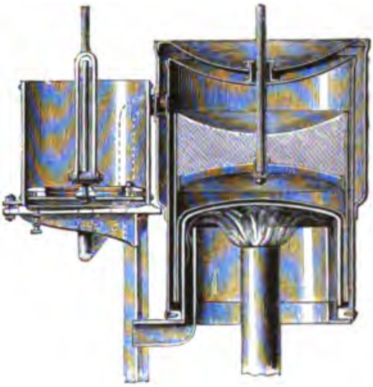
Fig. 316.—Section of Stirling's Air-engine.

364. The Steam-engine: its History.—As early as the seventeenth century, when Otto Guericke and Torricelli were investigating the pressure and the weight of air, attention had been given to the physical properties of steam, and the idea of employing it as a source of work had been entertained.

The first person who made steam drive a piston was Papin, a French philosopher, inventor of the digester and the safety-valve (born 1650; died 1710). About the year 1690 he constructed a working model, consisting of a cylinder open at the top, containing a piston and a little water below it. The water was converted into steam by the application of heat, and raised the piston. The machine being then allowed to cool, the steam lost its tension, and the pressure of the atmosphere forced the piston to descend. A backward and forward motion was thus obtained, which Papin proposed to convert into a rotatory motion by means of rack[1] and pinion work and ratchet-wheels.

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[1] A rack is a straight bar with teeth at one edge, which works with a toothed wheel called a pinion.

A description of Papin's machine is given in the *Acta Eruditorum* under date 1690.

The first steam-engine actually employed for doing useful work was invented by Savery about 1697, and was extensively used for draining mines. Steam from a separate boiler was admitted to press upon the surface of water in a vessel, and thus force it up through an ascending pipe; and on the condensation of the steam, water from a lower level was raised into the vessel by atmospheric pressure. The condensation was effected by applying cold water to the outside of the vessel.

Savery's engine, which was a steam-pump, and not an engine adapted for general purposes, was superseded by an engine jointly contrived by Newcomen, Savery, and Cawley, which combined the cylinder and piston with the separate boiler and with condensation by the injection of cold water into the cylinder. This engine is generally referred to as Newcomen's atmospheric engine,—so called because the descent of the piston was produced by atmospheric pressure, on the condensation of the steam beneath it.

James Watt (born 1736; died 1819), who effected the most important improvements in the steam-engine, had his attention called to the subject when engaged in repairing a model of Newcomen's engine, being at that time philosophical instrument maker to the University of Glasgow. His first improvement consisted in the introduction of a separate vessel for the condensation of the steam, so as to allow of keeping the cylinder always hot.

This first improvement, which immediately produced a great saving of fuel, was followed by another of scarcely less importance. This consisted in substituting the pressure of steam for the atmospheric pressure, which in Newcomen's engine caused the downward stroke of the piston. The upward stroke was effected by means of a counterpoise, the steam being admitted to press equally both above and below the piston. These two improvements, and a general perfecting of the details of the machinery, caused Watt's engine to supersede that of Newcomen. The engine thus contrived by Watt is called *single-acting*, because only the down-stroke of the piston is produced by the pressure of steam. This arrangement is particularly adapted for pumping, and is commonly employed at the present day for draining mines. It was not long before Watt perfected his engine by employing steam to produce both the up-stroke and the down-stroke. This is the characteristic of the *double-acting* engine, which was carried to a high degree of perfection by the inventor himself, and which is now most frequently adopted as the source of moving power. We may add that the improvements introduced in the steam-engine since Watt's time have been matters of detail rather than of principle. We proceed to describe Watt's engine.

365. Principle of the Double-acting Engine.—M (Fig. 317) is a boiler communicating with the top and bottom of the cylinder by means of two stop-cocks *a* and *b*. Connection can be established between the cylinder and the condenser I by two other cocks *c* and *d*. If now the cocks *a* and *c* are opened, and *b* and *d* shut, the steam from the boiler will arrive above the piston P, while that which was previously introduced below will, by communication with the condenser, be more or less condensed, and will thus lose its elastic force; the piston will accordingly descend to the bottom of the cylinder. The two cocks *b* and *d* are then opened, while the other two are shut; the steam above the piston is thus condensed, while that below the piston forces it up, thus causing the upward stroke, after which the piston may again be made to descend, and so on.

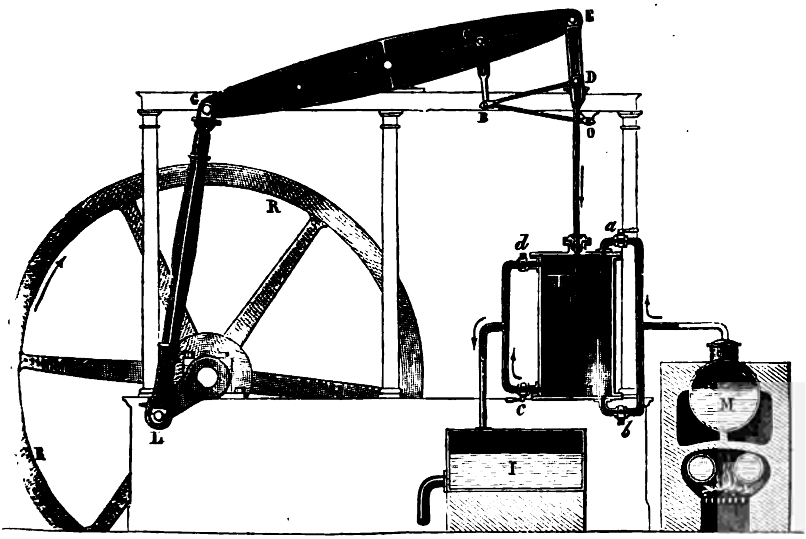

Fig. 317.—Principle of the Double-acting Engine.

We thus see that, by suitable manipulation of the stop-cocks *a*, *b*, *c*, *d*, we can give the piston a backward and forward motion, which may easily be transformed into one of rotation. For this purpose the piston-rod is connected with one end of the beam EG by the jointed parallelogram CBDE, while the other end of the beam is jointed to the connecting-rod GL, which is itself jointed to the crank of the fly-wheel RR.

It will be seen that, if the piston descends, the action of the crank will drive the wheel in the direction shown by the arrow. When the piston has completed its downward stroke, the connecting-rod and the crank will be in a straight line, and the action of the former upon the latter will have no tendency to turn the wheel either way. This position is called a *dead point*. But the momentum acquired by the fly-wheel will carry it past this position, and, the piston having then commenced its upward stroke, the rotatory movement will continue in the same direction until the rod and crank are at the other *dead point*, which occurs at 180° from the first, and is passed over in the same way. We thus see that, by means of the alternate motion of the piston, we can obtain a rotatory motion, which may be imparted to a horizontal shaft, and made to drive machinery of any kind.

The jointed parallelogram which connects the piston-rod with the beam is one of the most ingenious of the improvements introduced by Watt. Its use is evident. When the engine is at work, the end E of the beam describes an arc of a circle, while the end D of the piston-rod moves in a straight line; it is therefore impossible to joint them directly together. They are therefore connected through the medium of the short rod ED, which, with the two other rods, BD and BC, together with the part CE of the beam, form a *jointed parallelogram*, the angles of which can vary according to the position of the beam. The angle B is connected by a joint with the end of the *radius-rod* BO, movable about the fixed point O. The effect of this arrangement is as follows:—If we take the beam in a horizontal position, and suppose the end E to rise, the point D will be drawn towards the left by the action of the beam, and towards the right by the action of the radius-rod B O, which, from its checking the movement of the piston-rod to either side, is often called the *bridle-rod*. It will easily be understood that these two contrary actions may be made to balance each other almost exactly, and that, accordingly, the path of D will deviate very little from a straight line.

366. Arrangement for Admitting the Steam.—We have simplified the description of the steam-engine by supposing that the cocks *a* and *c*, *b* and *d* were alternately opened by hand. This, however, is not the actual arrangement, the opening and closing of the passages being really effected by automatic movements. The most usual arrangement for this purpose is the *slide-valve,* which we now proceed to describe.

The steam, instead of entering the cylinder directly, passes into it from a box in front of it (Fig. 318), which is called the *valve-chest.* In the opposite face of the box from that at which the steam enters, are three holes or *ports* near each other. The uppermost of these communicates with the upper part of the cylinder; the lowest one with the lower part; and the middle hole with *o,* which itself is in communication with the condenser. Upon this face of the box there slides a rectangular piece of metal, hollowed out on the side next these openings, and large enough to reach over two of the ports at once.

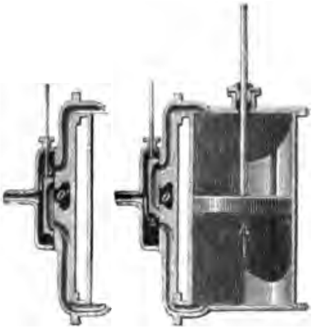

Fig. 318.—Slide-valve.

In the right-hand figure this *slide-valve* is supposed to be at the top of its upward stroke, thus admitting the steam below the piston, and pushing it in the direction indicated by the arrow; while the steam above the piston is put in communication with the condenser. In the left-hand figure, the opposite position is shown; the steam is admitted above the piston, while the lower part of the cylinder is in communication with the condenser.

367. Movement of the Sliding-valve.—It is desirable that the movement of the slide-valve should be automatic; for this purpose the following arrangement is employed. A circular piece of metal *e,* called the *eccentric,* is traversed by the shaft of the engine in a point which is not the centre of the piece, and is rigidly attached to the shaft. This eccentric is surrounded with a ring of metal, which can

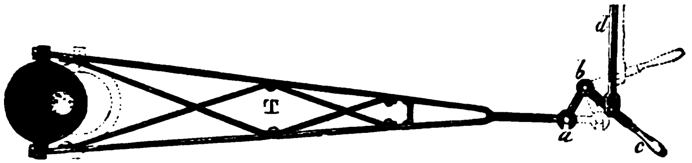

Fig. 319.—Eccentric for moving Slide-valve.

turn freely about it, and which forms part of the triangular frame T. The vertex of the triangle is fastened to a bent lever abc, which thus receives an oscillatory movement about the point b. This movement raises and lowers alternately the rod d, which is attached to the sliding-valve, and thus gives that valve its motion.

368. Air-pump of the Condenser.—The condenser is a cylinder into which a jet of cold water constantly plays, the quantity of which can be increased or diminished at pleasure. The steam, in its condensation, heats the cold water, and at the same time the air contained in the water is disengaged, owing to the small pressure in the condenser. It is thus found necessary to pump out both the air and the water; and the pump which does this is driven by the beam of the engine.

The warm water thus drawn out is conducted to a reservoir, whence a portion of it is raised by a second pump, and forced into the boiler. Finally, a third pump, usually of greater power than the other two, raises water from some external source, and discharges it into a bath called the *cold well*, which feeds the condenser. These last two pumps are also connected with the beam of the engine.

369. Governor-balls.—The apparatus called the governor-balls or the *centrifugal governor* was designed by Watt for the purpose of regulating the admission of steam in such a manner as to render the rate of the engine nearly constant through all variations of the resistance to be overcome.

It consists of a vertical axis y (Fig. 320), which receives a rotatory movement from the machine. To the top of this are jointed two rods $\alpha\beta$, $\alpha'\beta'$, carrying the heavy balls Z and Z'. Two other rods, $\beta\epsilon$, $\beta'\epsilon'$, are jointed to the first, so as to form with them a lozenge, the lower end of which is fastened to a sliding-ring m, which surrounds the axis of rotation. When the engine is at rest, the sides of the lozenge are as near as possible to the vertical, but when it begins to work, the balls are carried out from the vertical by centrifugal force, and the distance increases with the velocity of rotation. The sliding-ring is thus raised, and, by means of a system of levers, acts upon a throttle-valve (a disc turning about a diameter) in the steam-pipe, so as to diminish the supply of steam to the cylinder when the velocity increases.

370. Use of the Fly-wheel.—From the mode in which the motion of the piston is transmitted to the shaft, it is obvious that the driving couple undergoes great variations of magnitude. It is greatest (considered statically) when the crank is nearly at right angles to the connecting-rod, and diminishes in approaching the dead points, where it vanishes altogether. These variations in the driving couple tend to roduce corresponding variations in the velocity of rotation.

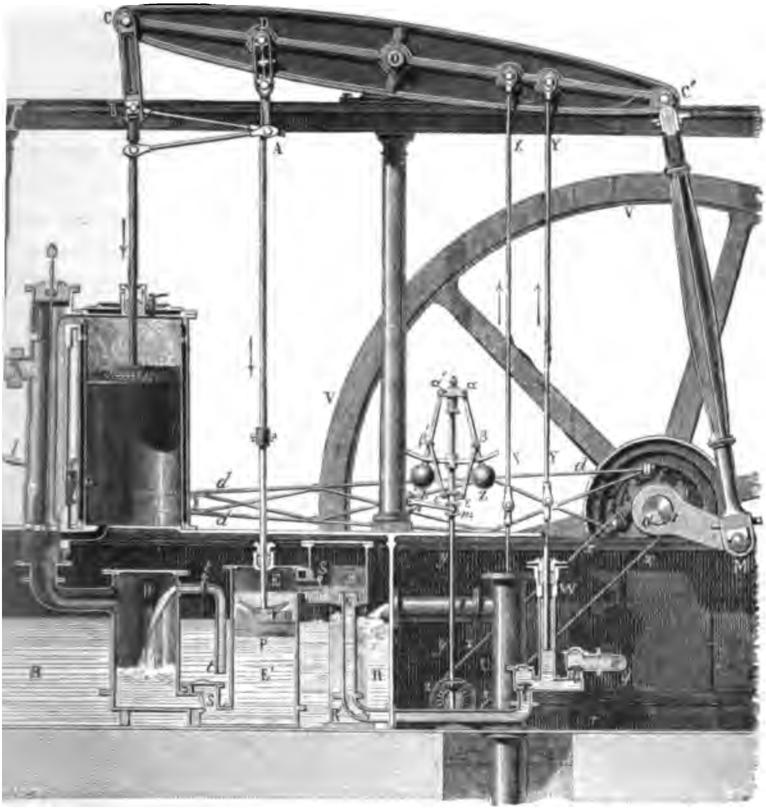

Fig. 320.—Watt's Engine.

CD, jointed parallelogram. CC′, beam, turning about O. C′M, connecting-rod. O′M, , attached to axis of fly-wheel. VV, fly-wheel. *c*, eccentric, which, by means of the frame oves the lever *el*, which moves the slide-valve. *xx*, an endless cord, passing over a pulley axis O′, and over a second pulley *s*, whose motion is transmitted by bevel-wheels to the of the centrifugal governor. *am*, a rod moved by the ring *m*, and transmitting its move- by means of levers to the throttle-valve. H, condenser. RR, cold well. *t*, tube through the water of the cold well under atmospheric pressure flows into the condenser. EE′, er of exhaust-pump. P, piston of ditto. S, valve. X, rod of the pump U which supplies d well. R′, bath which receives the water drawn from the condenser. Y, rod of the feed- W, which draws water from R′ and forces it into the boiler.

r causes also contribute to produce the same result, especially iations in the amount of resistance to be overcome. If, for instance, the engine drives a wheel with cams which raise a tilt-hammer, when the hammer falls the principal resistance is removed, and the engine immediately begins to quicken its speed. When the hammer is caught again by the next cam, the velocity is suddenly diminished, and so on. Similar results, though not of so marked a character, are produced in engines of all kinds. These sudden changes of velocity, if not mitigated, would be very injurious to the mechanism of the engine, by the shocks which they would produce.

The use of the fly-wheel is to remedy this evil. It is a wheel of considerable size and weight (the weight being collected as much as possible at the rim), and receives a rotatory movement from the engine. If the driving power increases, or the resistance diminishes, all the moving parts of the engine acquire increased velocities; but the greatest part of the additional energy of motion thus generated goes to the fly-wheel, which has such a large moment of inertia that a very slight change in its angular velocity represents a large amount of energy (§ 53 G). The energy thus absorbed by the fly-wheel is restored by it when the velocity is checked; and the rotation of the shaft is thus rendered nearly uniform in all parts of the stroke. The size of the fly-wheel is usually made such that the difference between the greatest and least velocities shall not exceed about $\frac{1}{30}$ of the mean velocity.

371. General Description of Watt's Engine.—The explanations above given will enable the reader to understand the general arrangement of Watt's engine as represented in Fig. 320. It is merely necessary to remark that the slide-valve is slightly different from that described above, but the modification is not of any importance.

372. Working Expansively.—Among the modifications introduced since Watt's time, we must notice in the first place what is called *expansive working.*[1] When the piston has performed a part of its stroke, the steam is shut off (or in technical phrase *cut off*) from the cylinder, and the expansive force of the steam already admitted is left to urge the piston through the remainder of its course. By this means a great economy of steam may be effected. The part of the stroke at which the cut-off occurs may be determined at pleasure. It is sometimes at half-stroke, sometimes at quarter-stroke, sometimes at one-fifth of stroke. In the latter case, the piston describes the remaining four-fifths of its stroke under the gradually diminishing pressure of the steam which entered the cylinder during the first fifth; and the work done during these four-fifths is so much work gained by working expansively.

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[1] This was one of Watt's inventions, but it has been much more fully developed in recent times.

373. Modification of Slide-valve for Expansive Working.—The cutting-off of the steam before the end of the stroke is usually effected by the contrivance represented in Fig. 321: *a d*, *a′d′*, are two plates forming part of the slide-valve and of much greater width than the openings L, L′. The excess of width is called *lap*. By this arrangement one of the apertures is kept closed for some time, so that the steam is shut off, and acts only by its expansion. The expansion increases with the lap, but not in simple proportion, as equal movements of the slide-valve do not correspond to equal movements of the piston. The amount of expansion can also be regulated by means of the *link-motion*, which will be described in § 390.

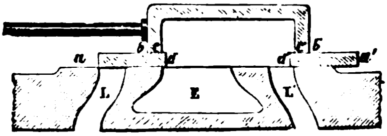

Fig. 321.—Slide-valve for Expansive Working.

374. Compound Engines.—This is the name given to engines in which the steam performs the greater part of its expansion in a second cylinder, of much larger cross-section than the first, the increased area of pressure on the piston serving to compensate for the diminished intensity of pressure which exists in the latter part of the stroke, and thus to produce greater steadiness of driving-power. Various methods have been adopted for connecting the two cylinders. One arrangement for this purpose is represented in Fig. 322. *p* is the smaller piston, working in the smaller cylinder A B C D. P is the larger piston, working in the larger cylinder A′B′C′D′. In the up-stroke, the passage D A′ is closed, and C B′ is open. The small piston is forced up by the high-pressure steam beneath, while the steam above it, instead of escaping to a condenser, expands into the large cylinder, and there raises the piston P, the upper part of the large cylinder being connected with the condenser. In the down-stroke, the passage C B′ is closed, and

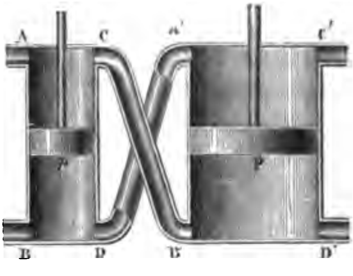

Fig. 322.—Compound-cylinder Arrangement.

D A′ is open. The two piston-rods are connected with the same end of the beam, and rise and fall together. The distribution of the steam is effected by means of two slide-valves, each governed by an eccentric.

Compound engines have been adopted for some lines of ocean-steamers, where it is of primary importance to obtain as much work as possible from a limited quantity of fuel. Engineers are, however, divided in opinion as to whether any advantage attends their use.

374A. Surface Condensation.—In several modern engines, the condenser consists of a number of vertical tubes of about half an inch diameter, connected at their ends, and kept cool by the external contact of cold water. The steam, on escaping from the cylinder, enters these tubes at their upper ends, and becomes condensed in its passage through them, thus yielding distilled water, which is pumped back to feed the boiler. The same water can thus be put through the engine many times in succession, and the waste which occurs is usually repaired by adding from time to time a little distilled water prepared by a separate apparatus.

375. Classification of Steam-engines.—The distinctions which exist between different kinds of stationary engines relate either to the pressure of the steam, or to its mode of action, or to the arrangement of the mechanism, especially as regards the mode in which the movement of the piston is transmitted to the rest of the machinery.

On the first of these heads, it must be remarked that the terms *low-pressure* and *high-pressure* are no longer equivalent to *condensing* and *non-condensing* as they once were, and merely express differences of degree.

When the pressures employed are very low there is little risk of explosion and little wear and tear; but the engine must be very large in proportion to its power, and expansive working cannot be employed. Low-pressure engines are always *condensing,* though the converse is not true.

With regard to the mode of action of the steam, engines may be classed as condensing or non-condensing, as expansive or non-expansive. Condensation increases the quantity of work obtainable from a given consumption of fuel, and is almost always employed for stationary engines where the supply of water is abundant, and also for marine engines, but it is dispensed with in locomotives and agricultural engines.

Expansive working is also conducive to efficiency, as is obvious from § 372. Assuming the temperature of the steam to remain constant during the expansion, the following table, calculated by an application of Boyle's law, exhibits the relative amounts of work

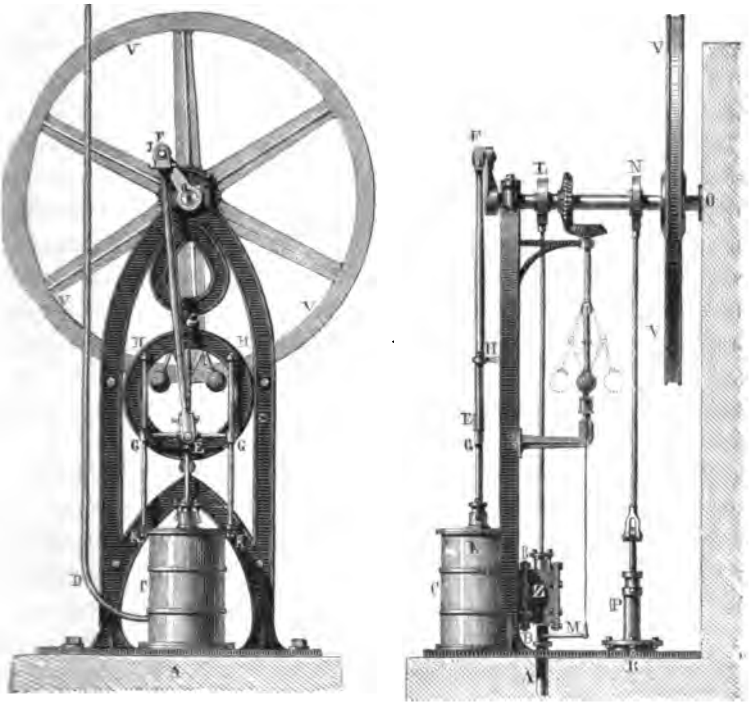

Fig. 323.—High-pressure Engine with Vertical Cylinder, working Expansively and without Condensation.

, steam-pipe through which the steam arrives from the boiler. Z, valve-chest. B, slide-valve. ylinder. GG, guides fixed to the cylinder and to the frame of the engine at K and H. E F, ecting-rod. J, crank. V V, fly-wheel. L, eccentric governing the slide-valve. N, eccentric e exhaust-pump P. D, outlet pipe for the steam. M, lever of throttle-valve, regulated by ifugal governor.

ed from the same weight of steam with different ranges of sion:—

Fraction of the completed before the tting-off of steam.	Work done.	Fraction of the stroke completed before the shutting-off of steam.	Work done.
1·0	1·000	·5	1·693
·9	1·105	·4	1·916
·8	1·223	·3	2·201
·7	1·357	·2	2·609
·6	1·509	·1	3·302

tion for another half-revolution, when the two sectors will have resumed the position represented in the figure.

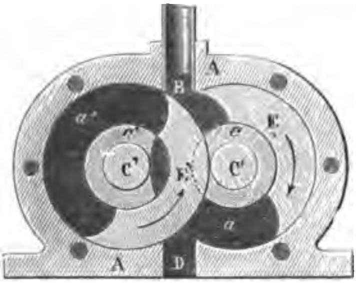

Fig. 326.—Section of Behren's Engine.

380. Boilers.—There are many forms of boiler in use. That which is represented in Fig. 327 is the favourite form in France, and is also extensively used in this country, where it is called the *French boiler*, or the *cylindrical boiler with heaters*. The main boiler-shell A is cylindrical with hemispherical ends. BB are two cylindrical tubes called *heaters*, of the same length as the main shell, and connected with it by vertical tubes *d, d*, of which there are usually three to each heater. A horizontal brick partition, a little higher than the centres of the heaters, extends along their whole length; and a vertical partition runs along the top of each heater, except where interrupted by the vertical tubes. The flame from the furnace is thus compelled to travel in the first instance backwards, beneath the heaters; then forwards, through the intermediate space between the heaters the vertical tubes and the main shell; and lastly, backwards, through the side passages CC, which lead to the chimney. By thus compelling the flame to travel for a long distance in contact with the boiler, the quantity of heat communicated to the water is increased.

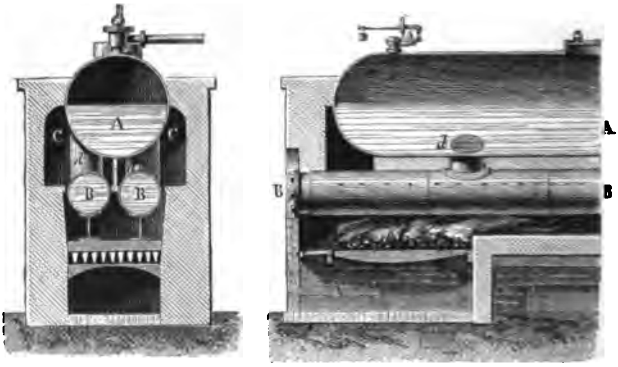

Fig. 327.—Boiler with Heaters.

The level of the water is shown at A in the left-hand figure. The relative spaces allotted to the steam and the water are not always the same; but must always be so regulated that the steam shall arrive in the cylinder as *dry* as possible, that is to say, that it shall not carry with it drops of water. Before being used, boilers should always be tested by subjecting them to much greater pressures than they will have to bear in actual use. Hydraulic pressure is commonly employed for this purpose, as it obviates the risk of explosion in case of the boiler giving way under the test.

381. Boilers with the Fire Inside.—When it is required to lessen the weight of the boiler, without much diminishing the surface exposed to heat, as in the case of marine engines, the method adopted is to place the furnace inside the boiler, so that it shall be completely surrounded with water except in front. The flame passes from the furnace, which is in the front of the boiler, into one or two large tubes, leading to a cavity near the back, whence it returns through a number of smaller tubes traversing the boiler, and finally escapes by the chimney.

382. Bursting of Boilers: Safety-valves.—Notwithstanding the tests to which boilers are subjected before being used, it too often happens that, owing either to excessive pressure or to weakening of the boiler, very disastrous explosions occur.

Excess of pressure is guarded against by gauges, which show what the pressure is at any moment, and by safety-valves, which allow steam to escape whenever the pressure exceeds a certain limit.

Various kinds of manometer or *pressure-gauge* have been described in Chap. xiv. That which is most commonly employed in connection with steam-boilers is Bourdon's (§ 126).

A thermometer, specially protected against the pressure and contact of the steam, is also sometimes employed, under the name of *thermo-manometer*, on the principle that the pressure of saturated steam depends only on its temperature.

The *safety-valve*, represented in the upper part of Fig. 327, consists of a piece of metal, having the form either of a truncated cone or of a flat plate, fitting very truly into or over an opening in the boiler. The valve is pressed down by a weighted lever; the weight and the length of the lever being calculated, so that the force with which the valve is held down shall be exactly equal to the force with which the steam would tend to raise it when at the limiting pressure. In movable engines, the weighted lever is replaced by a spring, the tension of which can be regulated by means of a screw.

Safety-valves afford ample protection against the danger arising from gradual increase of pressure; but they are liable to fail in cases where there is a sudden generation of a large quantity of steam. This explosive generation of steam may occur from various causes.

If, for instance, the water in the boiler is allowed to fall too low, the sides of the boiler may be heated to so high a temperature that, when fresh water is admitted, it will be immediately converted into steam on coming in contact with the metal.

Hence it is of great importance to provide that the water in the boiler shall not fall below a certain level, depending on the shape of the boiler and furnace.

The following are the means employed for securing this end:—

1. Two cocks are placed, one a little below the level at which the water should stand, and the other a little above it; these are opened from time to time, when water should issue from the first, and steam from the second.

2. The *water-gauge* is a strong vertical glass tube, having its ends fitted into two short tubes of metal, proceeding one from the steam-space and the other from the water-space. The level of the water is therefore the same in the gauge as in the boiler, and is constantly visible to the attendant. The metal tubes are furnished with cocks, which can be closed if the glass tube is accidentally broken.

384. Causes of Explosion.—Another cause of the explosive generation of steam is the incrustation of the boiler with a hard deposit, due to the impurities of the water employed. This crust is a bad conductor, and allows the portion of the boiler covered with it to become overheated; when, if water should find its way past the crust, and come in contact with the hot metal, there is great danger of explosion.

The best preventive of incrustation is the employment of distilled water in connection with surface condensation (§ 374 A). In default of this, portions of the water in the boiler must be blown off from time to time so as to prevent it from becoming too highly concentrated. This is especially necessary when the boiler is fed with salt water.

Among the causes of the bursting of boilers, we may also notice undue smallness of the vertical tubes in boilers with heaters (§ 380). When this fault exists, the steam which is generated is not immediately replaced by water, and overheating is liable to occur.

Another cause of explosions is probably to be found in a property of water which has only recently been recognized. It has been shown that, when water is deprived of air, it does not begin boiling till it has acquired an abnormally high temperature, and then bursts into steam with explosive violence (§ 263, Donny's experiment). This danger is to be apprehended when a boiler, which has been allowed to cool after being for some time in use, is again brought into action without the addition of a fresh supply of water.

But it appears that the most frequent cause of boiler explosions is the gradual eating away of some portion of the boiler by rust, so as to render it at last too weak to withstand the pressure of the steam within it. The only general remedy for this danger is periodical and enforced inspection.

385. Feeding of the Boiler: Giffard's Injector.—The feeding of the boiler is usually effected by means of a pump driven by the engine itself. Of late years this plan has been largely superseded by Giffard's invention of an apparatus by means of which the boiler is supplied with water by the direct action of its own steam.

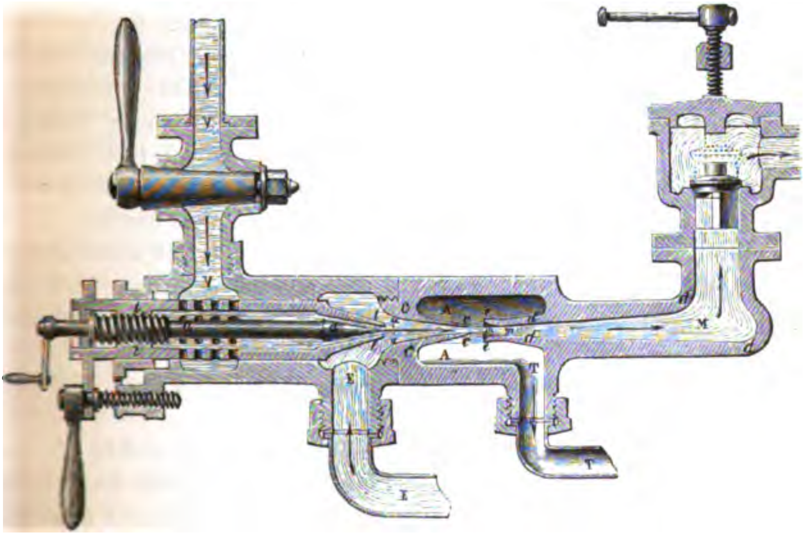

Fig. 328.—Giffard's Injector.

This very curious apparatus contains a conical tube *tt*, by which the steam issues when the injector is working; the steam from the boiler comes through the tube V V, and enters the tube *tt* through small holes in its circumference. On issuing from the cone *tt*, the steam enters another cone *c c*, where it meets the water which is to feed the boiler, and which comes through the tube EE. The contact of the water and the steam produces two results: (1) the steam, which possesses a great velocity due to the pressure of the boiler, communicates part of its velocity to the water; (2) at the same time the steam is condensed by the low temperature of the water, so that at the extremity of the cone as far as *e e* the entire space is occupied by water only, with the exception of a few bubbles of steam which remain in the centre of the liquid vein.

The liquid, on issuing from the cone *c c*, traverses an open space for a little distance before entering the divergent opposite cone *d d*, through which it is conducted to the boiler by the pipe M. The water will not enter the boiler unless it possess a sufficient velocity to produce in the divergent cone a greater pressure than that which exists in the boiler; when this is the case, the excess of pressure opens a valve, and water enters the boiler from the injector.

We may complete this brief description by pointing out one or two arrangements by which the action of the apparatus is regulated. It is useful to be able to vary the volume of steam issuing through the cone *t t*, as required by the pressure in the boiler; this is easily effected by means of the pointed rod *a a*, which is called the *needle*, and is screwed forwards or backwards by turning a handle. It is also necessary to be able to regulate the volume of water which enters the cone *c c* from the supply-pipe E; this is done by means of a lever, which is not shown in the figure, and which moves the tube and cone *t t* forwards or backwards.

The tube E dips into a bath containing the feed-water; and AT is the overflow pipe.

It appears at first sight paradoxical that steam should be able, as in Giffard's injector, to overcome its own pressure, and force water into the boiler against itself; but it must be remembered that the water which is forced in is less bulky than the steam which issues, so that the exchange, though it produces an increase of mass in the contents of the boiler, involves a diminution of pressure, as well as a fall of temperature.

386. Locomotive: History.—The following sketch of the history of the locomotive is given by Professor Rankine.[1] "The application of the steam-engine to locomotion on land was, according to Watt, suggested by Robison in 1759. In 1784 Watt patented a locomotive-engine, which, however, he never executed. About the same time Murdoch, assistant to Watt, made a very efficient working model of a locomotive-engine. In 1802 Trevithick and Vivian patented a locomotive-engine, which was constructed and set to work in 1804 or 1805. It travelled at about 5 miles an hour, with a net load of ten tons. The use of fixed steam-engines to drag trains on railways by ropes, was introduced by Cook in 1808.

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[1] *Manual of the Steam-engine*, pp. xxv–xxvii, edition 1866.

"After various inventors had long exerted their ingenuity in vain to give the locomotive-engine a firm hold of the track by means of rackwork-rails and toothed driving-wheels, legs and feet, and other contrivances, Blackett and Hedley, in 1813, made the important discovery that no such aids are required, the adhesion between smooth wheels and smooth rails being sufficient. To adapt the locomotive-engine to the great and widely-varied speeds at which it now has to travel, and the varied loads which it now has to draw, two things are essential—that the rate of combustion of the fuel, the original source of the power of the engine, shall adjust itself to the work which the engine has to perform, and shall, when required, be capable of being increased to many times the rate at which fuel is burned in the furnace of a stationary engine of the same size; and that the surface through which heat is communicated from the burning fuel to the water shall be very large compared with the bulk of the boiler. The first of these objects is attained by the *blast-pipe*, invented and used by George Stephenson before 1825; the second by the tubular boiler, invented about 1829, simultaneously by Séguin in France and Booth in England, and by the latter suggested to Stephenson. On the 6th October, 1829, occurred that famous trial of locomotive-engines, when the prize offered by the directors of the Liverpool and Manchester Railway was gained by Stephenson's engine the 'Rocket,' the parent of the swift and powerful locomotives of the present day, in which the blast-pipe and tubular boiler are combined. Since that time the locomotive engine has been varied and improved in various details, and by various engineers. Its weight now ranges from five tons to fifty tons; its load from fifty to five hundred tons; its speed from ten miles to sixty miles an hour."

389. Description of a Locomotive.—A section of a locomotive is represented in Fig. 329. The boiler is cylindrical. Its forward end abuts on a space beneath the chimney, called the *smoke-box*. At its other end is a larger opening, surrounded above and on the two sides by the boiler, and called the *fire-box.* The fuel is heaped up on the bars which form the bottom of the fire-box, and the cinders fall on the line. The fire-box and smoke-box are connected by brass tubes, firmly rivetted to the ends of the boiler; and the products of combustion escape by traversing these from end to end. The tubes are very numerous, usually from 150 to 180, thus affording a very large heating surface. The water in the boiler stands high enough to cover all the tubes, as well as the top of the fire-box. Its level is indicated in the same way as in stationary engines; and water is pumped in from the tender as required; its amount being regulated by means of a stop-cock in the pipe *e'*.

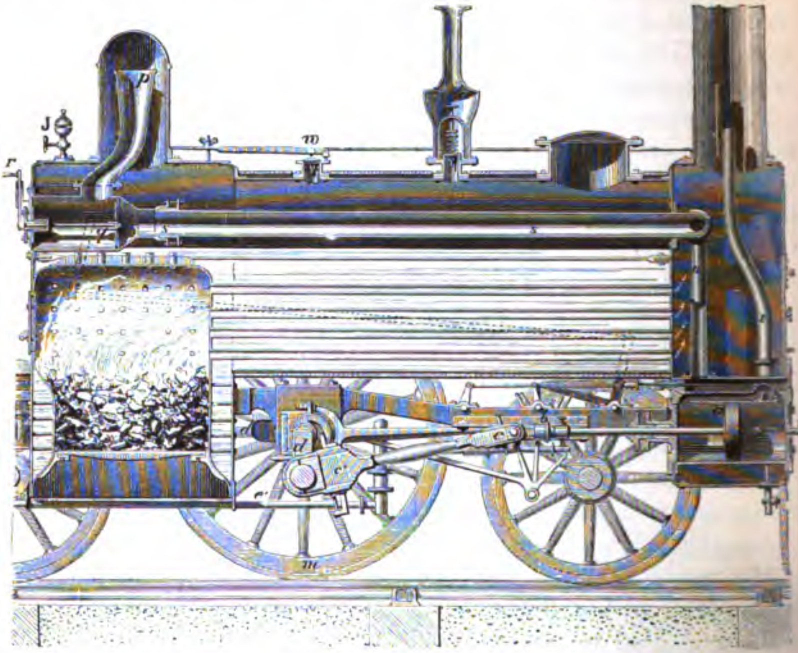

Fig. 329.—Section of Locomotive.

The steam escapes from the boiler by ascending into a dome, which forms its highest part, and thence descending the tube *p*, this arrangement being adopted in order to free the steam from drops of water. It then passes through a *regulator q*, which can be opened to a greater or less extent, into the pipe *s*, which leads to the valve-chests and traverses the whole length of the boiler. There are two cylinders, one on each side of the engine, each having a valve-chest and slide-valve, by means of which steam is admitted alternately before and behind the pistons. The steam escapes from the cylinder, through the blast-pipe *v*, up the chimney, and thus increases the draught of the fire. *a* is one of the pistons, *b* the piston-rod, *cc′* the connecting-rod, which is jointed to the crank *d* on the axle of the driving-wheel *m*. The cranks of the two driving-wheels, one on each side of the engine, are set at right angles to each other, so that, when one is at a dead point, the other is in the most advantageous position. *w* is a spring safety-valve, and J the steam-whistle.

390. Apparatus for Reversing: Link-motion.—The method usually employed for reversing engines is known as Stephenson's link-motion, having been first employed in locomotives constructed by Robert Stephenson, son of the maker of the "Rocket." The merit of the invention belongs to one or both of two workmen in his employ—Williams, a draughtsman, who first designed it, and Howe, a pattern-maker, who, being employed by Williams to construct a model of his invention, introduced some important improvements.

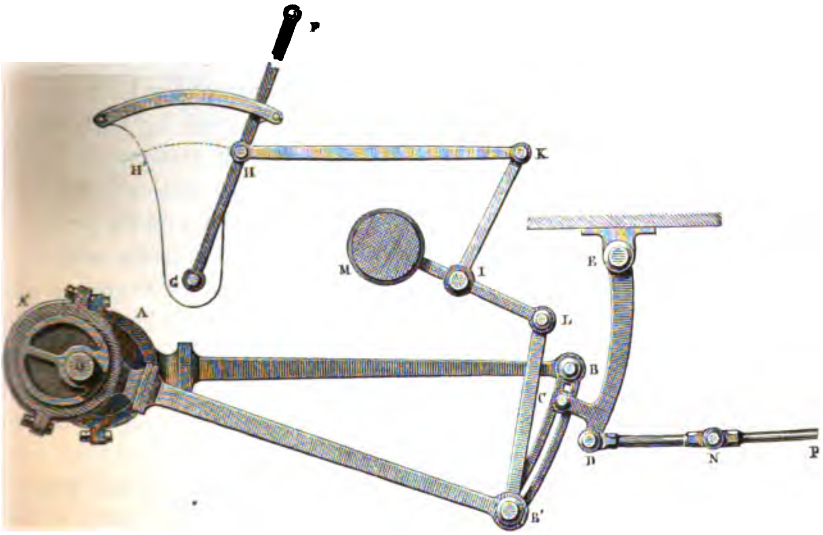

Fig. 330.—Link-motion.

The link-motion, which is represented in Fig. 330, serves two purposes; first, to make the engine travel forwards or backwards at pleasure; and, secondly, to regulate the amount of expansion which shall take place in the cylinder. Two oppositely placed eccentrics, A and A′, have their connecting-rods jointed to the two extremities of the *link* B B′, which is a curved bar, having a slit, of uniform width, extending along nearly its whole length. In this slit travels a stud or button C, forming part of a lever, which turns about a fixed point E. The end D of the lever D E is jointed to the connecting-rod D N, which moves the rod P of the slide-valve. The link itself is connected with an arrangement of rods L I K H,[1] which enables the engine-driver to raise or lower it at pleasure by means of the handle G H F. When the link is lowered to the fullest extent, the end B of the connecting-rod, driven by the eccentric A, is very near the runner C which governs the movement of the slide-valve; this valve, accordingly, which can only move in a straight line, obeys the eccentric A almost exclusively. When the link is raised as much as possible, the slide-valve obeys the other eccentric A′, and this change reverses the engine. When the link is exactly midway between the two extreme positions, the slide-valve is influenced by both eccentrics equally, and consequently remains nearly stationary in its middle position, so that no steam is admitted to the cylinder, and the engine stops. By keeping the link near the middle position, steam is admitted during only a small part of the stroke, and consequently undergoes large expansion. By moving it nearer to one of its extreme positions, the travel of the slide-valve is increased, the ports are opened wider and kept open longer, and the engine will accordingly be driven faster, but with less expansion of the steam. As a means of regulating expansion, the link-motion is far from perfect, but its general advantages are such that it has come into very extensive use, not only for locomotives but for all engines which need reversal.

393. Gas-engines.—This name includes engines in which work is obtained by the expansion of a mixture of coal-gas and air, on ignition or explosion. In Lenoir's engine a piston is driven alternately in opposite directions by successive ignitions of such a mixture on opposite sides of it, the proportions of gas and air being such as not to yield a true explosion.

In the engine of Otto and Langen (Fig. 331), a true explosive mixture is introduced beneath the piston, and is exploded by means

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[1] I is a fixed centre of motion, and the rods K I, M L are rigidly connected at right angles to each other. M is a heavy piece, serving to counterpoise the link and eccentric rods.

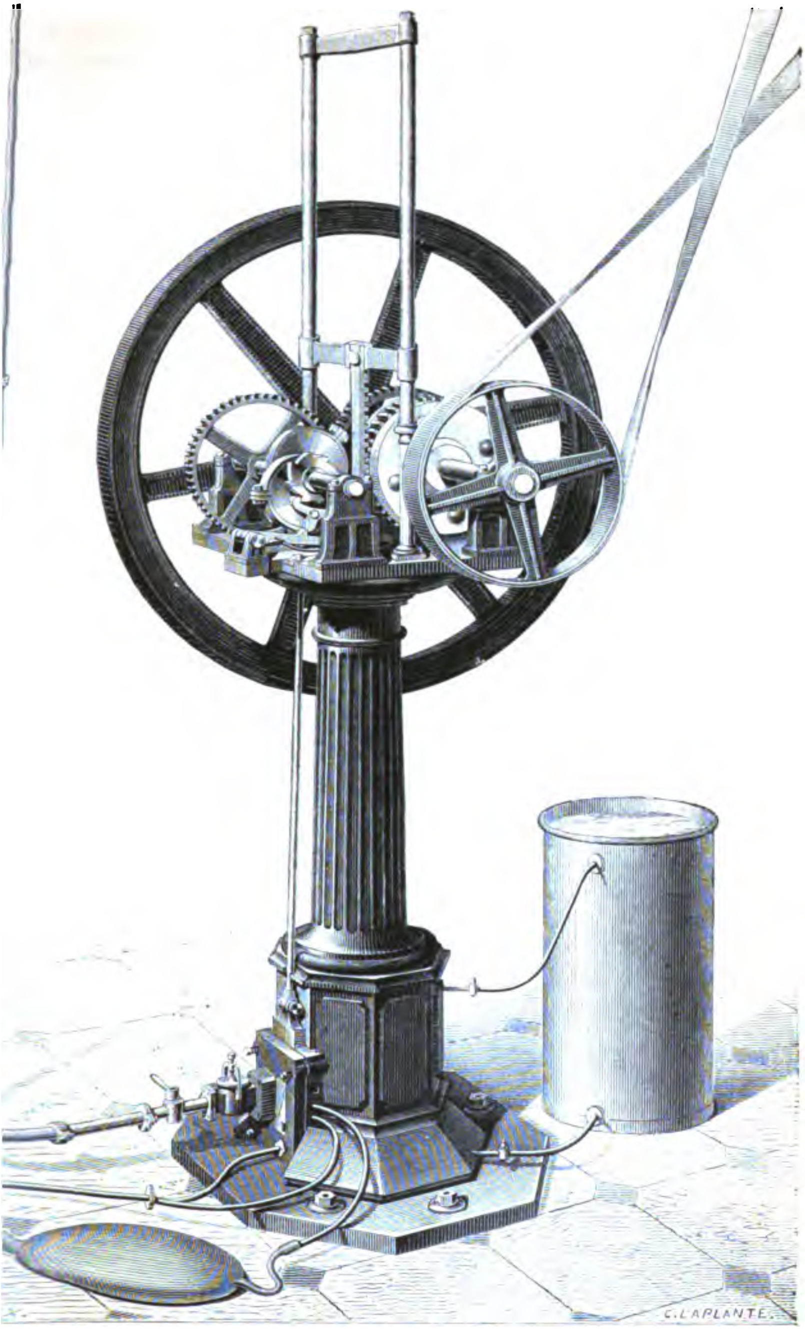

Fig. 331.—Gas-engine of Otto and Langen.

of a lighted jet, which is brought into contact with the mixture by means of a hole in a movable plate of metal, driven by an eccentric. The upward movement of the piston thus produced is too violent to admit of being directly communicated to machinery. The piston-rod is a rack, working with a pinion, which is so mounted that it can slip round on the shaft when the piston ascends, but carries the shaft with it when it turns in the opposite direction during the descent of the piston, this descent being produced by the pressure of the atmosphere, as the steam resulting from the explosion condenses, and the unexploded gases cool. The vessel shown on the right contains cold water, which is employed to cool the cylinder by circulating round the lower part of it. The quantity of water required for this purpose is much smaller, and the consumption of gas is also much less, than in Lenoir's engine.

CHAPTER XXXIV.

TERRESTRIAL TEMPERATURES.

394. **Temperature of the Air.**—By the *temperature of a place* eteorologists commonly understand the *temperature of the air* at noderate distance (5 or 10 feet) from the ground. This element is ily determined when there is much wind; but in calm weather, l especially when the sun is shining powerfully, it is often difficult avoid the disturbing effect of radiation. Thermometers for erving the temperature of the air must be sheltered from rain sunshine, but exposed to a free circulation of air.

95. **Mean Temperature of a Place.**—The *mean temperature of a* is obtained by making numerous observations at equal intervals ime throughout the day (24 hours), and dividing the sum of the rved temperatures by their number. The accuracy of the deter- ition is increased by increasing the number of observations; as nean temperature, properly speaking, is the mean of an infinite ber of temperatures observed at infinitely short intervals.

the curve of temperature for the day is given, temperature being sented by height of the curve above a horizontal datum line, iean temperature is the height of a horizontal line which gives akes equal areas; or is the height of the middle point of any ht line (terminated by the extreme ordinates of the curve) gives and takes equal areas.

empts have been made to lay down rules for computing the temperature of a day from two, three, or four observations at hours; but such rules are of very limited application, owing to ferent character of the diurnal variation at different places; best they cannot pretend to give the temperature of an ual day, but merely results which are correct in the long run. ations at 9 A.M. and 9 P.M. are very usual in this country; and f-sum of the temperatures at these hours is in general a good approximation to the mean temperature of the day. The half-sum of the highest and the lowest temperature in the day, as indicated by maximum and minimum thermometers, is often adopted as the mean temperature. The result thus obtained is usually rather above the true mean temperature, owing to the circumstance that the extreme heat of the day is a more transient phenomenon than the extreme cold of the night. The employment of self-registering thermometers has, however, the great advantage of avoiding errors arising from want of punctuality in the observer. The correction which is to be added or subtracted in order to obtain the true mean from the mean of two observations is called a *correction for diurnal range*. Its amount differs for different places, being usually greatest where the diurnal range itself (§ 113) is greatest.

The *mean temperature of a calendar month* is computed by adding the mean temperatures of the days which compose it, and dividing by their number.

The *mean temperature of a year* is usually computed by adding the mean temperatures of the calendar months, and dividing by 12; but this process is not quite accurate, inasmuch as the calendar months are of unequal length. A more accurate result is obtained by adding the mean temperatures of all the days in the year, and dividing by 365 (or in leap-year by 366).

396. Isothermals.—The distribution of temperature over a large region is very clearly represented by drawing upon the map of this region a series of *isothermal lines;* that is, lines characterized by the property that *all places on the same line have the same temperature.* These lines are always understood to refer to mean annual temperature unless the contrary is stated; but isothermals for particular months, especially January and July, are frequently traced, one serving to show the distribution of temperature in winter, and the other in summer. The first extensive series of isothermals was drawn by Humboldt in 1817, on the basis of a large number of observations collected from all parts of the world; and the additional information which has since been collected has not materially altered the forms of the lines traced by him upon the terrestrial globe. They are in many places inclined at a very considerable angle to the parallels of latitude; and nowhere is this deviation from parallelism more observable than in the neighbourhood of Great Britain, Norway, and Iceland—places in this region having the same mean annual temperature as places in Asia or America lying from 10° to 20° further south.

399. Insular and Continental Climates.—The difference between the temperatures of summer and winter is greatest in the interior of large continents, and smallest in small islands in the midst of the ocean; large masses of water being exceedingly slow in changing their temperature, and powerfully contributing to prevent extremes of temperature from occurring in their neighbourhood. It is common to distinguish in this sense between *continental* climates on the one hand and *insular* or *marine* climates on the other.

Some examples of both kinds are given in the following table. The temperatures are Centigrade:—

MARINE CLIMATES.

	Winter.	Summer.	Difference.
Faroe Islands, ...	3°·90	11°·60	7°·70
Isle of Unst (Shetland), . .	4 ·05	11 ·92	7 ·87
Isle of Man, ...	5 ·59	15 ·08	9 ·49
Penzance, ...	7 ·04	15 ·83	8 ·79
Helston, ...	6 ·19	16 ·00	9 ·81
CONTINENTAL CLIMATES.			
St. Petersburg, ...	− 8°·70	15°·96	24°·66
Moscow, ...	−10 ·22	17 ·55	27 ·77
Kasan, ...	−13 ·66	17 ·35	31 ·01
Slatoust, ...	−16 ·49	16 ·08	32 ·57
Irkutsk, ...	−17 ·88	16 ·00	33 ·88
Jakoutsk, ...	−38 ·90	17 ·20	56 ·10

400. Temperature of the Soil at Different Depths.—By employing thermometers with their bulbs buried in the earth, and their stems projecting above, numerous observations have been made of the temperature from day to day at different depths from 1 inch to 2 or 3 feet; and at a few places observations of the same kind have been made by means of gigantic spirit-thermometers with exceedingly strong bulbs, at depths extending to about 25 feet. It is found that variations depending on the hour of the day are scarcely sensible at the depth of 2 or 3 feet, and that those which depend on the time of year decrease gradually as the depth increases, but still remain sensible at the depth of 25 feet, the range of temperature during a year at this depth being usually about 2° or 3° Fahrenheit.

It is also found that, as we descend from the surface, the seasons lag more and more behind those at the surface, the retardation amounting usually to something less than a week for each foot of descent; so that, at the depth of 25 feet in these latitudes, the lowest temperature occurs about June, and the highest about December.

Theory indicates that 1 foot of descent should have about the same effect on diurnal variations as $\sqrt{365}$ that is 19 feet on annual variations; understanding by *sameness of effect* equal *absolute amounts* of lagging and equal *ratios* of diminution.

As the annual range at the surface in Great Britain is usually about 3 times greater than the diurnal range, it follows that the diurnal range at the depth of a foot should be about one-third of the annual range at the depth of 19 feet.

The variations of temperature at the surface are, as every one knows, of a very irregular kind; so that the curve of surface temperature for any particular year is full of sinuosities depending on the accidents of that year. The deeper we go, the more regular does the curve become, and the more nearly does it approach to the character of a simple curve of sines, whose equation can be written

$$y = a \sin. x.$$

Neglecting the departures of the curve from this simple character, theory indicates that, if the soil be uniform, and the surface plane, the annual range (which is equal to 2 a) goes on diminishing in geometrical progression as the depth increases in arithmetical; and observation shows that, if 10 feet be the common difference of depth, the ratio of decrease for range is usually about $\frac{1}{2}$ or $\frac{1}{3}$.

To find a range of a tenth of a degree Fahrenheit, we must go to a depth of from 50 to 80 feet in this climate. At a station where the surface range is double what it is in Great Britain, we should find a range of about two-tenths of a degree at a depth and in a soil which would here give one-tenth.

These remarks show that the phrase "stratum of invariable temperature," which is frequently employed to denote the supposed lower boundary of the region in which annual range is sensible, has no precise significance, inasmuch as the boundary in question will vary its depth according to the sensitiveness of the thermometer employed.

401. Increase of Temperature Downwards.—Observations in all parts of the world show that the temperature at considerable depths, such as are attained in mining and boring, is much above the surface temperature. In sinking a shaft at Rose Bridge Colliery, near Wigan, which is the deepest mine in Great Britain, the temperature of the rock was found to be 94° F. at the depth of 2440 feet. In cutting the Mont Cénis tunnel, the temperature of the deepest part, with 5280 feet of rock overhead, was found to be about 85° F.

The rate of increase downwards is by no means the same everywhere; but it is seldom so rapid as 1° F. in 40 feet, or so slow as 1° F. in 100 feet. The observations at Rose Bridge show a mean rate of increase of about 1° in 55 feet; and this is about the average of the results obtained at other places.

This state of things implies a continual escape of heat from the interior of the earth by conduction, and the amount of this loss per annum can be approximately calculated from the absolute values of conductivity of rock which we have given in Chap. xxx.

There can be no reasonable doubt that the decrease of temperature upwards extends to the very surface, when we confine our attention to mean annual temperatures, for all the heat that is conducted up through a stratum at any given depth must also traverse all the strata above it, and heat can only be conducted from a warmer to a colder stratum. Professor Forbes found, at his three stations near Edinburgh, increases of 1°·38, 0°·96, and 0°·19 F. in mean temperature, in descending through about 22 feet, that is, from the depth of 3 to the depth of 24 French feet. The mean annual temperature of the surface of the ground is in Great Britain a little superior to that of the air above it, so far as present observations show. The excess appears to average about 1° F. At Trevandrum in India the excess is in the same direction, and amounts to 5° or 6° F.

402. Decrease of Temperature Upwards in the Air.—In comparing the mean temperatures of places in the same neighbourhood at different altitudes, it is found that temperature diminishes as height increases, the rate of decrease for Great Britain, as regards mean annual temperature, being about 1° F. for every 300 feet. A decrease of temperature upwards is also usually experienced in balloon ascents, and numerous observations have been taken for the purpose of determining its rate. Mr. Glaisher's observations, which are the most numerous as well as the most recent, show that, upon the whole, the decrease becomes less rapid as we ascend higher; also, that it is less rapid with a cloudy than with a clear sky. The following table exhibits a few of Mr. Glaisher's averages:—

	Decrease of Temperature Upwards.	
Height.	With clear sky.	With cloudy sky
From 0 to 1000 feet, . . .	1° F. in 139 feet.	1° F. in 222 feet.
From 0 to 10,000 ft. . . .	1° F. in 288 feet.	1° F. in 331 feet.
From 0 to 20,000 ft. . . .	1° F. in 365 feet.	1° F. in 468 feet.

These rates may be taken as representing the general law of decrease which prevails in the air over Great Britain in the daytime during the summer half of the year; but the results obtained on different days differ widely, and alternations of increase and decrease are by no means uncommon in passing upwards through successive strata of air. Still more recent observations by Mr. Glaisher, relating chiefly to the first 1000 feet of air, show that the law varies with the hour of the day. The decrease upwards is most rapid soon after midday, and is at this time, and during daytime generally, more rapid as the height is less. About sunset there is a uniform *decrease* at all heights if the sky is clouded, and a uniform *temperature* if the sky is clear. From a few observations which have been taken after sunset, it appears that, with a clear sky, there is an *increase* upwards at night.

That an extremely low temperature exists in the interplanetary spaces, may be inferred from the experimental fact recorded by Sir John Herschel, that a thermometer with its bulb in the focus of a reflector of sufficient size and curvature to screen it from lateral radiation, falls lower when the axis of the reflector is directed upwards to a clear sky than when it is directed either to a cloud or to the snow-clad summits of the Alps. The atmosphere serves as a protection against radiation to these cold spaces, and it is not surprising that, as we increase our elevation, and thus diminish the thickness of the coating of air above us, the protection should be found less complete.

403. Cooling of an Ascending Column of Air.—Whenever a body of air ascends, it expands, in consequence of the diminution of pressure; and the work which it does in expanding consumes a portion of its heat, and lowers its temperature. In like manner, when air descends, the work done upon it in compressing it raises its temperature. The amount of this change of temperature can be at once inferred from § 347 A, by putting $t = 1°$ C., $\alpha = \cdot 00366$, $\beta = \cdot 41$. In fact, a compression or expansion amounting to ·00366 of the volume which the air would occupy at zero, alters its temperature by 0·41° C. Consequently, for an alteration of 1° C., we must have a change of volume amounting to ·00893, which, in conjunction with the change of temperature, implies a change of pressure amounting to $\cdot 00893 + \cdot 00366 = \cdot 0126 = \frac{1}{80}$ of the actual pressure. This corresponds to an ascent or descent through $\frac{1}{80}$ of the height of a homogeneous atmosphere (§ 111 A), that is, through about 330 feet. For a change of 1° F., the required height will be 183 feet. These numbers are computed on the assumption that the air is sufficiently dry to behave like a permanent gas. If ascending air contains vapour which is condensed by the loss of heat, this condensation greatly retards the cooling; and if descending air contains mist which is dissipated by the gain of heat, this dissipation retards the warming.

It is obvious that the ascent of warm air will not occur, unless the actual decrease of temperature upwards is more rapid than the cooling due to ascent; for air will not rise if the process of rising would make it colder and heavier than the air through which it would have to pass.

404. Causes of Winds.—The influences which modify the direction and intensity of winds are so various and complicated that anything like a complete account of them can only find a place in treatises specially devoted to that subject. There is, however, one fundamental principle which suffices to explain the origin of many well-known winds. This principle is plainly illustrated by the following experiment, due to Franklin. A door between two rooms, one heated, and the other cold (in winter), is opened, and two candles are placed, one at the top, and the other at the bottom of the doorway. It is found that the flame of the lower candle is blown towards the heated room, and that of the upper candle away from it.

From this experiment we can deduce the following general principle:—*When two neighbouring regions are at different temperatures, a current of air flows from the warmer to the colder in the upper strata of the atmosphere; and in the lower strata a current flows from the colder to the warmer.* We proceed to apply this principle to the land and sea breezes, the monsoons, and the trade-winds.

405. Land and Sea Breezes.—At the sea-side during calm weather a wind is generally observed to spring up at about eight or nine in the morning, blowing from the sea, and increasing in force until about two or three in the afternoon. It then begins gradually to die away, and shortly before sunset disappears altogether. A few hours afterwards, a wind springs up in the opposite direction, and lasts till nearly sunrise. These winds, which are called the sea-breeze and land-breeze, are exceedingly regular in their occurrence, though they may sometimes be masked by other winds blowing at the same time. Their origin is very easily explained. During the day the land grows warmer than the water; hence there results a wind blowing towards the warmer region, that is, towards the land. During the night the land and sea both grow colder, but the former more rapidly than the latter; and, accordingly, the relative temperatures of the two elements being now reversed, a breeze blowing from the land towards the sea is the consequence.

Monsoons.—The same cause which, on a small scale, produces the diurnal alternation of land and sea breezes, produces, on a larger scale, the annual alternation of monsoons in the Indian Ocean, and the seasonal winds which prevail in some other parts of the world. The general direction of these winds is towards continents in summer, and away from them in winter.

406. Trade-winds: General Atmospheric Circulation.—The trade-winds are winds which blow constantly from a north-easterly quarter over a zone of the northern hemisphere extending from a little north of the tropic of Cancer to within 9 or 10 degrees of the equator; and from a south-easterly quarter over a zone of the southern hemisphere extending from about the tropic of Capricorn to the equator. Their limits vary slightly according to the time of year, changing in the same direction as the sun's declination. Between them is a zone some 5° or 6° wide, over which calms and variable winds prevail.

The cause of the trade-winds was first correctly indicated by Hadley. The greater power of the sun over the equatorial regions causes a continual ascent of heated air from them. This flows over to both sides in the upper regions of the atmosphere, and its place is supplied by colder air flowing in from both sides below. If the earth were at rest, we should thus have a north wind sweeping over the earth's surface on the northern side of the equatorial regions, and a south wind on the southern side. But, in virtue of the earth's rotation, all points on the earth's surface are moving from west to east, with velocities proportional to their distances from the earth's axis. This velocity is nothing at the poles, and increases in approaching the equator. Hence, if a body on the earth's surface, and originally at rest relatively to the earth, be urged by a force acting along a meridian, it will not move along a meridian, but will outrun the earth, or fall behind it, according as its original rotational velocity was greater or less than those of the places to which it comes. That is to say, it will have a relative motion from the west if it be approaching the pole, and from the east if it be approaching the equator.

This would be true, even if the body merely tended to keep its original rotational velocity unchanged, and the reasoning becomes still more forcible when we apply the principle of conservation of angular momentum (§§ 53 F, G), in virtue of which the body tends to increase[1] its absolute rotational velocity in approaching the pole, and to diminish it in approaching the equator.

Thus the currents of air which flow in from both sides to the equatorial regions, do not blow from due north and due south, but from north-east and south-east. There can be little doubt that, notwithstanding the variable character of the winds in the temperate and frigid zones, there is, upon the whole, a continual interchange of air between them and the intertropical regions, brought about by the permanent excess of temperature of the latter. Such an interchange, when considered in conjunction with the difference in the rotational velocities of these regions, implies that the mass of air over an equa- orial zone some 50° or 60° wide, must, upon the whole, have a rela- ive motion from the east; and that the mass of air over all the rest f the earth must, upon the whole, have a relative motion from the est. This theoretical conclusion is corroborated by the distribution f barometric pressure. The barometer stands highest at the two rallels which, according to this theory, form the boundaries between sterly and westerly winds, while at the equator and poles it stands w. This difference may be accounted for by the excess of centri- gal force possessed by west winds, and the defect of centrifugal force east winds. If the air simply turned with the earth, centrifugal ce combined with gravity would not tend to produce accumulation air over any particular zone, the ellipticity of the earth being pre- ly adapted to an equable distribution. But if a body of air or er fluid is moving with sensibly different rotational velocity from earth, the difference in centrifugal force will give a tendency to ve towards the equator, or from it, according as the differential ion is from the west or from the east. The easterly winds over equatorial zone should therefore tend to remove air from the tor and heap it up at the limiting parallels; and the westerly ds over the remainder of the earth should tend to draw air away the poles and heap it up at the same limiting parallels. This retical consequence exactly agrees with the following table of barometric heights in different zones given by Maury:[1]—

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[1] The tendency is for velocity to vary inversely as distance from the axis of rotation.

Latitude.	Barometer.	South Latitude.	Barometer.
° to 5°	29·915	0° to 5°	29·940
° to 10°	29·922	5° to 10°	29·981
° to 15°	29·964	10° to 15°	30·028
° to 20°	30·018	15° to 20°	30·060
° to 25°	30·081	20° to 25°	30·102
° to 30°	30·149	25° to 30°	30·095
° to 35°	30·210	30° to 36°	30·052
to 40°	30·124	42° 53′	29·90
to 45°	30·077	45° 0′	29·66
to 50°	30·060	49° 8′	29·47
29′	29·99	51° 33′	29·50
51′	29·88	54° 26′	29·35
37′	29·759	55° 52′	29·36
		60° 0′	29·11
		66° 0′	29·08
		74° 0′	28·93

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ysical Geography and Meteorology of the Sea, p. 180, art. 362, edition 1860.

This table shows that the barometric height falls off regularly on both sides from the two limiting zones 30° to 35° N. and 20° to 25° S., the fall continuing towards both poles as far as the observations extend, and continuing inwards to a central minimum between 0° and 5° N.[1]

If the bottom of a cylindrical vessel of water be covered with saw-dust, and the water made to rotate by stirring, the saw-dust will be drawn away from the edges, and heaped up in the middle, thus showing an indraught of water along the bottom towards the region of low barometer in the centre. It is probable that, from a similar cause (a central depression due to centrifugal force), there is an indraught of air along the earth's surface towards the poles, underneath the primary circulation which our theory supposes; the diminution of velocity by friction against the earth, rendering the lowest portion of the air obedient to this indraught, which the upper strata are enabled to resist by the centrifugal force of their more rapid motion. This, according to Professor James Thomson,[2] is the explanation of the prevalence of south-west winds in the north temperate zone; their southerly component being due to the barometric indraught and their westerly component to differential velocity of rotation. The indraught which also exists from the limiting parallels to the region of low barometer at the equator, coincides with the current due to difference of temperature; and this coincidence may be a main reason of the constancy of the trade-winds.

406 A. Origin of Cyclones.—In the northern hemisphere a wind which would blow towards the north if the earth were at rest, does actually blow towards the north-east; and a wind which would blow towards the south blows towards the south-west. In both cases, the earth's rotation introduces a component towards the right with reference to a person travelling with the wind. In the southern hemisphere it introduces a component towards the left.

Again, a west wind has an excess of centrifugal force which tends to carry it towards the equator, and an east wind has a tendency to move towards the pole; so that here again, in the northern hemisphere the deviation is in both cases to the right, and in the southern hemisphere to the left.

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[1] The explanation here given of the accumulation of air towards the limiting parallels, as due to excess and defect of centrifugal force, appears to have been first published by Mr. W. Ferrel, a gentleman connected with the American Nautical Almanack. His later treatise (1860), reprinted from vols. i. ii. of the *Mathematical Monthly*, is the most complete exposition we have seen of the theory of general atmospheric circulation.

[2] *Brit. Assoc. Report*, 1857.

We have thus an explanation of cyclonic movements. In the rthern hemisphere, if a sudden diminution of pressure occurs over y large area, the air all round for a considerable distance receives impetus directed towards this area. But, before the converging eams can meet, they undergo deviation, each to its own right, so t, instead of arriving at their common centre, they blow tangenlly to a closed curve surrounding it, and thus produce an eddy m right to left with respect to a person standing in the centre. is is the universal direction of cyclonic rotation in the northern isphere; and the opposite holds for the southern isphere. The former is osite to, the latter the e as the direction of motion he hands of a watch lying its face up. In each case motion is opposite to the rent diurnal motion of the r the hemisphere in which urs.

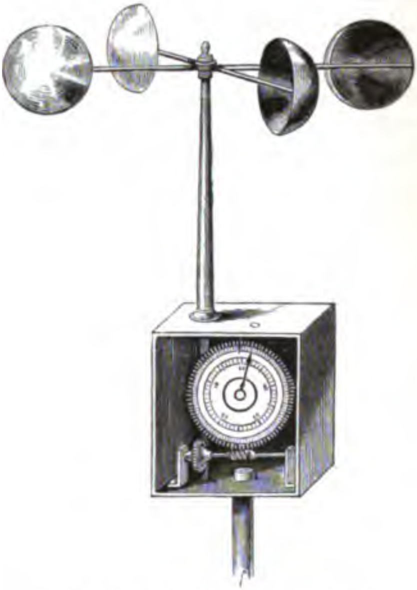

Fig. 331 A.—Robinson's Anemometer.

7. Anemometers.—Instru- for measuring either the r the velocity of the wind alled *anemometers.* Its is usually measured by anemometer, in which ressure of the wind is d upon a square plate d to one end of a spiral (with its axis horizon- hich yields more or less according to the force of the wind, ansmits its motion to a pencil which leaves a trace upon moved by clock-work. It seems that the force received by te is not rigorously proportional to its size, and that a plate square receives rather more than 9 times the pressure ate a foot square. The anemometer which has yielded the tisfactory results is that invented by the Rev. Dr. Robinson gh, which is represented in Fig. 331 A, and which indicates city of the wind. It consists of four hemispherical cups to the ends of equal horizontal arms, forming a horizontal cross, which turns freely about a vertical axis. By means of an endless screw carried by the axis, a train of wheel-work is set in motion; and the indication is given by a hand which moves round a dial; or, in some instruments, by several hands moving round different dials like those of a gas-meter. The anemometer can also be made to leave a continuous record on paper, for which purpose various contrivances have been successfully employed. It was calculated by the inventor, and confirmed by his own experiments both in air and water, as well as by experiments conducted by Prof. C. Piazzi Smyth at Edinburgh, and more recently by the astronomer-royal at Greenwich, that the centre of each cup moves with a velocity which is almost exactly one-third of that of the wind. This is the only velocity-anemometer whose indications are exactly proportional to the velocity itself. Dr. Whewell's anemometer, which resembles a small windmill, is very far from fulfilling this condition, its variations of velocity being much less than those of the wind.

The direction of the wind, as indicated by a vane, can also be made to leave a continuous record by various contrivances; one of the most common being a pinion carried by the shaft of the vane, and driving a rack which carries a pencil. But perhaps the neatest arrangement for this purpose is a large screw with only one thread composed of a metal which will write on paper. A sheet of paper is moved by clock-work in a direction perpendicular to the axis of the screw, and is pressed against the thread, touching it of course only in one point, which travels parallel to the axis as the screw turns, and comes back to its original place after one revolution. When one end of the thread leaves the paper, the other end at the same instant comes on. The screw turns with the vane, so that a complete revolution of the screw corresponds to a complete revolution of the wind. This is one of the many ingenious contrivances devised and executed by Mr. Beckley, mechanical assistant in Kew Observatory.

ELECTRICITY.

CHAPTER XXXV.

INTRODUCTORY PHENOMENA.

408. Fundamental Phenomena.—If a glass tube be rubbed with a silk handkerchief, both tube and rubber being very dry, the tube will be found to have acquired the property of attracting light bodies. If the part rubbed be held near to small scraps of paper, pieces of cut straw, sawdust, &c., these objects will move to the tube; sometimes they remain in contact with it, sometimes they are alternately attracted and repelled, the intensity as well as the duration of these effects varying according to the amount of friction to which the tube has been subjected.

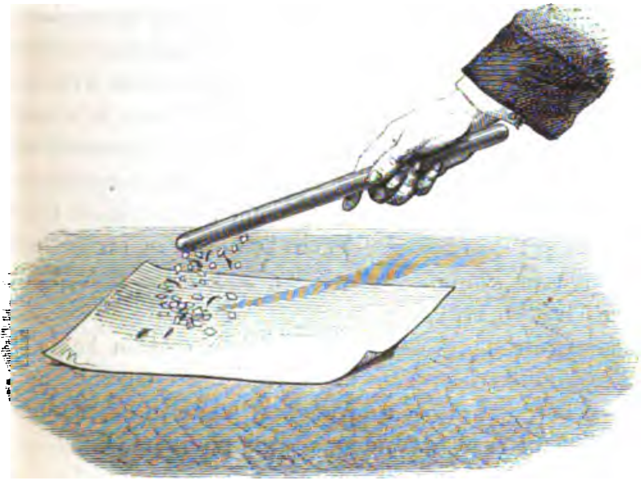

Fig. 332.—Attraction of Light Bodies by an Electrified Body.

If the tube be brought near the face, the result is a sensation similar to that produced by the contact of a cobweb. If the knuckle be held near the tube, a peculiar crackling noise is heard, and a bright spark passes between the tube and knuckle. The tube then has acquired peculiar properties by the application of friction. It is said to be *electrified*, and the name of *electricity* is given to the agent to which the various phenomena just described are attributed.

Glass is not the only substance which can be electrified by friction; the same property is possessed also by resin, sulphur, precious stones, amber, &c. The Greek name of this last substance (ἤλεκτρον) is the root from which the word *electricity* is derived.

At first sight it appears that this property of becoming electrified by friction is not common to all bodies; for if a bar of metal be held in the hand and rubbed with wool, it does not acquire the properties of an electrified body. But we should be wrong in concluding that metals cannot be electrified by friction; for if the bar be fitted on to a glass rod, and, while held by this handle, be struck with flannel or catskin, it may be very sensibly electrified. There is therefore no basis for the distinction formerly made between electrics and non-electrics, that is, between substances capable and incapable of being electrified by friction; for all bodies, as far as at present known, are capable of being thus excited. There is, however, an important difference of another kind between them, which was first pointed out by Stephen Grey in 1729.

Fig. 333.—Electrification of a Metal by Friction.

409. Conductors and Non-conductors.—In certain bodies, such as glass and resin, electricity does not spread itself beyond the parts of the surface where it has been developed; while in other bodies, such as metals, the electricity developed at any point immediately spreads itself over the whole body. Thus, in the last-mentioned experiment, the signs of electricity are immediately manifested at the end of the metal bar which is farthest from the glass rod, if the end next the rod be submitted to friction. Bodies of the former kind, such as glass, resin, &c, are said to be *non-conductors*. Metals are said to be good *conductors*. A non-conductor is often called an *insulator* and a conductor supported by a non-conductor is said to be *insulated*. The appropriateness of these expressions is evident. No substance is perfectly non-conducting, but the difference in conduct-

power between what are called non-conductors and good con-
tors, is enormous. The following are lists of conductors and
-conductors, arranged, at least approximately, in order of their
ducting powers. In the list of conductors, the best conductors
put first; in the list of non-conductors, the worst conductors (or
insulators) are put first.

CONDUCTORS.

ll metals.
ell-burned charcoal.
lumbago.
oncentrated acids.
ilute acids.
line solutions.
Metallic ores.
Animal fluids.
Sea water.
Spring water.
Rain water.
Snow.
Living vegetables.
Flax.
Hemp.
Living animals.
Flame.
Moist earth and stones.

NON-CONDUCTORS.

Shellac.
Amber.
Resins.
ulphur.
Vax.
et.
lass.
ica.
iamond.
Gems.
Ebonite.
Gutta-percha.
Silk.
Wool.
Feathers.
Dry paper.
Parchment.
Leather.
Baked wood.
Porcelain.
Marble.
Camphor.
Caoutchouc.
Chalk.
Lime.
Oils.
Metallic oxides.

man body is a good conductor of electricity. If a person
g on a stool with glass legs be struck with a catskin, he
s electrified in a very perceptible degree, and sparks may be
rom any part of his body.

an insulated and electrified conductor is allowed to touch
conductor insulated but not electrified, it is observed that,
contact, both bodies possess electrical properties, electricity
een communicated to the second body at the expense of the
the second body be much the larger of the two, the electri-
he first is greatly diminished, and may become quite insen-
his explains the disappearance of electricity when a body is
nnection with the earth, which, together with most of the
its surface, may be regarded as constituting one enormous
. On account of its practically inexhaustible capacity for
or absorbing electricity, the earth is often called *the com-*
voir.

now be easily understood why it is not possible to electrify
d by rubbing it while it is held in the hand; since the

electricity, as fast as it is generated, passes off through the body into the earth.

Air, when thoroughly dry, is an excellent insulator; and electrified conductors exposed to it, and otherwise insulated, retain their charge with very little diminution for a considerable time. Dampness in the air is, however, a great obstacle to insulation, partly from the impaired insulating power of the air itself, and still more from the moisture which condenses on the insulating supports. Electrical experiments are accordingly very difficult to perform in damp weather. The difficulty is sometimes met by employing a stove to heat the air in the neighbourhood of the supports, and thus diminish its relative humidity. Sir W. Snow Harris employed heating-irons, which were heated in a fire, and then fixed near the insulating supports; and thus succeeded in exhibiting electrical experiments to an audience in the most unfavourable weather. Sir W. Thomson, by keeping the air in the interior of his electrometers dry by means of sulphuric acid, causes them to retain their charge with only a small percentage of loss in twenty-four hours. Dry frosty days are the best for electrical experiments, and next perhaps to these, is the season of dry cutting winds in spring.

410. Duality of Electricity.—The elementary phenomena which we have mentioned in the beginning of this chapter may be more accurately studied by means of the electric pendulum, which consists of a pith-ball suspended by a silk fibre from an insulated support. When an electrified glass rod is brought near the insulated ball, the latter is attracted; but as soon as it touches the glass tube, the attraction is changed to repulsion, which lasts as long as the ball retains the electricity which it has acquired by the contact. A similar experiment can be shown by employing, instead of the glass tube, any other body which has been electrified by friction, for example, a piece of resin which has been rubbed with flannel.

If, while the pendulum exhibits repulsion for the glass, the electrified resin is brought near, it is attracted by the latter; and conversely, when it is repelled by the resin, it is attracted by the glass. These phenomena clearly show that the electricity developed on the resin is not of the same kind as that developed on the glass. They exhibit opposite forces towards any third electrified body, each attracting what the other repels. They have accordingly received names which indicate opposition. The electricity which glass acquires when rubbed with silk, is called *positive;* and that which resin acquires by friction with flannel, *negative.* The former is also called *vitreous,* and the latter *resinous.* On repeating the experiment with other substances, it is found that all electrified bodies behave either like the glass or like the resin.

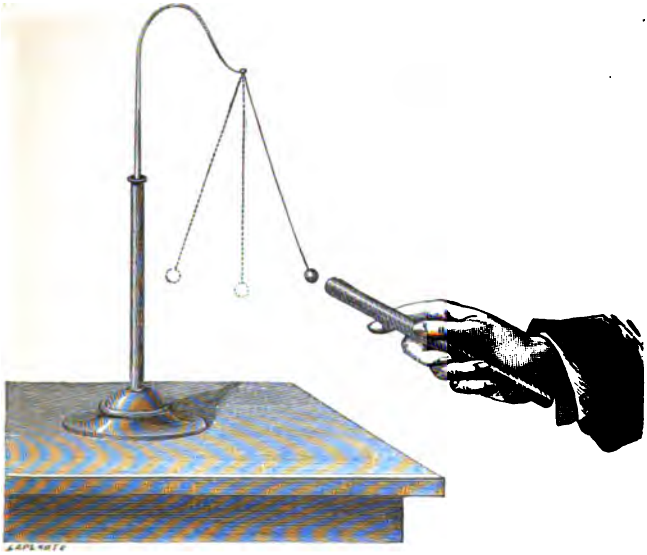

Fig. 334.—Electric Pendulum.

410 A.—Without making any assumption as to what electricity is, we may speak of an electrified body as being *charged with electricity,* and we may compare quantities of electricity by means of the attractions and repulsions exerted. Bodies oppositely electrified must then be spoken of as charged with *electricities of opposite kind,* or *of opposite sign ;* and experiment shows that, whenever electricity of the one kind is developed, whether by friction or by any other means, electricity of the opposite sign is always developed in exactly equal quantity. If a conductor receives two charges of electricity of equal quantity but opposite sign, it is found to exhibit no traces of electricity whatever.

Electricities of like sign repel one another and those of unlike sign attract one another.—The magnitude of the force exerted upon each other by two electrified bodies, is not altered in amount by reversing the sign of the electricity of one or both of them, provided that the quantities of electricity, and their distribution over the two bodies, remain unchanged. If the sign of one only be changed, the mutual force is simply reversed, and if the signs of both be changed, the force is not changed at all.

411.—The simultaneous development of both kinds of electricity is illustrated by the following experiment:—Two persons stand on stools with glass legs, and one of them strikes the other with a catskin. Both of them are now found to be electrified, the striker positively, and the person struck negatively, and from both of them sparks may be drawn by presenting the knuckle.

The kind of electricity which a body obtains by friction with another body, evidently depends on the nature of their surfaces. If, for example, we take two discs, one of glass, and the other of metal, and, holding them by insulating handles, rub them briskly together, we shall find that the metal becomes negatively, and the glass positively electrified; but if the metal be covered with a catskin, and the experiment repeated, it will be the glass which will this time be negatively electrified. In the subjoined list, the substances are arranged in such order that, generally speaking, each of them becomes positively electrified by friction with those which follow it, and negatively with those which precede it.

Fur of cat.
Polished glass.
Woollen stuffs.
Feathers.
Wood.
Paper.
Silk.
Shellac.
Rough glass.

411 A. Hypotheses regarding the Nature of Electricity.—Two theories regarding the nature of electricity must be described on account of the historical interest attaching to them.

The two-fluid theory, originally propounded by Dufaye, reduced to a more exact form by Symmer, and still very extensively adopted. maintains that the opposite kinds of electricity are two fluids. Positive electricity is called the *vitreous fluid,* and negative electricity the *resinous fluid.* Fluids of like name repel, and those of unlike name attract each other. The union of equal quantities of the two fluids constitutes the neutral fluid which is supposed to exist in very large quantity in all unelectrified bodies. When a body is electrified, it gains an additional quantity of the one fluid, and loses an equal quantity of the other, so that the total amount of electric fluid in a body is never changed; and (as a consequence of this last condition) when a current of either fluid traverses a body in any direction, an equal current of the other fluid traverses it in the opposite direction.

This theory is in complete agreement with all electrical phenomena
far as at present known; but as it is conceivable that the two
tricities, instead of being two kinds of matter, may be two kinds
otion, or, in some other way, may be opposite states of one and
same substance, it is more philosophical to avoid the assumption
lved in speaking of *two electric fluids*, and to speak rather of
opposite electricities. They may be distinguished indifferently
he names *vitreous* and *resinous*, or *positive* and *negative.*

he one-fluid theory, as originally propounded by Franklin, main-
d the existence of only one electric fluid, which unelectrified
s possess in a certain normal amount. A positively electrified
has more, and a negatively electrified body less than its normal
nt. The particles of this fluid repel one another, and attract
articles of other kinds of matter, at all distances. Æpinus, in
oping this theory more accurately, found it necessary to intro-
the additional hypothesis that the particles of matter repel one
er. Thus, according to Æpinus, the absence of sensible force
en two bodies in the neutral condition, is due to the equilibrium
r forces, two of which are attractive, and the other two repul-
Calling the two bodies A and B, the electricity which A pos-
in normal amount, is repelled by the electricity of B, and
ed by the matter of B. The matter of A is attracted by the
ity of B, and repelled by the matter of B. These four forces
equal, and destroy one another; but, without the supplemen-
pothesis of Æpinus, one of the four forces is wanting, and the
ium is not easily explained. To reconcile Æpinus's addition
e Newtonian theory of gravitation, it is necessary to suppose
equality between the four forces is not exact, the attractions
reater by an infinitesimal amount than the repulsions.

ne-fluid theory in this form is, like the two-fluid theory, con-
with the explanation of all known phenomena. But it is to
rked that there is no sufficient reason, except established
r deciding which of the two opposite electricities should be
as corresponding to an excess of the electric fluid.

lin was the author of the terms positive and negative to
he two opposite kinds of electrification; but the names can
ely be retained without accepting the one-fluid theory,
nding that opposite signs imply forces in opposite directions,
the connection between the *positive* sign and the forces
by *vitreous* electricity is merely conventional.

411 B.—In speaking of electric currents, the language of the one-fluid theory is almost invariably employed. Thus, if A is a conductor charged positively, and B a conductor charged negatively; when the two are put in connection by a wire, we say that the direction of the current is from A to B; whereas the language of the two-fluid theory would be, that a current of vitreous or positive electricity travels from A to B, and a current of resinous or negative from B to A.

CHAPTER XXXVI.

ELECTRICAL INDUCTION.

12. **Induction.**—In the preceding chapter we have spoken of move- ts of material bodies caused by electrical attractions and repul- s. We have now to treat of the movement of electricity itself edience to the attractions or repulsions exerted upon it by other *ricity.* This kind of action is called *induction.*

may be illustrated by means of the arrangement shown in Fig. The apparatus consists of a sphere C which is electrified posi- y, suppose, and of ducting insulated der A B placed it. From this are suspended at distances a few of pith-balls. the cylinder ught near the , the balls are ed to diverge. vergence of the it pairs is not ne, but goes on decreasing from the pair nearest the cylinder point M is reached, where there is no divergence. Beyond divergence goes on increasing. The neutral point M does ctly bisect the length of the cylinder, but is nearer the end the end B, and the former end is found to be more strongly d than the latter.

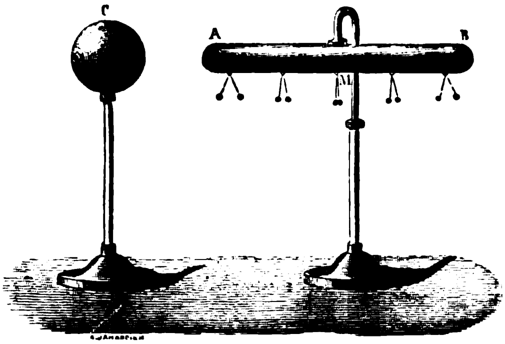

Fig. 336.—Electrification by Influence.

easy to show that the two ends of the cylinder are charged posite kinds of electricity; the end A being negatively, and the end B positively electrified. We have only to bring an electrified stick of resin near the pith-balls at A, when these will be found to be repelled; if, on the contrary, it be held near those at B, they will be attracted.

The explanation is, that the positive electricity with which C is charged attracts the negative electricity of AB to the end A, and repels the positive to the end B. This action is more powerful at A than at B, on account of the greater proximity of the influencing body, and for the same reason the effect falls off more rapidly in the portion AM than in MB.

If the cylinder be brought closer to the sphere, the divergence of the balls increases; if it be removed farther from it, the divergence diminishes. Finally, all signs of electricity disappear if the sphere be taken away, or connected with the earth.

If, while the cylinder is under the influence of the electricity of C, the end B is connected with the earth, the pith-balls at this end immediately collapse, while the divergence of those at A increases. The explanation is that the electricity which was repelled to the end B escapes to the earth, and thus affords an opportunity for a fresh exercise of induction on the part of the sphere, which increases the accumulation of negative electricity at A. We may also remark that the whole of the cylinder is now negatively electrified, the neutral line being pushed back to the earth. If the earth-connection be now broken, and the sphere C be then removed. the cylinder will remain negatively electrified, and will be in the same condition as if it had been touched by a negatively-electrified body. This mode

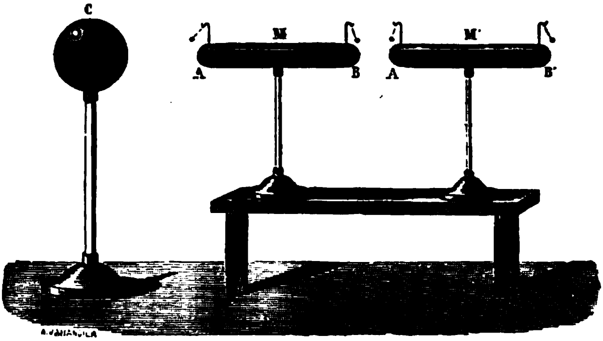

Fig. 337.- -Successive Induction.

f giving a charge to a conductor is called *charging by induction*,
nd the charge thus given is always opposite to that of the *inducing*
dy C.

If a series of such conductors as AB be placed in line, without
ntact, and the positively-electrified body C be placed opposite to
e end of the series, all the conductors will be affected in the same
nner as the single conductor in the last experiment. They will
be charged with negative electricity at the end next C, and with
sitive electricity at the remote end, the effect, however, becoming
bler as we advance in the series. In this experiment each of the
ductors acts inductively upon those next it; for example, if there
two conductors AB, A′B′, as in Fig. 337, the development of
tricity at A′ and B′ is mainly due to the action of the positive
tricity in MB. If the conductor AB be removed, the pith-balls
′ and B′ will diminish their divergence.

he molecules of a body may be regarded as such a series of con-
ors, or rather as a number of such series. When an electrified
is brought near it, each molecule may thus become positive on
side and negative on the other. In the case of good conductors,
polarization is only instantaneous, being destroyed by the dis-
ge of electricity from particle to particle. Good insulators are
ances which are able to resist this tendency to discharge, and
intain a high degree of polarization for a great length of time.
is Faraday's theory of "induction by contiguous particles."

Electrical Attraction and Repulsion.—The attraction which is
ved when an electrified is brought near to an unelectrified body,
endent upon induction. Suppose, for
ce, that a body C, which is positively
fied, is brought near to an insulated and
rged pith-ball. Negative electricity is
d on the near side of the pith-ball, and
al quantity of positive on the further
he former, being nearer to the body C,
e strongly attracted than the other is
d. The ball is therefore upon the whole attracted.

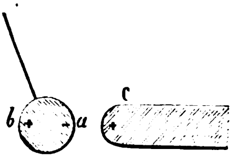

Fig. 338.—Electrical Attraction.

e pith-ball, instead of being insulated, is suspended by a con-
thread from a support connected with the earth, it will be
rongly attracted than before, as it is now entirely charged
gative electricity.

e case of any insulated conductor, the algebraic sum of the electricities induced upon it by the presence of a neighbouring electrified body must be zero. If the pith-ball be insulated, and have an independent charge of either kind of electricity, the total force exerted on the pith-ball is the algebraic sum[1] of the two following quantities:—

(1) The force which the ball would experience, if it had no independent charge. This force, as we have just seen, is always attractive.

(2) The force due to the independent charge when distributed over the ball as it would be if C were removed. This second force is attractive or repulsive, according as the independent charge is of unlike or like sign to that of C. In the latter case, repulsion will generally be observed at distances exceeding a certain limit and attraction at nearer distances, the reason being that the force (1) due to the induced distribution increases more rapidly than the other as the distance is diminished.

It is important to remember this in testing, by the electric pendulum, or by any other electroscope, the kind of electricity with which a body is charged. In bringing the body towards the electroscope, the first movement produced is that which is to be observed, and repulsion is in general a more reliable test of kind of electricity than attraction.

415. Electroscopes.—An electroscope is an apparatus for detecting the presence of electricity, and determining its sign. The insulated electric pendulum is an electroscope. If the pith-ball, when itself uncharged, is attracted by a body brought near it, we know that the body is electrified. To determine the kind of electricity, the body is allowed to touch the pith-ball, which is then repelled. At this moment an excited glass tube is brought near. If it repels the ball, this latter, as well as the body which touched it, must be electrified positively. If the glass tube attracts it, or, still more decisively, if excited resin or sealing-wax repels it, the ball and the body which touched it are electrified negatively. The loss of electricity from the pith-ball is often so rapid as to render this test of sign somewhat uncertain.

The *gold-leaf electroscope* (Fig. 339) is constructed as follows:—

---

[1] We here suppose C to be a non-conductor, so that the distribution of its electricity is not affected by the presence of the pith-ball. If C be a conductor, the effect of induction upon it will be to favour attraction, so that an attractive force must be added to the two forces specified in the text.

'wo small gold-leaves are attached to the lower end of a metallic
od, which passes through an opening in the top of a bell-glass, and
rminates in a ball. The metallic rod is sometimes, for the sake of
etter insulation, inclosed in a glass tube secured by sealing-wax or
me other non-conducting cement,
id, for the same purpose, the upper
rt of the bell-glass is often varnished
th shellac, which is less apt than
ss to acquire a deposit of moisture
m the air. The bell-glass is attached
low to a metallic base, which ex-
des the external air. For the gold-
ves are sometimes substituted two
ıws, or two pith-balls suspended
linen threads; we have thus the
ıw-electroscope and the *pith-ball*
troscope.

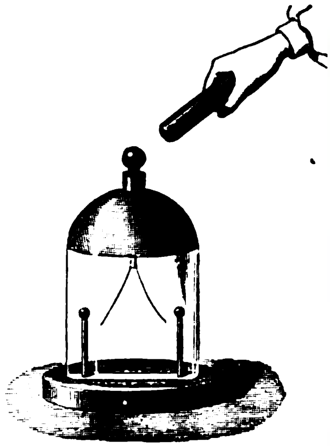

Fig. 339.—Gold-leaf Electroscope.

'o test whether a body is electri-
it is brought near the ball at
top of the electroscope. The like
ricity is repelled into the leaves,
makes them diverge, while the unlike is attracted into the
The sign of the body's charge may be determined in the
wing manner:—While the leaves are divergent under the in-
ce of the body, the operator touches the ball with his finger.
causes the leaves to collapse, and gives to the insulated con-
r composed of leaves, rod, and ball, a charge opposite to that
e influencing body. The finger must be removed while the
ncing body remains in position, as the amount of the induced
e depends upon the position of the influencing body at the
ıt of breaking connection. On now withdrawing the influencing
the charge of unlike electricity is no longer attracted to the
ut spreads over the whole of the conductor, and causes the
to diverge. If, while this divergence continues, an excited
ube, when gradually brought towards the ball, diminishes the
ence, we know that the body in question was electrified posi-
If it increases the divergence, the body was electrified nega-

t caution must be used in bringing electrified bodies near the
af electroscope, as the leaves are very apt to be ruptured by

quick movements. If they diverge so widely as to touch the sides of the bell-glass, it is often difficult to detach them from the glass without tearing. To prevent this contact, two metallic columns are interposed, communicating with the ground. If the leaves diverge too widely, they touch these columns and lose their electricity.

CHAPTER XXXVII.

MEASUREMENT OF ELECTRICAL FORCES.

16. **Coulomb's Torsion-balance.**—Coulomb, who was the first to e electricity an accurate science, employed in his researches an rument which is often called after his name, and which is still nsively employed. It depends on the principle that the torsion wire is simply proportional to the twisting couple. We shall describe it, and then point out some of its applications.

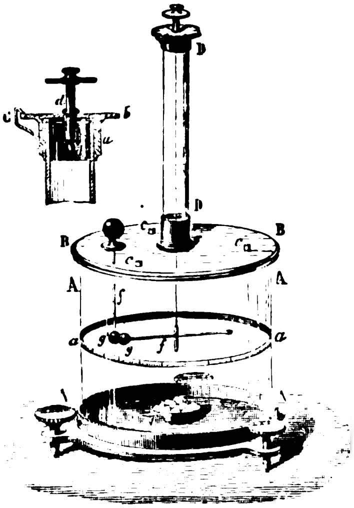

Fig. 340.—Coulomb's Torsion-balance.

consists of a cylindrical glass AA (Fig. 340), from the upper B of which rises another glass der DD of much smaller dia- r. This small cylinder is fitted e top with a brass cap *a*, carry- n index C. Outside of this, apable of turning round it, is er cap *b*, the top of which is d into 360 equal parts. In ntre of the cap *b* is an opening h which passes a small metal er *d*, capable of turning in pening with moderate fric- nd having at its lower end h or slit. When the cap *b* ed, the cylinder *d* turns with t the latter can also be separately, so as not to change the reading. These parts com- e *torsion-head.* A very fine metallic wire is held by the notch, pports a small piece of metal, through which passes a light f shellac *f*, carrying at one end a small gilt ball *g*. A circular scale runs round the outside of the large cylinder in the plane of the needle. Finally, opposite the zero of this scale, there is a fixed ball g' of some conducting material, supported by a rod f of shellac, which passes through a hole in the cover of the cylindrical case.

417. Laws of Electric Repulsion.—To illustrate the mode of employing this apparatus for electrical measurements, we shall explain the course followed by Coulomb in investigating the law according to which electrical repulsions and attractions vary with the distance. The index is set to the zero of the scale. The inner cylinder d is then turned, until the movable ball just touches the fixed ball without any torsion of the wire. The fixed ball is then taken out, placed in communication with an electrified body, and replaced in the apparatus. The electricity with which it is charged is communicated to the movable ball, and causes the repulsion of this latter through a number of degrees indicated by the scale which surrounds the case. In this position the force of repulsion is in equilibrium with the force of torsion tending to bring back the ball to its original position. The graduated cap b is then turned so as to oppose the repulsion. The movable ball is thus brought nearer to the fixed ball, and at the same time the amount of torsion in the wire is increased. By repeating this process, we obtain a number of different positions in which repulsion is balanced by torsion. But we know, from the laws of elasticity, that the force (strictly the couple[1]) of torsion is proportional to the angle of torsion. Hence we have only to compare the total amounts of torsion with the distances of the two balls. By such comparisons Coulomb found that the force of electrical repulsion varies *inversely as the square of the distance.*

The following are the actual numbers obtained in one of the experiments. The original deviation of the movable ball being 36°, it was found that, in order to reduce this distance to 18°, it was necessary to turn the head through 126°, and, for a farther reduction of the deviation to 8°·5, an additional rotation through 441° was required. It will thus be perceived that at the distances of 36°, 18°, and 8°·5, which may be practically considered as in the ratio of 1, ½, and ¼, the forces of repulsion were equilibrated by torsions of 36°,

---

[1] The repulsive force on the movable ball is equivalent to an equal and parallel force acting at the centre of the needle (the point of attachment of the wire), and a couple whose arm is the perpendicular from this centre on the line joining the balls. This couple must be equal to the couple of torsion. The other component produces a small deviation of the suspending wire from the vertical.

$6^\circ+18^\circ=144^\circ$, and $441+126+8{\cdot}5=575^\circ{\cdot}5$ respectively. Now 144 36×4, and 575·5 may be considered as 576, or 36×16. Hence perceive that, as the distance is divided by 2, or by 4, the force epulsion is multiplied by 4 or by 16, which precisely agrees with law enunciated above.

18. **Equation of Equilibrium.**—We must, however, observe that his mode of reducing the obserons two inaccurate assumptions made. First, the distance been the balls is regarded as being l to the arc which lies between , whereas it is really the chord at arc. Secondly, the force pulsion is regarded as acting ys at the same arm, whereas rm, being the perpendicular the centre on the chord, dimis as the distance increases. The following investigation is more us.

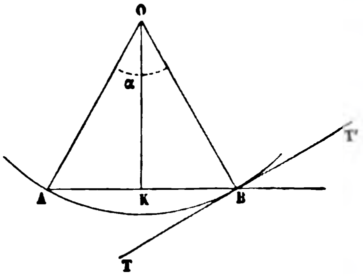

Fig. 341.

AOB (Fig. 341), the angular distance of the balls, be denoted and let l be the length of the radius OA. Then the chord $2l \sin \frac{1}{2} \alpha$, and the arm OK is $l \cos \frac{1}{2} \alpha$. Let f denote the f repulsion at unit distance, and n the couple of torsion for 1°. he force of repulsion in the given position is $\frac{f}{4l^2 \sin^2 \frac{1}{2} \alpha}$ if the inverse squares be true, and the moment of this about the is $\frac{f \cos \frac{1}{2} \alpha}{4l \sin^2 \frac{1}{2} \alpha}$, which must be equal to nA, if A be the number ees of torsion. Hence we have

$$\frac{f}{4nl} = \mathrm{A} \sin \tfrac{1}{2} \alpha \tan \tfrac{1}{2} \alpha,$$

he first member of this equation is constant, the second member t be constant also for different values of A and α, if the law se squares be true. The degree of constancy is shown by the g table:—

	α	A	A sin ½ α tan ½ α
st experiment, ...	36	36	3·614
l experiment, ...	18	144	3·568
l experiment, ...	8·5	575·5	3·169
pposed case, ...	9	576	3·557

fference between the first and second numbers of the last column is insignificant. That between the second and third is more considerable,[1] but in reality only corresponds to an error of half a degree in the measurement of the arc.

419. Case of Attraction.—The law of attractions may be investigated by a similar method. The index is set to zero, and the central piece is turned so as to place the movable ball at a known distance from the fixed ball. The two balls are then charged with electricity of different kinds. The movable ball is accordingly attracted towards the other, and settles in a position in which attraction is balanced by torsion. By altering the amount of torsion, different positions of the ball can be obtained. On comparing the distances with the corresponding torsions, it is found that the same law holds as in the case of repulsion. The experiment, however, is difficult, and is only possible when the balls are very feebly electrified. To prevent the contact of the two balls, Coulomb fixed a silk thread in the instrument, so as to stop the course of the movable ball.

420. Law of Attraction and Repulsion as depending on Amount of Charge.—We may assume as evident, that when an electrified ball is placed in contact with a precisely equal and similar ball, the charge will be divided equally between them, so that the first will retain only half the charge which it had before contact.

Suppose that an observation on repulsion has just been made with the torsion-balance, and that we touch the fixed ball with another exactly equal insulated ball, which we then remove. It will be found that the amount of torsion requisite for keeping the movable ball in its observed position is just half what it was before. The e result will be obtained by touching the movable ball with a of its own size. We conclude that, if the charge of either body ltered, the attractive or repulsive force between the bodies at n distance will be altered in the same ratio. The law is not ously true for bodies of finite size, unless the distribution of the ricity on the two bodies remains unchanged. When the two s are very small in all their dimensions in comparison with istance between them, their mutual force is represented by the ssion

---

[1] We have already seen that the mutual induction of two conductors tends to diminish their mutual repulsion, and that this inductive action becomes more important as the distance is diminished. Hence the repulsion at distance 9 should be less than a quarter of that at distance 18. The apparent error thus confirms the law.

Many persons have adduced, as tending to overthrow Coulomb's law of inverse squares, experimental results which really confirm it. Except when the dimensions of the charged bodies are very small in comparison with the distance, the observed attraction or repulsion is the resultant of an infinite number of forces acting along lines drawn from the different points of the one body to the different points of the other. The law of inverse squares applies directly to these several components, and not to the resultant which they yield. The latter can only be computed by elaborate mathematical processes.

It is incorrectly assumed in the text that the law ought to apply directly to two spheres, when by their distance we understand the distance between their nearest points. It is not obvious that the distance of the nearest points should give a better result than the distance between the centres.

The strongest evidence for the rigorous exactness of the law of inverse squares is indirect; see § 421 c.

$$\frac{q q'}{D^2},$$

l q' denoting their charges, and D the distance. If this ssion has the positive sign, the force is repulsive, if negative, tive.

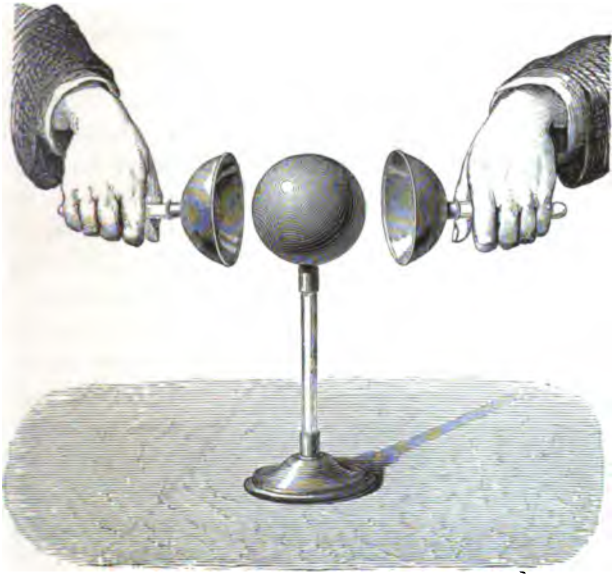

Fig. 342.—Biot's Experiment.

. **Electricity resides on the Surface.**—Electricity (subject to the is mentioned below) resides exclusively on the external f a conductor. This is perhaps implied in the experimental ently observed by Coulomb, that when a solid and a hollow equal external diameter are allowed to touch each other, e possessed by either is divided equally between them. A direct demonstration is afforded by the following experiment of Biot:—

We take an insulated sphere of metal, charge it with electricity, and cover it with two hemispheres furnished with insulating handles, which fit the sphere exactly (Fig. 342). If the two hemispheres be quickly removed, and presented to an electric pendulum, they will be found to be electrified, while the sphere itself will show hardly any traces of electricity. We must, however, remark that this experiment is rarely successful, and that generally the sphere remains very sensibly electrified. The reason of this is, that it is very difficult to remove the hemispheres so steadily, as not to permit their edges to touch the sphere after the first separation.

The following is a much surer form of the experiment:—

A hollow insulated sphere, with an orifice in the top, is charged with electricity (Fig. 343). A *proof-plane*, consisting of a small disc of gilt paper insulated by a thin handle of shellac, is then applied to the interior surface of the sphere, and, when tested by an electric pendulum or an electroscope, is found to exhibit no trace of electricity. But if, on the contrary, the disc be applied to the external surface of the sphere, it will be found to be electrified, and capable of attracting light bodies. Faraday varied this experiment, by substituting a cylinder of wire-gauze for the sphere. This cylinder rested on an insulated disc of metal. The disc was charged with electricity, and it was found that no trace of the electricity could be detected by applying the proof-plane to the interior surface of the cylinder.

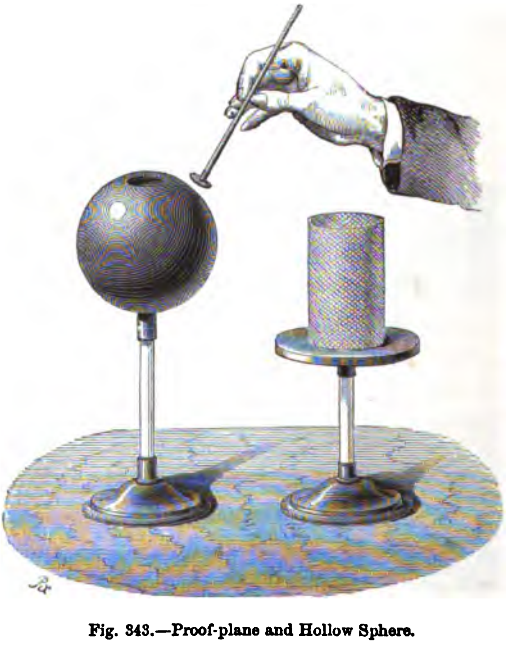

Fig. 343.—Proof-plane and Hollow Sphere.

The following experiment is also due to Faraday. A metal ring
ixed upon an insulating stand (Fig. 344). To this ring is attached
one-shaped bag of fine linen, which is a conductor of electricity.
ilk thread, attached to the apex of the cone, and extending both

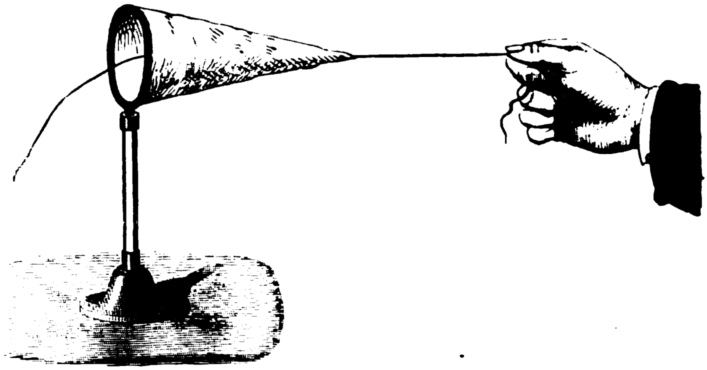

Fig. 344.—Faraday's Experiment.

enables the operator to turn the bag inside out as often as
ed, without discharging it. When the bag is electrified, the
tion of the proof-plane always shows that there is electricity
outer, but not on the inner surface. When the bag is turned
out, the electricity therefore passes from one surface of the
o the other.

Limitations of the Rule.—There are two exceptions to the
at electricity is confined to the external surface of a conductor.
does not hold for electric currents. We shall see hereafter in
ion with galvanic electricity, that the resistance which a wire
n length opposes to the passage of electricity through it,
not upon its circumference but upon its sectional area. A
wire will not conduct electricity so well as a solid wire of the
ternal diameter.

ctricity may be induced on the inner surface of a hollow
r by the presence of an electrified body insulated from the
r itself. If an insulated body charged with electricity be
ed into the interior of a hollow conductor, so as to be com-
urrounded by it, but still insulated from it, it induces upon
surface a quantity equal to its own charge, but of opposite
the conductor is insulated, an equal quantity, but of the same
e charge of the inclosed body, is repelled to the outside, and

this is true whether the conductor has an independent charge of its own or not. In this case, then, we have electricity residing on both the external and the internal surfaces of a hollow conductor, but it still resides only on the surfaces.

If a conducting body connected with the earth be introduced into the interior of a hollow charged conductor, so as to be partially surrounded by it, the body thus introduced will acquire an opposite charge by induction, and, by the reciprocal action of this charge, electricity will be induced on the inner at the expense of the outer surface of the hollow conductor, just as in the preceding case.

421 B. Ice-pail Experiment.—The effect of introducing a charged body within a hollow conductor is well illustrated by the following experiments of Faraday. Let A (Fig. 344 A) represent an insulated pewter ice-pail, ten and a half inches high and seven inches in diameter, connected by a wire with a delicate gold leaf electroscope E, and let C be a round brass ball insulated by a dry thread of white silk, three or four feet in length, so as to remove the influence of the hand holding it from the ice-pail below. Let A be perfectly discharged, and let C, after being charged at a distance, be introduced into A as in the figure. If C be positive, E also will diverge positively; if C be taken away, E will collapse perfectly, the apparatus being in good order. As C enters the vessel A, the divergence of E will increase until C is about three inches below the edge of the vessel, and will remain quite steady and unchanged for any greater depression. If C be made to touch the bottom of A, all its charge is communicated to A, and C, upon being withdrawn and examined, is found perfectly discharged. Now Faraday found that at the moment of contact of C with the bottom of A, not the slightest change took place in the divergence of the gold-leaves. Hence the charge previously developed by induction on the outside of A must have been precisely equal to that acquired by the contact, that is, must have been equal to the charge of C.

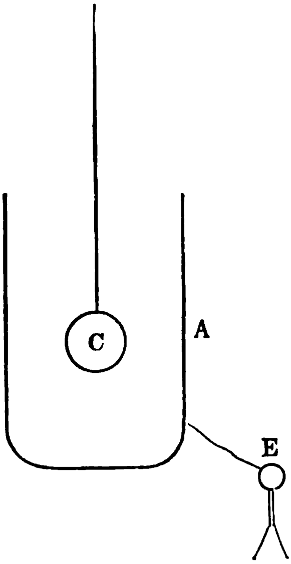

Fig. 344 A.—Ice-pail Experiment.

He then employed four ice-pails (Fig. 344 B), arranged one within the other, the smallest innermost, insulated from each other by plates of shellac at the bottom, the outermost pail being connected with the electroscope. When the charged carrier-ball C was introduced within the innermost pail, and lowered until it touched the bottom, the electrometer gave precisely the same indications as when the outermost pail was employed alone. When the innermost was lifted out by a silk thread after being touched by C, the gold-leaves collapsed perfectly. When it was introduced again, they opened out to the same extent as before. When 4 and 3 were connected by a wire let down between them by a silk thread, the leaves remained unchanged, and so they still remained when 3 and 2 were connected, and finally when all four pails were connected.

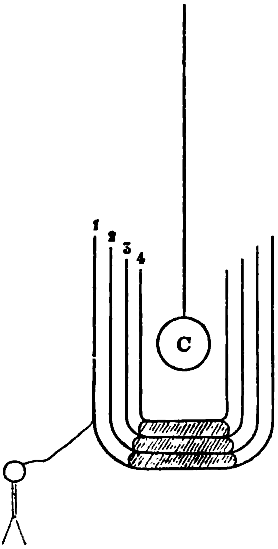

Fig. 344 B.—Experiment with Four Ice-pails.

421c. No Force within a Conductor.—When a hollow conductor is electrified, however strongly, no effect is produced upon pith-balls, gold-leaves, or any other electroscopic apparatus in the interior, whether connected with the hollow conductor, or insulated from it, provided, in the latter case, that they have no communication with bodies external to the hollow conductor. Faraday constructed a cubical box, measuring 12 feet each way, covered externally with copper wire and tin-foil, and insulated from the earth. He charged this box very strongly by outside communication with a powerful electrical machine; but a gold-leaf electrometer within showed no effect. He says, "I went into the cube and lived in it, using lighted candles, electrometers, and all other tests of electrical states. I could not find the least influence upon them, or indication of anything particular given by them, though all the time the outside of the cube was powerfully charged, and large sparks and brushes were darting off from every part of its outer surface."

The fact that electricity resides only on the external surface of a conductor, combined with the fact that there is no electrical force in the space inclosed by this surface, affords a rigorous proof of the law of inverse squares. For if the conductor be a sphere removed from the influence of external bodies, its charge must be distributed uniformly over its surface. Now it admits of proof, and is well known to mathematicians, that a uniform spherical shell exerts no attraction at any point of the interior space, if the law of attraction be that of inverse squares, and that the internal attraction does not vanish for any other law.

421 D. **Electrical Density and Distribution.**—When the proof-plane is applied to different parts of the surface of a conductor, the quantities of electricity which it carries off are not usually equal. But the electricity carried off by the proof-plane is simply the electricity which resided on the part of the surface covered by it, for the proof-plane during the time of its contact is virtually part of the surface of the conductor. We must therefore conclude that equal areas on different parts of the surface of a conductor have not equal amounts of electricity upon them. It is also found that if the charge of the conductor be varied, the electricity resident upon any specified portion of the surface is changed in the same ratio. The ratio of the quantities of electricity on two specified portions of the surface is in fact independent of the charge, and depends only on the form of the conductor. This is expressed by saying that *distribution* is independent of charge, and that the distribution of electricity on the surface of a conductor depends on its form.

By the *average electrical density* on the whole or any specified portion of the surface of a conductor, is meant the quantity of electricity upon it, divided by its area. By the *electrical density at a specified point* on the surface of a conductor, is meant the average electrical density on an exceedingly small area surrounding it, in other words, the *quantity of electricity per unit area* at the point. The name is appropriate, from the analogy of ordinary material density, which is mass per unit volume, and is not intended to imply any hypothesis as to the nature of electricity. The name was introduced by Coulomb, who first investigated the subject in question, and is generally employed by the best electricians in this country. The term *thickness of electrical stratum,* which was introduced by Poisson, is much used in France, but is more open to objection from the coarse assumptions which it seems to involve.

The following are some of Coulomb's results. The dotted line in each of the figures is intended to represent, by its distance from the outline of the conductor, the electric density at each point of the latter. In all cases it is to be understood that the conductor is so far removed from external bodies as not to be influenced by them:—

1. *Sphere* (Fig. 345). The electric density is the same for all points on the surface of a spherical conductor.

2. *Ellipsoid* (Fig. 346). The density is greatest at the ends of the longest, and least at the ends of the shortest axis; and the densities at these points are simply proportional to the axes themselves.[1]

Fig. 345.—Distribution on Sphere.

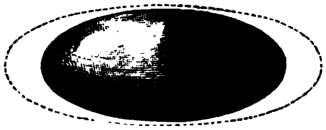

Fig. 346.—Distribution on Ellipsoid.

3. *Flat Disc* (Fig. 347). The density is almost inappreciable over the whole of both faces, except close to the edges, where it increases almost *per saltum.*

4. *Cylinder with Hemispherical Ends* (Fig. 348). The density is a minimum, and nearly uniform, at parts remote from the ends, and attains a maximum at the ends. The ratio of the density at the ends to that at the sides increases as the radius of the cylinder diminishes, the length of the cylinder remaining the same.

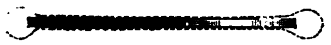

Fig. 347.—Distribution on Disc.

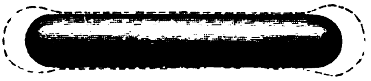

Fig. 348.—Distribution on Cylinder with rounded ends.

5. *Spheres in Contact.*—In the case of equal spheres, the charge, which is nothing at the point of contact, and very feeble up to 30° from that point, increases very rapidly from 30° to 60°, less rapidly from 60° to 90°, and almost insensibly from 90° to 180°. When the spheres are of unequal size, the charge at any point on the smaller

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[1] More generally, the density at any point on the surface of an ellipsoid is proportional to the length of a perpendicular from the centre of the ellipsoid on a tangent plane at the point.

If an ellipsoid, similar and nearly equal to the given one, be placed so that the corresponding axes of the two are coincident, we shall have a thin ellipsoidal shell, whose thickness at any point exactly represents the electric density at that point.

Such a shell, if composed of homogeneous matter attracting inversely as the square of the distance, would exercise no force at points in its interior.

sphere is greater than at the corresponding point on the larger one; and as the smaller sphere is continually diminished, the other remaining the same, the ratio of the densities at the extremities of the line of centres tends to become 2 : 1.

422. Method of Experiment.—The preceding results were obtained by Coulomb in the following manner. He touched the electrified body at a known point with the proof-plane, and then put the plane in the place of the fixed ball of the torsion-balance, the movable ball having previously been charged with electricity of the same sign. Repulsion was thus produced, and the amount of torsion necessary to keep the balls at a certain distance asunder was observed. He then repeated the experiment with electricity taken from a different point of the body under examination, and the ratio of the densities at the two points was given by the ratio of the torsions necessary to keep the balls at the same distance.

By way of checking the accuracy of this mode of experimentation, Coulomb electrified an insulated sphere, and measured the electric density on its surface by the method described above. He then touched the sphere with another precisely equal sphere, and on again applying the proof-plane he found that the charge carried off by the plane was just half what it had been before.

423. Alternate Contact.—The above experiments naturally require some time, during which the body under investigation is gradually losing its charge. The consequence is, that the densities indicated by the balance, if taken singly, do not correctly represent the electric distribution. This source of error was avoided by Coulomb in the following manner. He touched two points on the body successively, and determined the electric density at each; and then, after an interval equal to that between the two experiments, he touched the first point again, and obtained a second measure of its density, which was less than the first, on account of the dissipation of electricity. If the densities thus observed be denoted by A and A′, and the density observed at the second point by B, it is evident that $\frac{A}{B}$ is greater, and $\frac{A'}{B}$ less than the ratio required. Coulomb adopted, as the correct value, their arithmetic mean $\frac{1}{2}\,\frac{A+A'}{B}$.

424. Power of Points.—The distribution of electricity on a conductor of any form may be roughly described, by saying that the density is greatest on those parts of the surface which project most, or which have the sharpest convexity, and that in depressions or concavities it is small or altogether insensible. Theory shows that at a perfectly sharp edge, such, for example, as is formed by two planes meeting at any angle however obtuse, but *not rounded off*, the density must be infinite, and *a fortiori* it must be infinite at a perfectly sharp point, for example at the apex of a cone, however obtuse, *if not rounded off*. Practically, the points and edges of bodies are always rounded off; the microscope shows them merely as places of very sharp convexity (that is, of very small radius of curvature), and hence the electric density at those places is really finite; but it is exceedingly great in comparison with the density at other parts, and this is especially true of very acute points, such as the point of a fine needle. The consequence is, that if a pointed conductor is insulated and charged, the concentration of a large amount of repulsive force within an exceedingly small area produces very rapid escape of electricity at the points. Conductors intended to retain a charge of electricity must have no points or edges, and must be very smooth. If of considerable length in proportion to their breadth, they are usually made to terminate in large knobs.

425. Dissipation of Charge.—When an insulated conductor is charged and left to itself, its charge is gradually dissipated, and at length completely disappears. This loss takes place partly through the supports, and partly through the air.

As regards the supports, the loss occurs chiefly at their surface, especially if (as is usually the case) they are not perfectly dry. It is diminished by diminishing their perimeter, and by increasing their length; for example, a long fibre of glass or raw silk is an excellent insulator.

As regards the air, we must distinguish between conduction and convection. Moist air and highly rarefied air probably act as conductors; but with air that is relatively dry the loss is probably chiefly due to contact and convection. Successive layers of air become electrified by contact with the conductor, and are then repelled, carrying off the electricity which they have acquired. It is by an action of this kind that electricity escapes into the air from points, as is proved by the wind which passes off from them (§ 444). Particles of dust present in the air, in like manner, act as carriers, being attracted to the conductor, charged by contact with it, and then repelled. They also frequently adhere by one end to the conductor, and thus constitute pointed projections through which electricity is discharged into the air.

Coulomb deduced from his observations on dissipation of charge, a law precisely analogous to Newton's law of cooling, namely, that when all other circumstances remain the same, *the rate of loss is simply proportional to the charge*, so that the charges at equal intervals of time form a decreasing geometric series. Subsequent experience has confirmed this law, as approximately true for moderate charges of the same sign. Negative charges are, however, dissipated more rapidly than positive.

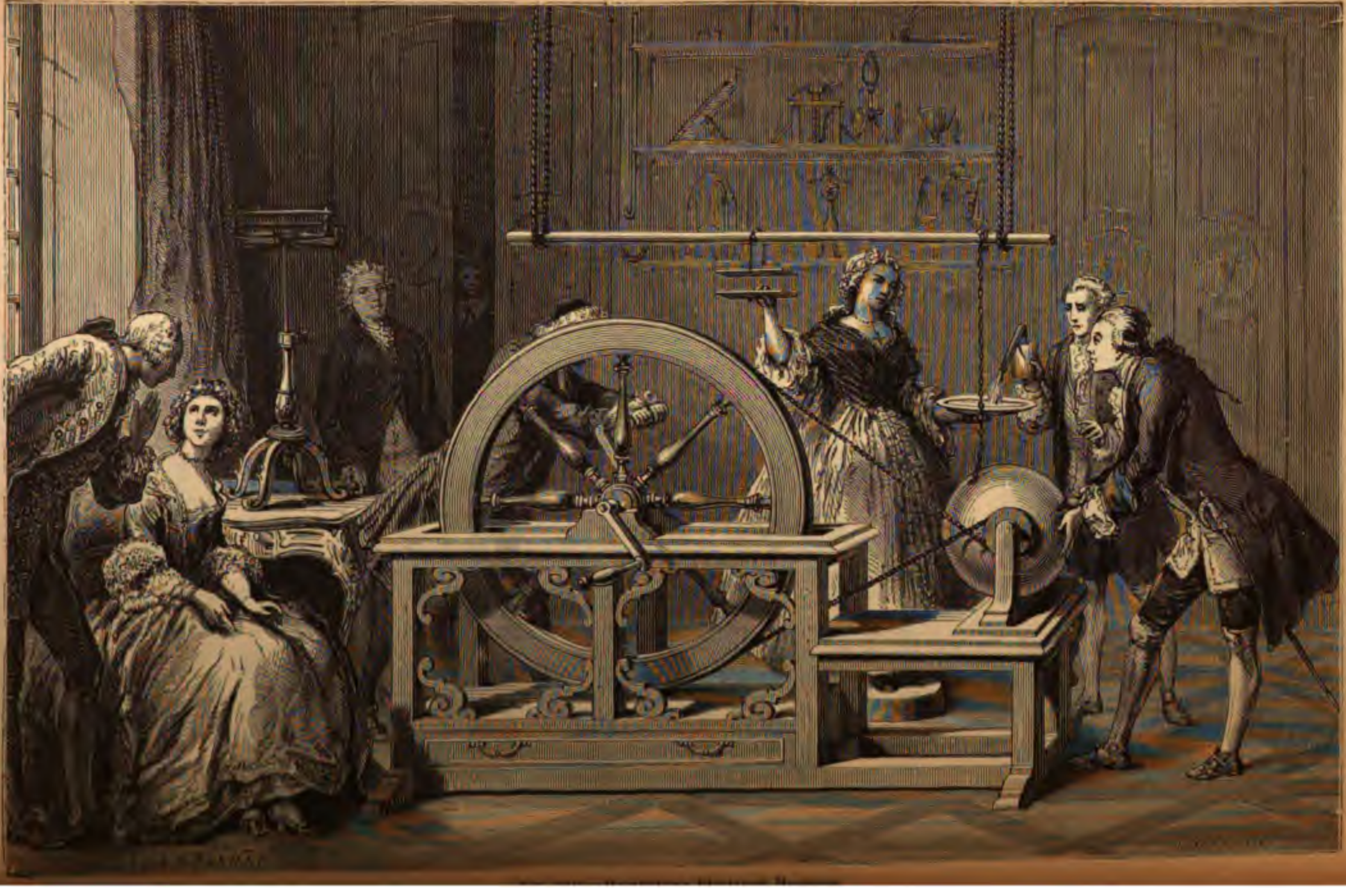
CHAPTER XXXVIII.

ELECTRICAL MACHINES.

426. Electrical Machines.—The first electrical machine was invented by Otto Guericke, to whom, as we have already seen (§ 129), science is indebted for the invention of the air-pump. It consisted of a ball of sulphur which was turned upon its axis by one person, while another held his hands upon the ball, thus causing the friction necessary for the production of electricity. The result was that the globe was negatively electrified, and the positive electricity escaped into the earth through the hands of the operator. This machine, however, was capable of producing only very feeble effects, and the sparks obtained from it were visible only in the dark. An English philosopher, Hawksbee, substituted a globe of glass for the globe of sulphur; the electricity thus obtained was positive, and the sparks obtained by the new machine were of considerable brightness. The machine, however, was for the time superseded by the use of glass tubes, which continued to be the favourite instruments for generating electricity until the middle of the eighteenth century, when a German philosopher, Boze, professor of physics at Wittemberg, revived and perfected Hawksbee's machine, which became universally adopted.

Fig. 349, which is taken from the *Leçons de Physique* of the Abbé Nollet, published in 1767, shows the arrangement of the machine adopted by this celebrated philosopher. It consists of a large wheel, round which is passed an endless cord, which, passing also round a pulley, serves to turn a glass globe when the wheel is set in motion. The electricity thus produced is collected on a conductor suspended from the ceiling by silk cords.

It will be observed that, in the figure, the friction is produced by the hand. This mode of applying friction, which is evidently rude



and defective, was nevertheless long used for want of a better, though many attempts were made to replace it by the use of rubbers of leather, stuffed with hair, and pressed against the globe by means of regulating screws. The shape of the globe rendered the use of these very difficult, and it was not until a cylinder was substituted for the globe that they were generally adopted.

427. Ramsden's Machine.—The kind of machine most commonly employed at present is the plate-machine, invented by Ramsden about 1768, and only slightly changed and improved since.

The most usual form of this machine is shown in Fig. 350. It has a circular plate of glass, which turns on an axis supported by two wooden uprights. On each side of the plate, at the upper and lower parts of the uprights, are two cushions, which act as rubbers when the plate is turned. In front of the plate are two metallic conductors supported on glass legs, and terminating in branches, which are bent round the plate at the middle of its height,

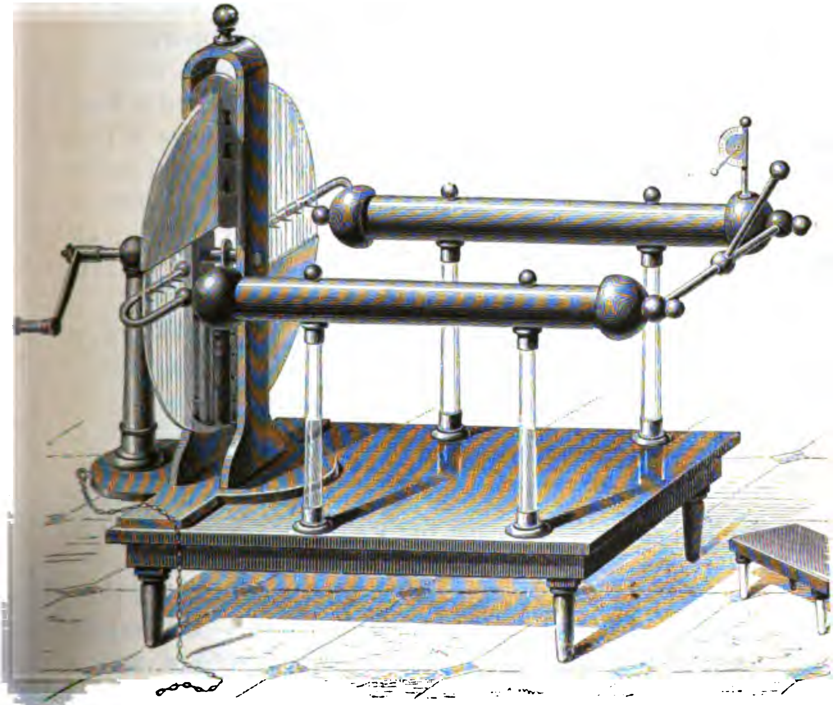


Fig. 350.—Ramsden's Electrical Machine.

and are studded with points projecting towards it. The plate becomes charged with positive electricity by friction against the cushions, and gives off its electricity through the points to the two conductors, or, what amounts to the same thing, the conductors give off negative electricity through the points to the positively-electrified plate. In order to avoid loss of electricity from that portion of the plate which is passing from the cushions to the points, sector-shaped pieces of oiled silk are placed so as to cover it on both sides. The cushions become negatively electrified by the friction; and the machine will not continue working unless this negative electricity is allowed to escape. The cushions are accordingly connected with the earth by means of metal plates let into their supports.

428. Limit of Charge.—As the conductors become more highly charged, they lose electricity to the air more rapidly, and a time soon arrives when they lose electricity as fast as they receive it from the plate. After this, if the machine continues to be worked uniformly, their charge remains nearly constant. This limiting amount of charge depends very much upon the condition of the air; and in damp weather the machine often refuses to work unless special means are employed to keep it dry.

The rubbers are covered with a metallic preparation, of which several different kinds are employed. Sometimes it is the compound called *aurum musivum* (bisulphide of tin), but more frequently an amalgam. Kienmeier's amalgam consists of one part of zinc, one of tin, and two of mercury. The amalgam is mixed with grease to make it adhere to the leather or silk which forms the face of the cushion.

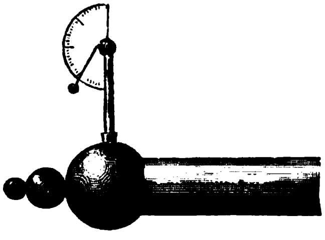


Fig. 351.—Quadrant Electroscope.

Before using the machine, the glass legs which support the conductors should be wiped with a warm dry cloth. The plate must also be cleaned from any dust or portions of amalgam which may adhere to it, and lastly, dried with a hot cloth or paper. When these precautions are taken, the machine, if standing near a fire, will always work; but the charging of Leyden jars, and especially of batteries, may be rendered impossible by bad weather.

The variations of charge are indicated by the *quadrant electroscope* (Fig. 351), which is attached to one of the conductors. It consists of an upright conducting stem, supporting a quadrant, or more commonly a semicircle, of ivory, at whose centre a light needle of ivory is jointed, carrying a pith-ball at its end. When there is no charge in the conductor, this pendulum hangs vertically, and as the charge increases it is repelled further and further from the stem. In damp weather it will be observed to return to the vertical position almost immediately on ceasing to turn the machine, while in very favourable circumstances it gives a sensible indication of charge after two or three minutes.

429. Nairne's Machine.—Ramsden's machine furnishes only positive electricity. In order to obtain negative electricity, it is necessary to insulate the cushions from the ground, and to place them in communication with an insulated conductor. An arrangement of this kind is adopted in Nairne's machine.

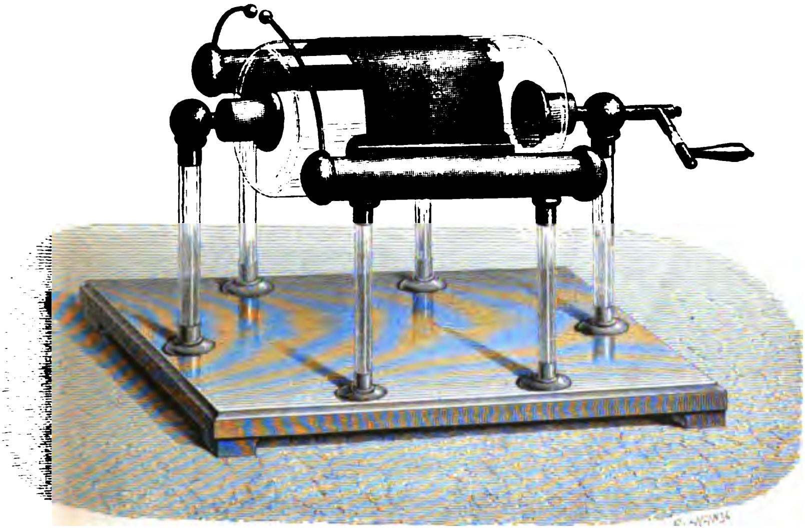


Fig. 352.—Nairne's Electrical Machine.

In this machine a large cylinder of glass revolves between two separately insulated conductors. One of these has a row of points projecting towards the glass, and collects positive electricity. The other is connected with the rubber, and collects negative. If one kind of electricity only is required, the conductor which furnishes the other must be connected with the ground.

430. Winter's Machine.—Winter, of Vienna, has introduced some modifications in Ramsden's machine.

Instead of four cushions, there are, as will be seen by the figure (Fig. 353), only two, which are in communication with a spherical conductor, supported on a glass pillar. This may be used to collect negative electricity, in the same way as the negative conductor in Nairne's machine. The chief or positive conductor consists of an insulated sphere, on the top of which is often another sphere of smaller size. The positive electricity is collected from the plate by means of two rings opposite to each other, one on each side of the plate. On the side next the plate, they have a groove, which is lined with metal, and studded with points. They are supported by an arm which is inserted in the positive conductor. The size of the positive conductor is often increased by the addition of a very large ring (3 or 4 feet in diameter) which is supported on the top of the large sphere. The ring consists of very stout brass wire inclosed in well-polished mahogany.

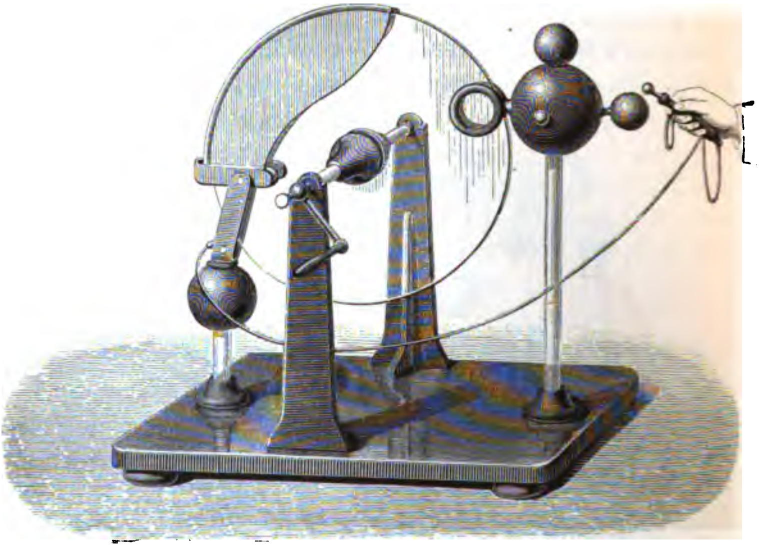


Fig. 353.—Winter's Electrical Machine.

Winter's machine appears to give longer sparks than the ordinary machine under the same circumstances. This circumstance is owing, partly at least, to the considerable distance between the rubber and the positive conductor, which prevents the occurrence of discharges between them.

431. Hydro-electric Machine.—About the year 1840, Mr. (now Sir) W. Armstrong invented an electric machine, in which electricity was generated by the friction of steam against the sides of orifices, through which it is allowed to escape under high pressure. It consists of a

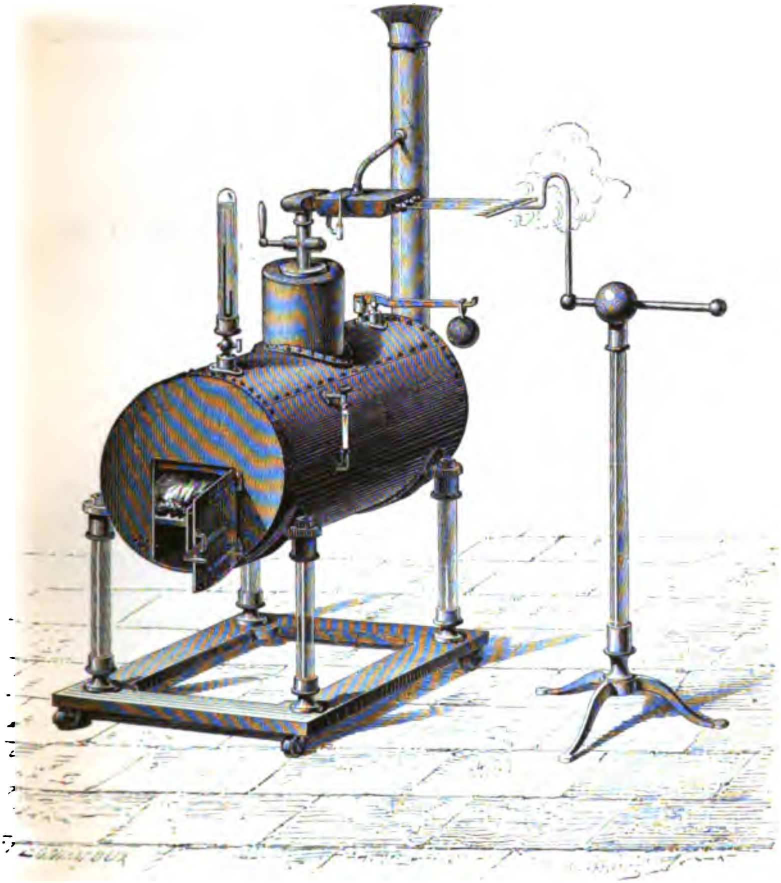


Fig. 354.—Armstrong's Hydro-electric Machine.

boiler with the fire inside, supported on four glass legs. The steam, before escaping, passes through a number of tubes which traverse a cooling-box containing water, into which dip meshes of cotton, which are led over the tubes, and passed round them. The cooling thus produced in the tubes, causes partial condensation of the steam. This has been found to be an indispensable condition, the friction of perfectly dry steam being quite inoperative. Speaking strictly, it is the friction of the drops of water against the sides of the orifice, which generates the electricity, and the steam merely furnishes the means of applying the friction. The jet of steam is positively, and the boiler negatively electrified. The positive electricity is collected by directing the jet of steam upon a metal comb communicating with an insulated conductor.

The form of the outlet by which the steam escapes is shown in Fig. 355. The steam is checked in its course by a tongue of metal, round which it has to pass, before it can enter the wooden tube through which it escapes into the air. This machine, in order to work well, requires a pressure of several atmospheres. The water in the boiler should be distilled water. If a saline solution be introduced into the tube through which the steam escapes, all traces of electricity immediately disappear. The generation of electricity varies both in sign and degree, according to the substance of which the escape-tube is composed, and according to the liquid whose particles are carried out by the steam. Thus, when a small quantity of oil of turpentine is introduced into the jet of steam, the boiler becomes positively, and the steam negatively electrified.

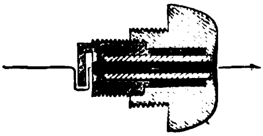


Fig. 355.—Outlet of Steam.

The hydro-electric machine is exceedingly powerful. At the Polytechnic Institution in London, there was one with a boiler 78 inches long and 42 in diameter, and with 46 jets. Sparks were obtained from the conductor at the distance of 22 inches. The machine is, however, very inconvenient to manage. A long time is required to get up the requisite pressure of steam. The boiler must be carefully washed with a solution of potash, after each occasion of its use; and, finally, the working of the machine is necessarily accompanied by the disengagement of an enormous quantity of steam which, besides causing a deafening noise, has the mischievous effect of covering with moisture everything within reach. Accordingly, though very interesting in itself, it is by no means adapted to the general purposes of an electrical machine.

432. **Holtz's Machine.**—In the machines just described, electricity is produced by the friction of one substance against another. Quite recently, several machines have been invented of quite a different kind, in which a body is electrified once for all, and made to act by

induction upon a movable system, in such a way as to produce a continual generation of electricity. The most successful of these is that invented by Holtz of Berlin in 1865.

It contains two thin circular plates of glass, one of which, A, is fixed, while the other, B, which is rather smaller, can be made to revolve very near it. In the fixed plate there are two large holes or windows near the extremities of its horizontal diameter. Across these, and partly covering them, are glued two paper bands or armatures, having points *f*, *f'* directed the opposite way to that in which the movement takes place. Two metallic combs P, P' are placed opposite the windows, on the other side of the revolving plate, and are connected with two insulated conductors terminating in the knobs *n*, *m*, which may be called the poles or electrodes of the machine. These knobs can be set at any distance asunder. In starting the machine, they are placed in contact, and one of the armatures, suppose *f*, is electrified by holding against it a sheet of vulcanite which has been charged with negative electricity by fric-

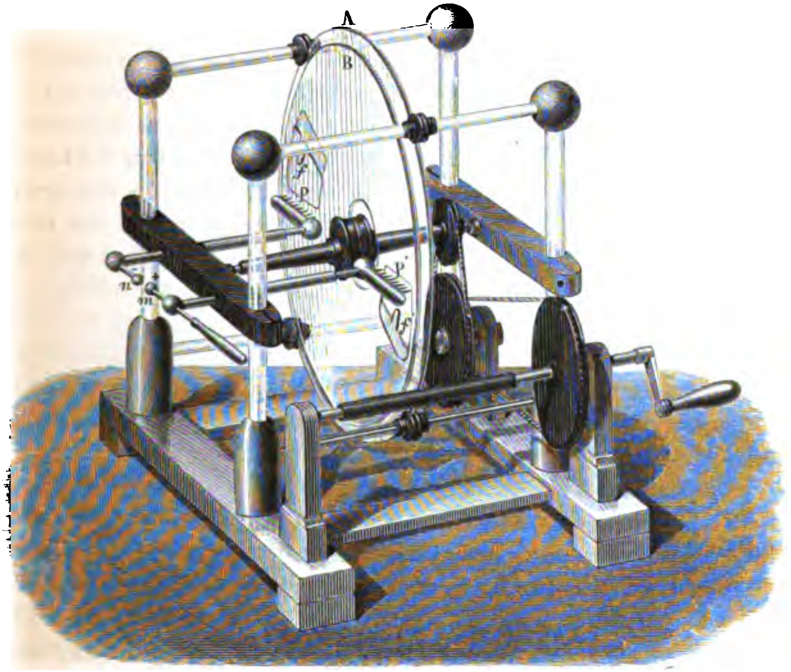


Fig. 356.—Holtz's Electrical Machine.

tion. The plate is then turned for a few seconds, and the two knobs are gradually separated.

A continuous crackling noise is immediately produced at the place of separation, resulting from electric discharge across the interval. In the circumstances supposed, the knob *n* is the negative, and the knob *m* the positive electrode. In dry weather, the machine, when once started, will continue in action for a long time if the motion is kept up, but it soon ceases to act if the air is damp, being even more sensitive to moisture than the ordinary machine.

The action of the machine is as follows:—The negative electricity of the armature *f*, acting inductively on the opposed conductor, from which it is separated by the revolving plate, causes this conductor to discharge positive electricity, through the comb, upon the face of the plate, and thus to acquire a negative charge; when the part of the plate which has been thus affected comes opposite the other armature, the latter is affected inductively, and discharges negative electricity through its point *f′* upon the back of the plate, thus becoming itself positively electrified. Positive electricity from the front of the plate is at the same time collected by the comb P′, an equal quantity of negative being of course discharged from the comb upon the plate. In the subsequent stages of the process, the negative electricity thus discharged upon the face of the plate exceeds the positive which was previously there, so that the face of the plate passes on with a negative charge. When the portion of the plate which we are considering again comes opposite *f*, it increases the negative electrification both of the armature and the conductor, inasmuch as it has more of negative or less of positive electricity upon both its surfaces than it had when it last moved away from that position. Both armatures thus become more and more strongly electrified, until a limit is attained which depends on the goodness of the insulation; and as the electrification of the armatures increases, the conductors also become more powerfully affected, and are able to discharge to each other by the knobs *mn* at a continually increasing distance.

The inventor has recently introduced a modified form of his machine. The plates are placed horizontally (Fig. 357), they have neither windows nor armatures, and they both revolve, but in opposite directions. Two conductors furnished with rows of points are placed above the upper plate at the extremities of one diameter, and two others below the lower plate at the extremities of another diameter perpendicular to the former. Each of the upper conductors is connected with one of the lower, so that there are virtually only two conductors. In starting the machine, a sector of electrified vulcanite is held over the upper plate, opposite one of the lower combs.

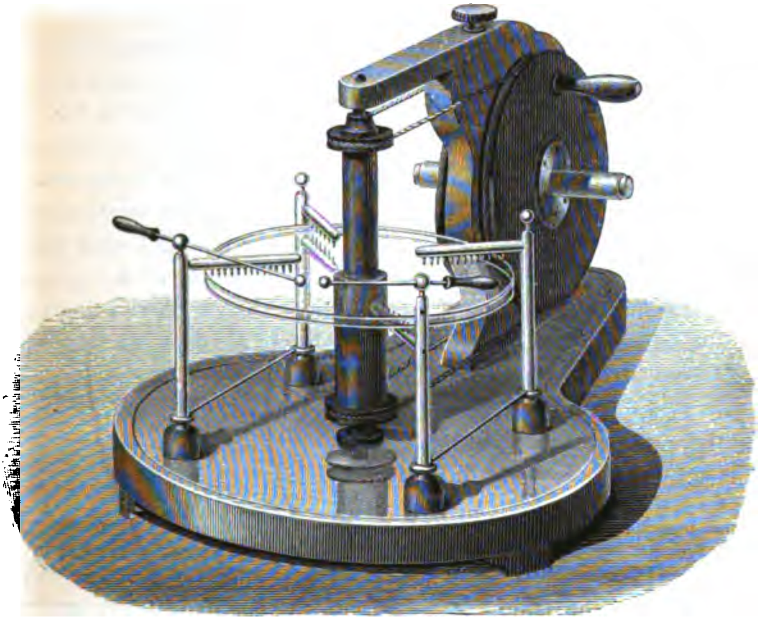


Fig. 357.—Holtz's Machine with Horizontal Plates.

When the machine has been turned for a few seconds, the sector may be removed, and a continual discharge of sparks takes place between the two knobs which are connected with the two conducting systems. Frequently, as in the figure, a comb is placed above,. opposite to the lower comb, and this arrangement appears to increase the efficiency of the machine.

The action in this form of the machine also depends upon induction, the conductors performing the duty of armatures as well. We shall not enter into details, but merely remark that, in both forms of the machine, work is spent in turning the plates in opposition to electrical attractions and repulsions; and that the mechanical energy thus consumed produces an equivalent in the form of electrical energy.

433. Electrophorus.—When electricity is required in comparatively small quantities, it is readily supplied by the simple apparatus called the *electrophorus*. This consists (Fig. 358) of a disc of resin, or some other material easily excited by friction, and of a polished metal disc B with an insulating handle CD. The resin disc is electrified by striking or rubbing it with catskin or flannel, and the metal plate is then laid upon it. In these circumstances, the upper plate does not receive a direct charge from the lower, but, if touched with the finger (to connect it with the earth), receives an opposite charge by induction. On lifting it away by its insulating handle, it is found to be charged, and will give a spark. It may then be replaced on the lower plate (touching it at the same time with the finger), and the process repeated an indefinite number of times, without any fresh excitation, if the weather is favourable.

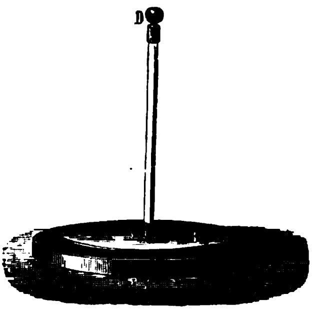


Fig 358.—Electrophorus.

The resinous plate has usually a base or *sole* of metal, which is in connection with the earth while the electrophorus is being worked. This sole, by the mutual induction which takes place between it and the upper plate or *cover*, increases the capacity of the latter (see Chap. xl.), and thus increases the charge acquired. When the cover receives its positive charge on being connected with the earth, the sole at the same time receives from the earth a negative charge, and as the cover is gradually lifted this negative charge gradually returns to the earth.

The most convenient form of the electrophorus is that of Professor Phillips, in which the cover, when placed upon the resinous plate, comes into metallic connection with the metal plate below. That this arrangement is allowable is evident, when we reflect that, when the upper plate is touched with the finger, it is in fact connected with the lower plate, since both are connected with the earth; and it effects a great saving of time when many spark are required in quick succession, for the cover may be raised and lowered as fast as we please, coming alternately into contact with the resinous plate and the body which we wish to charge.

434. Bertsch's Electrical Machine.—A machine which has been called a rotatory electrophorus has recently been invented by Bertsch, and is represented in Fig. 360. A circular plate of ebonite D can be made to revolve rapidly. A sector of the same material, previously cited by friction, is fixed opposite the lower portion of the plate; d on the other side, immediately opposite to this, is a metallic nb N forming the extremity of a nductor connected with the earth. the upper part is another comb connected with the conductor A. der the influence of the electrified or, the conductor C discharges posi- electricity on the plate through comb N. In passing the comb M, ortion of this electricity is col- d by the points, and charges conductor A. The effect is in- ed by connecting A with another conductor E of very large nsions.

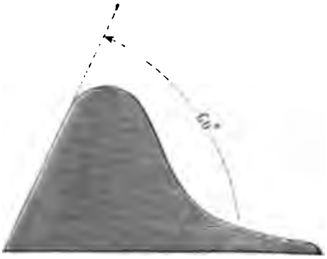


Fig. 359.—Electrified Sector.

is machine differs from that of Holtz in furnishing no means for

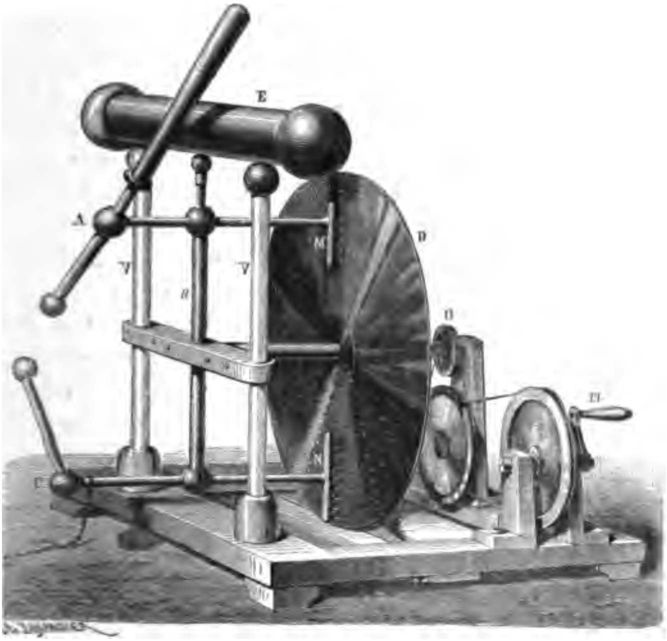


Fig. 360.—Bertsch's Electrical Machine.

ig, or even sustaining, the charge of the armature. In this t resembles the ordinary electrophorus.

CHAPTER XXXIX.

VARIOUS EXPERIMENTS WITH THE ELECTRICAL MACHINE

435. Electric Spark.—The spark furnished by an electrical machine of small dimensions is short, and usually straight. Powerful machines sometimes give sparks of the length of a foot. Such sparks have usually a zig-zag form, like flashes of lightning. One of the readiest means of obtaining long sparks consists in placing, opposite to one of the small knobs of the conductor of the machine, a large conductor, having good earth connection, and presenting a polished and slightly convex surface towards the knob. A more powerful effect will be obtained by connecting this conductor with the rubber or the negative conductor of the machine, instead of with the earth. Very frequently, when the spark is a foot or more in length, finer ramifications proceed from its main track, as shown in Fig. 362.

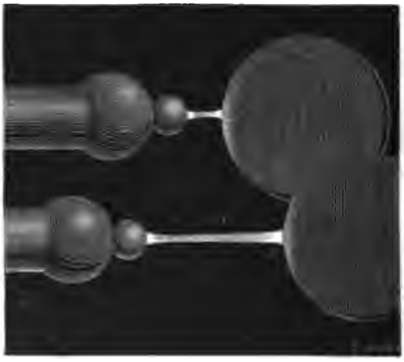


Fig. 361.—Electric Spark.

436. Brush.—When a powerful machine is working in a very dry atmosphere, the rubbers being in good order, and the machine being turned rapidly, a characteristic sound is heard, which is an indication of continuous discharge into the air. In the dark, luminous appearances called *brushes* are seen on the projecting parts of the conductors. They may be rendered very conspicuous by presenting a large conducting surface at a distance a little too great for a spark to pass. It will then be observed that the brush consists (Fig. 363, of a short foot-stalk, with a multitude of rays diverging from it like a fan, and with other smaller ramifications proceeding from these. Positive electricity gives larger and finer brushes than negative. We may add, that, when the machine is working well, brilliant sparks continually leap across the plate, consisting of discharges between the cushions and the nearest part of the conductor. The conductor itself is also surrounded with luminosity. In the dark, the brilliant spectacle presented by these combined appearances, with the continual crackling which accompanies them, is very impressive, and furnished an inexhaustible subject of curiosity to the electricians of last century.

It is probable that the passage of a spark is always preceded by a very high degree of polar tension in all the particles of air in and about its track, and that the spark occurs when this tension anywhere exceeds what the particles are able to bear. The frequent crookedness of the spark is probably

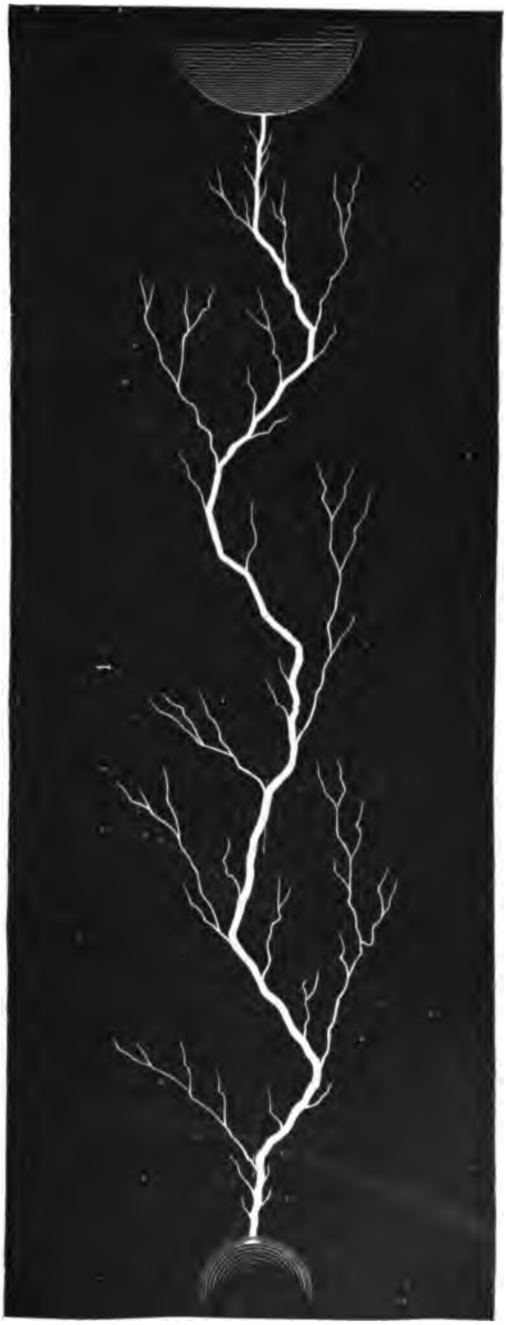


Fig. 362.—Spark with Ramifications.

due to the presence of conducting particles of dust, which serve as stepping-stones, and render a crooked course the easiest.

437. Duration of the Spark.—We can form no judgment of the duration of the electric spark from what we see with the unaided

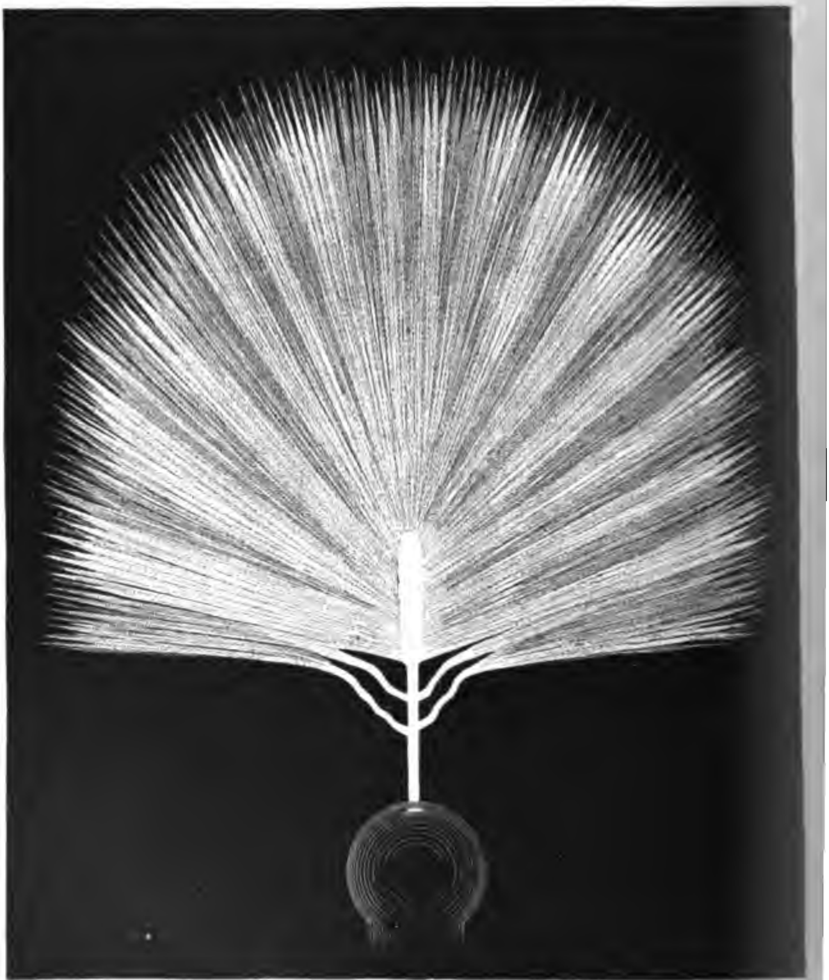


Fig. 363.—Electric Brush, after Van Marum.

eye; for impressions made upon the retina remain uneffaced for something like $\frac{1}{10}$ of a second, and the duration of the spark is incomparably less than this. Wheatstone, in a classical experiment, succeeded in measuring its duration by means of a revolving mirror; an expedient which has since been employed with great advantage in many other researches, especially in determining the velocity of light.

Let *mn* (Fig. 364) be a mirror revolving with great velocity about an axis passing through *c*, and suppose that, during the rotation, an electric spark is produced at *a*. An eye stationed at *o* will see an image in the symmetrical position *a'*. If the spark is strictly instantaneous, its image will be seen as a luminous point at *a'*, notwithstanding the rotation of the mirror; but if it has a finite duration, the image will move from *a'* to *a''*, while the mirror moves from *ee'* to *tt'*, the latter being its position when the spark ceases. What is actually seen in the mirror will therefore not be a point, but a luminous track *a'a''*.

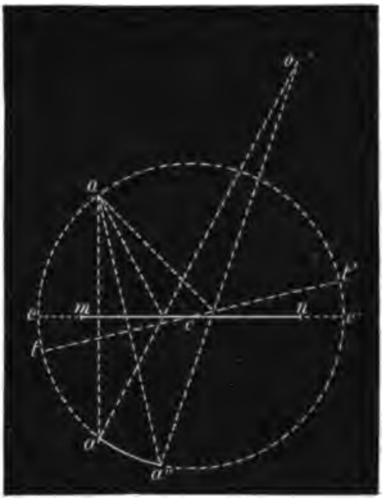


Fig. 364.—Duration of Spark.

The length of this image will be double of the arc *et*; for the angle *ect* at the centre is equal to the angle *a'aa''* at the circumference, the sides of the one being perpendicular to those of the other. In Wheatstone's experiment, the mirror made 800 turns in a second, and the image *a'a''* was an arc of 24°; the mirror therefore turned through 12°, or $\frac{1}{30}$ of a revolution, while the spark lasted. The duration of the spark was therefore $\frac{1}{30}$ of $\frac{1}{800}$, that is, $\frac{1}{24000}$ of a second. [illegible] examining the brush in the same way, Wheatstone found it to consist of a succession of sparks.

[illegible]. **Spark in Rarefied Gases.**—The appearance of the spark is greatly modified by rarefying the air in which it is taken. To show this, an apparatus is employed which is called the *electric egg*. It is an oval glass vessel, which can be exhausted by means of a stop-cock at its lower end. Its upper end is closed by a cap, in which slides a brass rod terminated by a knob, which can be adjusted to any distance from another knob connected with a cap at the lower end.

When the egg contains air at atmospheric pressure, a spark passes in the ordinary way between the two knobs; but, as the pressure is diminished, the aspect of the spark changes. At a pressure of six centimetres of mercury ($\frac{1}{13}$ of an atmosphere), a sort of ramified sheaf proceeds from the positive knob, some of the rays terminating at a small distance from their origin, while others extend to the negative knob. The latter is surrounded with a violet glow; the rays are also violet, but with a reddish tinge. The light at the positive knob is of a reddish purple.

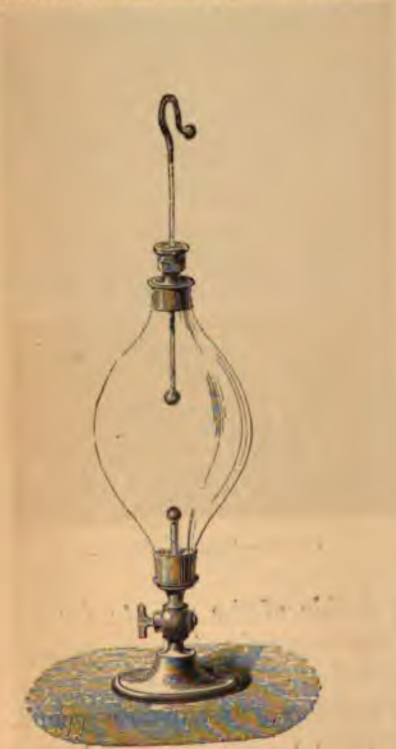


Fig. 365.—Electric Egg.

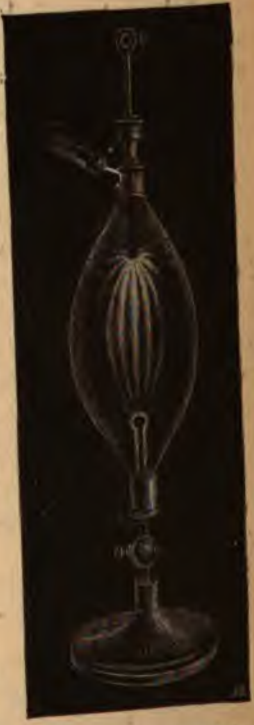


Fig. 366.—Spark in Rarefied Air.

As the pressure is gradually reduced to a few millimetres, the rays become less distinct, and finally coalesce into an oval cloud of pale violet light, extending from one knob to the other, with a reddish tint at the positive and a deep violet at the negative end.

In performing this experiment with the ordinary electrical machine, the upper knob is connected with the conductor, and the lower one with the ground. Holtz's machine can be very advantageously

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employed in experiments of this kind, its two poles being connected with the two knobs.

When, instead of the electric egg, we employ a long tube, such as is employed for showing the fall of bodies *in vacuo*, the whole length of the tube is filled with violet light, which exhibits continual flickering, and suggests the idea of undulations travelling in the same direction as the positive electricity. In all these experiments, as we diminish the density of the air, we diminish the resistance to discharge, and at the same time diminish the intrinsic brightness of the spark.

In the Torricellian vacuum, electric discharge is accompanied by a perceptible though very feeble luminosity, as may be shown by an arrangement due to Cavendish, and represented in Fig. 367. Two barometric tubes, united at the top, are plunged in two cups of mercury. The mercury in one cup is connected with the conductor of the machine, while that in the other is connected with the earth. In these circumstances, the vacuum-space is filled with luminosity, which is brighter as the temperature is higher, probably on account of the greater density of the mercurial vapour which serves as the medium of discharge.

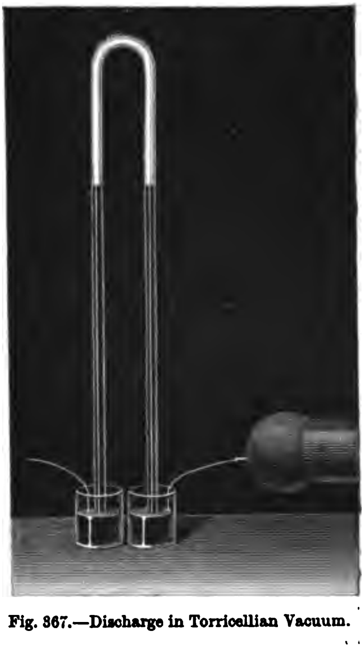

Fig. 367.—Discharge in Torricellian Vacuum.

The experiments of Gassiot and others have shown that electricity traverses a space occupied by a gas with continually increasing facility as the density of the gas is diminished, until a certain limit is attained; but that when special means are employed to render the vacuum as nearly perfect as possible, this limit can be exceeded, and the resistance may increase so much as to prevent discharge.

This latter point is illustrated by the apparatus represented in Fig. 368, which is constructed by Alvergniat. T is a tube which has been exhausted as completely as possible by a Geissler's pump. It has then been heated, and maintained for some time near the temperature of fusion of glass, in order to produce absorption of the remaining air. Two platinum wires have been previously sealed in the ends of the tube, and approach within $\frac{1}{10}$ of a millimetre of each other. The two poles of a Holtz's machine are connected with the binding-screws B and B′, which are in communication with these two wires, and also with two rods whose extremities *p p′* are at a moderate striking distance from each other in air. As long as the machine works, sparks pass between these latter, while, in spite of the very much closer proximity of the platinum wires, no luminosity is perceptible between them. Instead of being placed a small distance apart in air, *p* and *p′* may be fitted into the ends of a tube of considerable length containing rarefied air. It will be found that discharge can take place at greater distance as the air is more rarefied, till we attain a limit far beyond the reach of ordinary air-pumps.

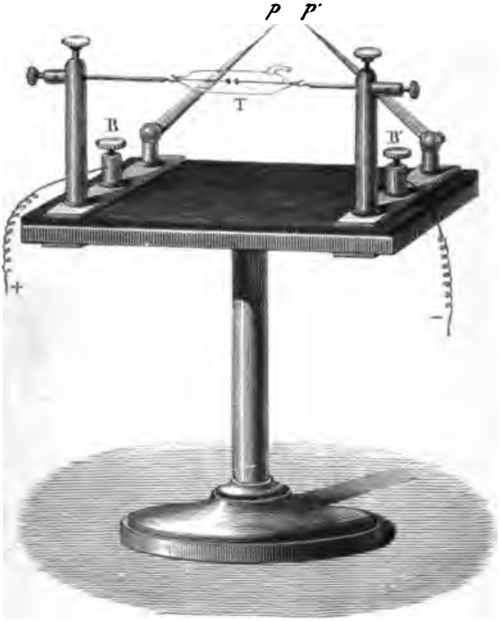

Fig. 368.—Non-conductivity of Perfect Vacuum.

439. Colour of the Spark.—The colour of the spark or other luminous discharge depends partly on the material of the conductors between which it passes, and partly on the gaseous medium which it traverses. The former influence predominates when the spark is strong, the latter when it is weak. The effect of the metal seems to depend upon the vaporization of a portion of it, for, on examining the spark by the spectroscope, bright lines are seen which are known to indicate the presence of metallic vapour. For studying the effect of the gaseous medium, the discharge is taken between two platinum wires sealed into the ends of glass tubes, containing the gases in a rarefied condition. The wires are connected either with the poles of a Holtz's machine, or of a Ruhmkorff's coil, which we shall describe in Chap. lii. It is found that the colour in air or oxygen is white with a tinge of blue, in nitrogen blue, in hydrogen red, and in carbonic acid green.

440. Multiplication of the Electric Spark.—The old electricians contrived several pieces of apparatus for multiplying the electric spark. The principle of all is the same. Small squares of tin-foil are arranged in series at a small distance from each other on an insulating surface. The first of the series is connected with a metallic knob which can be brought near the electrical machine; and the last of them is connected with another knob which is in communication with the earth. By allowing a discharge to pass through the series, sparks can be simultaneously obtained at all the intervals between the successive squares.

In the *spangled tube* (Fig. 370) the squares of tin-foil are arranged spirally along a cylindrical glass tube which has a brass cap at each end. One cap is put in communication with the machine, and the other with the earth.

Sometimes a glass globe is substituted for the cylinder. We have thus the *spangled globe* (Fig. 371).

In the *sparkling pane* a long strip of tin-foil is disposed in one continuous crooked line (consisting of parallel strips connected at alternate ends) from a knob at the top to another knob at the bottom of the pane. A pattern is then traced by scratching away the tin-foil in numerous places with a point, and when the spark passes, it is seen at all these places, so as to render the pattern luminous (Fig. 372).

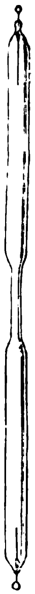
Fig. 369.—Tube for Rarefied Gases.

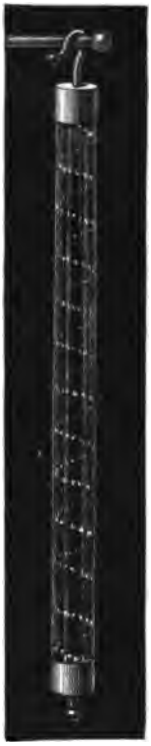
Fig. 370. Spangled Tube.

441. Physiological Effects of the Spark: Electric Shock.—When a strong spark is drawn by presenting the hand to the conductor of a very large and powerful machine, a peculiar sensation is experienced. With ordinary machines the same effect can be obtained by employing a Leyden jar. The sensation is difficult to describe, and only capable of being produced by electrical agency. It is a painful shock, felt especially in the arm, and causing an involuntary bending of the elbow.

At the distance of a few feet from a machine in powerful action, a tickling sensation is felt on the exposed parts of the body, due to the movement of the hairs in obedience to electrical force. These phenomena are exhibited in a still more marked manner when a person stands on a stool with glass legs, and keeps his hand upon the conductor. He thus becomes highly charged with electricity. His hair stands on end, and is luminous if seen in the dark. If a conductor connected with the earth is presented to him, a spark passes, and his hair falls again.

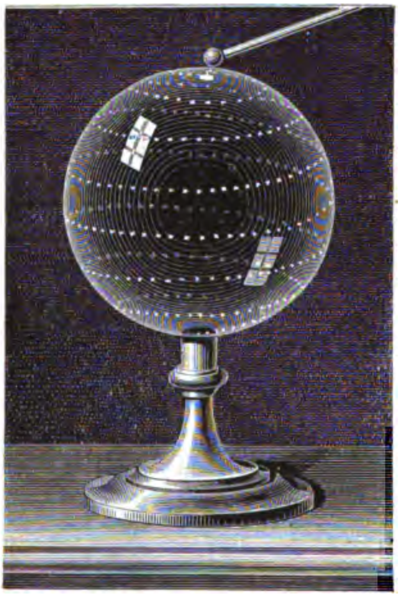

Fig. 371.—Spangled Globe.

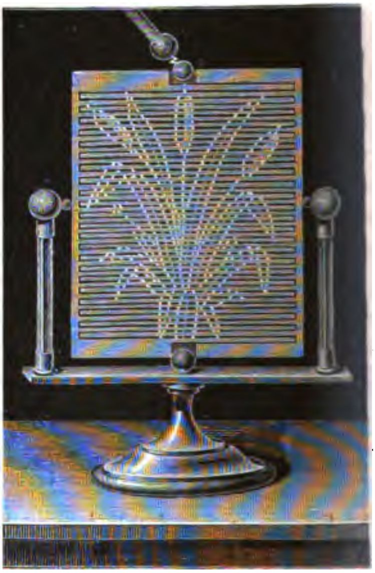

Fig. 372.—Spangled Pane.

Electricity has frequently been resorted to for medical purposes. The electrical machine was first employed, and afterwards the Leyden jar, but both have now been abandoned in favour of magneto-electric machines and induction coils, which we shall describe in a later chapter (Chap. lii.)

442. Mechanical and Physical Properties of the Spark.—The electric spark produces a violent commotion in the medium in which it occurs. This is easily shown by means of Kinnersley's thermometer (Fig. 373), which consists of two glass tubes of unequal diameters, the smaller being open at the top, while the larger is completely closed, with the exception of a side passage, by which it communicates with the smaller. The caps which close the ends of the large tube are traversed by rods terminating in knobs, and the upper one can be raised and lowered to vary the distance between the knobs. Both tubes are filled, to a height a little below the lower knob, with a very mobile liquid such as alcohol. When the spark passes between the knobs, the liquid is projected with great violence, and may rise to a height of several yards if the spark is very strong. The same property of the spark is exhibited in the experiment of the electric mortar, which is sufficiently explained by the figure (Fig. 374).

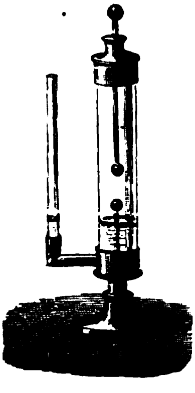
Fig. 373.—Kinnersley's Thermometer.

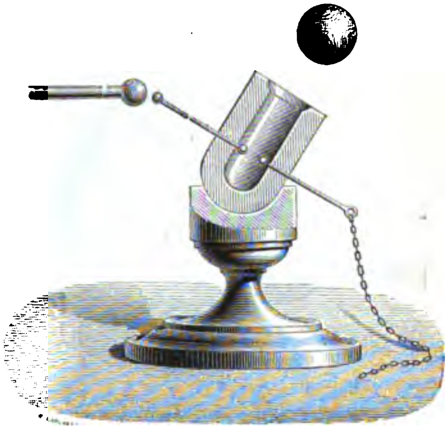
Fig. 374.—Electric Mortar.

The spark may be obtained in the interior of a non-conducting liquid, which it agitates in a similar manner. If the liquid is contained in a closed vessel, this is often broken. The spark can also traverse thin non-conducting plates, producing in this case perforation of the plates; but the experiment usually requires very powerful discharges, such as can only be obtained by means of apparatus described in the next chapter.

The luminosity of the electric spark is probably due to the very high temperature which is produced in the particles traversed by the discharge. Coal-gas is easily inflamed, by a person standing on a stool with glass legs holding one hand on the conductor of the machine, and giving sparks from a finger of the other hand to the burner from which the gas is issuing. Kinnersley regarded elevation of temperature as the cause of the movement of the liquid in his apparatus; hence the name which it bears.

Heating may also occur in the case of conductors. This is shown by the influence of the metal upon the colour of the spark, and it may be more directly proved by arranging a conductor in communication with the earth, and connected by an exceedingly fine metallic wire with another conductor. When the latter is presented to a very powerful electrical machine, so that a strong spark passes, the fine wire is sometimes heated to redness.

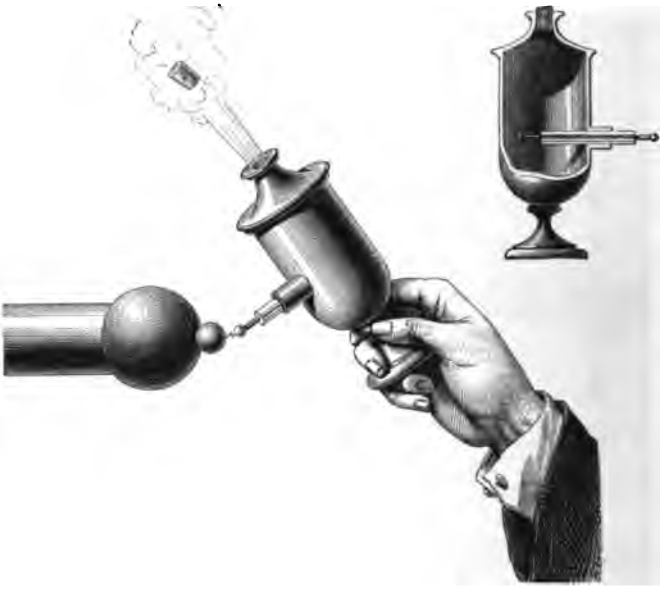

Fig. 375.—Volta's Pistol.

443. Chemical Properties of the Spark.—The electric spark is able to produce very important chemical effects. When it occurs in an explosive mixture of two parts of hydrogen with one of oxygen, it causes these gases instantly to combine. This experiment is usually shown by means of Volta's pistol (Fig. 375), which is a metallic vessel, containing the mixture, and closed by a cork. Through one side passes an insulated metallic rod with a knob at each end, that at the inner end being at a short distance from the opposite side of the vessel, so that, if a spark is given to the exterior knob, a spark also passes in the interior, and inflames the mixture. This effect is accompanied by a violent detonation, and the cork is projected to a distance.

The electric spark often produces a reverse effect—that is to say, the decomposition of a compound body; but the action in this case is gradual, and a great number of sparks must be passed before the full effect is obtained. Thus, if a succession of sparks be passed in the interior of a mass of ammonia, contained in a vessel inverted over mercury, the volume of the gas is observed to undergo a gradual increase, until at length, if kept at constant pressure, the volume is exactly doubled. It then consists of a mechanical mixture of nitrogen and hydrogen, the constituents of ammonia.

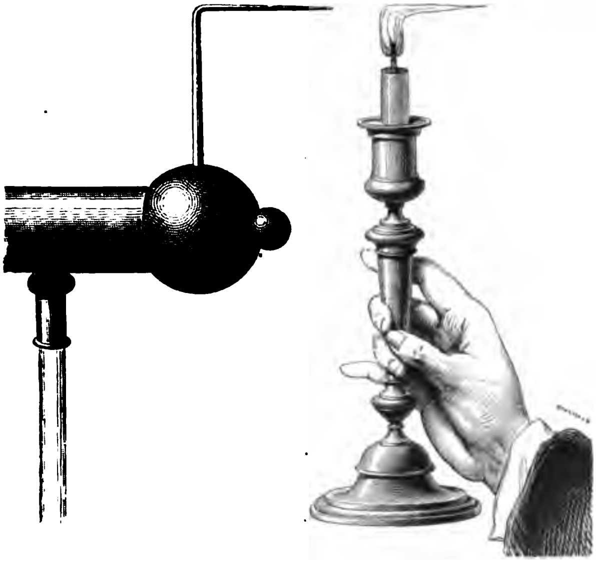

Fig. 376.—Wind from Points.

Composition and decomposition are often both produced at once. Thus, if a spark is passed in a mixture of carburetted hydrogen and a certain proportion of oxygen, the former gas is decomposed, its hydrogen combining with a portion of the oxygen to form water, and its carbon combining with another portion to form carbonic acid.

Vessels intended for taking the electric spark in gases are extensively used in chemistry, and are called *eudiometers.*

444. Wind from Points.—If a metallic rod terminating in a point be attached to the conductor of the electrical machine, electricity escapes in large quantity from the point, which, accordingly, when viewed in the dark, is seen to be crowned with a tuft of light. A layer of air in front of the point is electrified by contact, and then repelled, to make way for other portions of air, which are in their turn repelled. A continuous current of air is thus kept up, which is quite perceptible to the hand, and produces a very visible effect on the flame of a taper (Fig. 376).

The *electric whirl* (Fig. 377) consists of a set of metallic arms, radiating horizontally from a common centre about which they can turn freely, and bent, all in the same direction, at the ends, which are pointed. When the central support is mounted on the conductor of the machine, the arms revolve in a direction opposite to that in which their ends point. This effect is due to the mutual repulsion between the pointed ends and the electrified air which flows off from them.

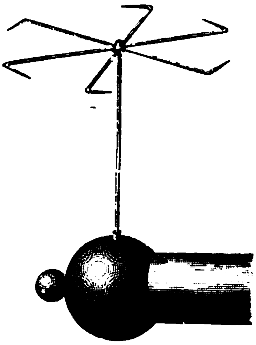

Fig. 377.—Electric Whirl.

It is instructive to remark that if, by a special arrangement, the rotating part be inclosed in a well-insulating glass case, the rotation soon ceases, because, in these circumstances, the inclosed air quickly attains a state of permanent electrification.

445. Electric Watering-pot.—Let a vessel containing a liquid, and furnished with very fine discharge tubes, be suspended from the conductor of the machine. When the vessel is not electrified, the liquid comes out drop by drop; but when the machine is turned, it issues in continuous fine streams. It has, however, been observed that the quantity discharged in a given time is sensibly the same in both cases. This must be owing to the equality of action and reaction between different parts of the issuing stream.

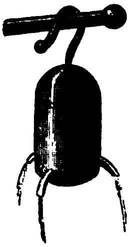

Fig. 378.—Electric Bucket.

CHAPTER XXXIX^A.

ELECTRICAL POTENTIAL, AND LINES OF ELECTRIC FORCE.

445A. Introductory Remarks on Potential.—Electrical reasonings are, in many cases, greatly facilitated by employing the conception denoted by the name *electrical potential.*

Potential essentially depends upon forces (whether attractive or repulsive) mutually exerted upon each other by particles at a distance, and has been advantageously employed in the theories of gravitation and magnetism, as well as of electricity. We shall at present confine our attention to *electrical* potential.

All the space in the neighbourhood of electrified bodies is in a certain sense pervaded by their influence. This influence is completely specified by stating the numerical values, with proper sign, of electrical potential, at the different points of the space.

Electrical potential never changes its value *per saltum* in passing from one point to the next. Moreover, if it is constant in value throughout any finite portion of a space not containing electricity, it is constant throughout the whole of this space.

445B. Relation of Potential to Force.—When electrical potential is constant throughout a given space, there is no electrical force[1] in that space; and, conversely, if there be an absence of electrical force in a given space, the potential throughout that space must be uniform. These propositions apply to the space within a hollow conductor. They also apply to the whole substance of a solid conductor, and to the whole space inclosed within the outer surface of a hollow conductor. Whenever a conductor is in electrical equilibrium, it has the same potential throughout the whole of its substance, and also through any completely inclosed hollows which it may contain.

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[1] DEFINITION.—There is electrical force at a point in the air, if an electrified particle placed there would experience force tending to move it in virtue of electrical attraction or repulsion. There is electrical force at a point in a conductor, when electricity flows through the point. A conductor is said to be in electrical equilibrium, when there is no electrical force at any part of it; in other words, when it is completely free from currents of electricity.

When a conductor is not in electrical equilibrium, currents set in, tending to restore equilibrium; and the direction of such currents is always from places of higher to places of lower potential. In like manner, when a small positively electrified body experiences electrical force tending to move it, the direction of this force is from higher to lower potential.

When flow of electricity is compared with flow of heat, potential is the analogue of temperature. Heat flows from places of higher to places of lower temperature.

The precise direction of the force exerted upon a positively electrified particle (or upon an element of positive electricity), when brought to a place where potential has not a constant value from point to point, is the direction in which potential diminishes most rapidly. A negatively electrified particle (or an element of negative electricity) will be urged in the opposite direction, which is the direction in which potential increases most rapidly. We here use the words *increase* and *decrease* in the algebraical sense.

445c. Line of Force.—The direction thus defined (especially by reference to the force on the *positive* particle) is called *the direction of resultant electrical force* at the point where the particle is placed. If a line be traced such that every small portion of it (small enough to be regarded as straight) is the direction of resultant electrical force at the points which lie upon it, it is called a *line of force;* in other words, a line of force is *a line whose tangent at any point is the direction of resultant electrical force at that point.* We may express this briefly by saying that lines of electrical force are *the lines along which resultant electrical force acts.*

It is evident that lines of force cannot cut one another, since we cannot have two different directions of resultant force at a point.

445d. Intensity of Force is Equal to Rate of Variation of Potential.—The intensity of resultant force at a point is equal to the rate at which potential diminishes in the direction in which the diminution is most rapid, namely, along a line of force at the point. Let V denote the potential at the point, and $V - \delta V$ the potential at a neighbouring point on the same line of force, at a distance δs from the first point; then $\frac{\delta V}{\delta s}$ is the intensity of force at either point, or, more strictly, is the average intensity along the short line δs, and the direction of the force (for a positive particle) is from the first point towards the second.

A similar proposition applies to two neighbouring points not situated on the same line of force; the component force in the direction of a line joining them, is equal to $\frac{\delta V}{\delta x}$, where δx denotes the length of the joining line, and δV the difference of the potentials of the two points. This proposition is usually expressed by saying that *the rate of variation of potential in any direction is equal to the component force in that direction.*

445E. Relation between Potential and Work.—The work done by or against electrical force in carrying a unit of electricity through this distance δx is the product of force by distance, and is therefore simply δV. More generally, the work done by or against electrical force in carrying a unit of electricity from one point to another, is equal to the difference of potentials of the two points; and the work done in carrying any quantity of electricity is the product of this quantity by the difference of potentials.

An analogy is thus suggested between different potentials and different *levels.* Positive electricity tends to run down from higher to lower potential, and, when it does so run down, there is a loss of *potential energy* equal to the *product of the quantity* which runs down, *and the difference of potential* through which it runs down. When the quantity which runs down is unity, the loss of potential energy is equal to the loss of potential. It is usual to assume, as the zero of potential, the potential of the earth at the place of observation; but this assumption is not rigorously consistent with itself, since the existence of earth-currents demonstrates that different potentials may exist at different parts of the earth. Electrical potential is rigorously zero at places infinitely distant from all electricity.

445F. Equipotential Surfaces.—An equipotential surface is a surface over the whole of which there is the same value of potential. It is obvious, from the latter part of § 445D, that there is no tangential force at any point of such a surface. The direction of resultant force is everywhere normal to the surface, or *equipotential surfaces everywhere cut lines of force at right angles.* An equipotential surface is the analogue of a *level surface.* If two equipotential surfaces are given, their potentials being V_1 and V_2, the work done in carrying a unit of electricity from any point of the one to any point of the other, is constant, and equal to the difference of V_1 and V_2.

If we consider two equipotential surfaces very near to one another, so that the portions which they intercept on the lines of force may be regarded as straight, the intensity of force at different points of the intermediate space will vary inversely as the distance between the two equipotential surfaces; for, when equal amounts of work are done in travelling unequal distances, the forces must be inversely as the distances.

445a. Tubes of Force.—If we conceive a narrow tube bounded on all sides by lines of force, and call it a *tube of force*, we can lay down the following remarkable rules[1] for the comparison of the forces which exist at different points in its length. (1) *In any portion of a tube of force not containing electricity, the intensity of force varies inversely as the cross-section of the tube,* or the product of *intensity of force by section of tube,* is constant.[2] (2) *When a tube of force cuts through electricity, this product changes, from one side of this electricity to the other, by the amount* $4\pi q$, where q denotes the quantity of the electricity inclosed by the tube.

The following are particular cases of (1):—

When the electricity to which the force is due is collected in a point, the lines of force are straight, the tubes of force are cones (in the most general sense), and the law of force becomes the law of inverse squares, since the section of a cone varies as the square of the distance from the vertex. These results also apply to the case of electricity uniformly distributed over the surface of a sphere, the common vertex of the cones being now at the centre of the sphere.

When the electricity consists of the charges of two oppositely electrified parallel plates, whose length and breadth exceed the distance between them (the plates being conductors, and placed opposite to each other), the lines of force between their central portions are sensibly straight and parallel, the tubes of force are therefore cylinders (in the most general sense), and the force is constant, being equal to the difference of the potentials of the plates divided by the distance between them. The same thing holds if, instead of being oppositely electrified, the plates are similarly electrified, but not to the same potential.

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[1] For the proof of these rules, as mathematical deductions from the law of inverse squares, see Thomson and Tait's *Natural Philosophy*, §§ 490, 492.

[2] This is obviously analogous to the law which applies to the comparison of the velocities of a liquid in different parts of a tube through which it flows, since the product of area by velocity is the volume of liquid which flows past any section in unit time. The tube may be an imaginary one, bounded by lines of flow in a large body of liquid flowing steadily Lines of flow are thus the analogues of lines of force.

445H. Force Proportional to Number of Tubes which cut Unit Area.—The cross-sections of tubes of force are portions of equipotential surfaces. If one equipotential surface be divided into portions, such that the product of *area by force-intensity* shall be the same for all, then, if all neighbouring space not containing electricity be cut up into tubes, springing from these portions as their respective bases, the product of any cross-section of any one of these tubes by the force-intensity over it will be constant. The force-intensities at any points in this space are therefore inversely as the cross-sections of the tubes at these points, or are directly as the number of tubes per unit area of equipotential surfaces at the points.

445I. Force just Outside an Electrified Conductor.—Since there is no force in the interior of a conductor, the lines and tubes of force become indeterminate; but proposition (2) of § 445G can be shown to hold when we give them any shape not discontinuous. Let ρ denote the electric density at a point on the surface, and a a small area around this point, which area we shall regard as a section of a tube of force cutting through the surface. Let F denote intensity of force just outside the surface opposite this point, then, since the intensity inside is zero, we have

$$F a = 4 \pi q = 4 \pi \rho a \quad \therefore F = 4 \pi \rho ;$$

that is, *the intensity of force just outside any part of the surface of a charged conductor, is equal to the product of* 4π *into the density at the nearest part of the surface.*

445J. Relation of Induction to Lines and Tubes of Force.—Lines of force are also the lines along which induction takes place. On Faraday's theory of induction by contiguous particles, the line of poles, for any particle, is coincident with the line of force which passes through the particle. Apart from all theory, it is matter of fact that *a tube of force extending from an influencing to an influenced conductor, and not containing any electricity in the interval between, has equal quantities of electricity on its two ends, these quantities being of opposite sign.* This equality follows at once from § 445G (2), if we consider the tube as penetrating the two conductors; for the product of force by section, which is constant for the portion of the tube in air, is zero in both conductors; and the quantity of electricity on *either* end of the tube must be the quotient of this constant product by 4π. In connection with this reasoning, it is to be remarked that the surface of a conductor is an equipotential surface, and is cut at right angles by lines of force.

In Faraday's ice-pail experiment, a tube of force springing from the upper side of the charged ball, and of such small section at its origin as to inclose only an insensible fraction of the charge of the ball, opens out so fast, as it advances, that it fills the whole opening at the top of the pail.

In every case of induction, *the total quantities of inducing and induced electricity are equal, and of opposite sign.*

When the inducing electricity resides in or upon a non-conductor, for example on the surface of a glass rod, or in the substance of a mass of air, the quantity of electricity induced on the base of a tube of force is equal and opposite to the quantity contained within the tube. In the simplest case, all the tubes will have a common apex, which will be a point of maximum or minimum potential.

445K. Potential defined as $\Sigma \frac{q}{r}$.

The resultant potential at a point, due to several different quantities of electricity, is the algebraical sum of the potentials due to the different quantities separately considered. This would follow at once from the relation above indicated as existing between potential and work.

If the unit of quantity of electricity be defined as that quantity which, at unit distance, produces unit intensity of force, it can be shown[1] that the whole work done by or against the force of an element q, in bringing a unit of electricity from infinite distance to a point at a distance r from the element, is $\frac{q}{r}$. This expression therefore denotes the potential due to q at the distance r, if we adopt the very natural convention that the potential at infinite distance shall be reckoned zero.

The resultant potential at a point, due to the presence of any quantity of electricity distributed in any way, is to be computed by dividing the electricity into elements, each occupying so little space that all parts of it may be regarded as at the same distance from the point, and summing all the quotients $\frac{q}{r}$. The distances denoted by r are essentially positive. If the electricity is not all of one sign, some of the quotients $\frac{q}{r}$ will be positive, and others negative, and their algebraical sum is to be taken.

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[1] See *Thomson and Tait*, § 491, first paragraph.

445 L. Application to Sphere.—In the case of a charged conducting sphere, all the elements q are equally distant from the centre of the sphere, and the sum of the quotients $\frac{q}{r}$, when we are computing the potential at the centre, will be $\frac{Q}{R}$, Q denoting the charge, and R the radius of the sphere. But the potential is the same at all points in a conductor. $\frac{Q}{R}$ is therefore the potential of a sphere of radius R, with charge Q, when uninfluenced by any other electricity than its own.

445 M. Capacity.—The electrical capacity of a conductor is *the quantity of electricity required to charge it to unit potential,* when it is not influenced by any other electricity besides its own charge and the electricity which this induces in neighbouring conductors. Or, since, in these circumstances, potential varies directly as charge, capacity may be defined as the *quotient of charge by potential.* Let C denote capacity, V potential, and Q charge, then we have

$$C = \frac{Q}{V} \quad ; \quad V = \frac{Q}{C} \quad ; \quad Q = VC.$$

But we have seen that, for a sphere of radius R, at a distance from other conductors or charged bodies, $V = \frac{Q}{R}$. Hence $C = R$; that is, the *capacity of a sphere is numerically equal to its radius.*

This is a particular instance of the general proposition that the capacities of similar conductors are as their linear dimensions; which may be proved as follows:—

Let the linear dimensions of two similar conductors be as $1 : n$. Divide their surfaces *similarly* into very small elements, which will of course be equal in number. Then the areas of corresponding elements will be as $1 : n^2$, and, if the electrical densities at corresponding points be as $1 : x$, the charges on corresponding elements are as $1 : n^2x$. The potential at any selected point of either conductor is the sum of such terms as $\frac{q}{r}$ (§ 445 K). Selecting the corresponding point in the other conductor, and comparing potentials, the values of q are as $1 : n^2x$, and the values of r are as $1 : n$; therefore the values of $\frac{q}{r}$ are as $1 : nx$. Hence the potentials of the two conductors are as $1 : nx$. If they are equal, we have $nx = 1$, and therefore $n^2x = n$; that is, the charges on corresponding elements, and therefore also on the whole surfaces, are as 1 : n.

We shall see, in the chapter on Condensers, that the capacity of a conductor may be greatly increased by bringing it near to another conductor connected with the earth.

445 N. Connection between Potential and Induced Distribution.—In the circumstances represented in Fig. 336 (§ 412), if we suppose the influencing body C to be positively charged, the potential due to this charge will be algebraically greater at the near end A of the influenced conductor than at the remote end B. The induced electricity on AB must be so distributed as to balance this difference, in fact the potential due to this induced electricity is negative at A and positive at B. All cases of induced electricity upon conductors fall under the rule that *the potential at all parts of a conductor must be the same, and hence, wherever the potential due to the influencing electricity is algebraically greatest, the potential due to the electricity on the influenced conductor must be algebraically least.*

As there can be no force in the interior of a conductor, the force at any point in the interior, due to the influencing electricity, must be equal and opposite to the force due to the electricity on the surface of the conductor. This holds, whether the conductor be solid or hollow. A hollow conductor thus completely screens from external electrical forces all bodies placed in its interior.

445 O. Electrical Images.—If a very large plane sheet of conducting material be connected with the earth, and an electrified body be placed in front of it near its middle, the plate will completely screen all bodies behind it from the force due to the electrified body. The induced electricity on the plate therefore exerts, at all points behind the plate, a force equal and opposite to that of the electrified body, or, what is the same thing, a force identical with that which the electrified body would exert if its electricity were reversed in sign. But electricity distributed over a plane surface must act symmetrically towards both sides. Hence the force which the induced electricity exerts in front, is identical with that which would be exerted by a body precisely similar to the given electrical body, symmetrically placed behind the plane, and charged with the opposite electricity. The total force at any point in front of the plane is the resultant of the force due to the given electrified body, and the force due to this imaginary image. The name and the idea of *electrical images*, of which this is one of the simplest examples, are due to Sir W. Thomson.

CHAPTER XL.

ELECTRICAL CONDENSERS.

45p. **Condensers.**—The process called *condensation of electricity* sists in increasing the capacity of a conductor by bringing near nother conductor connected with the earth. The two conductors usually thin plates or sheets of metal, placed parallel to one her, with a larger plate of non-conducting material between n.

et A and B (Fig. 379) be the two conducting plates, of which A, d the *collecting plate*, is connected the conductor of the machine, and lled the *condensing plate*, with the ; and let C be the non-conducting (or dielectric) which separates them. , if the machine has been turned the limit of charge is attained, the e of B which faces towards A is ed with negative electricity, drawn the earth, and held by the attrac- f the positive electricity of A; and, rsely, the surface of A which faces ds B, is covered with positive elec- , held there by the attraction of the ve of B, in addition to the charge would reside upon it if the conductor were at the existing ial, and B and C were absent. In fact, the electrical density face of A, as well as the whole charge of A, would, in this ase, be almost inappreciable, in comparison with those which n the actual circumstances. By condensation of electricity, re are to understand *increase*—usually enormous increase—

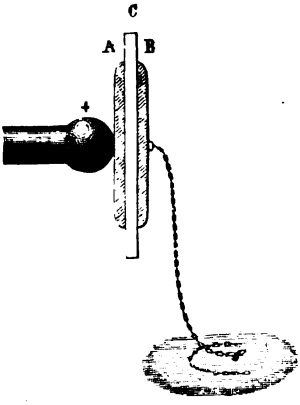

Fig. 379.—Theoretical Condenser.

of electrical density on a given surface, attained without increase of potential. If two conducting plates, in other respects alike, but one with, and the other without a condensing plate, be connected by a wire, and the whole system be electrified, the two plates will have the same potential, but nearly the whole of the charge will reside upon the face of that which is accompanied by a condensing plate.

445Q. Calculation of Capacity of Condenser.—The lines of force between the two plates A and B are everywhere sensibly straight and perpendicular to the plates, with the exception of a very small space round the edge, which may be neglected. The tubes of force (§ 445G) are therefore cylinders, and the intensity of force is constant at all parts of their length. Also, since the potential of the plate B is zero, if we take V to denote the potential of the plate A, which is the same as the potential of the conductor, and t to denote the thickness of the intervening plate C, the rate at which potential varies along a line of force is $\frac{V}{t}$, which is therefore (§ 445D) the expression for the force at any point between the plates A, B. The whole space between the plates may be regarded as one cylindrical tube of force of cross-section S equal to the area of either plate, the two ends of the tube being the inner faces of the plates. The quantities of electricity $\pm Q$ residing on these faces are therefore equal, but of opposite sign (§ 445J); and as the force changes from nothing to $\frac{V}{t}$ in passing from one side to the other of the electricity which resides on either of these surfaces, we have (§ 445G)

$$\frac{V}{t} . S = 4\pi Q.$$

Hence the capacity of the plate A, being, by definition, equal to $\frac{Q}{V}$, is equal to

$$\frac{S}{4\pi t}.$$

We should, however, explain that, if the intervening plate C is a solid or liquid, we are to understand by t not the simple thickness, but the thickness reduced to an equivalent of air, in a sense which will be explained further on (§ 453). This reduced thickness is, in the case of glass, about half the actual thickness.

If s denote an element of area of A, and q the charge residing upon it, it is evident, from considering the tube of force which has s for one of its ends, that

$$\frac{V}{t} . s = 4\pi q;$$

and the electric density $\frac{q}{s}$ on the element is equal to $\frac{V}{4\pi t}$, which is constant over the whole face of the plate.

To give a rough idea of the increase of capacity obtained by the employment of a condensing plate, let us compare the capacity of a circular disc of 10 inches diameter, accompanied by a condensing plate at a reduced distance of $\frac{1}{20}$ of an inch, with the capacity of a globe of the same diameter as the disc. The capacity of the globe is equal to its radius, and may therefore be denoted by 5. The capacity of the disc is $\frac{25\pi}{4\pi \times \frac{1}{20}} = 125$, or 25 times the capacity of the globe. It is, in fact, the same as the capacity of a globe 250 inches (or 20 ft. 10 in.) in diameter.

446. Discharge of Condenser.—If, by means of a jointed brass discharger (Fig. 380) with knobs M N at the ends, and with glass handles, we put the two plates A and B in communication, a brilliant spark is obtained, resulting from the combination of the positive charge of A with the negative of B, and the condenser is discharged. When the quantity of electricity is small, the glass handles are unnecessary, and the simpler apparatus represented in Fig. 381 may be employed, consisting simply of two brass rods jointed together, and with knobs at their ends, care being taken to touch the plate B, which is in communication with the earth, before the other. The operator will then experience no shock, as the electricity passes in preference through the brass rods, which are much better conductors than the human body. If, however, the operator discharges the

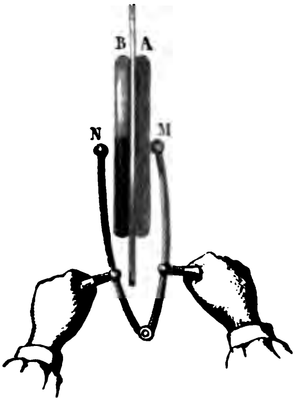

Fig. 380.—Discharge of Condenser.

Fig. 381.—Discharger without Handles.

condenser with his hands by touching first the plate B, and then also the plate A, the whole discharge takes place through his arms and chest, and he experiences a severe shock. If he simply touches the plate A, while B remains connected with the earth by a chain, as in Fig. 379, he receives a shock, but less violent than before, because the discharge has now to pass through external bodies which consume a portion of its energy. If, instead of a chain, B is connected with the earth by the hand of an assistant touching it, he too will receive a shock when the operator touches A.

447. Discovery of Cuneus.—The invention of the Leyden jar was brought about by a shock accidentally obtained. Some time in the year 1746, Cuneus, a pupil of Muschenbroeck, an eminent philosopher of Leyden, wishing to electrify water, employed for this purpose a wide-mouthed flask, which he held in his hand, while a chain from the conductor of the machine dipped in the water (Fig. 382). When the experiment had been going on for some time, he wished to disconnect the water from the machine, and for this purpose was about to lift out the chain; but, on touching the chain, he experienced a shock, which gave him the utmost consternation, and made him let

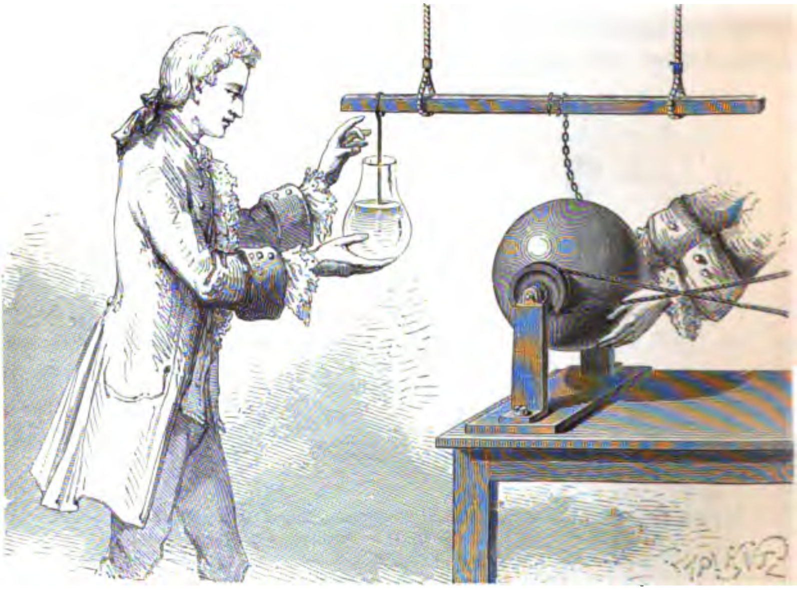

Fig. 382.—Experiment of Cuneus.

fall the flask. He took two days to recover himself, and wrote to Réaumur that he would not expose himself to a second shock for the crown of France. The news of this extraordinary experiment spread over Europe with the rapidity of lightning, and it was eagerly repeated everywhere. Improvements were soon introduced in the arrangement of the flask and its contents, until it took the present form of the *Leyden Phial* or *Leyden Jar.* It is easy to see that the effect obtained by Cuneus depended on condensation of electricity, the water in the vessel serving as the collecting plate, the hand as condensing plate, and the vessel itself as the dielectric. When he touched the chain, the two oppositely charged conductors were put in communication through the operator's body, and he received a shock.

448. Leyden Jar.—The Leyden jar, as now usually constructed, consists of a glass jar coated, both inside and out, with tin-foil, for about four-fifths of its height. The mouth is closed by a cork, through which passes a metallic rod, terminating above in a knob,

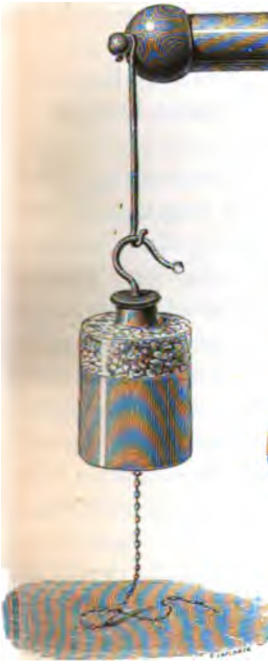

Fig. 383.—Leyden Jar.

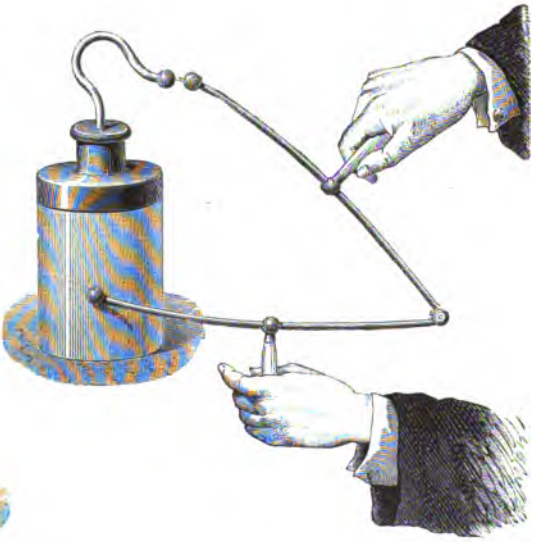

Fig. 384.—Discharge of Leyden Jar.

and connected below with the inner coating, either by a chain depending from it, or by pieces of metallic foil with which the jar is filled. The interior of the jar must be thoroughly dry before it is closed, and the cork and neck are usually covered with sealing-wax, and shellac varnish, which is less hygroscopic than glass. The Leyden jar is obviously a condenser, its two coatings of tin-foil performing the parts of a collecting plate and a condensing plate. If the inner coating is connected with the electrical machine, and the outer coating with the earth, the former acquires a positive, and the latter a negative charge. On connecting them by a discharger, as in Fig. 384, a spark is obtained, whose power depends on the potential of the inner coating, and on its electrical capacity. If these be denoted respectively by V and C, and if Q denote the quantity of electricity residing on either coating, the amount of electrical energy which runs down and undergoes transformation when the jar is discharged, is $QV = CV^2 = \frac{Q^2}{C}$.

The quantities Q, V, C, which are, properly speaking, the charge, potential, and capacity of the *inner coating*, are usually called the charge, potential, and capacity of the jar.

449. Residual Charge.—When a Leyden jar has been discharged by connecting its two coatings, if we wait a short time we can obtain another but much smaller spark by again connecting them, and other sparks may sometimes be obtained after further intervals. These are called secondary discharges, and the electricity which thus remains after the first discharge is called the *residual charge*. Faraday's experiments leave little room for doubt that they depend mainly upon a gradual penetration of electricity from both sides into the substance of the glass, to a very small depth, but sufficient to prevent the electricity which has so entered from at once escaping to the earth when connection is made. Faraday appeals to this phenomenon as strongly confirming his view that the difference between conductors and non-conductors is only one of degree, this penetration being only an extremely slow process of conduction. A small part of the residual charge also consists of electricity spread over the surface of the glass beyond the edges of the coatings.

The whole charge of the outer coating, and all except an insignificant portion of the charge of the inner coating, resides on the side of the foil which is in contact with the glass, or, more probably, on the surfaces of the glass itself, the mutual attraction of the two opposite electricities causing them to approach as near to each other as the glass will permit. This is illustrated by Franklin's experiment of the *jar with movable coatings* (Fig. 385). The jar is charged in the ordinary way, and placed on an insulating stand. The inner coating is then lifted out by a glass hook, and touched with the hand to discharge it of any electricity which it may retain. The glass is then lifted out, and the outer coating also discharged. The jar is then put together again, and is found to give nearly as strong a spark as it would have given originally.

450. Discharge by Alternate Contacts.—Instead of discharging a Leyden jar at once by connecting its two coatings, we may gradually discharge it by alternate contacts. To do this, we must set it on an insulating stand (or otherwise insulate both coatings from the earth), and then touch the two coatings alternately. At every contact a small spark will be drawn. The coating last touched has always rather less electricity upon it than the other, but the difference is an exceedingly small fraction of the whole charge, and, after a great number of sparks have been drawn by these alternate contacts, we may still obtain a powerful discharge by connecting the two coatings.

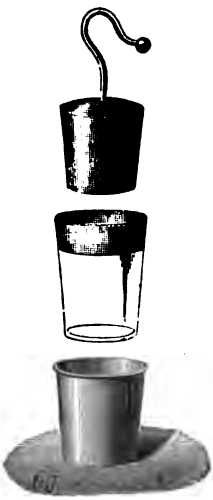
Fig. 385.—Jar with Movable Coatings.

The quantities of electricity thus alternately discharged from the two coatings form two decreasing geometric series, one for each coating. In fact, if m and m' be two proper fractions such that, when the outer coating is connected with the earth, the ratio of its charge to that of the inner is $-m$; and, when the inner coating is connected with the earth, the ratio of its charge to that of the outer is $-m'$, we have the following series of values:—

	On inner coating.		On outer coating.
Original charges, ...	$+Q$	...	$-mQ$
After 1st contact, ...	$+m'mQ$	...	$-mQ$
2d „	$+m'mQ$	...	$-m'm^2Q$
3d „	$+m'^2m^2Q$	...	$-m'm^2Q$
	&c.		&c.

The quantities discharged from the inner coating are, successively $(1-m'm)Q$, $m'm(1-m'm)Q$, $m'^2m^2(1-m'm)Q$, &c.; and the quantities successively discharged from the outer, neglecting sign, are, $m\ (1-m'm)\ Q$, $m'm^2\ (1-m'm)\ Q$, $m'^2m^3\ (1-m'm)\ Q$, &c.

The quantity $(1-m'm)\ Q$ discharged at the first contact, represents that portion of the charge[1] which is not due to condensation; so that the actual capacity of the Leyden jar is to the capacity of the inner coating if left to itself as $1 : 1-m'm$.

The discharge by alternate contacts can be effected by means of a carrier suspended between two bells, as in Fig. 386. The rod from the inner coating terminates in a bell, and the outer coating is connected, by means of an arm of tin, with another bell supported on a metallic column. An insulated metallic ball is suspended between the two. This is first attracted by the positive bell. Then, being repelled by this and attracted by the other, it carries its charge of positive electricity to the negative bell, and receives a charge of negative, which it carries to the positive bell, and so on alternately. The whole apparatus stands upon an insulating support. It is not, however, necessary that the carrier should be insulated from the earth, but it must be insulated from both coatings.

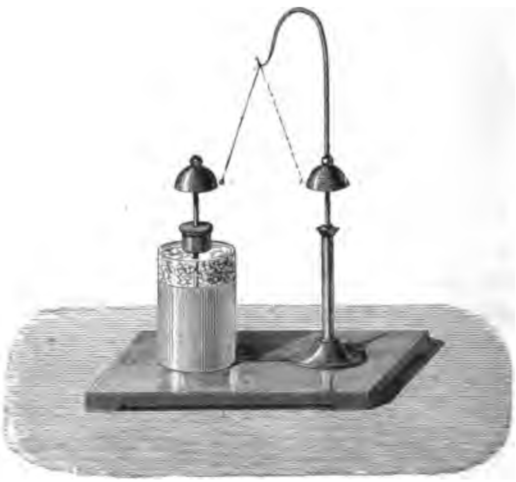

Fig. 386.—Alternate Discharge.

451. Condensing Power.—By the condensing power of a given arrangement is meant the ratio in which the capacity of the collecting plate is increased by the presence of the condensing plate, which ratio, as we have seen in last section, is equal to the fraction $\frac{1}{1-m'm}$. Riess has investigated its amount experimentally under varying conditions, by means of the apparatus represented in Fig. 387, which is a modification of the condenser of Æpinus. It consists of two metallic plates A and B, supported on glass pillars, and travelling on a rail, so that they can be adjusted at different distances. Between them is a large glass plate C. A is charged from the machine, B being at the same time touched to connect it with the ground. The electrical density on the anterior face of A was observed by means of Coulomb's proof-plane and torsion-balance. Riess' experiments are completely in agreement with the theory laid down in the preceding sections of this chapter; for example, he found, among other results, that the condensing power was independent of the absolute charge, and that it varied nearly in the inverse ratio of the distance.

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[1] This portion of the original charge is said to be *free*, and the remaining portion to be *bound*, *dissimulated*, or *latent*. These terms are applicable to all cases of condensation.

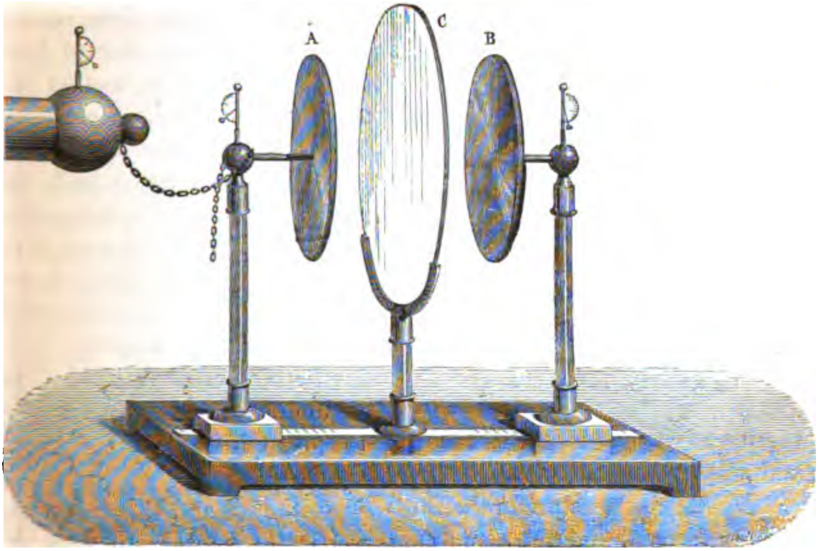

Fig. 387.—Condenser of Æpinus.

453. Influence of the Dielectric.—Faraday discovered that the amount of condensation obtained in given positions of the two conducting plates, depended upon the material of the *intervening non-conductor* or *dielectric.* The annexed figure (Fig. 388) represents a modification of one of Faraday's experiments. A is an insulated metallic disc, with a charge, which we will suppose to be positive. B and C are two other insulated metallic discs at equal distances from A, each having a small electric pendulum suspended at its back. Let B and C be touched with the hand; they will become negatively electrified by induction, but their negative electricity will reside only on their sides which face towards A, and the pendulums will hang vertically. If, while matters are in this condition, we move B nearer to A, we shall see both the pendulums diverge, and, on testing, we shall find that the pendulum B diverges with positive, and C with negative electricity. The reason is obvious. The approach of B to A causes increased induction between them, so that more negative is drawn to the face of B, and positive is driven to its back; at the same time the symmetrical distribution of electricity on A is disturbed, a portion being accumulated on the side next B at the expense of the side next C. The inductive action of A upon C is thus diminished, and a portion of the negative charge of C is left free to spread itself over the back, and affect the pith-ball.

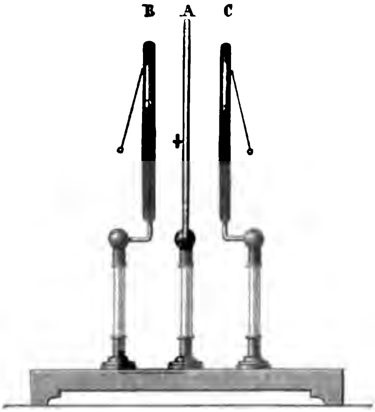

Fig. 388.—Change of Distance.

If, while the discs are in their initial position, B and C being equidistant from A, and the pendulums vertical, we interpose between B and A a plate of sulphur, shellac, or any other good insulator, the same effect will be produced as if B had been brought nearer to A. We see, then, that the insulating plate of a condensing arrangement serves not only to prevent discharge, but also to increase the inductive action and consequent condensation, as compared with a layer of air of the same thickness; inductive action through a plate of sulphur or shellac of given thickness, is the same as through a thinner plate of air. The numbers in the subjoined table denote the thickness of each material which is equivalent to unit thickness of air. For example, the mutual induction through 2·24 inches of sulphur is the same as through 1 inch of air. These numbers are called

SPECIFIC INDUCTIVE CAPACITIES.

Air or any gas, ...	1·00	Pitch, ...	1·80
Spermaceti, ...	1·45	Wax, ...	1·86
Glass, ...	1·76	Shellac, ...	2·00
Resin, ...	1·77	Sulphur, ...	2·24

The quotient of the actual thickness of the plate by the specific inductive capacity of its material may appropriately be called the *thickness reduced to its equivalent of air*, or simply the *reduced thickness*.

454. Faraday's Determinations.—Faraday, to whom the name and discovery of specific inductive capacity are due, operated by comparing the capacities of condensers, alike in all other respects, but differing in the materials employed as dielectrics. One of his condensers is represented in Fig. 389. It is a kind of Leyden jar, containing a metallic sphere A, attached to the rod M, and forming with it the inner conductor. The outer conductor consists of the hollow sphere B, divided into two hemispheres which can be detached from each other. The interval between the outer and inner conductor can be filled, either with a cake of solid non-conducting material, or with gas, which can be introduced by means of the cock R. The method of observation and reduction will be best understood from an example.

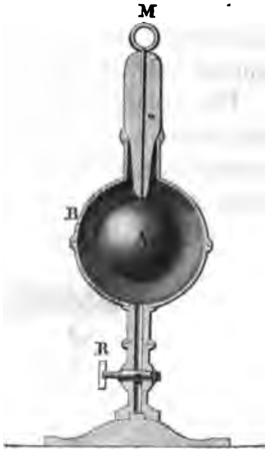

Fig. 389.—Apparatus for Specific Inductive Capacity.

The interval being occupied by air, the apparatus was charged, and a carrier-ball, having been made to touch the summit of the knob M, was introduced into a Coulomb's torsion-balance, and found to be charged with a quantity of electricity represented by 250° of torsion. When the second apparatus was precisely similar to the first, it was found that, on contact of the two knobs, the charge divided itself equally, and the carrier-ball, if applied to either knob, took a charge represented very nearly by 125°.

The conditions were then changed in the following way. The first jar still containing air, the interval between the two conductors in the second was filled with shellac. It was then found that the air-jar, being charged to 290°, was reduced, by contact of its knob with that of the shellac-jar, to 114°, thus losing 176°. If no allowance be made for dissipation, the capacities of the air-jar and shellac-air would therefore be as 114 : 176, or as 1 : 1·54, and the specific inductive capacity of shellac would be 1·54.

455. Polarization of the Dielectric.—As the interposed non-conductor, or dielectric, modifies the mutual action of the two electricities which it separates, and does not play the mere passive part which was attributed to it before Faraday's experiments, it is natural to conclude that the dielectric must itself experience a peculiar modification. According to Faraday, this modification consists in a polarization of its particles, which act inductively upon each other along the lines of force, and have each a positive and a negative side, the positive side of each facing the negative side of the next. This polarization is capable of being sustained for a great length of time in good non-conductors; but in good conductors it instantly leads to discharge between successive particles, and the opposite electricities appear only at the two surfaces.

The polarization of dielectrics is clearly shown in the following experiment. In a glass vessel (Fig. 390) is placed oil of turpentine, containing filaments of silk 2 or 3 millimetres long. Two metallic rods, A, B, each terminating within in a point, are connected, one with the ground, and the other with an electric machine. On working the machine, the little filaments are seen to arrange themselves in a line between the points, and, on endeavouring to break the line with a glass rod, it will be found that they return to this position with considerable pertinacity. On stopping the machine, they immediately fall to the bottom.

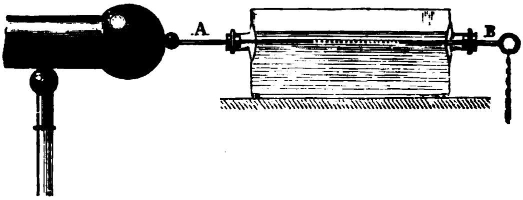

Fig. 390.—Polarization of Dielectric.

An experiment of Matteucci's demonstrates this polarization still more directly. A number of thin plates of mica are pressed strongly together between two metallic plates. One of the metallic plates is charged, while the other is connected with the ground; and, on removing the metallic plates by insulating handles, it is found that all the mica plates are polarized, the face turned towards the positive metal plate being covered with positive electricity, and the other face with negative.

459. Limit to Thinness of Interposed Plate.—We have seen (§ 445 Q) that the capacity of a condenser varies inversely as the distance between the collecting and the condensing plate. But if this distance is very small, the resistance of the interposed dielectric (which varies directly as its thickness) may be insufficient to prevent discharge, and it will not be practicable to establish a great difference of potential between the two plates. We may practically distinguish two sorts of condensers, one sort having a very thin dielectric and very great condensing power, but only capable of being charged to feeble[1] potential; the other having a dielectric thick enough to resist the highest tensions attainable by the electrical machine. The Leyden jar comes under the second category. The first includes the electrophorus (except in so far as its action is aided by the metallic sole), and the condenser of Volta's electroscope.

460. Volta's Condensing Electroscope.—This instrument, which has rendered very important services to the science of electricity, differs from the simple gold-leaf electroscope previously described (§ 415), in having at its top two metal plates, of which the lower one is connected with the gold-leaves, and is covered on its upper face with insulating varnish, while the upper is varnished on its lower face, and furnished with a glass handle. These two plates constitute the condenser. In using the instrument, one of the two plates (it matters not which) is charged by means of the body to be tested, while the other is connected with the earth. They thus receive opposite and sensibly equal charges. The upper plate is then lifted off, and the higher it is raised the wider do the gold-leaves diverge. The separation of the plates diminishes the capacity, and strengthens the potential of both, one becoming more strongly positive, and the other more strongly negative. This involves increase of potential energy, which is represented by the amount of work done against electrical attrac-

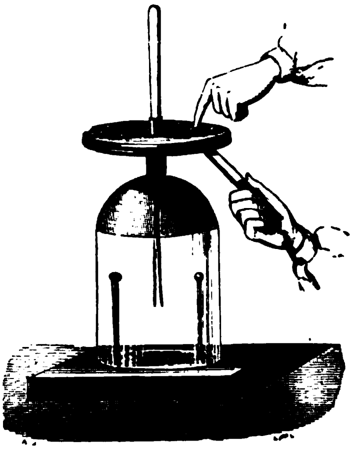

Fig. 391.—Condensing Electroscope.

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[1] *Strong* potential is potential differing very much from zero either positively or negatively. *Feeble* potential is potential not differing much from zero. *Tension* is measured by difference of potential; and when the earth is one of the terms of the comparison, tension becomes identical with potential.

tion in separating the plates. No increase in quantity of electricity is produced by the separation; hence the instrument is chiefly serviceable in detecting the presence of electricity which is available in large quantity but at weak potential. The glass handle of the upper plate is by no means essential, as it is only necessary that the lower plate should be insulated. The charge may be given by induction; in which case one plate must be connected with the earth while the inducing body is held near it, and the other plate must be kept connected with the earth while the influencing body is withdrawn. The plates will then be left charged with opposite electricities, that which was more remote from the influencing body having acquired a charge similar to that of the body. For inductive charges, however, the condensing arrangement serves no useful purpose, beyond enabling the electroscope to retain its charge for a longer time, the effect finally obtained on separating the plates being no greater than would have been obtained by employing only the lower plate.

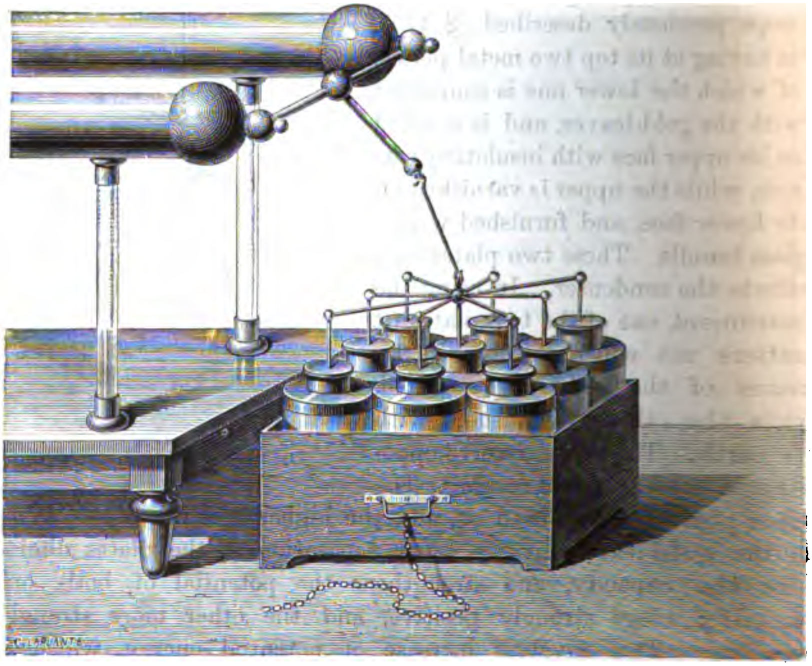

Fig. 392.—Battery of Leyden Jars.

461. Leyden Battery.—The Leyden battery consists of a number of

Leyden jars, placed in compartments of a box lined with tin-foil, which serves to establish good connection between their outer coatings, while their inner coatings are connected by brass rods. It is advisable that the outer coatings should have very free communication with the earth. For this purpose a metallic handle, which is in metallic communication with the lining of the box, should be connected, by means of a chain, with the gas or water pipes of the building.

The capacity of a Leyden battery is the sum of the capacities of ars which compose it. The charge is given in the ordinary way, nnecting the inner coatings with the conductor of the machine. d weather this is usually a very difficult operation, on account

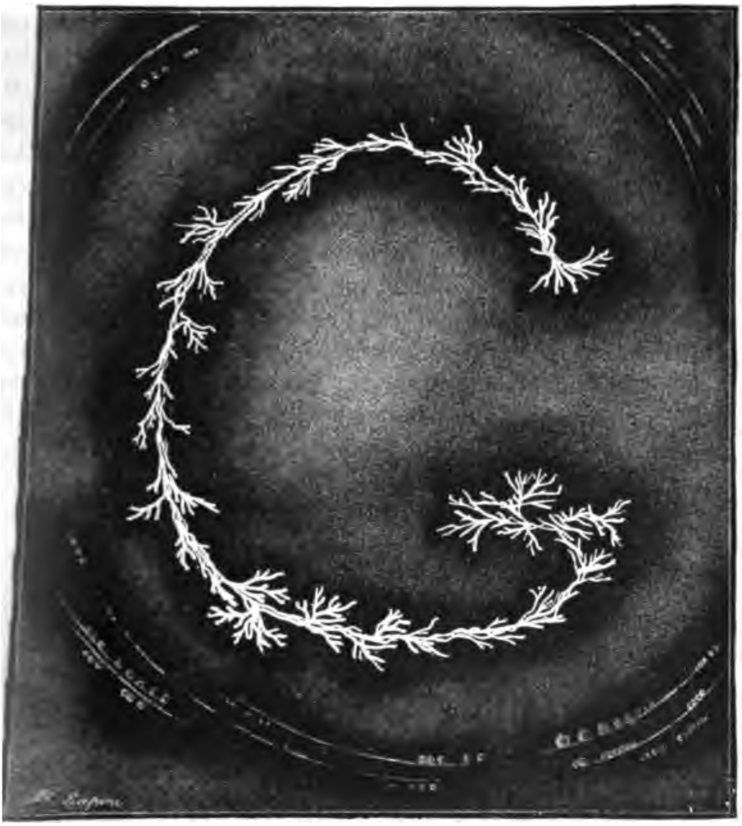

Fig. 393.—Lichtenberg's Figures

of the large quantity of electricity required for a full charge, and the large surface from which dissipation goes on.

Holtz's machine can be very advantageously employed for charging a battery, one of its poles being connected with the inner, and the other with the outer coatings. In dry weather it gives the charge with surprising quickness.

462. Lichtenberg's Figures.—An interesting experiment devised by Lichtenberg serves to illustrate the difference between the physical properties of positive and negative electricity.

A Leyden jar is charged, and the operator, holding it by the outer coating, traces a design with the knob on a plate of shellac or vulcanite. He then places the jar on an insulating stand, to enable him to transfer his hold to the knob, and traces another pattern on the cake with the outer coating. A mixture of flowers of sulphur and red-lead, which has previously been well dried and shaken together, is then sprinkled over the cake. The sulphur, having become negatively electrified by friction with the red-lead, adheres to the pattern which was traced with positive electricity, while the red-lead adheres to the other. The yellow and red colours render the patterns very conspicuous. The particles of sulphur (represented by the inner pattern in Fig. 393) arrange themselves in branching lines, while the red-lead (shown in the outer pattern) forms circular spots; whence it would appear that positive electricity travels along the surface more easily than negative. A similar difference has already been pointed out between positive and negative brushes.

CHAPTER XLI.

EFFECTS PRODUCED BY THE DISCHARGE OF CONDENSERS.

463. Discharge of Batteries.—The effects produced by the discharge of a Leyden jar or battery differ only in degree from those of an ordinary electric spark. The shock, which is smart even with a small jar, becomes formidable with a large jar, and still more with a battery of jars.

If a shock is given to a number of persons at once, they must form a chain by holding hands. The person at one end of the chain must place his hand on the outer coating of a charged jar, and the person at the other end must touch the knob. The shock will be felt by all at once, but somewhat less severely by those in the centre.

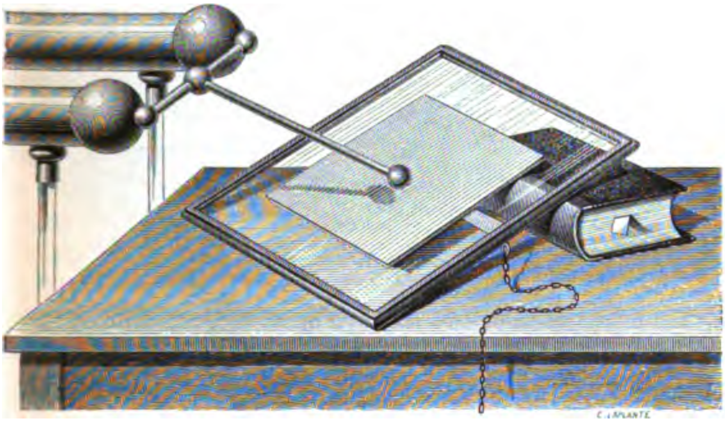

Fig. 394.—Coated Pane.

The *coated pane*, represented in Fig. 394, is simply a condenser, consisting of a pane of glass, coated on both sides, in its central portion, with tin-foil. Its lower coating is connected with the earth by a chain, and a charge is given to its upper coating by the machine. When it is charged, if a person endeavours to take up a coin laid upon its upper face, he will experience a shock as soon as his hand comes near it, which will produce involuntary contraction of his arm, and prevent him from taking hold of the coin.

464. Heating of Metallic Threads.—The discharge of electricity through a conducting system produces elevation of temperature, the amount of heat generated being the equivalent of the potential energy which runs down in the discharge, and which is jointly proportional to quantity of electricity and difference of potential. The incandescence of a fine metallic thread can be easily produced by the discharge of a battery. The thread should be made to connect the knobs *a b* of an apparatus called a *universal discharger;* these knobs being the extremities of two metallic arms supported on glass stems. One of the arms is connected with the external surface of the battery, and the other arm is then brought into connection with the internal surface by means of a discharger with glass handles. At the instant of the spark passing, the thread becomes red-hot, melts, burns, or volatilizes, leaving, in the latter cases, a coloured streak on a sheet of paper *c* placed behind it. When the thread is of gold, this streak is purple, and exactly resembles the marks left on walls when bell-pulls containing gilt thread are struck by lightning.

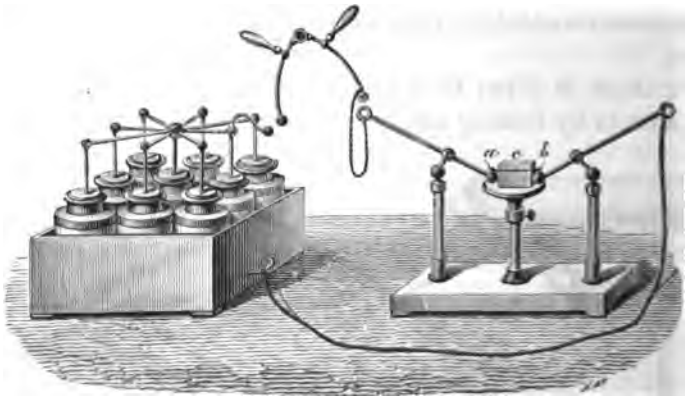

Fig. 395.—Universal Discharger.

465. Electric Portrait.—The volatilization of gold is employed in producing what are called electric portraits. The outline of a portrait of Franklin is executed in a thin card by cutting away narrow strips. Two sheets of tin-foil are gummed to opposite edges of the card, which is then laid between a gold-leaf and another card. The whole is then placed in a press (Fig. 396), the tin-foil being allowed to protrude, and strong pressure is applied. The press is placed on the table of the universal discharger, and the two knobs of the latter are connected with the two sheets of tin-foil. The discharge is then passed, the gold is volatilized, and the vapour, passing through the slits to the white card at the back, leaves purple traces which reproduce the design.

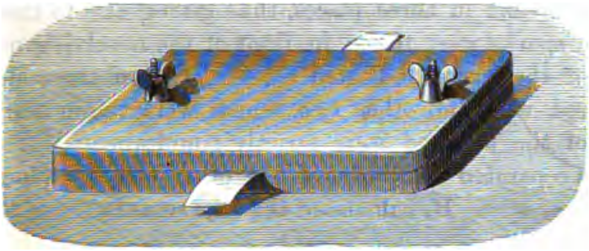

Fig. 396.—Press for Portrait.

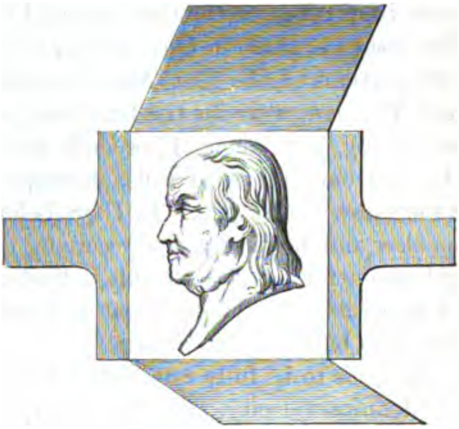

Fig. 397.—Arrangement for Portrait.

466. Velocity of Electricity.—Soon after the invention of the Leyden jar, various attempts were made to determine the velocity with which the discharge travels through a conductor connecting the two coatings. Watson, about 1748, took two iron wires, each more than a mile long, which he arranged on insulating supports in such a way that all four ends were near together. He held one end of each wire in his hands, while the other ends were connected with the two coatings of a charged jar. Although the electricity had more than a mile to travel along each wire before it could reach his hands, he could never detect any interval of time between the passage of the spark from the knob of the jar and the shock which he felt. The velocity was in fact far too great to be thus measured.

Wheatstone, about 1836, investigated the subject with the aid of the revolving mirror of which we have spoken above (§ 437). He connected the two coatings of a Leyden jar by means of a conductor which had breaks in three places, thus giving rise to three sparks. When the sparks were taken in front of the revolving mirror, the positions of the images indicated a retardation of the middle spark, as compared with the other two, which were taken near the two coatings of the jar, and were strictly simultaneous. The middle break was separated from each of the other two by a quarter of a mile of copper wire. He calculated that the retardation of the middle spark was $\frac{1}{1,152,000}$ of a second, which was therefore the time occupied in travelling through a quarter of mile of copper wire. This is at the rate of 288,000 miles per second, a greater velocity than that of light, which is only about 184,000 miles per second.

Since the introduction of electric telegraphs, several observations have been taken on the time required for the transmission of a signal. For instance, trials in Queenstown harbour, in July, 1856, when the two portions of the first Atlantic cable, on board the *Agamemnon* and *Niagara*, were for the first time joined into one conductor, 2500 miles long, gave about $1\frac{3}{4}$ seconds as the time of transmission of a signal from induction coils, corresponding to a velocity of only 1400 miles per second. In 1858, before again proceeding to sea, a quicker and more sensitive receiving instrument—Thomson's mirror galvanometer—gave a sensible indication of rising current at one end of 3000 miles of cable about a second after the application of a Daniell's battery at the other.

It seems to be fully established by experiment that electricity has no definite velocity, and that its apparent velocity depends upon various circumstances, being greater through a short than through a long line, greater (in a long line) with the greater intensity and suddenness of the source, greater with a copper than with an iron wire, and much greater in a wire suspended in air on poles than in one surrounded by gutta-percha and iron sheathing, and buried under ground or under water. In a long submarine line, a short sharp signal sent in at one end, comes out at the other as a signal gradually increasing from nothing to a maximum, and then gradually dying away.

467. Unit-jar.—For quantitative experiments on the effects of discharge, Lane's *unit-jar* has frequently been employed. One of its forms is represented in Fig. 398. It consists of a very small Leyden phial, having two knobs *a*, *b*, one connected with each coating, the distance between them being adjustable by means of a sliding rod. To measure the charge given to a jar or battery, the latter is placed upon an insulating support, its inner coating is connected with the conductor of the machine, and its outer coating is connected with the inner coating of the unit-jar. The outer coating of the unit-jar must be in connection with the ground. When the machine is worked, sparks pass between *a* and *b*, each spark being produced by the escape of a definite quantity of electricity from the outer coating of the battery, and indicating the addition of a definite amount to the charge of the inner coating. The charge is measured by counting the sparks.

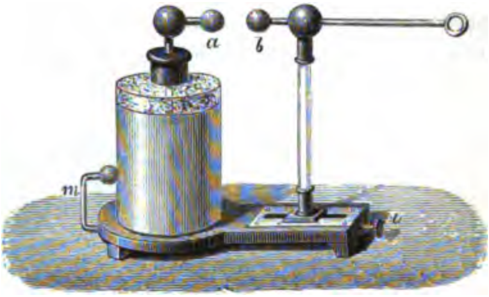

Fig. 398.—Unit-jar.

Snow Harris modified the arrangement by insulating the unit-jar instead of the battery. One coating of his unit-jar is connected with the battery, and the other with the conductor of the machine. The battery thus receives its charge through the unit-jar[1] by a succession of discharges between the knobs *a*, *b*, each representing a definite quantity of electricity.

Both arrangements, as far as their measuring power is concerned, depend upon the assumption that discharge between two given conductors, in a given relative position, involves the transfer of a definite quantity of electricity. This assumption implies a constant condition of the atmosphere. It may be nearly fulfilled during a short interval of time in one day, but is not true from one day to another. Moreover, it is to be remembered that, as dissipation is continually going on, the actual charge in the battery at any time is less than the measured charge which it has received.

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[1] Lane's arrangement might have been described by saying that the *outer coating* of the battery receives its negative charge from the earth through the unit-jar.

468. Mechanical Effects.—The effects of discharge through bad conductors are illustrated by several well-known experiments.

1. *Puncture of card.* A card is placed (Fig. 399) between two points connected with two conductors which are insulated from one another by means of a glass stem. The lower conductor having been connected with the outer coating of a Leyden jar which is held in the hand, the knob of the jar is brought near the upper conductor. A spark passes, and another spark at the same instant passes between the two points, and punctures the card. In performing this experiment it is observed that, if the points are not opposite each other, the perforation is close to the negative point. This want of symmetry appears to be due to the properties of the air. When arrangements are made for exhausting the air, it is found that, as the density of the air is diminished, the perforation takes place nearer to the centre.

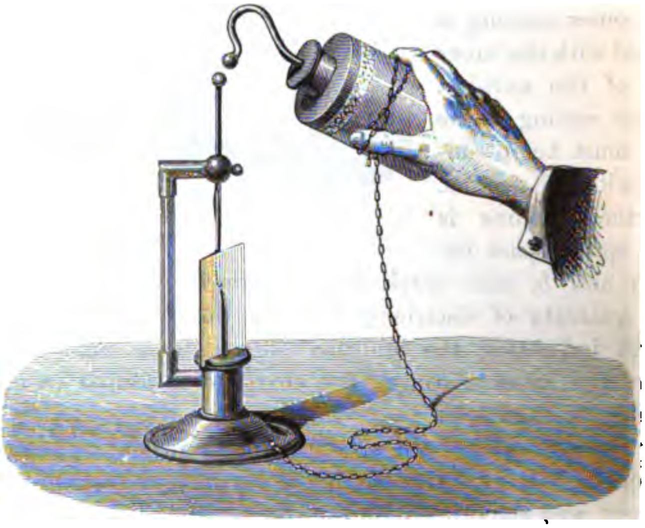

Fig. 399.—Puncture of Card.

The piercing of a card can very easily be effected by Holtz's machine. Its two conductors are connected with the two coatings of a small Leyden jar. The discharges between the poles will then consist of powerful detonating sparks in rapid succession; and if a sheet of paper or card be interposed, every spark will puncture a minute hole in it.

2. *Perforation of glass.* To effect the perforation of glass, a pane

of glass is supported on one end of a glass cylinder in whose axis there is a metallic rod terminating in a point which just touches the pane. Another pointed rod exactly over this, and insulated from it, is lowered until it touches the upper face of the pane. A powerful spark from a Leyden jar or battery is passed between the two points, and, if the experiment succeeds, a hole is produced by pulverization of the glass.

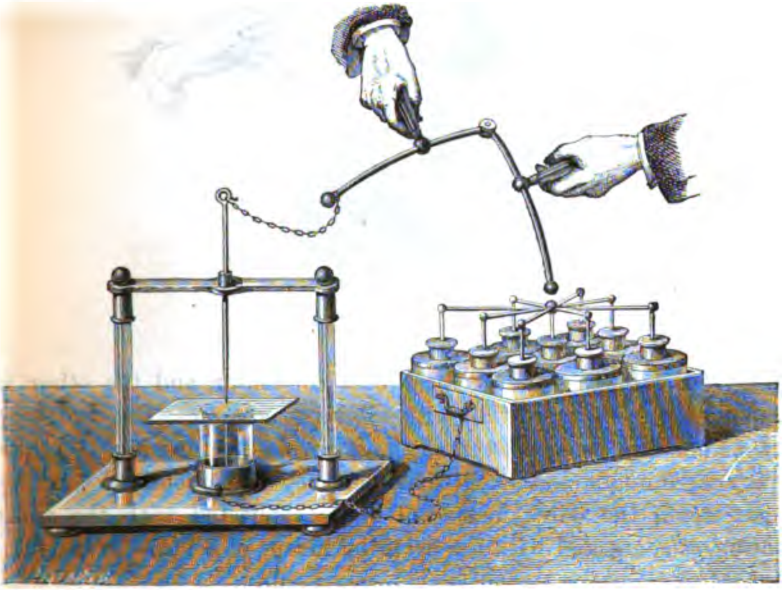

Fig. 400.—Puncture of Glass.

The experiment sometimes fails, by discharge taking place round the edge of the glass instead of through its substance. To prevent this, a drop of oil is placed on the upper face of the pane at the point where the hole is to be made; but this precaution does not always insure success, and, when the experiment has once failed, it is useless to try it again with the same piece of glass, for the electricity is sure to follow in the course which the first discharge has marked out for it.

469. Explosion of Mines.—If a strongly charged Leyden jar be discharged by means of a jointed discharger which has one of its knobs covered with gun-cotton, when the spark passes between the jar and this knob, the gun-cotton will be inflamed. Ordinary cotton mixed with powdered resin can be kindled in the same way.

A similar arrangement is often used for exploding mines. A fuse is employed containing two wires embedded in gutta-percha, but with their ends unprotected and near together. One of these wires is connected with the outer coating of a condenser, and the other is brought into communication with the inner coating. The discharge is thus made to pass between the ends within the fuse, and to ignite a very inflammable compound by which they are surrounded. Sometimes one of the wires, instead of being connected with the outer coating, is connected with the earth by means of a buried wire.

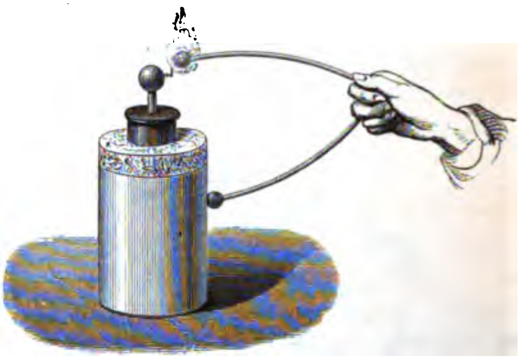

Fig. 401.—Gun-cotton Fired.

CHAPTER XLI^{A}.

ELECTROMETERS.

469 A. Object of Electrometers.—Electrometers are instruments for the measurement of differences of electrical potential. The gold-leaf electroscope, the straw-electroscope, and other instruments of the same type, afford rough indications of the difference of potential between the diverging bodies and the air of the apartment, and more measurable indications are given by the electrometers of Peltier and Dellmann; but none of these instruments are at all comparable in precision to the various electrometers which have been invented from time to time by Sir Wm. Thomson.

469 B. Attracted-disc Electrometers, or Trap-door Electrometers.—We shall first describe what Sir Wm. Thomson calls "Attracted-disc Electrometers." These instruments, one of which is represented in Figs. 401 A, 401 B, contain two parallel discs of brass *g*, *h*, with an aperture in the centre of one of them, nearly filled up by a light trap-door of aluminium *f*, which is supported in such a way as to admit of its electrical attraction towards the other disc being resisted by a mechanical force which can be varied at pleasure. The trap-door and the perforated plate surrounding it must have their faces as nearly as possible in one plane when the observation is taken, and, as they are electrically connected, they may then be regarded as forming *one disc of which a small central area is movable.* There is always attraction between the two parallel discs, except when they are at the same potential.

Let their potentials be denoted by V and V′, the electrical densities on their faces by ρ and ρ', and their mutual distance by D. We have seen (§ 445 Q) that, in such circumstances, ρ and ρ' are constant (except near the edges of the discs), opposite in sign, and equal, and that the intensity of force in the space between them is everywhere the same, and equal at once to $\frac{V-V'}{D}$ and to $4\pi\rho$. This force is jointly due to attraction by one plate and repulsion by the other, each of these having the intensity $2\pi\rho$, or half the total intensity.

Let A denote the area of the trap-door. The quantity of electricity upon it will be ρA, and the force of attraction which this experiences will be $\rho A \times 2\pi\rho = 2\pi\rho^2 A$, which we shall denote by F. Then from the equations

$$F = 2\pi\rho^2 A \quad , \quad \frac{V-V'}{D} = 4\pi\rho, \qquad (1)$$

we find, by eliminating ρ,

$$F = \frac{A}{8\pi}\left(\frac{V-V'}{D}\right)^2, \text{ or } V-V' = \pm D\sqrt{\frac{8\pi F}{A}}. \qquad (2)$$

469 C. Absolute Electrometer.—In the *absolute electrometer*, which somewhat resembles Fig. 401 B turned upside down, the force of electrical attraction on the trap-door is measured by direct comparison with the gravitating force of known weights. This is done by first observing what weights must be placed on the trap-door to bring it into position when no electrical force acts (the plates being electrically connected), and by then removing the weights, allowing electrical force to act, and adjusting the plates at such a distance from one another, by the aid of a micrometer screw, that the trap-door shall again be brought into position. Then, in equation (2), F, A and D are known, and the difference of potentials V—V′ can be determined. In the absolute electrometer, the perforated disc *h* is uppermost, so that the direction of electrical attraction on the trap-door is similar to the direction of the gravitating force of the weights. The reverse arrangement is usually adopted in the portable electrometer, which we shall next describe. In both instruments, the trap-door constitutes one end of a very light lever *fil* of aluminium, balanced on a horizontal axis.

469 D. Portable Electrometer.—In the *portable electrometer* (Figs. 401 A, 401 B) this axis passes very accurately through the centre of gravity of the lever, the suspension being effected by means of a fine platinum wire *w w* tightly stretched, which is secured at its centre to the lever in such a manner that, when the trap-door comes into position, the wire is under torsion tending to draw back the disc from the attracting plate *g*. This torsion (except in so far as it is affected by causes of error such as temperature and gradual loss of elasticity) is always the same when the disc is in position, and as it is to be balanced in every observation by electrical attraction, the latter must also be always the same; that is to say, the quantity F in equations (2) is constant for all observations with the same instrument; whence it is obvious that V—V′ is directly proportional to D, the distance between the plates. The observation for difference of potential therefore consists in altering this distance until the trap-door comes into position. This is done by turning the micrometer screw, by means of the milled head *m*. The divided circle of the micrometer indicates the amount of turning for small distances, and whole revolutions are read off on the vertical scale traversed by the index carried by the arm *d*. The correct position is very accurately identified by means of two sights, one of them being attached to a fixed portion of the instrument, and the other to one end *l* of the lever. One of these sights moves up and down close in front of the other, and they are viewed through a lens *o* in front of both. This arrangement is also adopted in the absolute electrometer.

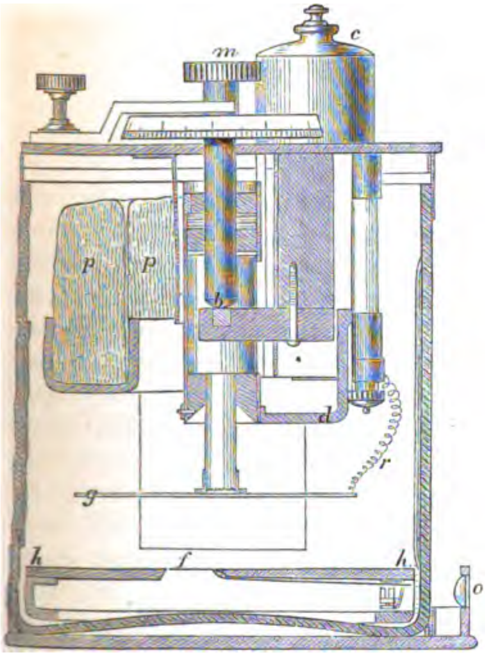

Fig. 401 A.—Portable Electrometer.

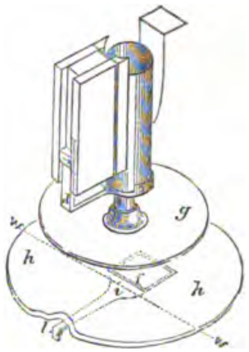

Fig. 401 B.—Parallel Discs.

One of the two parallel plates *h* is connected with the inner coating of a Leyden jar,[1] which, being kept dry within by means of pumice *p* wetted with sulphuric acid, retains a sufficient charge for some weeks. The other plate *g* is in communication, by means of the spiral wire *r*, with the insulated umbrella *c*, which can be connected with any external conductor; and, in order to determine the potential of any conductor which we wish to examine, two observations are taken, one of them giving the difference of potential between this conductor and the Leyden jar, and the other the difference between the earth and the jar. We thus obtain, by subtrac-

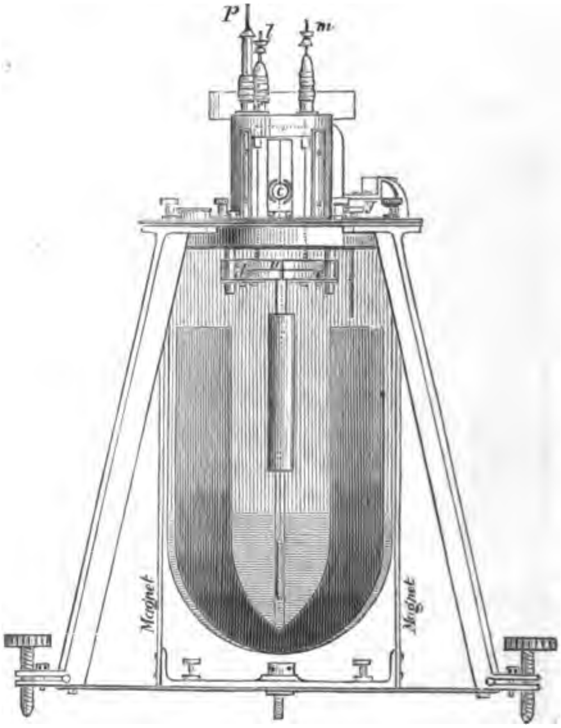

Fig. 401 c.—Quadrant Electrometer.

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[1] The use of the Leyden jar is to give constancy of potential. Its capacity is so much greater than that of the disc with which it is connected that the electricity which enters or leaves the latter in consequence of the inductive action of the other disc is no sensible fraction of its whole charge, and produces no sensible change in its potential. Its great capacity in comparison with the extent of surface exposed likewise tends to prevent rapid loss of potential by dissipation of charge.

tion, the difference of potential between the conductor in question and the earth.

469 E. Quadrant Electrometer.—The most sensitive instrument yet invented for the measurement of electrical potential is the *quadrant electrometer*, which is represented in front view in Fig. 401 C, some of its principal parts being shown on a larger scale in Figs. 401 D, 401 E.

In this instrument, the part whose movements give the indications is a thin flat piece of aluminium *u*, narrow in the middle and broader towards the ends, but with all corners rounded off. This piece, which is called the *needle*, and is represented by the dotted line in Fig. 401 D, is inclosed almost completely in what may be described as a shallow cylindrical box of brass, cut into four quadrants, *c*, *d*, *c′ d′*. These parts are shown in plan in Fig. 401 D, and in front view in Fig. 401 C. The needle *u* is attached to a stiff platinum wire, which is supported by a silk fibre hanging vertically. The same wire carries a small concave mirror *t* (Fig. 401 C) for reflecting the light from an illuminated vertical slit. An image of the slit is thus formed at the distance of about a yard, and is received upon a paper scale of equal parts, by reference to which the movements of the image can be measured. The movements of the image depend upon the movements of the mirror, which are precisely the same as those of the needle. We have now to explain how the movements of the needle are produced.

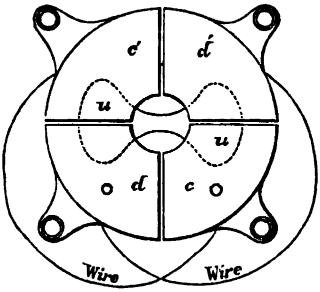


Fig. 401 D.—Needle and Quadrants.

One pair of opposite quadrants *c c′* are connected with each other, and with a stiff wire *l* projecting above the case of the instrument. The other quadrants *d d′* are in like manner connected with the other projecting wire *m*. The projecting parts *l m* are called the *chief electrodes*, and are to be connected respectively with the two conductors whose difference of potential is required, one of which is usually the earth. Suppose the needle to have a positive charge of its own, then if the potential of *c* and *c′* be higher (algebraically) than that of *d* and *d′*, one end of the needle will experience a force urging it from *c* to *d*, and the other end will experience a force urging it from *c′* to *d′*. These two forces constitute a couple tending to turn the needle about a vertical axis. If the potential of c and c' be lower than that of d and d', the couple will be in the opposite direction. To prevent the needle from deviating too far under the action of this couple, and to give it a definite position when there is no electrical couple acting upon it, a small light magnet is attached to the back of the mirror, and by means of controlling magnets outside the case the earth's magnetism is overpowered, so that, whatever position be chosen for the instrument, the needle can be made to assume the proper zero position. In some instruments recently constructed, the magnets are dispensed with, and a bifilar suspension is substituted for the single silk fibre. The permanent electrification of the needle is attained by connecting it, by means of a descending platinum wire, with a quantity of strong sulphuric acid, which occupies the lower part of the containing glass jar. The acid, being an excellent conductor, serves as the inner coating of a Leyden jar, the outside of the glass opposite to it being coated with tin-foil, and connected with the earth. The acid at the same time serves the purpose of keeping the interior of the apparatus very dry. The charge is given to the jar through the *charging electrode* p, which can be thrown into or out of connection at pleasure. As the sensibility of the instrument increases with the potential of the jar, a *gauge* and *replenisher* are provided for keeping this potential constant. The *gauge* is simply an "attracted-disc electrometer," in which the distance between the parallel discs is never altered, so that the aluminium square only comes into position when the potential of one of the discs, which is connected with the acid in the jar, differs by a certain definite amount from the potential of the other, which is connected with the earth. A glance at the gauge shows, at any moment, whether the potential of the jar has the normal strength. If it has fallen below this point, the *replenisher* is employed to increase the charge.

This apparatus, which is separately represented, dissected, in Fig. 401 E, and is for simplicity omitted in Fig. 401 C, consists of a vertical stem of ebonite s, which can be rapidly twirled with the finger by means of a milled head y, and which carries two metal wings or *carriers*, b, b, insulated from each other. In one part of their revolution, these come in contact with two light steel springs $f\ f$, which simply serve to connect them for the instant with each other. In another part of their revolution, they come in contact with two other springs $e\ e$, connected respectively with the acid of the jar and with the earth. The first of these contacts takes place just before the wings emerge from the shelter of the larger metallic sectors or *inductors a a*, of which one is connected with the acid, and the other with the earth. Suppose the acid to have a positive charge. Then, at the instant of contact, an inductive movement of electricity takes place, producing an accumulation of negative electricity in the carrier which is next the positive inductor, and an accumulation of positive in the other. The next contacts are effected when the carrier which has thus acquired a positive charge is well under cover of the positive inductor, to which accordingly it gives up its electricity, for, being in great part surrounded by this inductor, and being connected with it by the spring, the carrier may be regarded as forming a portion of the interior of a concave conductor, and the electricity accordingly passes from it to the external surface, that is to the inductor, and to the acid connected with it, which form the lining of the jar. The negative electricity on the other carrier is, in like manner, given off to the other inductor, and so to the earth.

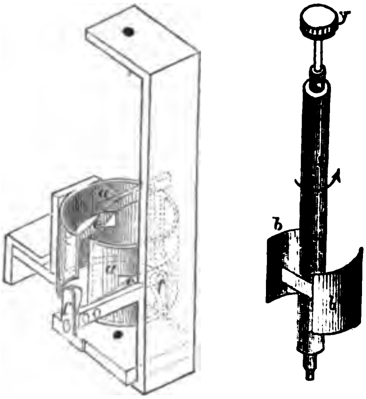


Fig. 401 E.—Replenisher.

The jar thus receives an addition to its charge once in every half-revolution of the replenisher; and, as these increments are very small, it is easy to regulate the charge so that the gauge shall indicate exactly the normal potential. If the charge is too strong, it can be diminished by turning the replenisher in the reverse direction.

469 F. Cage-electrometer.—In another form of electrometer, which has some advantages of its own, though now but little used, the observation for difference of potential consists in applying torsion to a glass fibre until the needle (a straight piece of aluminium wire) which it carries, is forced, against electrical repulsion, to assume a definite position marked by sights. The repulsion, which acts upon the two ends of the needle so as to produce a couple, is exerted by two vertical brass plates, which are connected with the needle by means of fine platinum wires dipping in sulphuric acid at the bottom of a Leyden jar. The needle and the plates which repel it are thus at the potential of the jar. The repulsion between them is modified by the influence of a cage of brass wire, which surrounds them, and which is connected with the conductor whose potential is to be examined. If this conductor has the same potential as the jar, there is no repulsion. If its potential differs either way from that of the jar, the couple of repulsion is proportional to the square of this difference of potentials.[1] The difference of potential is therefore obtained by taking the square root of the number of degrees of torsion of the fibre.

[1] In a given position of the needle, the quantities of electricity upon it and upon the plates which repel it are both proportional to this difference of potentials, and the distribution is invariable. Hence (§ 420) the force of repulsion is proportional to the product of the two quantities, that is to the square of either of them.

CHAPTER XLII.

ATMOSPHERIC ELECTRICITY.

470. Resemblance of Lightning to the Electric Spark.—The resemblance of the effects of lightning to those of the electric spark struck the minds of many of the early electricians. Lightning, in fact, ruptures and scatters non-conducting substances, inflaming those which are combustible; heats, reddens, melts, and volatilizes metals; and gives shocks, more or less severe, and frequently fatal, to men and animals; all of these being precisely the effects of the electric spark with merely a difference of intensity. We may add that lightning leaves behind it a characteristic odour precisely similar to that which is observed near an electrical machine when it is working, and which we now know to be due to the presence of ozone. Moreover, the form of the spark, its brilliancy, and the detonation which attends it, all remind one forcibly of lightning.

To Franklin, however, belongs the credit of putting the identity of the two phenomena beyond all question, and proving experimentally that the clouds in a thunder-storm are charged with electricity. This he did by sending up a kite, armed with an iron point with which the hempen string of the kite was connected. To the lower end of the string a key was fastened, and to this again was attached a silk ribbon intended to insulate the kite and string from the hand of the person holding it. Having sent up the kite on the approach of a storm, he waited in vain for some time even after a heavy cloud had passed directly over the kite. At length the fibres of the string began to bristle, and he was able to draw a strong spark by presenting his knuckle to the key. A shower now fell, and, by wetting the string, improved its conducting power, the silk ribbon being still kept dry by standing under a shed. Sparks in rapid succession were drawn from the key, a Leyden jar was charged by it, and a shock given.

Shortly before this occurrence, Dalibard, acting upon a published suggestion of Franklin, had erected a pointed iron rod on the top of a house near Paris. The rod was insulated from the earth, and could be connected with various electrical apparatus. A thunder-storm having occurred, a great number of sparks, some of them of great power, were drawn from the lower end of the rod.

These experiments were repeated in various places, and Richmann of St. Petersburg, while conducting an investigation with an apparatus somewhat resembling that of Dalibard, received a spark which killed him on the spot.

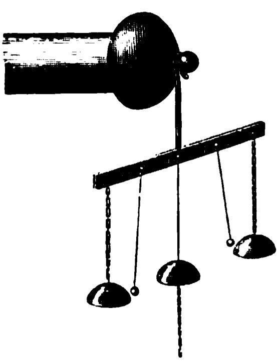


Fig. 402.—Electric Chimes.

472. Electric Chimes.—Franklin devised an apparatus for giving warning when the insulated rod is charged with electricity. It consists (Fig. 402) of a metal bar, carrying three bells with two clappers between them. The two extreme bells are hung from the bar by metallic chains. The middle one is hung by a silk thread, and connected with the ground. The clappers are also hung by silk threads. When the bar is electrified, the clappers are first attracted by the two extreme bells, and then repelled to the middle bell, through which they discharge themselves, to be again attracted and repelled, thus keeping up a continual ringing as long as the bar remains electrified.

473. Duration of Lightning.—It appears that thunder-clouds must be regarded as charged masses of considerable conducting power. The discharges which produce lightning and thunder occur sometimes between two clouds, and sometimes between a cloud and the earth. The duration of the illumination produced by lightning is certainly less than the ten-thousandth of a second. This has been established by observing a rapidly rotating disc (Fig. 403) divided into sectors alternately black and white. If viewed by daylight, the disc appears of a uniform gray; and if lightning, occurring in the dark, renders the separate sectors visible, the duration of its light must be less than the time of revolving through the breadth of one sector. The experiment has been tried with a disc divided into 60 sectors, and making 180 revolutions per second, so that the time of turning through the

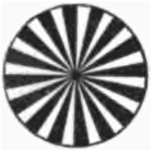


Fig. 403.—Duration of Flash.

space occupied by one sector is $\frac{1}{60}$ of $\frac{1}{180}$ of a second, that is, $\frac{1}{10800}$. When the disc, turning with this velocity, is rendered visible by lightning, the observer sees black and white sectors with gray ones between them. For the black and white sectors to be seen sharply defined, without intermediate gray, it would be necessary that the illumination should be absolutely instantaneous.

476. Thunder.—Thunder frequently consists of a number of reports heard in succession. This can be explained by supposing that (as in the experiment of the spangled tube, § 440) discharge occurs at several places at once. The reports of these explosions will be heard in the order of their distance from the observer. If, for example, the lines of discharge form the zig-zag M N (Fig. 404), an observer at O will hear first the explosion at *a*, then, a little later, the five explosions at *m*, *n*, *r*, *s*, *t*; he will consequently observe an increase in the intensity of the sound.

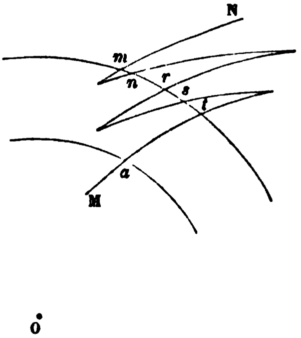


Fig. 404.—Simultaneous Explosions.

477. Shock by Influence.—Persons near whom a flash of lightning passes, frequently experience a severe shock by induction. This is analogous to the phenomenon, first observed by Galvani, that a skinned frog in the neighbourhood of an electrical machine, although dead, exhibits convulsive movements every time a spark is drawn from the conductor. In like manner, if Volta's pistol (§ 443) be placed on the wooden supports of an electrical machine, and its knob be connected with the ground by a chain, on drawing a spark from the machine, another spark will pass in the interior of the pistol, and fire it off.

478. Lightning-conductors.—Experience having shown that electricity travels in preference through the best conductors, it is easy to understand that, if a building be fitted with metallic rods terminating in the earth, lightning will travel through these instead of striking the building. But further, if these rods terminate above in a point, they may exercise a preventive influence by enabling the earth and clouds to exchange their opposite electricities in a gradual way, just as the conductor of a machine is prevented from giving powerful sparks by presenting to it a sharp point connected with the earth.

While the electrical machine is working powerfully, and the quadrant electroscope shows a strong charge, let a pointed metallic rod be presented, as in Fig. 405; the pith-ball will immediately fall back to the vertical position, and it will be found impossible to draw a spark from any part of the conductor. If the experiment is performed in the dark, the point will be seen to be tipped with light; and a similar appearance is sometimes observed on the tops of lightning-rods and of ships' masts. In the latter position it is known to sailors as *St. Elmo's fire.*

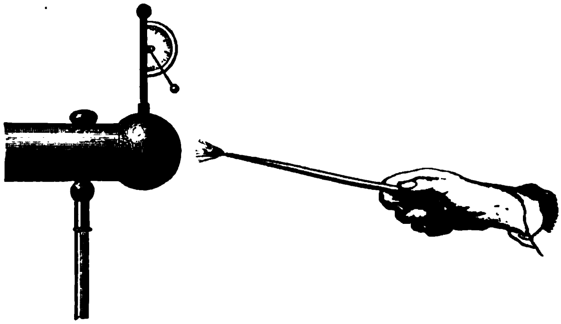


Fig. 405.—Conductor Discharged by presenting a Point.

479. Construction of Lightning-conductors.—A badly constructed lightning-conductor may be a source of danger, instead of a protection. The following conditions should always be complied with:—

1. The connection with the ground should be continuous.

2. The conductor must be everywhere of so large a section that it will not be melted by lightning passing through it. The French Academy of Sciences recommend that the section for iron rods should be nowhere less than 2·25 centimetres, or $\frac{8}{10}$ of an inch.

3. The earth contact must be good. The conductor may be connected at its base with the iron pipes which supply the neighbourhood with water or gas; or it may terminate in the water of a well or pond. Failing these, it should be provided with branches traversing the soil in different directions, and surrounded by coke, which is a good conductor.

4. At no part of its course above ground should it come near to the metal pipes which supply the house with water or gas, nor to any large masses of metal in the house. All large masses of metal on the outside of the house, such as lead roofing, should be well connected with the conductor.

xtreme point should be sharp. A former commission of ny recommended a platinum point, which should be connected with the iron by welding. But as this construction is both difficult and expensive, later directions have been issued recommending a gilded copper cone, screwed on to the iron, as shown in Fig. 407, which is half the actual size. This form of termination is better than a needle point, because less liable to fusion.

The general arrangement is represented in Fig. 406. The rod has a diameter of 2 or 3 inches at its base, and gradually tapers upwards to the place where the point is screwed on. The descending portion *b* is connected with the base of this rod by the broad band *ll'*.

480. Ordinary Electricity of the Atmosphere.—The presence of electricity in the upper regions of the air is not confined to thunder-clouds, but can be detected at all times. In fine weather this electricity is almost invariably positive, but in showery or stormy weather negative electricity is as frequently met with as positive; and it is in such weather that the indications of electricity, whether positive or negative, are usually the strongest.

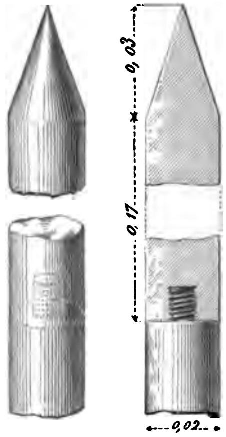
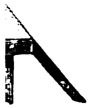


Fig. 407.—Gilded Copper Point.

480 A. Methods of obtaining Indications.—One of the early methods of observing atmospheric electricity consisted in shooting up an arrow, attached to a conducting thread, having at its lower end a



tning-

as laid upon the top of a gold-leaf electroscope. As the higher, the leaves diverge more and more with elec- same sign as that overhead; and they remain divergent after the ring has been lifted off by the movement of the arrow.

Sometimes, instead of the arrow, a point on the top of the electroscope is employed to collect electricity from the air, as in Fig. 408. Both these methods are very uncertain in their action.

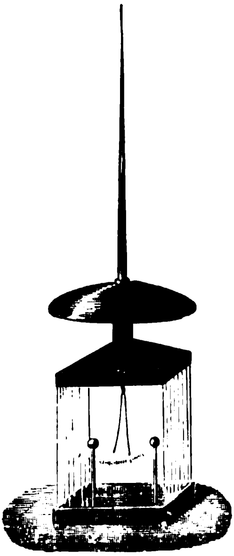


Fig. 408.—Early Form of Electrometer.

A better method of collecting electricity from the air was long ago devised by Volta, who employed for this purpose a burning match attached to the top of a rod connected with the gold-leaves or straws of his electroscope. If there is positive electricity overhead, its influence causes negative electricity to collect at the upper end of the rod, whence it passes off by convection in the products of combustion of the match, leaving the whole conducting system positively electrified. In like manner, if the electricity overhead be negative, the system will be left negatively electrified.

Another method which, in the hands of Peltier, Quetelet, and Dellmann, has yielded good results, consists in first exposing, in an elevated position such as the top of a house, a conducting ball supported on an insulating stand, and, while exposed, connecting it with the earth, then insulating it, and examining the charge which it has acquired. This charge, being acquired from the earth by the inductive action of the electricity overhead, is opposite in sign to the inducing electricity.

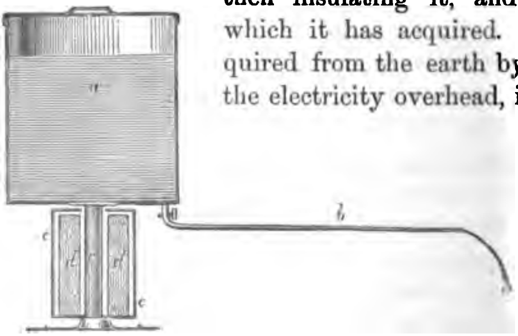


Fig. 408 A.—Water-dropping Collector.

Another method which in principle resembles that of Volta, but is speedier in its action, has been introduced by Sir W. Thomson. It consists in allowing a fine stream of water to flow, from an insulated metallic vessel, through a pipe, which projects through an open window or other aperture in the wall of a house, so that the nozzle from which the water flows is in the open air. The apparatus for this purpose, called the water-dropping collector, is represented in Fig. 408A. *a* is a copper can, containing water, which can be discharged through the brass pipe *b* by turning a tap. The mode of insulation is worthy of notice. The can is supported on a glass stem *c*, which is surrounded, without contact, by a ring or rings of pumice *dd*, moistened with sulphuric acid. These are protected by an outer case of brass *ee*, having a hole in its top rather larger than the glass stem, the brass being separated from the moist pumice by an inner case of gutta-percha. The acid needs renewal about once in two months.

In severe frost, burning matches can be used instead of water, and are found to give identical indications. Whether water or match be used, the principle of action[1] is that, as long as any difference of potential exists between the insulated conductor and the point of the air where the issuing stream (whether of water or smoke) ceases to be one continuous conductor, and begins to be a non-conductor or a succession of detached drops, so long will each drop or portion that detaches itself carry off either positive or negative electricity, and thus diminish the difference of potential. This is an application of the principle of § 445B, that electricity tends to travel from places of higher to places of lower potential. The time required to reduce the system to the potential which exists at the point above specified, is practically about half a minute with the water-jet, and from half a minute to a minute or more, according to the strength of the wind, with a match.

The water-dropper is the most convenient collecting apparatus when the observations are taken always in the same place. For portable service, Sir Wm. Thomson employs blotting-paper, steeped in solution of nitrate of lead, dried, and rolled into matches. The portable electrometer carries a light brass rod or wire projecting upwards, to the top of which the matches can be fixed.

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[1] The following quotation from an article by Sir W. Thomson puts the matter very clearly:—"If, now, we conceive an elevated conductor, first belonging to the earth, to become insulated, and to be made to throw off, and to continue throwing off, portions from an exposed part of its surface, this part of its surface will quickly be reduced to a state of no electrification, and the whole conductor will be brought to such a potential as will allow it to remain in electrical equilibrium in the air, with that portion of its surface neutral. In other words, the potential throughout the insulated conductor is brought to be the same as that of the particular equi-potential surface in the air, which passes through the point it from which matter breaks away. A flame, or the heated gas passing from a burning match, does precisely this: the flame itself, or the highly heated gas close to the match, is a conductor which is constantly extending out, and gradually becoming a non-conductor. The drops [into which the jet from the water-dropper breaks] produce the same effect with more pointed decision, and with more of dynamical energy to remove the removed matter, with the electricity which it carries, from the neighbourhood of the fixed conductor."—*Nichol's Cyclopædia*, second edition, art. "Electricity, Atmospheric."

480 B. Interpretation of Indications.—We have seen that the collecting apparatus, whether armed with water-jet or burning match, is merely an arrangement for reducing an insulated conductor to the potential which exists at a particular point in the air. An electrometer will then show us the difference between this potential and that of any other given conductor, for example the earth. The earth offers so little resistance to the passage of electricity, that any temporary difference of potential which may exist between different parts of its surface, must be very slight in comparison with the differences of potential which exist between different points in the non-conducting atmosphere above it. As there is no possible method of determining absolute potential, since all electric phenomena would remain unchanged by an equal addition to the potentials of all points, it is convenient to assume, as the zero of potential, that of the most constant body to which we have access, namely the earth; and under the name earth we include trees, buildings, animals, and all other conductors in electrical communication with the soil.

Now we find that, as we proceed further from the earth's surface, whether upwards from a level part of it, or horizontally from a vertical part of it, such as an outer wall of a house, the potential of points in the air becomes more and more different from that of the earth, the difference being, in a broad sense, simply proportional to the distance. Hence we can infer[1] that there is electricity residing on the surface of the earth, the density of this electricity, at any moment, in the locality of observation, being measured by the difference of potential which we find to exist between the earth and a given point in the air near it. Observations of so-called atmospheric electricity[2] made in the manner we have described, are in fact simply determinations of the quantity of electricity residing on the earth's surface at the place of observation. The results of observations so made are however amply sufficient to show that electricity residing in the atmosphere is really the main cause of the variations observed. A charged cloud or body of air induces electricity of the opposite kind to its own on the parts of the earth's surface over which it passes; and the variations which we find to occur in the electrical density at the parts of the surface where we observe, are so rapid and considerable, that no other cause but this seems at all adequate to account for them. We may therefore safely assume that the difference of potential which we find, in increasing our distance from the earth, is mainly due to electricity induced on the surface of the earth by opposite electricity in the air overhead.

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[1] By § 445 I, if ρ denote the quantity of electricity per unit area on an even part of the earth's surface, the force in the neighbouring air is $4\pi\rho$. This must be equal to the change of potential in going unit distance (§ 445 D). If potential increases positively, ρ is negative.

[2] No good electrical observations have yet been made in balloons, and very little is known regarding the distribution of electricity at different heights in the air. A method of gauging this distribution by balloon observations is suggested by the principles of § 445 G, which show that, when the lines of force are vertical, and the tubes of force consequently cylindrical, the difference of electrical force at different heights is proportional to the quantity of electricity which lies between them.

As electrical density is greater on projecting parts of a surface than on those which are plane or concave, we shall obtain stronger indications on hills than in valleys, if our collecting apparatus be at the same distance from the ground in both cases. Under a tree, or in any position excluded from view of the sky, we shall obtain little or no effect.

480c. **Results of Observation.**—The only regular series of observations which have as yet been taken[1] with Sir Wm. Thomson's instruments, consist of two years' continuous observations with self-recording apparatus at Kew Observatory; and two years' observations, at three stated times daily, and at other irregular times, at Windsor in Nova Scotia (lat. 45° N.) The electrometer used at Kew was an earlier form of the quadrant electrometer already described; and the autographic registration was effected by throwing the image of a bright point (a small hole with a lamp behind it) upon a sheet of photographic paper drawn upwards by clock-work, whereas the movements of the image, formed by means of the mirror attached to the needle, were horizontal. The curves thus obtained give very accurate information respecting the potential of the air at the point of observation, when of moderate strength; but fail to record it when of excessive strength, as the image on these occasions passed out of range. The Windsor observations were taken with the cage-electrometer, of which two forms were employed, one being much more sensitive than the other. The more sensitive form was usually employed. When the potential became inconveniently strong, the first step was to shorten the discharging pipe by screwing off some of its joints. This reduced the strength of potential in about the ratio of 3 : 1; but even this reduction was often not enough for the more sensitive instrument, and on such occasions the other (which was intended as a portable electrometer) was employed instead. As the ratio of the indications of the two instruments was known, a complete comparison of potentials in all weathers was thus obtained. The results are as follows.

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[1] The observations at Windsor, N.S., and at Kew, are described in three papers by the editor of this work, *Proc. R. S.*, June 1863, January 1865, and *Trans. R. S.*, December 1867. Dellmann's observations at Kreuznach, which were taken with apparatus devised by himself, are described in *Phil. Mag.* June 1858. Quetelet's observations (taken with Peltier's apparatus) are described in his volume *Sur le Climat de la Belgique* (Brussels, 1849).

Employing a unit in terms of which the average fine-weather potential for the year was +4, the potential was seldom so weak as 1, though on rare occasions it was for a few minutes as low as 0·1. In wet weather, especially with sudden heavy showers, the potential was often as strong as ±20 to ±30, and it was fully as strong during hail. With snow, the average strength was about the same as with heavy rain, but it was less variable, and the sign was almost always positive. Occasionally, with high wind accompanying snow, during very severe frost, it was from +80 to +100, or even higher. With fog, it was always positive, averaging about +10. In thunder-storms it frequently exceeded ±100, and on a few occasions exceeded −200. There was usually a great predominance of negative potential in thunder-storms. Change of sign was a frequent accompaniment of a flash of lightning or a sudden downpour of rain. At all times, there was a remarkable absence of steadiness as compared with most meteorological phenomena, wind-pressure being the only element whose fluctuations are at all comparable, in magnitude and suddenness, with those of electrical potential. Even in fine weather, its variations during two or three minutes usually amount to as much as 20 per cent. In changeable and stormy weather they are much greater; and on some rare occasions it changes so much from second to second that, notwithstanding the mitigating effect of the collecting process, which eases off all sudden changes, the needle of the electrometer is kept in a continual state of agitation.

480D. Annual and Diurnal Variations.—Observations everywhere[1] concur in showing that the average strength of potential is greater in winter than in summer; but the months of maxima and minima appear to differ considerably at different places. The chief maximum occurs in one of the winter months, varying at different places from the beginning to the end of winter; and the chief minimum occurs everywhere in May or June. Both Kew and Windsor show distinctly two maxima in the year, but Brussels, and apparently Kreuznach, show only one. The ratio of the highest monthly average to the lowest is at Kew about 2·5, at Windsor 1·9, and at Kreuznach 2·0.

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[1] The remarks in this section express the results of observation at places all of which are in the north temperate zone.

The Kew observations, being continuous, are specially adapted to throw light on the subject of diurnal variation. They distinctly indicate for each month two maxima, which in July occur at about 8 A.M. and 10 P.M., in January about 10 A.M. and 7 P.M., and in spring and autumn about 9 and 9. The result of the Brussels observations is about the same.

481. **Causes of Atmospheric Electricity.**—Various conjectures have been hazarded regarding the sources of atmospheric electricity; but little or no certain knowledge has yet been obtained on this subject. Evaporation has been put forward as a cause, but, as far as laboratory experiments show, whenever electricity has been generated in connection with evaporation, the real source has been friction, as in Armstrong's hydro-electric machine. The chemical processes involved in vegetation have also been adduced as causes, but without any sufficient evidence. It is perhaps not too much to say that the only natural agent which we know to be capable of electrifying the air is the friction of solid and liquid particles against the earth and against each other by wind. The excessively strong indications of electricity observed during snow accompanied by high wind, favour the idea that this may be an important source.

Without knowing the origin of atmospheric electricity, we may, however, give some explanation of the electrical phenomena which occur both in showers and in thunder-storms. Very dry air is an excellent non-conductor; very moist air has, on the other hand, considerable conducting power. When condensation takes place at several centres, a number of masses of non-conducting matter are transformed into conductors, and the electricity which was diffused through their substance passes to their surfaces. These separate conductors influence one another. If one of them is torn asunder while under influence, its two portions may be oppositely charged; and if rain falls from the under surface of a cloud which is under the influence of electricity above it, the rain which falls may have an opposite charge to the portion which is left suspended.

The coalescence of small drops to form large ones, though it increases the electrical density on the surfaces of the drops, does not increase the total quantity of electricity, and therefore (§ 445 K) cannot directly influence the observed potential.

Thunder-storms and other powerful manifestations of atmospheric electricity seem to be accompaniments of very sudden and complete condensation which gives unusually free scope to the causes of irregular distribution just indicated.

483. Hail.—Hail has sometimes been ascribed to an electrical origin, and a singular theory was devised by Volta to account for the supposed fact that hailstones are sustained in the air. He imagined that two layers of cloud, one above the other, charged with opposite electricities, kept the hailstones continually moving up and down by alternate attraction and repulsion. An experiment called *electric hail* is sometimes employed to illustrate this idea. Two metallic plates are employed (Fig. 409), the lower one connected with the earth, and the upper one with the conductor of the electrical machine; and pith-balls are placed between them. As the machine is turned, the balls fly rapidly backwards and forwards from one plate to the other.

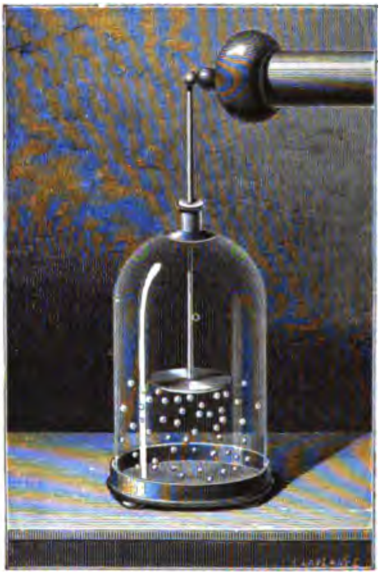

Fig. 409.—Electric Hail.

484. Waterspouts.—Waterspouts, being often accompanied by strong manifestations of electricity, have been ascribed by Peltier and

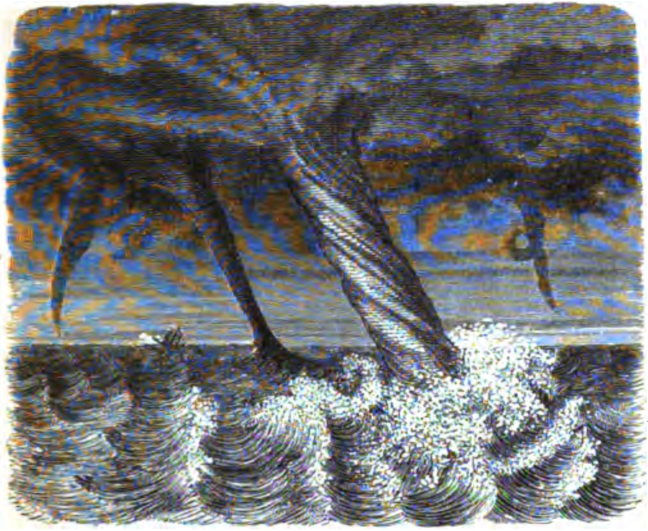

Fig. 410.—Waterspouts.

others to an electrical origin; but the account of them given in the subjoined note appears more probable.[1]

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[1] "On account of the centrifugal force arising from the rapid gyrations near the centre of a tornado, it must frequently be nearly a vacuum. Hence when a tornado passes over a building, the external pressure, in a great measure, is suddenly removed, when the atmosphere within, not being able to escape at once, exerts a pressure upon the interior, of perhaps nearly fifteen pounds to the square inch, which causes the parts to be thrown in every direction to a great distance. For the same reason, also, the corks fly from empty bottles, and everything with air confined within explodes. When a tornado happens at sea, it generally produces a waterspout. This is generally first formed above, in the form of a cloud shaped like a funnel or inverted cone. As there is less resistance to the motions in the upper strata than near the earth's surface, the rapid gyratory motion commences there first. . . . This draws down the strata of cold air above, which, coming in contact with the warm and moist atmosphere ascending in the middle of the tornado, condenses the vapour and forms the funnel-shaped cloud. As the gyratory motion becomes more violent, it gradually overcomes the resistances nearer the surface of the sea, and the vertex of the funnel-shaped cloud gradually descends lower, and the imperfect vacuum of the centre of the tornado reaches the sea, up which the water has a tendency to ascend to a certain height, and thence the rapidly ascending spiral motion of the atmosphere carries the spray upward, until it joins the cloud above, when the waterspout is complete. The upper part of a waterspout is frequently formed in tornadoes on land. When tornadoes happen on sandy plains, instead of waterspouts they produce the moving pillars of sand which are often seen in sandy deserts."—W. Ferrel, in *Mathematical Monthly*. See note § 406.

MAGNETISM.

CHAPTER XLIII.

GENERAL STATEMENT OF FACTS AND LAWS.

485. Magnets, Natural and Artificial.—Natural magnets, or *lodestones*, are exceedingly rare, although a closely allied ore of iron, capable of being strongly acted on by magnetic forces, and hence called *magnetic iron-ore*, is found in large quantity in Sweden and elsewhere. Artificial magnets are usually pieces of steel, which have been permanently endowed with magnetism by methods which we shall hereafter describe. Magnets are chiefly characterized by the property of attracting iron, and by the tendency to assume a particular orientation when freely suspended.

486. Force Greatest at the Ends.—The property of attracting iron is very unequally manifested at different points of the surface of a magnet. If, for example, an ordinary bar-magnet be plunged in iron-filings, these cling in large quantity to the terminal portions, and leave the middle bare, as in the lower diagram of Fig. 411. If the magnet is very thick in proportion to its length, we may have filings adhering to all parts of it, but the quantity diminishes rapidly towards the middle. The name *poles* is used, in a somewhat loose sense, to denote the two terminal portions of a magnet, or to denote two points, not very accurately defined, situated in these portions. The middle portion, to which the filings refuse to adhere, is called *neutral*.

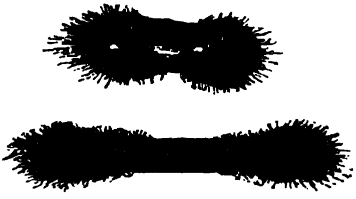

Fig. 411.—Magnets dipped in Filings.

487. Lines Formed by Filings.—If a sheet of card is laid horizontally upon a magnet, and wrought-iron filings are sifted over it, these will, with the assistance of a few taps given to the card, arrange s in a system of curved lines, as shown in Fig. 412. These very important indications both of the direction and intene force produced by the magnet at different points of the

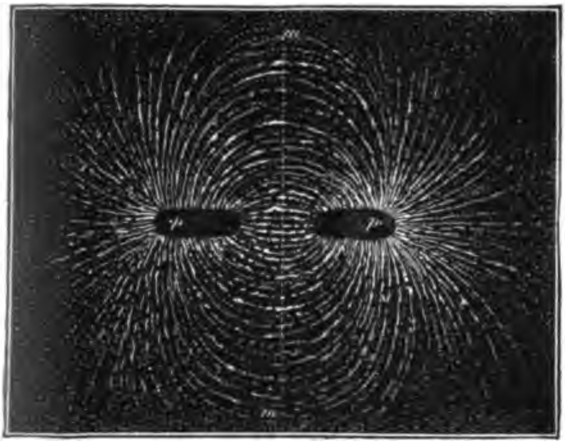

Fig. 412.—Magnetic Curves.

nd it.[1] They cluster very closely about the two poles p p, ıdicate the places where the force is most intense.

ve of Intensities.—Some idea may be obtained of the relaities of magnetic force at different points in the length of by measuring the weights of iron which can be supported Much better determinations can be obtained either by the torsion-balance, or by counting the number of vibrations mall magnetized needle when suspended opposite different ɜ bar, the bar being in a vertical position, and the vibra- ̧ needle being horizontal. The intensity of the force is ıe square of the number of vibrations; on the same prinıe force of gravity at different places is proportional to the ıe number of vibrations of a pendulum (§ 47). Both these determination were employed by Coulomb, who was the :e magnetism an accurate science; and the results which are represented by the curve of intensities AMB (Fig. the middle of the bar, O one end of it, and the ordinates

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rmed by the filings may be called the lines of *effective force for particles only the plane of the card.* The lines of total force cut the card at various at some places perpendicular to it, as shown by the filings standing on end. ɔn of lines of magnetic force, see § 494 A.

of the curve (that is, the distances of its points from the line OX) represent the intensities of force at the different points in its length. The curve was constructed from observations of the force at several points in the length; but in dealing with the observation made opposite the very end, the force actually observed was multiplied by 2. Perfect symmetry was found between the intensities over the two halves of the length. In the figure we have inverted the curve for one-half, in order to indicate an opposition of properties, which we shall shortly have to describe. The curves of intensities for two magnets of different sizes but of the same form are usually similar.

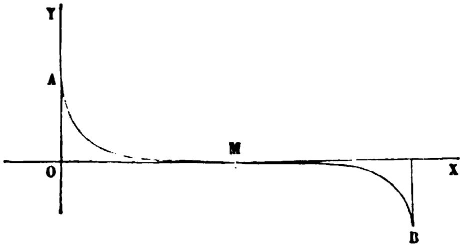

Fig. 413.—Curve of Intensities.

489. Magnetic Needle.—Any magnet freely suspended near its centre is usually called a *magnetic needle*, or more properly a *magnetized needle*. One of its most usual forms is that of a very elongated rhombus of thin steel, having, very near its centre, a concavity or cup by means of which it can be balanced on a point. When it is thus balanced horizontally, it does not, like a piece of ordinary matter, remain in equilibrium in all azimuths,[1] but assumes one particular direction, to which it always comes back after displacement. In this position of stable equilibrium, one of its ends points to magnetic north, and the other to magnetic south, which differ in general by several degrees from geographical (or true) north and south. This is the principle on which compasses are constructed.

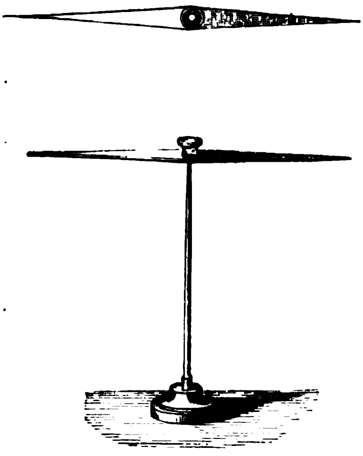

Fig. 414.—Magnetized Needle.

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[1] All lines in the same vertical plane are said to have the same *azimuth*. Azimuthal angles are angles between vertical planes, or between horizontal lines. The azimuth of a line when stated numerically, is the angle which the vertical plane containing it makes with a vertical plane of reference, and this latter is usually the plane of the meridian.

490. Declination.—The difference between magnetic and true north, or the angle between the magnetic meridian and the geographical meridian, is called *magnetic declination*.[1] It is very different at different places, and at a given place undergoes a gradual change from year to year, besides smaller changes, backwards and forwards, which are continually taking place. At Greenwich, at the present time, its value is about 20° W., that is, magnetic north is west of true north by this amount. For the British Isles generally its value is from 20° to 30° W.

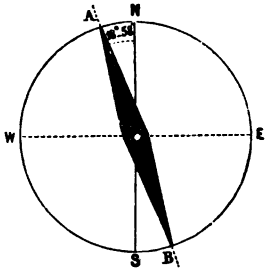

Fig. 415.—Declination.

491. Inclination or Dip.—If, before magnetizing a needle, we mount it on an axis passing through its centre of gravity, and support the ends of the axis, as in Fig. 416, by a thread without torsion, the needle will remain in equilibrium in any position in which it may be placed. If it be then magnetized, it will no longer be indifferent, but will place itself in a particular vertical plane called the magnetic meridian, and will take a particular direction in this plane. This direction is not horizontal, but inclined, generally at a considerable angle, to the horizon; and this angle is called *dip* or *inclination*. Its value at Greenwich is about 67°, the end which points to the north pointing at the same time downwards. In the northern hemisphere generally, it is the north end of the needle which dips, and in the southern hemisphere it is the end which points south.

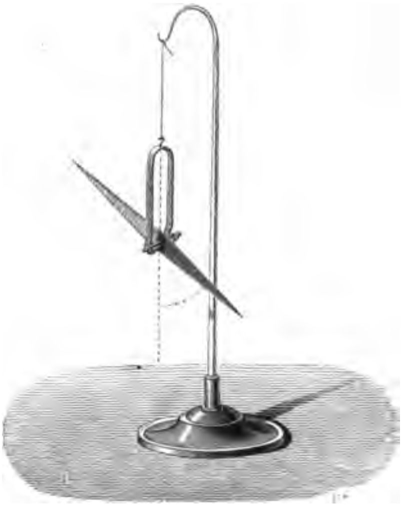

Fig. 416.—Dip.

Some readers may be glad to be reminded that by the plane of the *meridian* is meant a vertical plane passing through the place of observation, and through or parallel to the earth's axis. A horizontal line in this plane is a meridian line. The *magnetic meridian* is the vertical plane in which a magnetized needle, when freely suspended, tends to place itself.

[1] The nautical name for magnetic declination is *variation;* but it is most inconvenient and confusing to denote the element itself by the same name as the variations of the element.

It follows that, if a magnetized needle is to be balanced in a horizontal position, the point or axis of support must not be in the same vertical with the centre of gravity, but must be between the centre of gravity and the end which tends to dip. Needles thus balanced, as in the ordinary mariner's compass, are called *declination needles*.

492. Mutual Action of Poles.—On presenting one end of a magnet to one end of a needle thus balanced, we obtain either repulsion or attraction, according as the pole which is presented is similar or dissimilar to that to which it is presented. *Poles of contrary name attract each other; poles of the same name repel each other.*

This property furnishes the means of distinguishing a body which is merely magnetic (that is, capable of temporary magnetization) from a permanent magnet. The former, a piece of soft iron for example, is always attracted by either pole of a magnet; while a body which has received permanent magnetization has, in ordinary cases, two poles, of which one is attracted where the other is repelled. Magnetic attractions and repulsions are exerted without modification through any body which may be interposed, provided it be not magnetic.

492A. Names of Poles.—The phenomena of declination and inclination above described, evidently require us to regard the earth, in a broad sense, as a magnet, having one pole in the northern and the other in the southern hemisphere. Now since poles which attract one another are dissimilar, it follows that the magnetic pole of the earth which is situated in the northern hemisphere is *dissimilar* to that end of a magnetized needle which points to the north. Hence great confusion of nomenclature has arisen, the usage of the best writers being opposite to that which generally prevails. We shall call that end or pole of a needle which seeks the north, the *north-seeking* end or pole, and the other the *south-seeking* end or pole. Sir Wm. Thomson calls the north-seeking pole the *south* pole, and the other the *north* pole, because the former is similar to the south. and the latter to the north pole of the earth. In like manner most French writers call the north-seeking pole of a needle the *austral*, and the other the *boreal* pole. Popular usage in this country calls the north-seeking end the *north*, and the other the *south* pole, a nomenclature which introduces great confusion whenever we have to reason respecting the earth regarded as a magnet. Faraday, to avoid the ambiguity which has attached itself to the names north and south pole, calls the north-seeking end the *marked*, and the other the *unmarked* pole. Airy, for a similar reason, employs, in his recent *Treatise on Magnetism*, the distinctive names *red* and *blue* to denote respectively the north-seeking and south-seeking ends, these names, as well as those employed by Faraday, being purely conventional, and founded on the custom of marking the north-seeking end of a magnet with a transverse notch or a spot of red paint. Maxwell and Jenkin, in a report to the British Association,[1] call the south-seeking pole of a needle *positive*, and the north-seeking pole *negative*.

493. Magnetic Induction.—When a piece of iron is in contact with a magnet, or even when a magnet is simply brought near it, it becomes itself, for the time, a magnet, with two poles and a neutral portion between them. If we scatter filings over the iron, they will adhere to its ends, as shown in Fig. 417. If we take away the influencing magnet, the filings will fall off, and the iron will retain either no traces at all or only very faint ones of its magnetization. If we apply similar treatment to a piece of steel, we obtain a result similar in some respects, but with very important differences in degree. The steel, while under the influence of the magnet, exhibits much weaker effects than the iron; it is much more difficult to magnetize than iron, and does not admit of being so powerfully magnetized; but, on the other hand, it retains its magnetization after the influencing magnet has been withdrawn. This property of retaining magnetism when once imparted has been (somewhat awkwardly) named *coercive force*. Steel, especially when very hard, possesses great coercive force; iron, especially when very pure and soft, scarcely any.

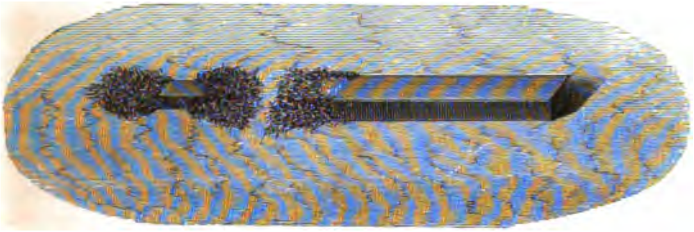

Fig. 417.—Induced Magnetism.

In magnetization by influence, which is also called *magnetic induction*, it will be found, on examination, that the pole which is next the inducing pole is of contrary name to it; and it is on account of the mutual attraction of dissimilar poles that the iron is attracted by the magnet. The iron can, in its turn, support a second piece of iron; this again can support a third, and so on through many steps. A magnetic chain can thus be formed, each of the component pieces having two poles. An action of this kind takes place in the clusters of filings which attach themselves to one end of a magnetized bar, these clusters being composed of numerous chains of filings.

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[1] *Report of Electrical Standards Committee*, Appendix C. 1863.

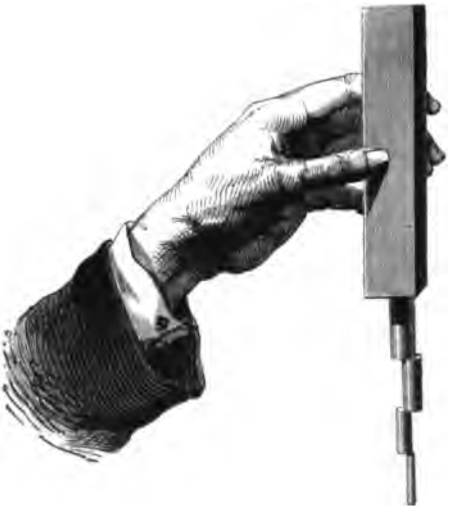

Fig. 418.—Magnetic Chain.

In comparing the phenomena of magnetic induction with those of electrical induction, we find both points of resemblance and points of difference. In the case of electricity, if the influencing and influenced body are allowed to come in contact, the former loses some of its own charge to the latter. In the case of magnetism there is no such loss, a magnet after touching soft iron is found to be as strongly magnetized as it was before.

494. Effect of Rupture on a Magnet.—If a magnet is broken into any number of pieces, every piece will be a complete magnet with poles of its own. In the case of an ordinary bar-magnet or needle, the similar poles of the pieces will all be turned the same way, as in Fig. 419, which represents a magnet AB broken into four pieces. The ends *a*, *a*, *a*, *a* are of one name, and the ends *b*, *b*, *b*, *b* of the opposite name.

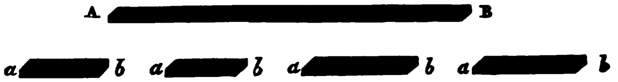

Fig. 419.—Broken Magnet.

494 A. Imaginary Magnetic Fluids: Magnetic Potential.—All mutual forces between magnets can be reduced to attractions and repulsions between different portions of two imaginary fluids,[1] one of which may be called *positive* and the other *negative*. Neither fluid can exist apart from the other; every magnet possesses equal quantities of both; quantity being measured by force of attraction or repulsion at given distance, just as in the case of electricity, like portions repelling, and unlike portions attracting each other inversely as the square of the distance. Equal quantities of the two fluids, when coexisting at the same place, produce no resultant effect, and may be regarded as destroying each other.

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[1] Poisson, following Coulomb, spoke of *two magnetic fluids*, and laid down a theory of

With reference to these imaginary fluids, *magnetic potential* can be defined in the same way as electrical potential, and magnetic *lines of force* possess the same properties as electrical lines of force (§ 445 A —445 K). The direction of magnetic force at a point can either be defined as the direction in which a pole of a magnet would be urged if brought to the point, or as the direction in which a small magnetized needle, if brought to the point and balanced at its centre of gravity, would place its line of poles; and lines of magnetic force are lines to which this direction is everywhere tangential. It is important to remark that a linear piece of soft iron, though it sets its length along a line of force, does not travel along a line of force, but deviates towards the concave side. This is easily shown by tapping the card represented in Fig. 412. It will be found that filings placed on the line *m m* move along that line, and therefore at right angles to the lines of force. The force which is specified by magnetic "lines of force" is the force which *one pole* of a permanent magnet would experience; and it is the same in intensity, but opposite in direction, for dissimilar poles.

494 B. Specification of Magnetization.—A piece of steel is said to be *uniformly magnetized*, if equal and similar portions, cut in parallel directions from all parts of it, are precisely alike in their magnetic properties.

If a piece of magnetized steel be suspended at its centre of gravity, so as to be free to turn all ways about it, the effect of the earth's magnetism upon it consists in a tendency for a particular line, through this centre of gravity, to take a determinate direction, which is the direction of terrestrial magnetic force. When the line is placed in any other position, the couple tending to bring it back is propor-

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their action. Sir W. Thomson, avoiding the hypothetical parts of Poisson's theory, speaks of *imaginary magnetic matter* of two dissimilar kinds. We have retained the more familiar name *fluid*, simply because it is more convenient to speak of *two fluids* than of *two kinds of matter*. It is to be noted that we cannot speak of two *magnetisms*, the name magnetism having been already appropriated in a different sense.

tional to the sine of the angle between the two positions, and is the same for all directions of deviation. The line which possesses this property is the *magnetic axis* of the body, and the name is sometimes given to all lines parallel to it. If the piece of steel be uniformly magnetized, this axis is the direction of magnetization; or *the direction of magnetization is the common direction of all those lines which tend to place themselves along lines of force* in a field[1] where the lines of force are parallel.

494 c. Ideal Simple Magnet: Thin Bar, uniformly and longitudinally Magnetized.—The mutual actions of magnets admit of very accurate expression when the magnets are very thin in comparison with their length, uniform in section, and uniformly magnetized in the direction of their length. Such bars, which may be called *simple magnets*, behave as if their forces resided solely in their ends, which may therefore in the strictest sense be called their poles. The two poles of any one such bar are equal in strength; that is to say, one of them attracts a pole of another simple magnet with the same force with which the other repels it at the same distance. In the language of the two-fluid theory, the two fluids destroy one another except at the two ends, and the quantities which reside at the ends are equal but of opposite sign. The same number which denotes the quantity of fluid at either pole, denotes the *strength of the pole*, or, as it is often called, the *strength of the magnet.* Its definition is best expressed by saying that the force between a pole of one simple magnet and a pole of another, is the product of their strengths divided by the square of the distance between them.[2]

The force which a pole of a simple magnet experiences in a magnetic field, is the *product of the strength of the pole and the intensity of the field.* This rule applies to the force which a pole experiences from the earth's magnetism, the intensity of the field being in this case the intensity of terrestrial magnetic force; and, from the uniformity of the field, the forces on the two poles are in this case equal, constituting a couple, whose arm is the line joining the poles multiplied by the sine of the angle which this line makes with the lines of force.

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[1] A *field of force* is any region of space traversed by lines of force; or, in other words, any region pervaded by force of attraction or repulsion. A *magnetic field* is any region pervaded by magnetic force. All space in the neighbourhood of the earth is a magnetic field, and within moderate distances the lines of force in it may be regarded as parallel, unless artificial magnets or pieces of iron are present to produce disturbance.

[2] We here, and throughout the remainder of this chapter, ignore the existence of induction, which, however, is not altogether absent even in the hardest steel. The effect of induction is always to favour attraction. The attractions will therefore be somewhat stronger, and the repulsions somewhat weaker, than our theory supposes.

The product of the line joining the two poles by the strength of either pole is called the *moment of the magnet*, and it is evident, from what has just been said, that the continued product of *the moment of the magnet, the intensity of terrestrial magnetic force, and the sine of the angle between the length of the magnet and the lines of force*, is equal to the moment of the couple which the earth's magnetism exerts upon the magnet.

494 D. Compound Magnet of Uniform Magnetization.—Any magnet which is not a simple magnet in the sense defined in § 494 C may be called a *compound magnet*. It is convenient to define the moment of a compound magnet by the condition stated in the concluding words of that section, so that the moments of different magnets, whether simple or compound, may be compared by comparing the couples exerted on them by terrestrial magnetism when their axes are equally inclined to the lines of force.

If a number of simple magnets of equal strength be joined end to end, with their similar poles pointing the same way, there will be mutual destruction of the two imaginary fluids at every junction, and the system will constitute one simple magnet of the same strength as any one of its components; but its moment will evidently be the *sum* of their moments.

If any number of simple magnets be united, either end to end or side to side, provided only that they are parallel, and have their similar poles turned the same way, the resultant couple exerted upon the whole system by terrestrial magnetism will (§ 14) be the sum of the separate couples exerted on each simple magnet, and the moment of the system will be the sum of the moments of its parts. But any piece of uniformly magnetized material may be regarded as being thus built up, and hence, if different portions be cut from the same uniformly magnetized mass, their moments will be simply proportional to their volumes. The quotient of moment by volume, for any uniformly magnetized mass, is called *intensity of magnetization*.

494 E. Actual Magnets.—The definitions and laws of simple magnets are approximately applicable to actual magnets, when magnetized in the usual manner.

If an actual bar-magnet in the form of a rectangular parallelopiped were magnetized with perfect uniformity, and in the direction of its length, it might be regarded as made up of a number of simple magnets laid side by side, and its behaviour would be represented by supposing a complete absence of magnetic fluid from all parts of it except its *ends* (in the strict mathematical sense). One of these terminal faces would be covered with positive, and the other with negative fluid, and if the magnet were broken across at any part of its length, the quantities of positive and negative fluid on the broken ends would be the same as on the ends of the complete magnet. The observed fact that magnets behave as if the fluids were distributed through a portion of their substance in the neighbourhood of the ends, and not confined to the ends strictly so called, indicates a falling off in magnetization towards the extremities, and is approximately represented by conceiving of a number of short magnets laid end to end, and falling off in strength towards the two extremities of the series.[1]

The resultant force due to the imaginary magnetic fluids which are distributed through the terminal portions of an actual bar-magnet is, in the case of actions at a great distance, sensibly the same as if the two portions of fluid were collected at their respective centres of gravity. These two centres of gravity are the poles of the magnet for all actions between the magnet and other magnets at a great distance, and more especially between the magnet and the earth.

The moment of any magnet, however irregular in its magnetization, may be defined by reference to the expression given in § 494c for the couple exerted on the body by terrestrial magnetism. This couple is M I *sin* a, where I denotes the intensity of terrestrial magnetic force, a the inclination of the magnetic axis of the body to the lines of the earth's magnetic force, and M the *moment* which we are defining.

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[1] Thus the last magnet at the positive end being weaker than its neighbour, its negative pole will be weaker than its neighbour's positive pole, so that there will be an excess of positive fluid at this junction. Similar reasoning applies to all the junctions near the ends. There will be an excess of positive fluid at all junctions near the positive end, and an excess of negative at all junctions near the negative end.

CHAPTER XLIV.

EXPERIMENTAL DETAILS.

The Earth's Force simply Directive.—The forces which pro-
e orientation of a magnet depend upon causes of which very
known. They are evidently connected in some way with
th, and are accordingly referred to TERRESTRIAL MAGNETISM.
e already stated (§ 494 B) that the combined effect of the forces
by terrestrial magnetism upon a magnetized needle is equi-
a couple tending to turn the needle into a particular direc-
(§ 494 E) that in the case of needles magnetized in the
way, there are two definite points or poles (near the two
he needle) which may be regarded as the points of applica-
e two equal forces which constitute the couple.

t that terrestrial magnetic force simply tends to turn the
d not to give it a movement of translation, in other words,
sultant *force* (as distinguished from *couple*) is zero, is com-
ved by the two following experiments:—

bar of steel is weighed before and after magnetization, no
ound in its weight. This proves that the vertical com-
ro.

ar of steel, not magnetized, is suspended by a long and
the direction of the thread is of course vertical. If the
nagnetized, the direction of the thread still remains ver-
nost rigorous tests fail to show any change of its position.
that the horizontal component is zero, a conclusion which
ied by floating a magnet on water by means of a cork.
nd that there is no tendency to move across the water
ular direction.

ntal, Vertical, and Total Intensities.—If S denote the
nagnet, and I the intensity of terrestrial magnetic force,

each pole of the magnet experiences a force SI, and if L denote the distance between the poles (often called the length of the magnet), the distance between the lines of action of these two parallel and opposite forces may have any value intermediate between L and zero, according to the position in which the needle is held. It will be zero when the line of poles is that of the dipping-needle; it will be L when the line of poles is perpendicular to the dipping-needle; and will be L sin α when the line of poles is inclined at any angle α to the dipping-needle.

The force SI upon either pole of the magnet acts in the direction of the dipping-needle; in other words, in the direction of the lines of force due to terrestrial magnetism. Let δ denote the dip, that is the inclination of the lines of force to the horizon, then the force SI can be resolved into SI cos δ horizontal, and SI sin δ vertical. Hence the horizontal and vertical intensities H and V are connected with the total intensity and dip I and δ by the two equations

$$H = I \cos\delta \quad , \quad V = I \sin\delta \qquad (1)$$

which are equivalent to the following two

$$\frac{V}{H} = \tan\delta \quad , \quad V^2 + H^2 = I^2. \qquad (2)$$

497. Torsion-balance.—Coulomb, in investigating the laws of the mutual action of magnets, employed a torsion-balance scarcely differing from that which he used in his electrical researches. The suspending thread carried, at its lower end, a stirrup on which a magnetized bar was laid horizontally. The torsion-head was so adjusted that one end of the magnet was opposite the zero of the divisions on the glass case when the supporting thread was without torsion. In order to effect this adjustment, the magnet was first suspended by a thread whose torsional power was inconsiderable, so that the magnet placed itself in the magnetic meridian. The case was then turned till its zero came to this position. The torsionless thread was then replaced by a fine metallic wire, and the magnet was replaced by a copper bar of the same weight. The head was then turned till this bar came into the magnetic meridian, and lastly the magnet was put in the place of the bar.

Fig. 420 shows the arrangement adopted for observing the repulsion or attraction between one pole of the suspended magnet and one pole of another magnet placed vertically. Before the insertion of the latter, the suspended magnet was acted on by no horizontal

:es except the horizontal component of terrestrial magnetism and torsion of the wire. It was then found that the torsion requisite keeping the magnet in any position was proportional to the sine he displacement from the meridian.

his result is evidently in accordance with the principles stated

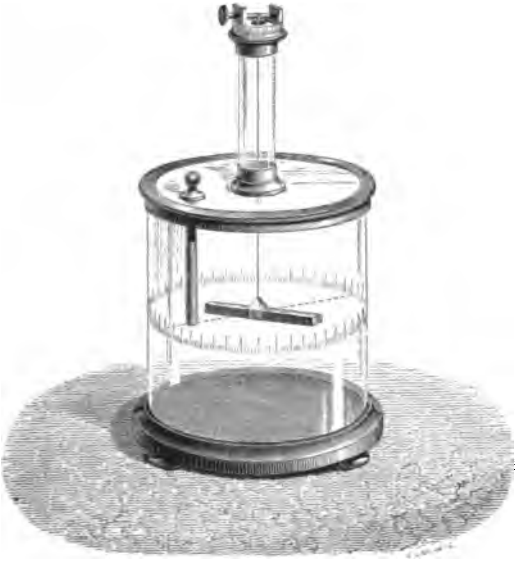

Fig. 420.—Torsion-balance.

›r the two equal horizontal forces on the two poles being for all positions, the couple which they compose is propor- the distance between their lines of action, and this distance tly L sin θ, L denoting the constant distance between the l θ the deviation of the needle from the meridian.

easurement of Declination.—Magnetic declination has been with several different forms of apparatus.

the most common method of determining it has consisted ng the magnetic bearing of the rising or setting sun, and this with its true bearing as calculated by a well-known :al method.

e accurate determination on land, the declination compass ion theodolite[1] (Fig. 422) has been frequently employed.

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te consists of a telescope mounted so as to have independent motions in

When the instrument is set by the help of astronomical observations, so that the vertical plane in which the telescope LL′ (or more accurately its line of collimation) moves, coincides with the geographical meridian, the ends of the needle indicate the declination on the graduated circle over which they move. This circle in fact turns with the telescope, the line of 0° and 180° *n s* being always in the same vertical plane with the line of collimation of the telescope. The external divided circle PQ is used for setting the instrument in the meridian.

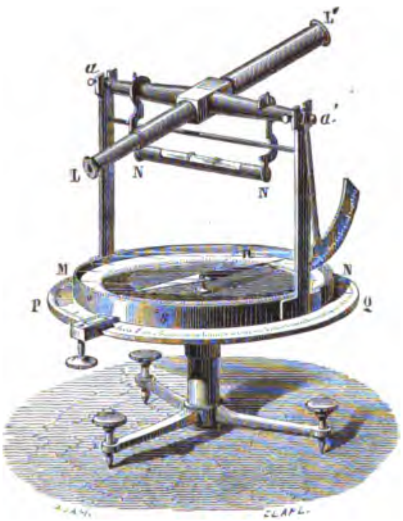

Fig. 422.—Declination Theodolite.

At fixed observatories more accurate methods of observation are employed. Fig. 422 A shows the arrangement adopted at Greenwich. A bar-magnet B carries at one end a cross of fine threads C, and

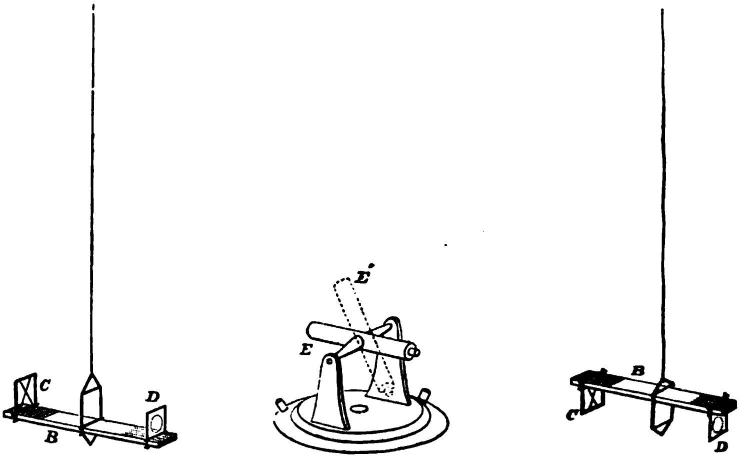

Fig. 422 A.—Declination Magnet.

azimuth and altitude, the amounts of these motions being indicated by divided circles or arcs of circles. It does not differ essentially from the larger instrument called the *altazimuth.*

the other a lens D, the distance between them being equal to the ıl length of the lens, thus forming a kind of inverted telescope, ɔse line of collimation is the line joining the cross to the optical re of the lens. The bar is suspended by means of a stirrup from a ionless thread, and sets its magnetic axis in the magnetic meridian. telescope E, with theodolite mounting, is stationed opposite the which carries the lens, and is so adjusted at each observation that ne of collimation is parallel to that of the inverted telescope d by the magnet, an adjustment which is identified by seeing oss C coincident with a similar cross fixed in the interior of the ɔpe E. When the observation has been made with the magnet position, it must be repeated with the magnet turned upside as shown in the figure. Error of parallelism between the ic axis of the bar and the line of collimation of the inverted ɔe which it carries, will affect these two observations to the tent in opposite directions, and will therefore disappear from ean. The readings are taken on a horizontal circle corre- g to the outer circle in Fig. 422, and astronomical observa- ıst be made once for all to determine what reading corre- ɔ the geographical meridian.

ər very accurate method consists in rigidly attaching to the ad of the lens and cross, a small vertical mirror. This can viewed through a telescope, so as to show the reflection of al scale of equal parts, which will appear to travel across ɔf view of the telescope as the magnet turns, or it can be to throw the image of a spot of light either upon a screen the observer, or still better upon photographic paper drawn ɔrk, which leaves a permanent record of continuous changes. methods of employing mirrors for the observation of small of rotation are now extensively employed in many appli- hey appear to have been first introduced by Gauss, who ıem for the purpose which we are now considering.

urement of Dip.—The dip-circle or inclination compass is in Fig. 423. It consists essentially of a magnetized accurately and delicately mounted on a horizontal axis centre of gravity, in the centre of a vertical circle on ɔsitions of the two ends of the needle can be read off. ın be turned with the needle into any azimuth, the tation being indicated by a horizontal circle. It is if the vertical circle is placed in the plane of the magnetic meridian, the needle, being free to move in this plane, will directly indicate the dip. On the other hand, if the vertical circle is placed in a plane perpendicular to the magnetic meridian, the horizontal component of terrestrial magnetism is prevented from moving the needle, which, accordingly, obeys the vertical component only, and takes a vertical position. In intermediate positions of the vertical circle, the needle will assume positions intermediate between the vertical and the true angle of dip. In fact, if θ be the angle which the plane of the vertical circle makes with the magnetic meridian, the component H $\sin\theta$ of terrestrial magnetism, being perpendicular to this plane, merely tends to produce pressure against the supports, and the horizontal component influencing the position of the needle is only H $\cos\theta$, which lies in the plane of the circle. As none of the vertical force is destroyed, the tangent of the apparent dip will be $\frac{V}{H\cos\theta}=\frac{\tan\delta}{\cos\theta}$. The most accurate method of setting the vertical circle in the magnetic meridian consists in first adjusting it so that the needle takes a vertical position, and then turning it through 90°.

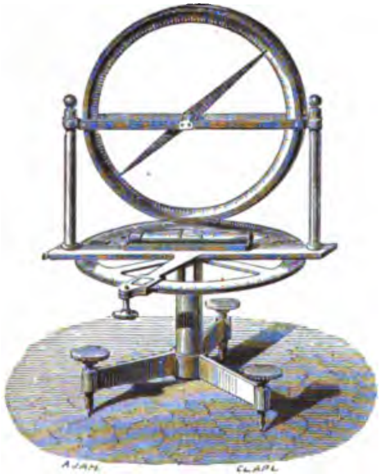

Fig. 423.—Dip-circle.

The instrument having thus been set, and a reading taken at each end of the needle, it should be turned in azimuth through 180°, and another pair of readings taken. By employing the mean of these two pairs of readings, several sources of error are eliminated, including non-coincidence of the axis of magnetization with the line joining the ends of the needle. One important source of error—deviation of the centre of gravity from the axis of suspension in a direction parallel to the length of the needle, is, however, not thus corrected. It can only be eliminated by remagnetizing the needle in the reverse direction so as to interchange its poles. The mean of the results obtained before and after the reversal of its magnetization will be the true dip.

A better form of instrument, known as the Kew dip-circle, is now employed. Its essential parts are represented in Fig. 423 A. There is no metal near the needle, and the readings are taken on a circle round which two telescopes travel. In each observation the telescopes are directed to the two ends of the needle.

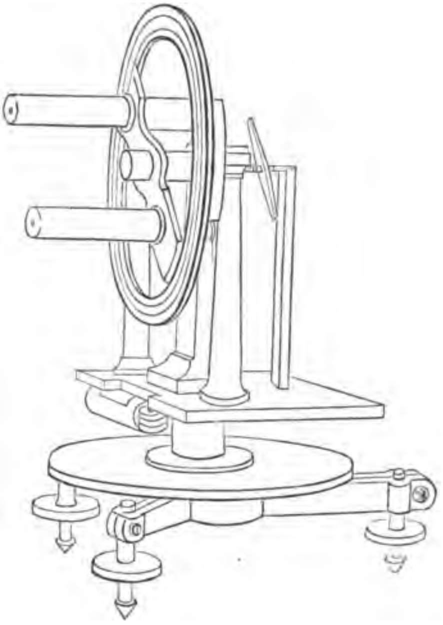

Fig. 423 A.—Kew Dip-circle.

500 A. Measurement of Intensity of Terrestrial Magnetic Force.—The complete specification of the earth's magnetic force at any place involves three independent elements. For example, if declination, dip, and horizontal force are determined by observation, vertical force and total force can be calculated by the formulæ of § 496.

Observations of magnetic force are made either by counting the number of vibrations executed in a given time, or by statical measurements. If a magnet executes small horizontal vibrations under the influence of the earth's magnetism, the square of the number of vibrations in a given time is proportional to $\frac{HM}{\mu}$, H denoting the horizontal intensity, M the moment of the magnet, and μ its moment of inertia about the centre of suspension. Hence it is easy to observe the *variations* of horizontal intensity which occur from time to time, if we can insure that our magnet itself shall undergo no change, or if we have the means of correcting for such changes as it undergoes. To obtain absolute determinations of horizontal intensity, the following method is employed.

First, observe the time of vibration of a freely-suspended horizontal magnet under the influence of the earth alone,—this will give the *product* of the earth's horizontal intensity and the moment of the magnet.

Secondly, employ this same magnet to act upon another also freely suspended, and thus compare its influence with that of the earth,—this will give the ratio of the same two quantities whose product was found before. Hence the two quantities themselves can easily be computed.

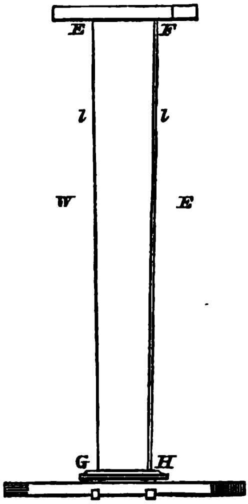

Fig. 423 B.—Bifilar Magnetometer.

500 B. Bifilar and Balance Magnetometers.—The changes of horizontal intensity are measured statically by means of the *bifilar magnetometer.* This consists of a bar-magnet (Fig. 423 B) suspended by two threads, which would be parallel if the bar were unmagnetized, but matters are so arranged that, under the combined action of the pull of the threads, the weight of the bar, and the earth's magnetism, the bar is kept in a position nearly perpendicular to the magnetic meridian. The only changes which occur in its position from time to time are those due to changes in the *intensity* of the earth's horizontal force, changes in the direction of this force, to the extent of a few minutes of angle, having no sensible effect, on account of the near approach to perpendicularity.

The changes of vertical intensity are measured by the *balance-magnetometer*, which consists of a bar-magnet placed in the magnetic meridian and suspended on knife-edges like the beam of an ordinary balance. Its deviations from horizontality are measures of the changes of vertical intensity.

Both these instruments have mirrors attached to the magnet, which produce a photographic record of the movements of the magnet, on principles above explained.

The moment of a magnet varies with temperature, being diminished by something like one ten-thousandth part of itself for each degree Fahr. of increase, and increasing again at the same rate when the temperature falls. Hence magnetic observatories must be kept at a nearly uniform temperature. They must also be completely free from iron. No iron nails are allowed to be used in their construction, copper being employed instead.

500 C. Results of Observation.—The annexed figures[1] contain an

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[1] For Figs. 422 A, 423 A, B, C, D, we are indebted to the publishers of Airy's *Treatise on Magnetism.*

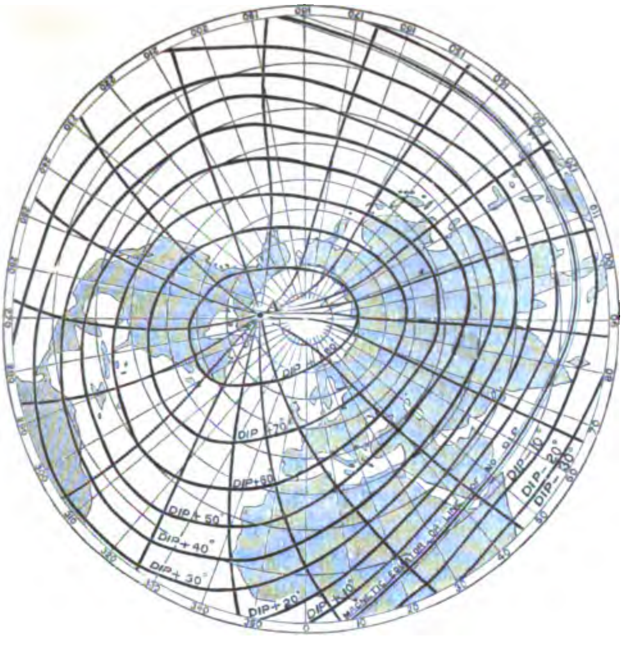

Fig. 423 c.—Northern Hemisphere.

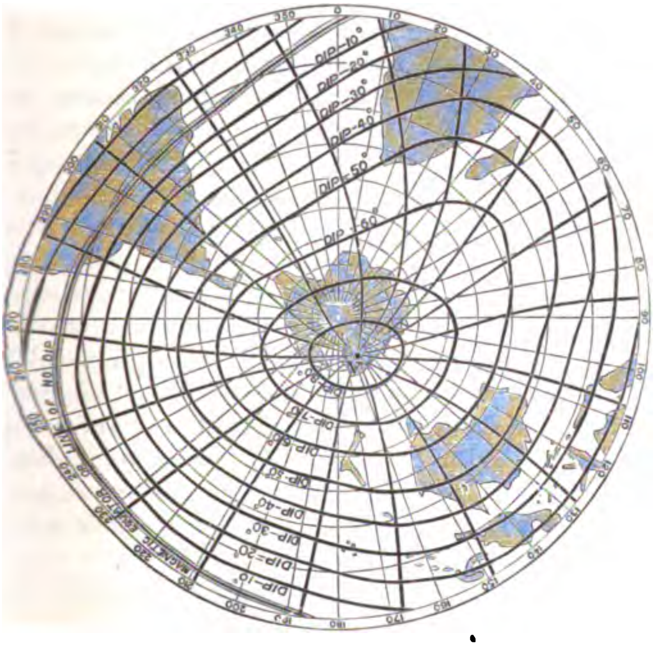

Fig. 423 d.—Southern Hemisphere.

Magnetic Meridians and Lines of Equal Dip.

approximate representation of the magnetic meridians and lines of equal dip over both hemispheres of the earth. These two systems of lines combined, furnish a complete specification of the *direction* of magnetic force at all parts of the earth's surface; but they indicate nothing as to *intensity*. The curves of equal total intensity have a general resemblance to the lines of equal dip, the intensity being greatest near the poles, and least near the equator; but their arrangement is somewhat more complicated, there being two north poles of greatest intensity, one in Canada, and the other in the northern part of Siberia. Speaking roughly, the intensity near the poles is about double of the intensity near the equator. Curves of equal total intensity are often called *isodynamic* lines; curves of equal dip are often called *isoclinic* lines; curves of equal declination are often called *isogonic* lines; curves cutting the magnetic meridians at right angles are often called *magnetic parallels*. They are the lines which would be traced by continually travelling in the direction of magnetic east or west.

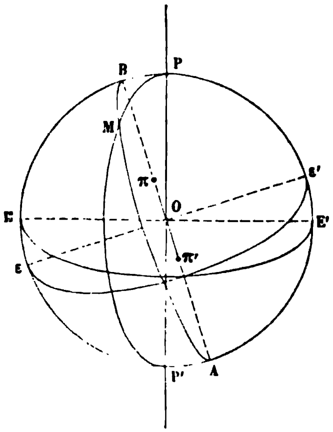

Fig. 421.—Biot's Hypothesis.

500 D. The Earth as a Magnet.[1]—The intensity and direction of terrestrial magnetic force at different places may be *roughly* represented by supposing that there is a magnet $\pi\pi'$ (Fig. 421) at the earth's centre, having a length very small in comparison with the earth's radius, and making an angle of about 20° with the earth's axis of rotation. The points A and B obtained by producing this magnet longitudinally to meet the surface, would be the magnetic poles, and at any other place the magnetic meridian would be the vertical plane containing the magnetic axis AB. At places situated on the great circle whose plane contains both the axis of rotation and the magnetic axis, the magnetic meridian would coincide with the geographical meridian, and the declination would be zero. At any other place M, the two meridians would cut each other at an angle which would be the angle of declination. At all places on the great circle $\epsilon\epsilon'$ whose plane is perpendicular to the magnetic axis, a needle suspended at its centre of gravity would place itself parallel to this axis, and consequently the dip would be zero. This circle would be the magnetic equator.[1] It would cut the geographical equator at an angle of 20°. Proceeding from the magnetic equator towards the north magnetic pole B, the needle would dip more and more, until at B it became vertical. A declination needle at B would remain indifferently in all positions. Similar phenomena would be observed at the other magnetic pole A. The end of the needle which would dip at B, and which at other parts of the earth would point to magnetic north, is that which is similar to the southern pole π' of the terrestrial magnet $\pi\pi'$, and the pole which is similar to π would dip at A.

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[1] This section corresponds to § 498 in the original. The hypothesis which it describes is known as *Biot's hypothesis*.

The actual phenomena of terrestrial magnetism are not in very close agreement with the results which would follow from the presence of such a magnet as we have described in the earth's interior, nor do they agree well with the hypothesis of two interior magnets inclined at an angle to each other, which has also been proposed. It would rather appear that the earth's magnetism is distributed in a manner not reducible to any simple expression.

501. Changes of Declination and Dip.—Declination and dip vary greatly not only from place to place, but also from time to time. Thus at the date of the earliest recorded observations at Paris, 1580, the declination was about 11° 30′ E. In 1663 the needle pointed due north and south, so that Paris was on the line of no declination. Since that time the declination has been west, increasing to a maximum of 22° 34′, which it attained in 1814. Since then it has gone on diminishing to the present time, its present value being about 19° W.

As to dip, its amount at Paris has continued to diminish ever since it was first observed in 1671. From 75° it has fallen to 66°, its present value. As its variations since 1863 have been scarcely sensible, it would seem to have now attained a minimum, to be followed by a gradual increase.

501A. Magnetic Storms.—Besides the gradual changes which occur in terrestrial magnetism, both as regards direction and intensity of force, in the course of long periods of time, there are minute fluctua-

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[1] If latitude reckoned from the magnetic equator be called magnetic latitude, and denoted by λ, it can be shown that we should have, on this theory,

$$\tan\delta = 2\tan\lambda\ ;\ I = E\sqrt{\cos^2\lambda + 4\sin^2\lambda},$$

E denoting the intensity at the magnetic equator.

tions continually traceable. To a certain extent these are dependent on the varying position of the sun, and, to a much smaller extent, of the moon, with respect to the place of observation; but over and above all regular and periodic changes, there is a large amount of irregular fluctuation, which occasionally becomes so great as to constitute what is called a *magnetic storm.* Magnetic storms "are not connected with thunder-storms, or any other known disturbance of the atmosphere; but they are invariably connected with exhibitions of aurora borealis, and with spontaneous galvanic currents in the ordinary telegraph wires; and this connection is found to be so certain, that, upon remarking the display of one of the three classes of phenomena, we can at once assert that the other two are observable (the aurora borealis sometimes not visible here, but certainly visible in a more northern latitude)."[1] They are sensibly the same at stations many miles apart, for example at Greenwich and Kew, and they affect the direction and amount of horizontal much more than of vertical force.

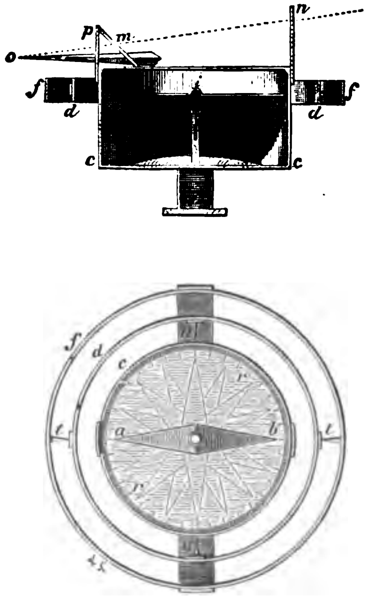

Fig. 424.—Ship's Compass.

502. Ship's Compass.—In a ship's compass, the box *cc* which contains the needle is weighted below, and hung on gimbals, which consist of two rings so arranged as to admit of motion about two independent horizontal axes *tt*, *uu* at right angles to each other. This arrangement prevents it from being tilted by the pitching and rolling of the ship. The needle *ab* is firmly attached to the compass-card, which is a circular card marked with the 32 points of the compass, as in Fig. 425, and also usually divided at its circumference into 360 degrees. The card with its attached needle is accurately balanced on a point at its centre. The needle, which, in actual use, is concealed from view, lies along the line NS. The box contains a vertical mark in its interior on the side next the ship's bow; and this mark serves as an index for reading off on the card the direction to which the ship's head is turned. Sometimes a reflector is employed, as *m* in the first figure, in such a position that an observer looking in from behind can read off the indicated direction by reflection, and can at the same time sight a distant object whose magnetic bearing is required. The origin of the compass is very obscure. The ancients were aware that the loadstone attracted iron, but were ignorant of its directing property. The instrument came into use in Europe some time in the course of the thirteenth century.

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[1] Airy on *Magnetism*, p. 204.

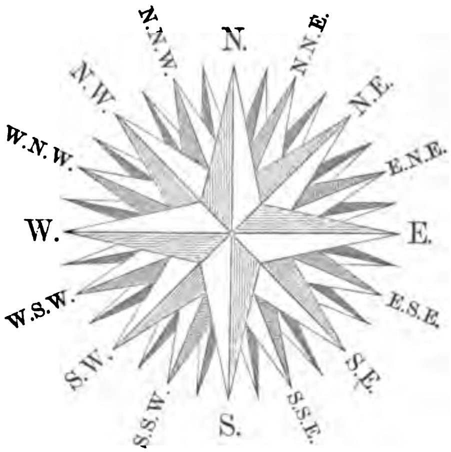
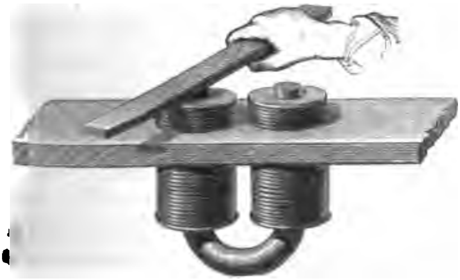


Fig. 425.—Compass-card.

503. Methods of Magnetization.—The usual process of magnetizing a bar consists in rubbing it with or against a bar already magnetized. Different methods of doing this, called single touch, double touch, &c., have been devised, in which magnetized bars of steel were the magnetizing agents. Much greater power can, however, be obtained by means of electro-magnetism; and the two following methods are now almost exclusively employed by the makers of magnets.

1. A fixed electro-magnet (Fig. 427) is employed, and the bar to be magnetized is drawn in opposite directions over its two poles. Each stroke tends to develop at the end of the bar at which the motion ceases, the opposite magnetism to that of the pole which is



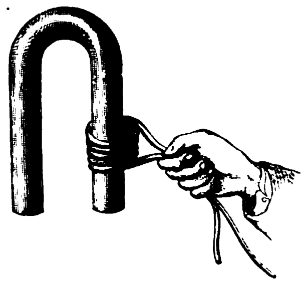


Fig.427. Methods of Magnetization. Fig. 428.

in contact with it. Hence strokes in opposite directions over the two contrary poles tend to magnetize the bar the same way.

2. When very intense magnetization is to be produced, the electromagnet must be very powerful, and the bar then adheres to it so strongly that the operation above described becomes difficult of execution, besides scratching the bar. Hence it is more convenient to move along the bar, as in Fig. 428, a coil of wire through which a current is passing. This was the method employed by Arago and Ampère.

A bar of steel is said to be magnetized to *saturation*, when its magnetization is as intense as it is able to retain without sensible loss. It is possible, by means of a powerful magnet, to magnetize a bar considerably above saturation; but in this case it rapidly loses intensity.

Pieces of iron and steel frequently become magnetized temporarily or permanently by the influence of the earth's magnetism, and this action is the more powerful as the direction of their length more nearly coincides with that of the dipping-needle. If fire-irons which have usually stood in a nearly vertical position be examined by their influence on a needle, they will generally be found to have acquired some permanent magnetism, the lower end being that which seeks the north.

It sometimes happens that, either from some peculiarity in the structure of a bar, or from some irregularity in the magnetizing process, a reversal of the direction of magnetization occurs in some part or parts of the length as compared with the rest. In this case the magnet will have not only a pole at each end, but also a pole at each point where the reversal occurs. These intermediate poles are called *consequent points.* Fig. 429 represents the arrangement of iron filings about a bar-magnet which has two consequent points a', b'

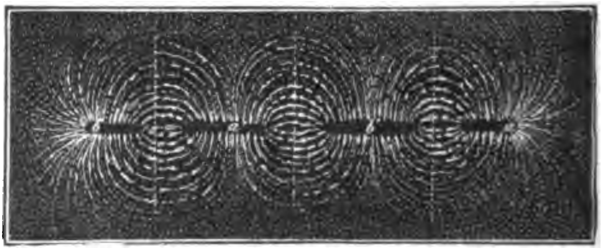


Fig. 429.—Consequent Points.

The whole bar may be regarded as consisting of three magnets laid end to end, the ends which are in contact being similar poles. Thus the two poles at a' and the one pole at a are of one kind, while the two poles at b' and the one pole at b are of the opposite kind.

The lifting power (or *portative* force) of a magnet generally increases with its size, but not in simple proportion, small magnets being usually able to sustain a greater multiple of their own weight than large ones. Hence it has been found advantageous to construct compound magnets, consisting of a number of thin bars laid side by side, with their similar poles all pointing the same way. Fig. 430 represents such a compound magnet composed of twelve elementary bars, arranged 4 × 3. Their ends are inserted in masses of soft iron, the extremities of which constitute the poles of the system.

Fig. 430.—Compound Magnet.

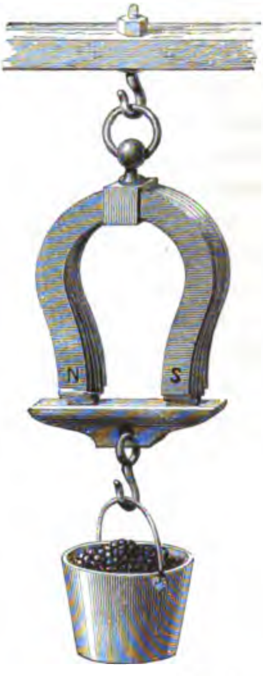


Fig. 431.—Compound Horse-shoe Magnet.

Fig. 431 represents a compound horse-shoe magnet, whose poles N and S support a keeper of soft iron, from which is hung a bucket for holding weights. By continually adding fresh weights day after day, the magnet may be made to carry a much greater load than it could have supported originally; but if the keeper is torn away from the magnet, the additional power is instantly lost, and the magnet is only able to sustain its original load.

Much attention was at one time given to methods of obtaining steel magnets of great power. These researches have now been superseded by electro-magnetism, which affords the means of obtaining temporary magnets of almost any power we please.

503 A. Molecular Changes accompanying Magnetization.—Joule has shown that, when a bar of iron is magnetized longitudinally, it acquires a slight increase of length, compensated, however, by transverse contraction, so that its volume undergoes no change.

If the magnetization is effected suddenly, by completing an electric circuit, an ear close to the bar hears a clink, and another clink is heard when the current is stopped.

These phenomena have been accounted for by the hypothesis that, when iron is magnetized, its molecules place their longest dimensions in the direction of magnetization.

The effect of heat in diminishing the strength of a magnet is another instance of the connection between magnetism and other molecular conditions. In ordinary cases, this diminution is merely transient; but if a steel magnet is raised to a white-heat, it is permanently demagnetized.

504. Action of Magnetism on all Bodies.—It has long been known that iron and steel were not the only substances which could be acted on by magnetism. Nickel and cobalt especially were known to be attracted by a magnet, though very much more feebly than iron, while bismuth and antimony were repelled. Faraday, by means of a powerful electro-magnet, showed that all or nearly all substances in nature, whether solid, liquid, or gaseous, were susceptible of magnetic influence, and that they could all be arranged in one or the other of two classes, characterized by opposite qualities. This opposition of quality is manifested in two ways.

1. As regards attraction and repulsion, iron and other *paramagnetic* bodies are attracted by either pole of a magnet, or more generally, they tend to move from places of weaker to places of stronger force. On the other hand, bismuth and other *diamagnetic* bodies are repelled by either pole of a magnet, and in general tend to move from places of stronger to places of weaker force.

2. As regards orientation, a paramagnetic body when suspended between the poles of a magnet tends to set *axially;* that is to say, tends to place its length along the line joining the poles, or more generally, when placed in any magnetic field, tends to place its length along the lines of force. Hence the name *para*magnetic.[1] A *dia*magnetic body, on the other hand, when suspended between the poles, sets *equatorially;* that is to say, places its length at right angles to

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[1] The nomenclature here adopted was proposed by Faraday in 1850 (*Researches*, § 2790), and is eminently worthy of acceptance. Many writers, however, continue to employ *magnetic* in the exclusive sense of *para*magnetic. To be consistent, they should call the other class *anti*magnetic, not *dia*magnetic. "The word *magnetic* ought to be general, and include *all* the phenomena and effects produced by the power."

the line joining the poles, or, more generally, tends to place its length at right angles to magnetic lines of force.

Fig. 432 represents the apparatus commonly employed for experiments on this subject. B, B are two large coils of stout copper wire, wound on massive hollow cylinders of soft iron. These latter form portions of the heavy frames F, F, which can be slid to or from each other, and fixed firmly at any distance by means of the screws E, E. The armatures P, which can be screwed on or off, have the form of rounded cones, and produce a great concentration of force at their extremities.

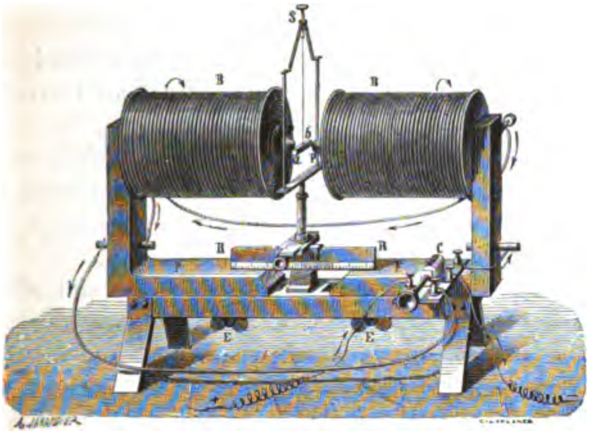

Fig. 432.—Apparatus for Diamagnetism.

The action of magnetism upon a solid can be examined by suspending a small bar of it *a b*, by means of a special support RS, between the poles P. When a current is passed through the coils, the bar immediately exhibits a preference either for the axial or the equatorial position. Attraction and repulsion are most easily tested by suspending a small ball of the substance at the level of the central line of poles, but a little beside it, the poles having first been brought very near together. On passing the current through the coil, the ball will move inwards towards the line of poles if paramagnetic, and outwards if diamagnetic.

It is important, however, to remark, that experiments of this kind, unless performed *in vacuo*, are merely differential—they merely indicate that the suspended body is, in the one case, more paramagnetic or less diamagnetic; in the other case more diamagnetic or less paramagnetic, than the medium in which it moves, the comparison being made between equal volumes. Oxygen is paramagnetic, and nitrogen is nearly or quite indifferent. Air is accordingly paramagnetic, and a body suspended in air appears less paramagnetic or more diamagnetic than it really is. If more feebly paramagnetic than air, it will appear to be diamagnetic. Thus heated air, in consequence probably of its rarefaction, appears diamagnetic when surrounded by cold air, and the flame of a taper is repelled downwards and outwards from the axial line.

If, on the other hand, the body under examination is suspended in water, it will appear more paramagnetic than it really is, by reason of the diamagnetism of water.

The following metals are paramagnetic: iron, nickel, cobalt, manganese, chromium, titanium, cerium, palladium, platinum, osmium.

The following are diamagnetic: bismuth, antimony, lead, tin, mercury, gold, silver, zinc, copper.

The following substances are also diamagnetic: water, alcohol, flint glass, phosphorus, sulphur, resin, wax, sugar, starch, wood, ivory, beef (whether fresh or dried), blood (whether fresh or dried), leather, apple, bread.

504 A. Magneto-crystallic Action.—The orientation of crystals in a magnetic field presents some remarkable peculiarities, which were extremely perplexing to investigators until Tyndall and Knoblauch discovered the principle on which they depend. This principle is, that crystals are susceptible of magnetic induction to different degrees in different directions. Every crystal (except those belonging to the cubic system) has either one line or one plane along which induction takes place more powerfully than in any other direction; and it is this line or plane which tends to place itself axially or equatorially according as the crystal is paramagnetic or diamagnetic. The directions of most powerful and least powerful induction are found to be closely related to the optic axes of crystals, and also to their planes of cleavage. When a sphere cut from a crystal is brought near to one pole of a magnet, it is attracted or repelled (according as it is para- or dia-magnetic) with the greatest force when the direction of most powerful induction coincides with the direction of the force.

Directions of unequal induction can be produced artificially in non-crystalline substances by applying pressure. "Bismuth is a brittle metal, and can readily be reduced to a fine powder in a mortar. Let a tea-spoonful of the powdered metal be wetted with gum-water, kneaded into a paste, and made into a little roll, say an inch long and a quarter of an inch across. Hung between the excited poles, it will set itself like a little bar of bismuth—equatorial. Place the roll, protected by bits of pasteboard, within the jaws of a vice, squeeze it flat, and suspend the plate thus formed between the poles. On exciting the magnet, the plate will turn, with the energy of a magnetic substance, into the axial position, though its length may be ten times its breadth.

"Pound a piece of carbonate of iron into fine powder, and form it into a roll in the manner described. Hung between the excited poles, it will stand as an ordinary [para]magnetic substance—axial. Squeeze it in the vice, and suspend it edgeways, its position will be immediately reversed. On the development of the magnetic force, the plate thus formed will recoil from the poles, *as if violently repelled*, and take up the equatorial position."[1]

In these experiments the direction of most powerful induction is a line transverse to the thickness, and this is also the direction in *which* pressure has been applied. Tyndall accordingly concludes that "if the arrangement of the component particles of any body be *such as* to present different degrees of proximity in different directions, then the line of closest proximity, other circumstances being equal, will be that chosen by the respective forces for the exhibition of their greatest energy. If the mass be [para]magnetic, this line will stand axial; if diamagnetic, equatorial."[2]

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[1] Tyndall on *Diamagnetism*, p. 18. [2] Ibid. p. 23.

CURRENT ELECTRICITY.

CHAPTER XLV.

GALVANIC BATTERY.

505. Voltaic Electricity.—Towards the close of last century, when the discovery of the various phenomena of frictional electricity had been followed by Coulomb's investigations, which first reduced them to an accurate theory, a new instrument was brought to light destined to effect a complete revolution in electrical science. In place of an element difficult to manage, capricious and uncertain in its behaviour, and constantly baffling investigation by the rapidity of its dissipation, the galvanic battery furnished a steady source of electricity, constantly available in all weathers, and requiring no special precautions to prevent its escape. Moreover, the electricity thus developed exhibited an entirely new set of phenomena, and opened up the way to such various and important applications, that frictional electricity at once fell into the second place, and the new agent became the main object of interest with all electrical investigators.

506. Galvanic Element.—The source of this new form of electricity is chemical action.

When a metal which is easily attacked by an acid is immersed in the acid, (for example, zinc in dilute sulphuric acid,) difference o potential is immediately established between them at the surface o contact, the acid becoming positive with respect to the metal. I either the acid or the metal is insulated (except where they touc each other) this difference of potential prevents chemical action fro taking place, at least when the metal is quite pure and homogeneou If, however, connection be established between the zinc and the aci by means of a conductor (such as platinum) which is not liable t be attacked by the acid, a continual current will flow through thi conductor in a direction tending to diminish the difference of pote l, and continual chemical action will take place between the acid l the zinc.

uch an arrangement constitutes a galvanic or voltaic *element* or and the difference of potential which exists between the acid the zinc at the surface of contact is called the *electro-motive* of the element. This difference appears to be the same when current is flowing as when it is prevented from flowing by ation.

instead of platinum, we establish the connection by means of al such as copper, which is liable to be acted on by the acid, n a much lower degree than zinc, there is probably a slight of potential in the acid above the copper, and the electro- e force of the element is diminished by this difference of poten- In every case, the electro-motive force is the difference between tentials of the two metals when both are immersed wholly or ly in the acid, and are otherwise insulated.

433 represents one of the simplest forms of such an arrange- -a zinc and copper cell. Two plates Z, C, one of zinc, and the f copper, are partially immersed in water acidulated by the 1 of sulphuric acid, and are ed by means of a wire M, ling-screws. The direction urrent through this wire is her to lower potential, and from the copper to the e. An equal current flows he acidulated water from e of contact with the zinc e of copper. The chemical ich takes place at the sur- e zinc plate, furnishes, as lasts, a continual supply e electricity to the acid ative electricity to the zinc. These opposite electricities te at the surface where they are produced, such union nted by the chemical relations which produced them, and w round to meet each other through the circuit composed per plate, wire, and zinc plate. For the sake of brevity, n in which the positive electricity flows is always spoken *ection of the current.*

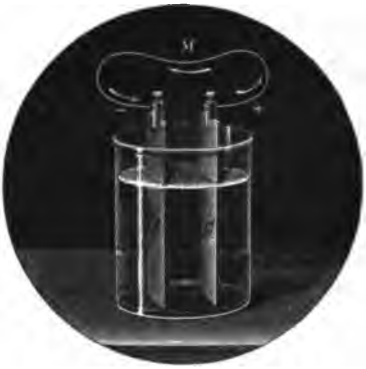

Fig. 433.—Voltaic Element.

508. Galvanic Battery.—By connecting the plates of successive elements in the manner represented in Fig. 434, we obtain a battery. The copper of the first cell on the left hand is connected with the zinc of the second, the copper of the second with the zinc of the third, and so on to the end of the series. The zinc of the first and the copper of the last cell (or wires proceeding from them) constitute the *electrodes* or *poles* of the battery, the zinc being the negative, and the copper the positive electrode. When these are insulated from each other, each copper is at the same potential[1] as the zinc of the next cell with which it is connected, and there is a sudden rise of potential in passing from each zinc to the liquid in contact with it. This rise occurs in four places in the arrangement represented in the figure, so that there are five distinct values of potential in different parts of the system. The highest value is for the copper on the extreme right, and the lowest is for the zinc on the extreme left, the difference between their values being four times as great as a single cell would give. This fact is usually expressed by saying that the electro-motive force of a battery of n similar cells arranged in series is n times the electro-motive force of a single cell. When the electrodes are connected, the current flows through the connecting wire from the positive electrode (copper) to the negative electrode (zinc).

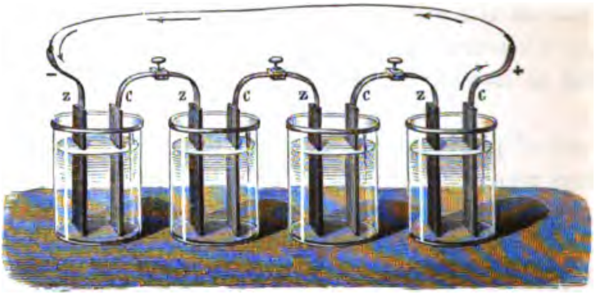

Fig. 434.—Battery of Four Elements.

509. Galvani's Discoveries.—About the year 1780, Galvani, professor of anatomy at Bologna, had his attention called to the circumstance that some recently skinned frogs, lying on a table near an electrical machine, moved as if alive, on sparks being drawn from the machine.

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[1] We neglect here, as relatively insignificant, the small difference of potential which probably always exists between two dissimilar metals in contact.

Struck with the apparent connection thus manifested between electricity and vital action, he commenced a series of experiments on the effects of electricity upon the animal system. In the course of these experiments, it so happened that, on one occasion, several dead frogs were hung on an iron balcony by means of copper hooks which were in contact with the lumbar nerves, and the legs of some of them were observed to move convulsively. He succeeded in obtaining a repetition of these movements by placing one of the frogs on a plate of iron, and touching the lumbar nerves with one end of a copper wire, the other end of which was in contact with the iron plate. Another mode of obtaining the result is represented in Fig. 435, two wires of different metals being employed which touch each other at one end, while their other ends touch respectively the lumbar nerves and the crural muscles. Every time the contact is completed, the limb is convulsed.

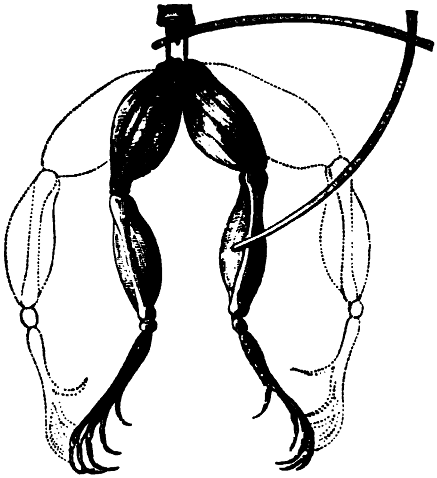

Fig. 435.—Experiment with Frog.

Galvani's explanation was, that at the junction of the nerves and muscles there is a separation of the two electricities, the nerve being positively, and the muscle negatively electrified, and that the convulsive movements are due to the establishment of communication between these two electricities by means of the connecting metals.

Volta, professor of physics at Pavia, disproved this explanation by showing that the movements could be produced by merely connecting two parts of a muscle by means of an arc of two metals; and he referred the source of electricity not to the junction of nerve and muscle, but to the junction of the two metals. Acting on this belief, he constructed in the year 1800 a voltaic pile.

511. Voltaic Pile.—This consisted of a series of discs of copper, zinc, and wet cloth, *c*, *z*, *d*, Fig. 436, arranged in uniform order, thus—copper, zinc, cloth, copper, zinc, cloth . . . the lowest plate of all being copper and the highest zinc. The wet cloth was intended merely to serve as a conductor, and prevent contact between each zinc and the copper above it. All the contacts between zinc and copper were between a copper below and a zinc above, so that they all tended, according to Volta's theory, to produce a current of electricity in the same direction. The effects obtained from the pile were so powerful as to excite extraordinary interest in the scientific world.

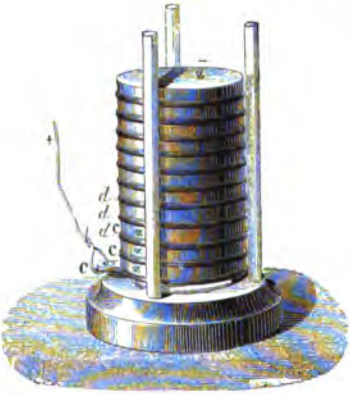

Fig. 436.—Structure of Pile.

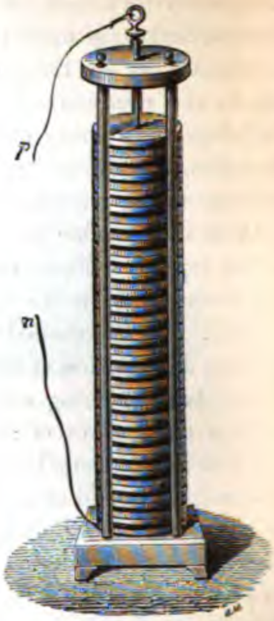

Fig. 437.—Complete Pile.

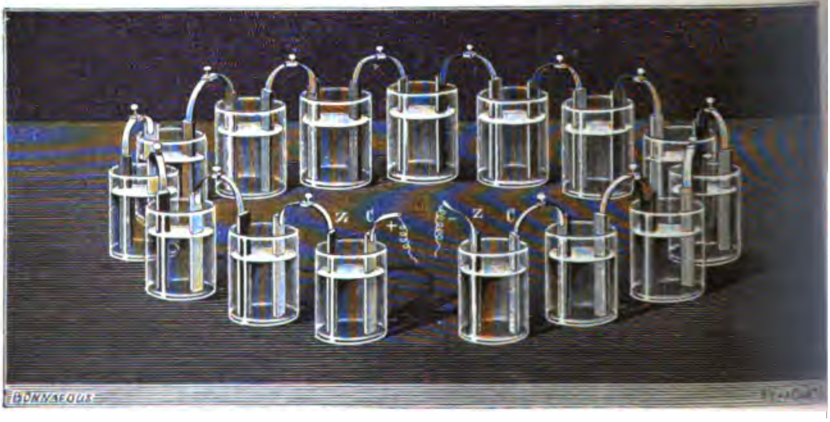

Fig. 438.—Couronne de Tasses.

513. Couronne de Tasses.—He shortly afterwards invented the *couronne de tasses* (crown of cups), consisting of a series of cups arranged in a circle, each containing salt water with a plate of silver or copper and a plate of zinc immersed in it, the silver or copper of each cup being connected with the zinc of the next, with the exception of the extreme plates. The last plate in liquid at each end of the series was connected with a plate of the other metal in air. These two plates in air are now known to be useless, and are omitted in the figure.

514. Trough Battery.—More convenient arrangements, equivalent to the couronne de tasses, were soon introduced. One of these, devised by Cruickshank, is represented in Fig. 439, consisting of a rectangular box, called a trough, of baked wood, which is a non-conductor of electricity, divided into compartments by partitions each consisting of a plate of zinc and a plate of copper soldered together. Dilute acid is poured into these compartments.

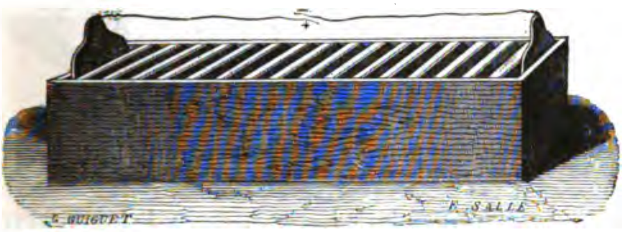

Fig. 439.—Cruickshank's Trough.

515. Wollaston's Battery.—In Wollaston's battery, the plates were suspended from a single horizontal bar, by means of which they could all be let down into the acid, or lifted out of it together. The liquid was contained either in compartments of a trough of glazed earthenware, with partitions of the same material, or in separate vessels as shown in Fig. 441. The plates were double-coppered; that is to say,

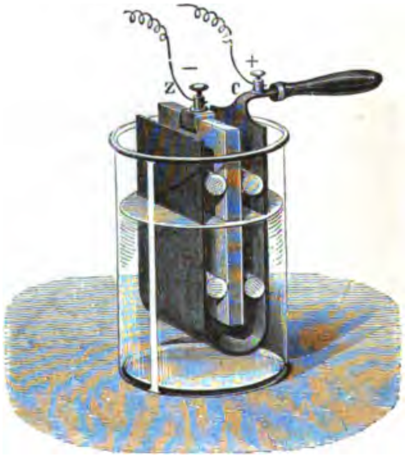

Fig. 440.—Wollaston's Cell.

they consisted of a zinc plate with a copper plate bent round it on both sides (Fig. 440), contact between them being prevented by pieces of wood or cork.

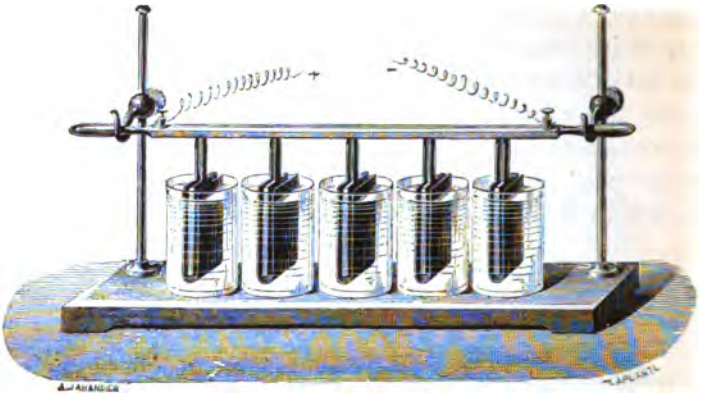

Fig. 441.—Wollaston's Battery.

517. Hare's Deflagrator.—For some purposes it is more important to diminish the resistance of a cell, or, in other words, to facilitate the conduction of electricity between the zinc and the copper plate, than to increase the electro-motive force by multiplying cells. The helical arrangement devised by Hare of Philadelphia (Fig. 443) is specially adapted to such purposes. It consists of two very large plates of zinc and copper rolled upon a central cylinder of wood, and prevented from touching each other by pieces of cloth or twine inserted between them. It is plunged in a tub of acidulated water, as represented in the figure. From the remarkably powerful heating effects which can be obtained by the use of this cell, it is called Hare's *deflagrator*.

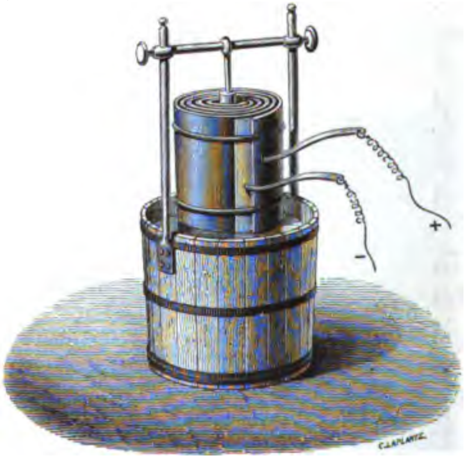

Fig. 443.—Hare's Deflagrator.

518. **Polarization of Plates.**—All the forms of battery which we 'e thus far described, are liable to a rapid decrease of power, owing auses which are partly chemical and partly electrical.

'he chemical action which takes place in each cell consists narily in the formation of sulphate of zinc, at the expense of the plate, the sulphuric acid, and the oxygen of the water with ch the acid is diluted, the hydrogen of the water being thus ated. As this action proceeds, the liquid becomes continually capable of acting powerfully on the zinc. Again, a portion of inc which has been dissolved becomes deposited on the copper , thus tending to make the two plates alike, and so to destroy urrent, which essentially depends on the difference between

t the most important cause of all is to be found in what is called *larization* of the copper plate; that is to say, in the deposition lm of hydrogen on the surface of the plate. This film not only oses resistance by its defect of conductivity, but also brings to n electro-motive force in the direction opposed to that of the t.

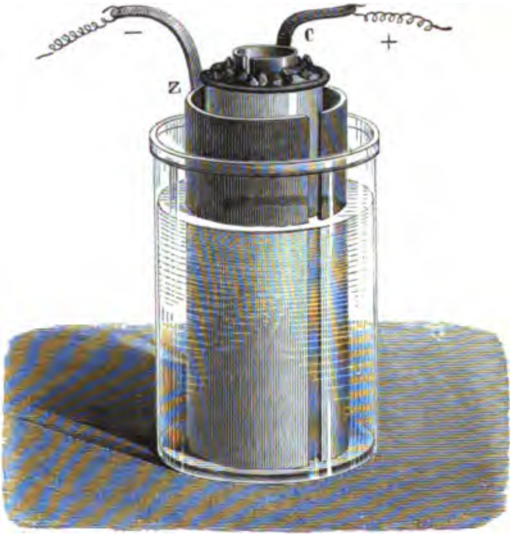

Fig. 444.—Daniell's Cell.

se obstacles to uintenance of a ıt current were overcome by

Daniell's Bat- In the cell de- y Daniell, there ous partition of d earthenware, ng the two li- hich are in con- with the zinc, other with the plate. These ıids are not alike, that s in contact copper being not simply dilute sulphuric acid like the t containing also as much sulphate of copper as it will For the purpose of keeping it saturated, crystals of sulphate of copper are suspended in it near its surface by means of a wire basket of copper. The effect of this arrangement is, that the hydrogen is intercepted before it can arrive at the copper plate, and the deposit which takes place on the copper plate is a deposit of copper, the hydrogen taking the place of this copper in the saturated solution.

The current given by a battery of these cells remains nearly constant for some hours.

In the figure, the copper plate C is represented as a cleft cylinder occupying the interior, with the crystals of sulphate of copper piled up round it. The entire cylinder surrounding these is the porous partition, outside of which is the cleft cylinder of zinc Z, the whole being contained in a vessel of glass.

It is more usual in this country to dispense with the glass vessel, and interchange the places of the zinc and copper in the figure, the copper plate being a cylindrical vessel of copper containing the saturated solution. In this is immersed the porous vessel containing the other fluid with the zinc plate immersed in it. The cells thus constructed are usually arranged in square compartments in a wooden box.

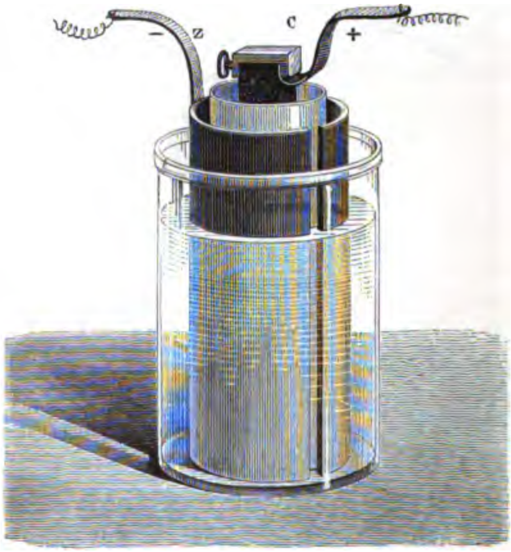

Fig. 445.—Bunsen's Cell.

520. Bunsen's Battery. — The battery which is now perhaps most extensively used for class experiments is that which was invented by Bunsen in 1843, being substantially identical with one previously invented by Grove, except that carbon is substituted for platinum.

The usual construction of its cells is very clearly represented in Fig. 445. The cleft cylinder is the zinc plate, which is immersed in dilute sulphuric acid. Within this is the porous cylinder, similar to Daniell's, containing *strong nitric acid,* in which is immersed a rectangular prism, of a very dense kind of charcoal, obtained from the interior of the retorts at gas-works, being deposited there in the manufacture of gas.

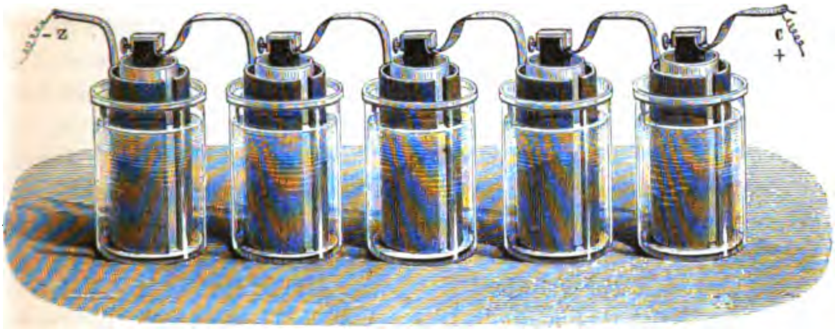

Fig. 446.—Bunsen's Battery.

In this cell the hydrogen is intercepted on its way to the carbon plate by the nitric acid, with which it forms nitrous acid.

Grove's battery possesses some advantages over Bunsen's; but its first cost is much greater.

521. Amalgamated Zinc.—When the poles of a battery are insulated from one another, there ought to be no chemical action in the cells. Any action which then goes on is wasteful, and is an indication that unproductive consumption of zinc goes on when the current is passing, in addition to the consumption which is necessary for producing the current. This wasteful action, which is called *local action*, goes on largely when the zinc plates are of ordinary commercial zinc, but not when they are of perfectly pure zinc. In this respect amalgamated zinc behaves like pure zinc, and it is accordingly almost universally employed. The amalgamation, which must be often renewed in the case of a battery in constant use, is performed by first cleaning the zinc plates with dilute acid, and then rubbing them with mercury.

522. Dry Pile: Bohnenberger's Electroscope.—For telegraphic purposes in this country, a battery is very commonly employed in which sand or sawdust, moistened with acidulated water, separates the zinc and copper plates of each cell.

The other forms of battery which have been devised are exceedingly numerous, and new forms are continually being introduced.

A *dry pile*, built up on the general plan of Volta's moist pile, was devised by De Luc, and improved by Zamboni. In Zamboni's construction, sheets of paper are prepared by pasting finely laminated zinc or tin on one side, and rubbing black oxide of manganese on the other. Discs are punched out of this paper, and piled up into a column, with their similar sides all facing the same way, to the number of a thousand or upwards, and are well pressed together. The difference of potential between the two ends is sufficient to produce sensible divergence of the gold-leaves of an electroscope, but the quantity of electricity which can be developed in a given time is exceedingly small. *No pile or battery can generate a sensible current, except by a sensible consumption of its materials in the shape of chemical action.*

A very delicate gold-leaf electroscope was devised by Bohnenberger, consisting of a single leaf suspended between the two poles of a dry pile, which for this purpose is arranged in two columns connected below, so that the poles are at the summits. If their lower ends, which form the middle of the series, be connected with the earth, one pole will always have positive, and the other negative potential. A very slight charge, positive or negative, given to the gold-leaf by means of the knob at the top of the case, suffices to make it move to the negative or the positive pole.

523. Thermo-electric Currents.—Electric currents can be produced by applying heat or cold to one of the junctions in a circuit composed of two different metals. This was first shown by Seebeck of Berlin in 1821. It may be illustrated by employing a rectangular frame (Fig. 448), having three sides formed of a copper plate, and the fourth of a cylinder of bismuth. It must be placed in the magnetic meridian. with a magnetized needle in its interior. On heating one of the junctions with a spirit-lamp, the needle will be deflected in such a direction as to indicate the existence of a current, which, in the copper portion of the circuit, flows from the hot to the cold junction, and in the bismuth portion from the cold to the hot. If cold instead of heat be applied to one junction, the direction of the current will still be from the warmer junction

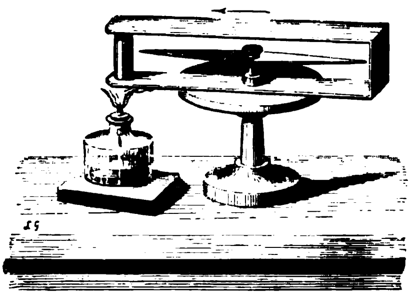

Fig. 448.—Thermo-electric Current.

rough the copper to the colder junction, and from this through the muth to the warmer junction. Antimony, if employed instead of per, gives a still more powerful effect.

524. Though a circuit composed of bismuth and antimony is cially susceptible of thermo-electric excitation, the property is sessed, in a more or less marked degree, by every circuit composed wo metals, and even by cir- s composed of the same metal lifferent states. If, for ex- le, a knot or a helix (as in 449), be formed in a piece of num wire, and heat applied ne side of it, a current will dicated by a delicate galvanometer. In metals which are usually ogeneous in their structure, such as bismuth, it is not uncom- to find currents produced by heating parts which appear quite m. If the ends of two copper wires be bent into hooks, and them be heated, on placing them in contact, a current will be ced due to the presence of a thin film of oxide on the heated With two platinum wires, no such effect is obtained.

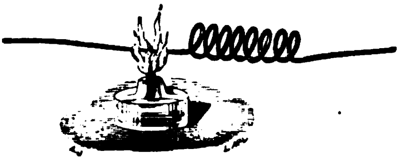

Fig. 449.—Current with one Metal.

Thermo-electric Order.—According to Becquerel's experiments, tals may be ranged in the following order, as regards the n of the current produced by heating a junction of any two n:—*Bismuth, platinum, lead, tin, copper, silver, zinc, iron, ny;* that is to say, if a junction of any two of these metals ed, the direction of the current at the junction in question from that which stands the list to the other. His ents have also established ortant fact that the cur- ained by heating all the s B, C, D, E, F, of a chain ilar metals to one com- perature, is the same as ined by uniting the two extreme bars AB, FG, directly to r, and heating their junction to the same temperature.[1]

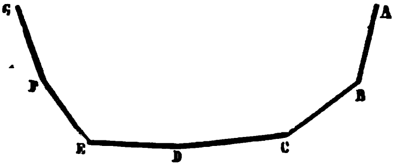

Fig. 451.

mparison of Electro-motive Forces.—By employing a chain composed of wires of different metals soldered together, with its two extremities connected with a galvanometer, and heating one junction to 20° C., while the rest were kept at 0° C., Becquerel obtained currents proportional to the following numbers:—

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[1] e accurate statement is, that the *electro-motive force* is the same in the two *current* will be sensibly the same if the resistance in BCDEF is insignificant n with the rest of the circuit. In order that there may be a current, the

Junction heated.	Current.	Junction heated.	Current.
Iron - silver,	26·20	Copper - platinum,	8·55
Iron - copper,	27·96	Copper - tin,	3·50
Iron - tin,	31·24	Silver - copper,	2·00
Iron - platinum,	36·07	Zinc - copper,	1·00

On comparing these numbers, it will be found that they are in approximate agreement with the law above stated. Thus the electromotive force of a silver-platinum circuit comes out 10·55 by adding 2·00 to 8·55, and 9·87 by subtracting 26·20 from 36·07. The electromotive force of copper-platinum is 8·55 as observed directly, and 8·11 as computed by taking the difference of iron-copper and iron-platinum. The deviation from precise agreement is not more than may fairly be ascribed to errors of observation.

527. Influence of Temperature.—When the junctions in a thermo-electric circuit are at only two temperatures, the lower of which remains constant while the other varies, the current will be sensibly proportional to the difference of temperature only as long as this difference is small. As the difference goes on increasing, the current increases less rapidly, then decreases, and at still greater differences of temperature is reversed.

528. Thermo-electric Pile.—If a thermo-electric chain be composed of two metals occurring alternately (as in Fig. 452), no effect will be obtained by equally heating *two consecutive* junctions; for the current which would be generated by heating the one is in the opposite direction to that due to the heating of the other. If we number the

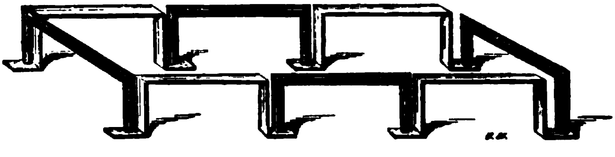

Fig. 452.—Pouillet's Thermo-pile.

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circuit must of course be completed, and not left open as in Fig. 451. In the case of an open circuit, the result of the heating will simply be to produce difference of potential between the extremities A, G. This difference of potential is the measure of the electromotive force, and will accordingly be the same in the two cases.

junctions in order, we shall obtain a current in one direction by heating any junction which bears an odd number, and in the opposite direction by heating any one that bears an even number. The thermo-electric pile, or *thermo-pile,* whose use has been already described in connection with experiments on radiant heat (§ 313), is an arrangement of this kind, in which all the odd junctions are presented together at one end, and all the even junctions at the other, the two metals composing the pile being antimony and bismuth. The electromotive force obtained with a given difference of temperature between the ends of the pile is proportional to the number of junctions, except in so far as accidental differences may exist between different junctions.

529. Application to Measurement of Temperature.—Thermo-electric currents may be employed either in testing equality of temperatures, or in comparing small differences of temperature. As an example of the former application, suppose a circuit to be formed of two long wires, one of iron and the other of copper, connected at both ends, and covered with gutta-percha or some other insulator except at the two junctions. Let one junction be lowered to the bottom of a boring, or any other inaccessible place whose temperature we wish to ascertain, and let the other junction be immersed in a vessel of water containing a thermometer. If one of the wires be carried round a galvanometer, the direction in which the needle is deflected will indicate whether the upper or lower junction is the warmer, and if we alter the temperature of the watér in the vessel till the deflection is reduced to zero, we know that the two junctions are at the same temperature, which we can read off by the thermometer immersed in the water.

CHAPTER XLVI.

GALVANOMETER.

530. Œrsted's Experiment.—The discovery by the Danish philosopher Œrsted, in 1819, that a magnetized needle could be deflected by an electric current, was justly regarded with intense interest by the scientific world, as affording the first indication of a definite relation existing between magnetism and electricity.

Œrsted's experiment can be repeated by means of the apparatus represented in Fig. 456. Two insulated metallic wires are placed in the magnetic meridian, one of them above, and the other below a magnetized needle. If a current be sent through one of these wires, the needle will be deflected; and if the current be strong, the deflection will nearly amount to a right angle. The direction of the deflection will be reversed if the current be passed through the lower instead of the upper wire. It will also be reversed by reversing the direction of the current. In the figure, the current is supposed to be passing above the needle from south to north. In this case the north end of the needle moves to the west, and the south end to the east. On making the current pass in various directions, either horizontally, vertically, or obliquely, near one pole of the needle, it will be found that deviation is always produced except when the plane containing the pole and current is perpendicular to the length of the needle.

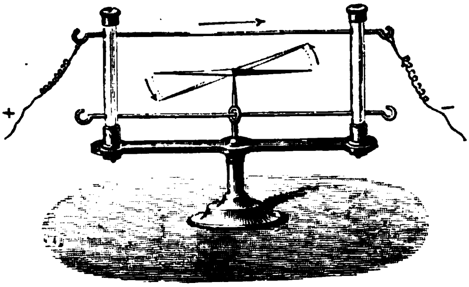

Fig. 456.—Œrsted's Experiment.

531. Ampère's Rule.—The direction in which either pole of a needle is deflected by a current, whatever their relative positions may be, is given by the following rule, which was first laid down by Ampère.

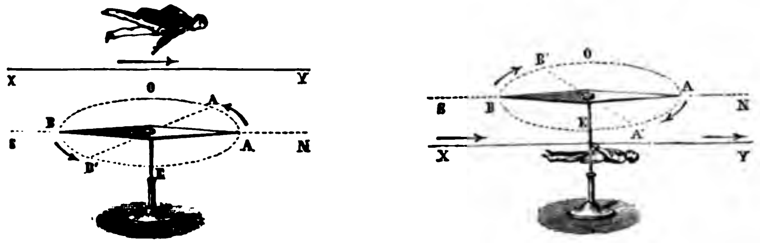

Fig 457. Ampère's Law. Fig. 458.

Imagine an observer to be so placed that the current passes through him, *entering at his feet* and leaving at his head, then the deflection of a *north-seeking pole* will be *to the left* as seen by him. The deflection of a south-seeking pole will be in the opposite direction. The two figures 457, 458 illustrate the application of this rule to the two cases just considered. The current is supposed, in both cases, to be flowing from south to north. A is the austral or north-seeking pole of the needle, and B the boreal or south-seeking pole.

531 A. Lines of Magnetic Force due to Current.—The relation between currents and magnetic forces may be more precisely expressed by saying that a current flowing through a straight wire produces circular lines of force, having the wire for their common axis. A pole of a magnet placed anywhere in the neighbourhood of the wire, experiences a force tending to urge it in a circular path round the wire, and the direction of motion round the wire is opposite for opposite poles. Fig. 458 A represents three of the lines of force for a north-seeking pole, due to a current flowing through a straight wire from the end marked + to the end marked −. The lines of force are circles (shown in perspective as ellipses), having their centre at a point C in the wire, and having their plane perpendicular to the length of the wire. The arrows indicate the direction in which a north-

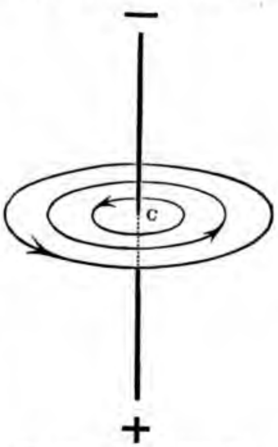

Fig. 458 A.—Lines of Force due to Current.

seeking pole will be urged. This direction is from right to left round the wire as seen from the wire itself by a person with his feet towards + and his head towards −, according to Ampère's rule. The figure may be turned upside down, or into any other position, and will still remain true.

531 B. Reaction of Magnet on Current.—While the wire, in virtue of the current flowing up through it, urges an austral pole from A towards A′ (Fig. 458 B), it is itself urged in the opposite direction CC′. If an observer be in imagination identified with the wire, the current being supposed, as in Ampère's rule, to enter at his feet, and come out at his head, the force which he will experience from a north-seeking pole directly in front of him will be a force to his right. It will be noted that the magnetic influence which thus urges him to the right, would urge a north-seeking pole from his front to his back. *A conductor conveying a current is not urged along lines of magnetic force, but in a direction which is at right angles to them, and at the same time at right angles to its own length.*

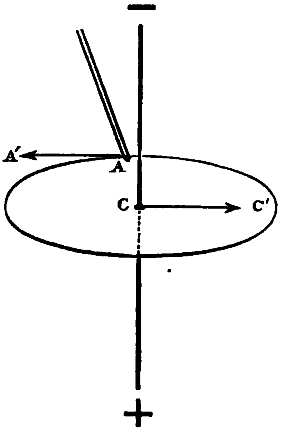

Fig. 458 B.—Reaction on Current.

532. Numerical Estimate of Currents.—The numerical measure of a current denotes the quantity of electricity which flows across a section of it in unit time. It is sometimes called *strength* of current, sometimes, especially by French writers, *intensity* of current, sometimes simply *current* or *amount* of current. If a thin and a thick wire are joined end to end, it has the same value for them both; just as the same quantity of water flows through the broad as through the contracted parts of the bed of a stream. Hence the name *intensity* is obviously inappropriate, for, with the same total quantity of electricity flowing through both, the current is, properly speaking, more *intense* in the thin than in the thick wire.

Currents may be measured experimentally by various tests, which are found to agree precisely. The most convenient of these for general purposes is the deflection of a magnetized needle. The force which a given pole experiences in a given position with respect to a wire conveying a current, is simply proportional to the current. Hence the name *strength* of current admits of being interpreted in a sense corresponding to that in which we speak of the strength of a pole. Instruments for measuring currents by means of the deflections which they produce in a magnetized needle are called *galvanometers.*

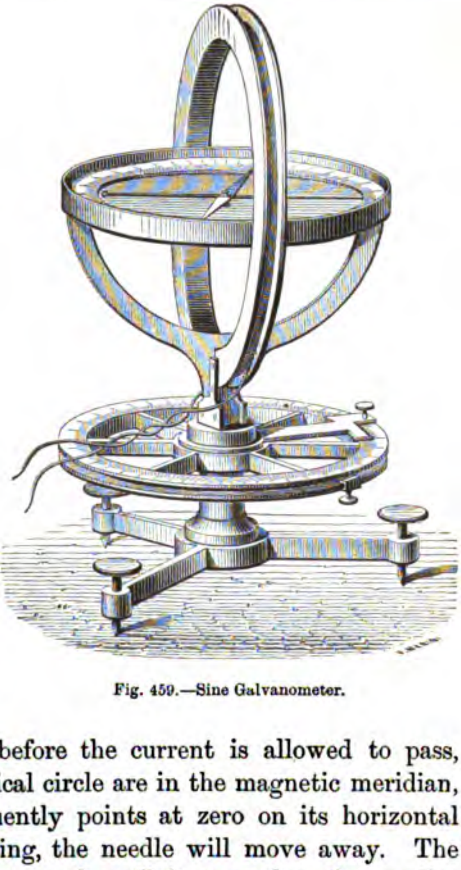

Fig. 459.—Sine Galvanometer.

533. Sine Galvanometer.—The sine galvanometer, which was invented by Pouillet, is represented in Fig. 459. The current which is to be measured traverses a copper wire, wrapped round with silk for insulation, which is carried either once or several times round a vertical circle; and this circle can be turned into any position in azimuth, the amount of turning being indicated on a horizontal circle. In the centre of the vertical circle, a declination needle is mounted, surrounded by a horizontal circle for indicating its position, this circle being rigidly attached to the vertical circle. Suppose that, before the current is allowed to pass, both the needle and the vertical circle are in the magnetic meridian, and that the needle consequently points at zero on its horizontal circle. On the current passing, the needle will move away. The vertical circle must then be turned until it overtakes the needle; that is, until the needle again points at zero. This implies turning the circles through an angle α equal to that by which the needle finally deviates from the magnetic meridian. In this position the terrestrial couple tending to bring back the needle to the meridian is proportional to $\sin \alpha$ (§ 498). The forces exerted upon the two poles by the current are perpendicular to the plane of the vertical circle, and are simply proportional to the current. Hence, in comparing different observations made with the same instrument, the amounts of current are proportional to the sines of the deviations.

534. Tangent Galvanometer.—The tangent galvanometer, which is simpler in its construction and use, and is much more frequently employed, consists of a declination needle mounted in the centre of a vertical circle whose plane always coincides with the magnetic meridian, the length of the needle being small in comparison with the radius of the circle.

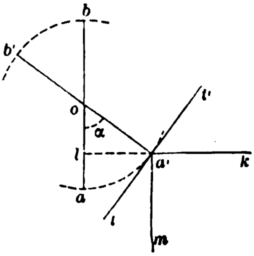

Fig. 460.—Principle of Tangent Galvanometer

Let *o* (Fig. 460) be the centre of suspension, *a b* the initial position of the needle, and *a' b'* its deflected position. The force F exerted on either pole by the current is sensibly the same at *a'* as at *a* on account of the smallness of the needle, and it acts in the direction *lk*, while the horizontal force of the earth upon the pole acts along *a'm*, and these two forces give a resultant along *oa'*. Hence, taking the triangle *ola'* as the triangle of forces,[1] the force exerted by the current is to the horizontal force exerted by the earth as *la'* to *ol*, or as *tan* α to unity; that is, the current is proportional to the tangent of the deflection.

In order to permit the deviations of the short needle to be accurately read, a long pointer is attached to it, usually at right angles, the two ends of which move along a fixed horizontal circle.

535. Multiplier.—The idea of carrying a wire several times round a needle in a vertical plane is due to Schweiger. The form of apparatus designed by him, called *Schweiger's multiplier*, is represented in Fig. 461. The difference between the rectangular and the circular form is merely a matter of detail. The name *multiplier* is derived from the fact that, if the current is not sensibly diminished by increasing the number of convolutions of wire through which it has to pass, the force exerted on the needle is *n* times as great with *n* convolutions as with only 1, since each convolution exerts its own force on the needle independent of the rest. Cases, however, frequently occur in which the increased *resistance* introduced by increasing the number of convolu-

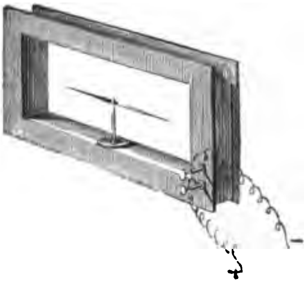

Fig. 461.—Schweiger's Multiplier.

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[1] The parallelogram of forces is divided by its diagonal into two triangles, either of which may be called the triangle of forces.

tions outweighs the advantage of multiplication, so that a short thick wire with few convolutions gives a more powerful effect than a long thin wire with many. This is especially the case with thermo-electric currents. The names *multiplier* and *galvanometer* are commonly used as equivalent.

The difference between the rectangular and the circular form is merely a matter of detail. Whichever form be adopted, all parts of the coil contribute to make the needle deviate in the same direction. For instance, in Fig. 462, if the current proceeds in the direction indicated by the arrows, the application of Ampère's rule to any one of the four sides of the rectangle shows that the austral pole *a* will be urged towards the front of the figure. When the coil is circular, and the needle so small that each pole is nearly in the centre, equal lengths of the current, in whatever parts of the circle they may be situated, exert equal forces upon the needle, and all alike urge the poles in directions perpendicular to the plane of the coil.

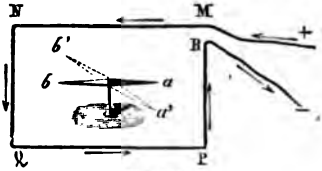

Fig. 462.

535 A. Differential Galvanometer.—The coil of a galvanometer sometimes consists of two distinct wires, having the same number of convolutions, and connected with separate binding-screws. This arrangement allows of currents from two distinct sources being sent at the same time round the coil either in the same or in opposite directions. In the latter case, the resultant effect upon the needle will be that due to the difference of the two currents; and if they are not exactly equal, the direction of the deflection will indicate which of them is the greater. An instrument thus arranged is called a *differential galvanometer*.

536. Astatic Needle.—The sensibility of the galvanometer is greatly increased by employing what is called an *astatic* needle. It consists of a combination of two magnetized needles *with their poles turned opposite ways.* The two needles are rigidly attached at different heights to a vertical stem, and the system is usually suspended by a silk fibre, which gives greater freedom of motion than support upon a point. On account of the opposition of the poles, the directive action of the earth on the system is very feeble. If the magnetic moments of the two needles were exactly equal, the resultant moment would be zero, and the system would remain indifferently in all azimuths.

One of the needles *ab* (Fig. 463) is nearly in the centre of the coil CDEF through which the current passes. The other *a′ b′* is just above the coil. When a current traverses the coil in the direction of the arrows, the action of all parts of the current upon the lower needle tends to urge the austral pole *a* towards the back of the figure, and the boreal pole *b* to the front. The upper needle *a′ b′* is affected principally by the current in the upper part CD of the coil, which urges the austral pole *a′* to the front of the figure and the boreal pole *b′* to the back. Both needles are thus urged to rotate in the same direction by the current, and as the opposing action of the earth is greatly enfeebled by the combination, a much larger deflection is obtained than would be given by one of the needles if employed alone.

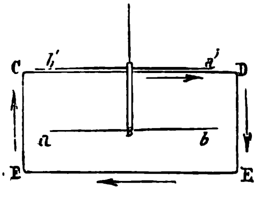

Fig. 463.

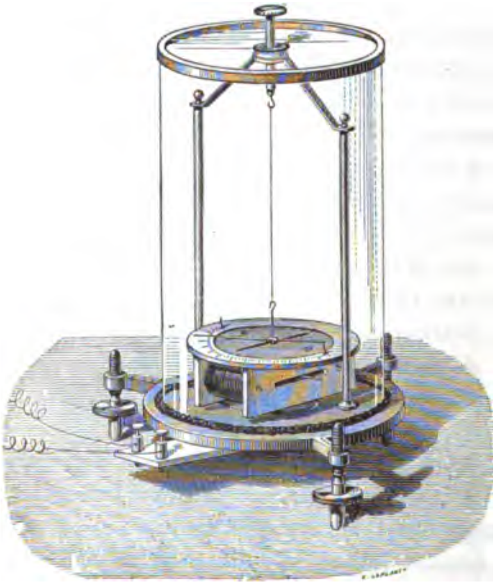

Fig. 464.—Astatic Galvanometer.

If the two needles had rigorously equal moments, the system would be said to be *perfectly astatic.* The smallest current in the coil would then suffice to set the needles at right angles to the meridian, and no measure would be obtained of the amount of current.

Fig. 464 represents an astatic galvanometer, as usually constructed. The coil is wound upon an ivory frame, which supports the divided circle in whose centre the upper needle is suspended. The ends of the coil are connected with two binding-screws for making connection with the wires which convey the current to be measured. The needles are usually two sewing-needles, and the upper one often carries a light pointer. The suspending fibre is attached at its upper end to a hook, which can be raised or lowered, and when the instrument is not in use this is lowered till the upper needle rests upon the plate beneath it, so as to relieve the fibre from strain. In using the instrument, care must be taken to adjust the three levelling-screws so that the needle swings free.

536 A. Thomson's Mirror Galvanometer.—The most sensitive galvanometer as yet invented is the mirror galvanometer of Sir W. Thomson. Its needle, which is very short, is rigidly attached to a small light concave mirror, and suspended in the centre of a vertical coil of very small diameter by a silk fibre. A movable magnet is provided for bringing the needle into the plane of the coil when the latter does not coincide with the magnetic meridian. A divided scale is placed in a horizontal position in front of the mirror, at the distance of about a yard, and the image of an illuminated slit, which is thrown by the mirror upon this scale, serves as the index. The arrangement of the mirror and scale, which is the same as in the case of the quadrant electrometer described in a previous chapter, is exhibited in Fig. 464 A. M is the mirror of silvered glass, slightly concave, with a small piece of magnetized watch-spring attached to its back, the two together weighing only a grain and a half, and suspended by a few fibres of unspun silk. AA is a divided scale forming an arc of a horizontal circle about the mirror as centre. Immediately below the centre of this scale is a circular opening S with a fine wire stretched vertically at the back of it. A paraffine lamp L is placed directly behind this opening, so as to shine through it upon the mirror, which is at such a distance as to throw upon the screen a bright image of the opening with a sharply-defined dark image of the wire in its centre. The image of the wire is employed as the index in taking the readings.

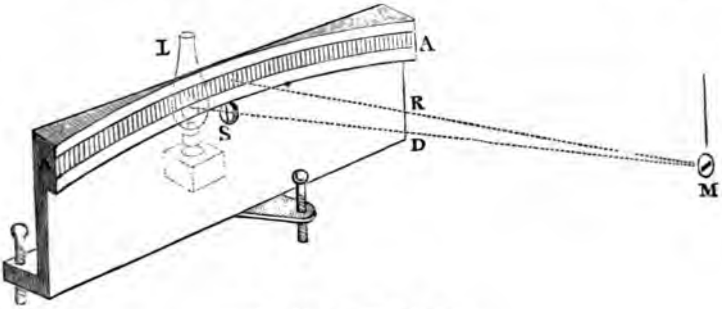

Fig. 464 A.—Mirror and Scale.

For use at sea, the galvanometer is modified by fastening the supporting fibre of silk at both ends, so as to keep it tight, with the needle and mirror attached at its centre, care being taken to make the direction of the fibre pass through the common centre of gravity of the needle and mirror, in order that the rolling of the ship may not tend to produce rotation. In this form it is called the *marine galvanometer.*

537. Calibration of Galvanometer.—The deviations of the needle of a galvanometer are not in general proportional to the currents which produce them. In order to be able to translate the indications of the instrument into proportional measure, a preliminary investigation must be made, and its results embodied in a table. This has been done in several ways. We shall merely indicate the method employed by Melloni for deducing from the deflections of his galvanometer the amounts of heat received by his thermo-pile.

He placed two sources of heat opposite the two ends of the pile, and allowed them to radiate to it, first one at a time, and then both together. One of them produced a deviation, say of 5°, and the other of 10°, and when the two were acting jointly the deviation was 5°. Since the latter number is the difference of the other two, the inference is that up to 10° the deflections are proportional to the amounts of heat received. Melloni thus established that the proportionality subsisted up to 20°. When the two sources separately produced deflections of 20° and 25°, and a deflection of 6°·5 jointly, he inferred that a deflection of 25° indicated an amount of heat represented by 26°·5; for the heat which produced the deflection of 25° was the sum of the two amounts represented separately by 20° and 6°·5. By a succession of steps of this kind, the calibration[1] (as this process is called) can be extended nearly to 90°.

This mode of investigation covers any want of proportionality which may exist in the production of thermo-electric currents, as well as in the proportionality of these currents to the deflections. Another method of calibrating a galvanometer will be described in the next chapter.

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[1] The application of the name *calibration* to this process is, we believe, due to Professor Tyndall. Its analogy to the calibration of a thermometer is obvious; the object in both cases being to reduce observed differences to proportional measure. It is often called the *graduation* of a galvanometer; but, in point of fact, the galvanometer is graduated, by dividing its circle into 360 degrees, before the process begins.

CHAPTER XLVII.

OHM'S LAW.

538. Statement of Ohm's Law.—The strength of the current which traverses a circuit depends partly on the electro-motive force of the source of electricity, and partly on the resistance of the circuit. For equal resistances, it is proportional to the whole electro-motive force tending to maintain the current, and for equal electro-motive forces it is inversely as the whole resistance in the circuit. Hence, when proper units are chosen for expressing the current C, the resistance R, and the electro-motive force E, we have

$$C = \frac{E}{R},$$

or *the current is equal to the electro-motive force divided by the resistance.* This is Ohm's law, so called from its discoverer.

539. Explanation of the term Electro-motive Force.—Whenever an acid is in contact with a metal liable to be acted upon by it, a difference of potential exists at the surface of contact, the acid having higher potential than the metal. In a galvanic circuit, the tendency of this difference of potential is to force a current round from the acid through the rest of the circuit to the metal. In a battery consisting of several cells arranged in a series, the total electro-motive force is the sum of the forces of the separate cells; and if the circuit be interrupted at any one place, and the potentials at the two sides of the interruption be examined by Thomson's quadrant electrometer, they will exhibit a difference which is the sum of the differences exhibited by the separate cells when similarly examined. A similar difference, but usually much smaller, is exhibited by the two poles of a thermo-electric pile when its two faces are at different temperatures, and is the sum of the differences due to the separate elements.

When the current is passing round a galvanic circuit there is, in each cell, a gradual fall of potential from the film of liquid in contact with the zinc to the film in contact with the carbon or copper plate, and there is also a gradual fall in the connecting wire from the last carbon (or the positive pole) to the first zinc (or the negative pole), the sum of all these gradual falls being precisely equal to the sum of all the sudden rises[1] which occur at the surfaces of contact between metal and liquid. The sum of all these sudden rises constitutes the whole electro-motive force of the circuit.

In like manner, in a thermo-electric circuit, there is difference of potential probably at each junction, whatever its temperature may be; and the algebraic sum of these sudden differences (a rise of potential being called positive and a fall negative, in travelling with the current) is the whole electro-motive force of the thermopile. When the faces of the pile are at equal temperatures, the opposite electro-motive forces are equal, and destroy one another; when the temperatures are unequal, the positive electro-motive forces exceed the negative, and the total or resultant electro-motive force is the measure of this excess.

540. Explanation of the term Resistance.—When the current of a circuit is taken through the coil of a galvanometer, it is found that, by introducing different lengths of connecting wire, very different amounts of deflection can be obtained. The longer the wire which connects either pole of the battery with the galvanometer, the smaller is the deflection; and a small deflection indicates a feeble current. The current is in like manner weakened by introducing a fine instead of a stout wire, if their length and material be the same, or by introducing an iron wire instead of a copper wire of the same dimensions. These differences in the properties of the different wires are expressed by saying that they have different resistances.

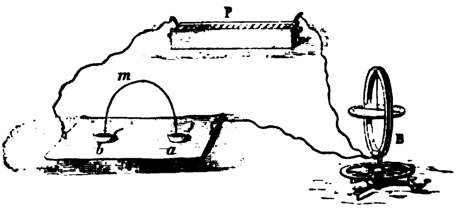

Fig. 465.—Comparison of Resistances.

The apparatus represented in Fig. 465 can be employed for comparing resistances in this way. The current given by a battery P passes through a wire to the galvanometer B, and after traversing its coil is led on by another wire to the cup of mercury *a*, thence through the connecting wire *m* to the other cup of mercury *b*, and back to the battery through another wire. The circuit can also be completed as shown in the figure, without passing through *m*, by means of a broad conducting plate whose resistance may be neglected.

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[1] If any sudden falls occur, it is the algebraic sum (that is the excess of sudden rises above sudden falls) that is to be here understood.

In changing the wire *m*, it is found that, to produce no change in the deflection, the length of the wire must vary directly as its cross-section; that is to say, if l, l', l'' .... be the lengths of different wires employed, and $s\ s'\ s''$ .... their sectional areas, their resistances will be equal, if

$$\frac{l}{s}=\frac{l'}{s'}=\frac{l''}{s''}\ldots\ldots$$

This is on the supposition that the wires are all of precisely the same material. Every substance has its own specific resistance, the reciprocal of which is its electrical conductivity and is precisely analogous to thermal conductivity. Denoting specific resistances by $r, r', r'', \ldots$ the condition of equal resistances, when the materials are different, is

$$\frac{rl}{s}=\frac{r'l'}{s'}=\frac{r''l''}{s''}\ldots\ldots$$

and the resistance of any wire is expressed by the formula $\frac{rl}{s}$, l denoting its length, s its sectional area, and r the specific resistance of its material.

541. Experimental Proofs of Ohm's Law.—Pouillet, who conducted numerous experiments bearing on Ohm's law, investigated the connection between currents and resistances in the following ways:—

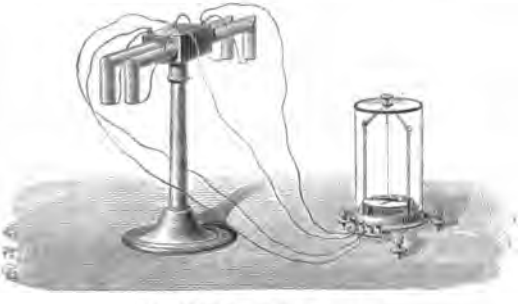

Fig. 466.—Pouillet's Comparison.

1. *For thermo-electric currents*, he employed two thermo-electric elements, each consisting of a stout cylinder of bismuth with its ends bent down and soldered to copper wires. The two elements were arranged side by side as in Fig. 466, and the junctions at one end were immersed in hot water, those at the other end being kept in melting ice. The hot and cold junctions of the one were connected

by a wire which was carried round a galvanometer needle. Those of the other were connected by a wire ten times as long, which made ten times as many turns round the same needle in the opposite direction, so that the two currents opposed each other in their action on the needle. It was found that the needle remained at zero, showing that the current in the short wire was ten times as strong as the other, for one of its convolutions was able to balance ten convolutions of the other. As the resistance in the stout bars of bismuth was inappreciable, it followed that the currents in the two circuits were inversely as the resistances.

2. *For voltaic currents*, he first sent the current of a battery through a galvanometer without any interposed resistance, and observed the strength of current C. He then introduced, successively, known lengths of uniform wire l_1, l_2, l_3, and observed the currents obtained. Denoting these by C_1, C_2, C_3, and taking x to denote the length of wire which would be equivalent to the unknown resistance of the original circuit consisting only of the battery and the galvanometer, we should have—

$$\frac{c}{c_1}=\frac{x+l_1}{x}\ ,\quad \frac{c}{c_2}=\frac{x+l_2}{x}\ ,\quad \frac{c}{c_3}=\frac{x+l_3}{x}.$$

From any one of these three equations x can be determined, and Ohm's law is verified if they all give the same value of x. This Pouillet found to be the case.

By repeating the experiment with a different kind of wire, a new value of x will be obtained, and thus the resistances of equal lengths of the two wires can be compared.

542. Reduced Length: Total Resistance of Circuit.—To express, in terms of the equivalent length of one wire, the resistance of a circuit composed of several, we can employ the relation (§ 540)

$$\frac{rl}{s}=\frac{r'l'}{s'}\ ;\ \text{whence } l=\frac{s}{s'}\,\frac{r'}{r}\,l',$$

l denoting the length of one kind of wire equivalent to the length l' of the other. The length l is called the reduced length of the wire whose actual length is l'.

543. Rheostat.—Wheatstone's *rheostat* is a very convenient instru- ment for the comparison of resistances. It consists (Fig. 467) of tw cylinders, one of brass, and the other of non-conducting material, s arranged that a copper wire can be wound off the one on to the oth by turning a handle. The surface of the non-conducting cylinder has a screw-thread cut in it, for its whole length, in which the wire lies, so that its successive convolutions are well insulated from each other. Two binding-screws are provided for introducing the rheostat into a circuit; and the resistance which is thus introduced depends on the length of wire which is wrapped upon the non-conducting cylinder, for the brass cylinder A has so large a section that its resistance may be neglected. The amount of resistance can thus be varied as gradually as we please by winding on and off. The handle can be shifted from one cylinder to the other. The figure shows it in the position for winding wire off A on to B. The number of convolutions of wire on B can be read off on a graduated bar provided for the purpose, and parts of a revolution are indicated on a circle at one end.

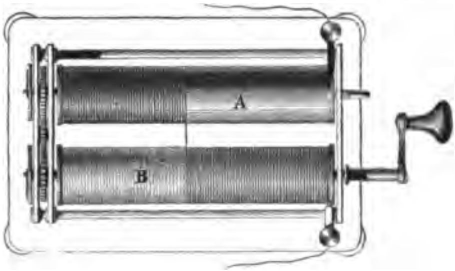

Fig. 467.—Rheostat.

Fig. 468 represents a very direct mode of measuring resistances by the rheostat. The current traverses a galvanometer B, a rheostat R, and the conductor *m*, whose resistance is to be measured, the whole of the wire of the rheostat being wound on the brass cylinder. The deflection of the galvanometer having been observed, the conductor *m* is taken out of circuit, the two wires at *a* and *b* are directly connected, and as much of the rheostat wire is brought into circuit as suffices to reduce the deflection to its former amount.

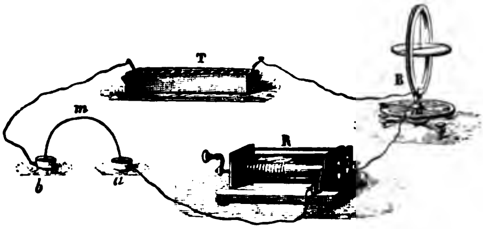

Fig. 468.—Measurement of Resistance.

543 A. Specific Resistances and Conductivities.—Numerous experimenters have compared the specific resistances of the different metals. Though the results thus obtained exhibit some diversity, they all agree in making silver, gold, and copper the three best conductors. Slight impurities, especially in the case of copper, have a very great effect in diminishing conductivity; or, in other words, in increasing resistance. Resistance is also increased, in the case of metals, by increase of temperature.

Forbes has pointed out that the order of the metals as regards their conductivity for heat is the same as for electricity. The effects of impurity and of change of temperature are also alike in the two cases, as has been recently shown by Professor Tait.

The following are E. Becquerel's determinations of specific electrical resistance at the temperature 15° C., the resistance of silver at 0° C. being denoted by 100:—

Specific Resistances at 15° C.

Silver,	107
Copper,	112
Gold,	155
Cadmium,	407
Zinc,	414
Tin,	734
Palladium,	715
Iron,	825
Lead,	1213
Platinum,	1243
Mercury,	5550

On comparing this list with the list of thermal conductivities, § 333, it will be observed that the order is precisely the same as far as the comparison extends, and that the numerical values are nearly in inverse proportion, showing that electrical and thermal *conductivities* are nearly in direct proportion.

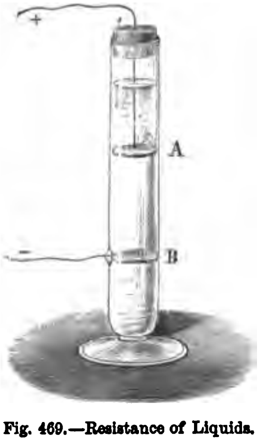

Fig. 469.—Resistance of Liquids.

544. Resistance of Liquids.—The resistance of liquids can be determined on similar principles, the current being transmitted between two parallel plates of metal immersed in the liquid. One form of apparatus for this purpose is represented in Fig. 469. Care must be taken to employ metals which will not give rise to electro-motive force by chemical action.

The resistance even of the best conducting liquids, except mercury, is enormously greater than that of metals. For instance, in round numbers, the resistance of dilute sulphuric acid is a million times, and that of solution of sulphate of copper ten million times greater than that of pure silver. The resistance of pure water is very much greater than either of these.

In the cells of a galvanic battery, the current has to traverse liquid conductors, and the resistance of these is sometimes a large part of the whole resistance in circuit. It is diminished by bringing the plates nearer together, and by increasing their size, since the former change involves diminution of length, and the latter increase of sectional area in the liquid conductor to be traversed. This is the only advantage of large plates over small ones, the electro-motive force being the same for both. The advantage of the double coppers in Wollaston's battery (§ 515) is similarly explained, the resistance with this arrangement being about half what it would be with copper on only one side of the zinc, at the same distance.

545. Calibration of Galvanometer by the Rheostat.—The rheostat can be employed for determining the relative values of the deflections of a galvanometer. For this purpose the two instruments are to be introduced into the circuit of a battery, and in the first instance all the wire of the rheostat is to be on the non-conducting cylinder. The deflection, which will then be comparatively small, is to be noted. Successive lengths of the rheostat wire are then to be wound off, so as to diminish the resistance, and the deflections are to be noted in each case. If the current, when the whole length l of rheostat wire was in circuit, be denoted by C, and the currents with lengths $l_1 \, l_2$ . . . in circuit by $C_1 \, C_2$ . . . , and if r denote the resistance of the battery and galvanometer, which can be determined by methods already explained, we shall have

$$\frac{C_1}{C} = \frac{r+l}{r+l_1} \;, \quad \frac{C_2}{C} = \frac{r+l}{r+l_2} \; \ldots\ldots$$

Hence the ratios of the currents C, C_1, C_2 . . . corresponding to the observed deflections are known.

546. Arrangement of Cells in Battery.—Suppose that we have a number n of precisely similar cells, each having electro-motive force e and resistance r, and that we connect them in a series, as in Figs. 434, 446, with a conductor of resistance R joining their poles. The whole electro-motive force in the circuit will then be ne, and the whole resistance will be $nr + R$; hence the strength of current will be

$$C = \frac{ne}{nr+R}.$$

This formula shows that, if the external resistance R is much greater than the resistance in the battery nr, any change in the number of cells will produce a nearly proportional change in the current; but that when the external resistance is much less than that of one cell, as is the case when the poles are connected by a short thick wire, a change in the number of cells affects numerator and denominator almost alike, and produces no sensible change in the current. It is impossible, by connecting any number of similar cells *in a series*, to obtain a current exceeding $\frac{e}{r}$, which is precisely the current which one of the cells would give alone if its plates were well connected by a short thick wire.

It is possible, however, by a different arrangement of the cells, to obtain a current about n times stronger than this, namely, by connecting all the zinc plates to one end of a conductor, and all the carbons or coppers to the other end, as in Fig. 470. In the arrangement of three cells here figured, the current which passes through the spiral connecting wire is the sum of the currents which the three cells would give separately. The arrangement is equivalent to a single cell with plates three times as large superficially, and at the same distance apart. The electro-motive force with n cells so arranged is simply e, but the resistance is only $\frac{r}{n}+\mathrm{R}$, so that the current is

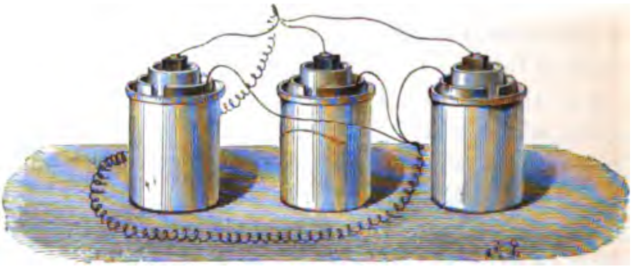

Fig. 470.—Cells with similar Plates connected.

$$\mathrm{C}=\frac{e}{\frac{r}{n}+\mathrm{R}}=\frac{n e}{r+n\mathrm{R}}.$$

This system of arrangement may be called *arranging the cells as one element*. It has sometimes been called the *arrangement for quantity*, the arrangement in a series being called the *arrangement for intensity*.

If in Fig. 470 we substitute for each of the three cells a *series* consisting of four cells, the electro-motive force in circuit will be $4e$, and the resistance in circuit will be $\frac{4r}{3}+\mathrm{R}$, for each series has a sistance of $4r$, and three parallel series connected at the ends are quivalent to a single series, of the same electro-motive force as one the component series, and of one-third the resistance. The current ill therefore be

$$C = \frac{4e}{\frac{4r}{3} + R} = \frac{12e}{4r + 3R} = \frac{e}{\frac{r}{3} + \frac{R}{4}}.$$

The question often arises, What is the best manner of grouping a ven number of cells in order to give the strongest possible current ough a given external conductor? The answer is, they should be grouped that the internal and external resistance should be as rly as possible equal; for example, if we have 12 cells as above, the resistance R in the given conductor is $\frac{4}{3}$ of the resistance of of these cells, the arrangement just described is the best.[1]

47. **Divided Circuits.**—When two or more wires are connected *in* , that is so as to form one continuous wire, *the resistance of the le is the sum of the resistances of the wires composing it.*

n the other hand, when two or more wires are arranged side by and connected at each end, so as to constitute so many indepen- channels of communication between the ends, the joint resist- is evidently less than the resistance of any one of the wires. n such an arrangement occurs in any part of a circuit, the circuit d to be *divided.* If the several wires are of the same length naterial, they act as one wire having a section equal to the sum ir sections, and the joint resistance is the quotient of the resist- of one of the wires by the number of wires. More generally, reciprocal of the resistance of a conductor be called its con- g power, *the conducting power of a system of wires thus con- at both ends is the sum of the conducting powers* of the l wires which compose it. Thus, in Fig. 471, if r_1, r_2 denote sistances of the wires *acb*, *adb*, their joint resistance R will n by the equation

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ad of 3 and 4, put x for the number of series, and y for the number of cells in a hen the current will be $\frac{e}{\frac{r}{x} + \frac{R}{y}}$, and will vary inversely as $\frac{r}{x} + \frac{R}{y}$. Now the pro- and $\frac{R}{y}$ is given, being the quotient of r R by the whole number of cells; and when t of two variables is given, their sum is least when they are equal, and increases e made more and more unequal. As x and y must be integers, exact equality erally be obtained.

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}, \quad \text{whence} \quad R = \frac{r_1 r_2}{r_1 + r_2}$$

547 A. Wheatstone's Bridge.—In any wire through which a current is flowing steadily, without leakage or lateral offshoots, the amount of the current is equal to the *difference of potential between the ends of the wire, divided by the resistance of the wire,* the units employed being the same as those which make $C = \frac{E}{R}$ for the whole circuit. The same thing is true for *any portion* of the length of such a wire, and, still more generally, for *any portion* of a circuit, whether single or divided, *terminated by equipotential cross-sections,* provided that no source of electro-motive force occurs in it. It follows that, in travelling along such a wire with the current, the fall of potential is proportional to the resistance travelled over, or *equal falls of potential occur in traversing equal resistances.* This rule does not apply to the comparison of the two independent channels of a divided circuit, unless equal currents are passing through them. It applies to the comparison of any two wires which are conveying equal currents, and it is not applicable to the comparison even of different portions of the same wire if, owing to leakage, the current is unequal at different parts of its length.

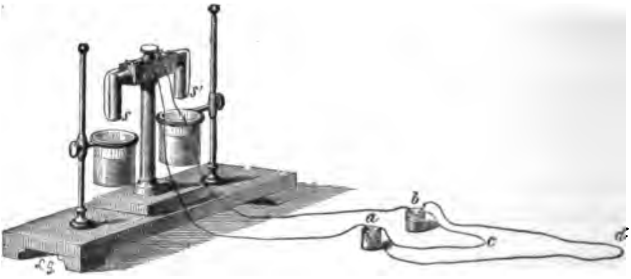

Fig. 471.—Divided Circuit.

Equality of potential in two points of a divided circuit can be tested by observing whether, when they are connected by a cross-channel, any current passes between them. This principle has been applied by Wheatstone, Thomson, and others, to the measurement of resistances, and the apparatus employed for the purpose is generally known as *Wheatstone's bridge.* It is typically represented in Fig. 471 A.

The poles P, N of a battery are connected by two independent channels of communication ACB, ADJEB. The former is a uniform wire; the latter consists of the wire D, whose resistance is to be determined, and of a standard resistance-coil E. The observation has for its immediate object to find what point in the uniform wire AB has the same potential as the junction J of the other two. When this point C is found, and connected with J through a galvanometer G, no current will pass across, and the needle of the galvanometer will not move. If a point C_1 on the positive side of C were connected with J, a current would run from C_1 to J, and if a point C_2 on the negative side were connected, the current would be from J to C_2. The deflection diminishes as the right point C is approached, and becomes reversed in passing it. When it is found, we know that the resistances in AC and CB have the same ratio as those of D and E, each of those ratios being in fact equal to the fall of potential between A and JC divided by the fall between JC and B. As the resistance of E is known, and the resistances of AC, CB are as their lengths, which are indicated on a divided scale, the resistance of D can be computed by simple proportion.

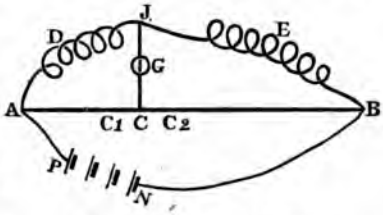

Fig. 471 A.—Wheatstone's Bridge.

In Wheatstone's original arrangement, the resistances of the two portions AC, CB were equal, and the resistances of the other two portions ADJ, JEB were made equal by the help of a rheostat.

547 B. Difference of Potential of Poles of a Battery.—We have already stated that, when the poles of a battery are not connected, the difference of their potentials is a measure of the electro-motive force of the battery. On connecting them, this difference will be diminished, the diminution being greater as the resistance of the connecting wire is less. In fact, if the original difference of potential be divided into two parts, in the ratio of the external to the internal resistance, the first of these parts will be the fall of potential in the connecting wire (in other words, the difference of potential between the poles), and the second part will be the sum of the falls which occur in the liquids of the battery.

The annexed diagram (Fig. 471 B) represents the potentials in the different parts of a circuit consisting of a battery of four cells arranged as in Fig. 434, with its poles connected by a long wire whose resistance is *double* that of the battery.

The base-line AC represents the total resistance in circuit, AB being the resistance of the battery, and BC that of the connecting wire. We suppose the negative pole to be connected with the earth (which will not affect the current), so that its potential is zero. If any other part of the circuit be connected with the earth instead of this, the potentials here indicated must all be diminished by a constant quantity.

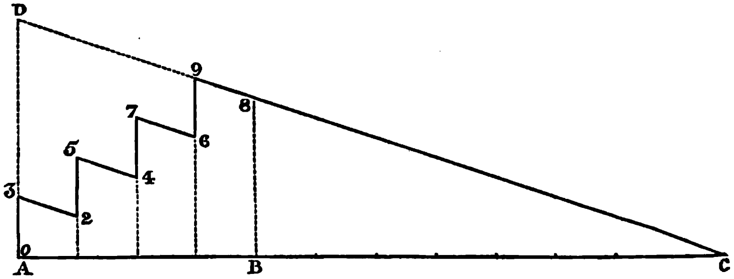

Fig. 471 B.—Curve of Potential for Closed Circuit.

The potential rises suddenly in passing from the first zinc to the acid in contact with it. Thence it falls gradually to the copper, which may be regarded as having the same potential with the part of the acid in contact with it, and also with the zinc of the second cell which is in metallic connection with it. These two suppositions are probably not quite correct, as there is reason to believe that some difference of potential always exists between dissimilar substances in contact, but for our present purpose these small differences may be neglected. Similar changes of potential occur in travelling through the other cells, and in each instance the gradual fall is one-third of the sudden rise (if the internal resistance of the battery be half the external). The distribution of potential may be stated in tabular form as follows, the electro-motive force of each cell being called 3:—

		Potential.			Potential.
1st cell	Zinc,	0	3d cell	Zinc,	4
	Acid,	3 to 2		Acid,	7 to 6
	Copper,	2		Copper,	6
2d cell	Zinc,	2	4th cell	Zinc,	6
	Acid,	5 to 4		Acid,	9 to 8
	Copper,	4		Copper,	8
			Connecting wire,		8 to 0

AD or twelve represents the whole electro-motive force of the battery; and if the external resistance were infinite, or if the poles were disconnected, the sloping lines marked 3,2; 5,4; 7,6; 9,8; would not be sloping but horizontal, and would have to be marked 3,3; 6,6; 9,9; 12,12.

547c. Measurement of Resistance of Battery.—The resistance of a battery may be measured in various ways, of which we shall only describe one.

Let the poles of the battery be directly connected with a galvanometer whose resistance is either very small or accurately known, and let the deflection be noted. Then let a wire of known resistance be introduced into the circuit, and the deflection again noted. The two currents thus measured will be inversely as the resistances, since the electro-motive force is the same in both cases. Let the resistance of the galvanometer coil be denoted by G, that of the wire introduced in the second case by W, and that of the battery by x. Then if the amounts of current be denoted by C_1, C_2, we have $\frac{C_1}{C_2} = \frac{x + G + W}{x + G}$; whence x can be determined.

548. Choice of Galvanometer.—The circumstances which should influence the choice of a galvanometer coil for a particular purpose, will now be intelligible. If stout wire is employed, the resistance is small, but it is not practicable to multiply convolutions to any great extent. Short coils of thick wire are accordingly employed in connection with thermo-piles, the resistance in the pile itself being so small that the total resistance in circuit is nearly proportional to the number of convolutions.

When, on the other hand, the resistance in the other parts of the circuit is very considerable, the resistance of the galvanometer coil becomes comparatively immaterial, so that, within moderate limits, the deflection of the needle is nearly proportional to the number of convolutions, and a coil composed of a great length of wire will give the maximum effect.

In both cases, for a given length and diameter of wire, the sensibility increases with the conductivity of the metal composing the wire. Copper is the metal universally employed, and its purity is of immense importance for purposes of delicacy, as impurities often increase its resistance by 50 or even 100 per cent.

549. Measurement of Electro-motive Force.—The most direct mode of comparing the electro-motive forces of cells of different kinds, would be to observe how many cells of the one kind arranged in series must be opposed to a given number of the other kind, in order that the resultant electro-motive force may be nil as indicated by the absence of deflection in a galvanometer forming part of the circuit. For example, if two Daniell's cells and one Grove's cell be connected with each other and with a galvanometer, in such a manner that the current due to the Daniell is in one direction, and that due to the Grove is in the opposite direction, the current actually produced will be in the direction of the greater electro-motive force. It will thus be shown whether the electro-motive force of a Grove's cell is more or less than double that of a Daniell's. This method has not been much used.

Another method of comparison consists in first connecting the two cells to be compared, so that their electro-motive forces tend the same way, and then again connecting them, so that they tend opposite ways, the resulting current being observed in both cases with the same galvanometer. The resistance in circuit is the same in both cases, being the resistance of the galvanometer plus the sum of the resistances of the cells; hence the currents will be simply as the electro-motive forces, that is to say, as $E_1 + E_2$ to $E_1 - E_2$, if E_1 and E_2 denote the electro-motive forces of the cells. Hence the ratio of E_1 to E_2 is easily computed.

Another method, which has been employed by Jules Regnault, is illustrated by Fig. 472. It consists in balancing the electro-motive force of the cell P which is to be tested, by that of a series of thermo-

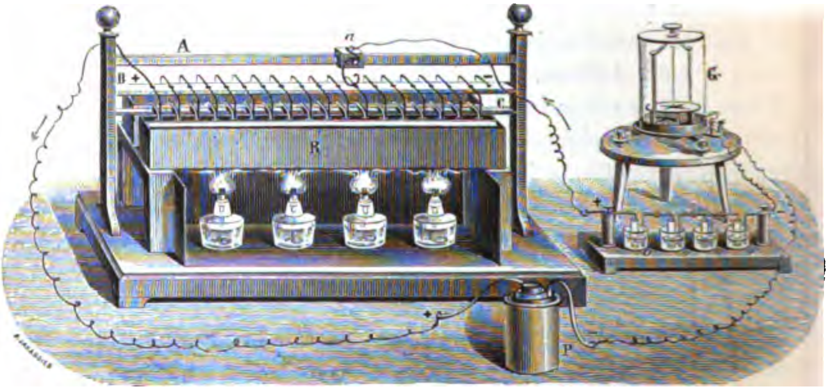

Fig. 472.—Jules Regnault's Apparatus.

electric elements, the number of which can be varied at pleasure. A is a thermo-electric pile, consisting of sixty elements of bismuth and copper, with their opposite junctions maintained at 0° and 100° C. Any number of these can be included in the circuit by moving the slider *a*, and the direction of the current which they tend to produce is opposite to that due to the cell P. As sixty thermo-electric elements would not be enough to balance one ordinary cell, some auxiliary cells *o o* of feeble electro-motive force, which has been previously determined, are employed to assist in opposing the cell P. It has thus been found that one Daniell's cell has the electro-motive force of about 174 of these thermo-electric elements.

Electro-motive force may also be measured statically by means of Thomson's quadrant electrometer, the poles of the battery being connected with the two chief electrodes of the instrument, in which arrangement no current will pass, and the electro-motive force will be directly indicated by the difference of potential observed.

According to Latimer Clark, the electro-motive forces of a cell of Grove, Bunsen, Daniell, and Wollaston are approximately as 100, 98, 56, and 46; but the last of these, being a one-fluid battery, is liable to fall off 50 per cent. or more, owing to the deposition of hydrogen on the copper plate.

CHAPTER XLVIII.

ELECTRO-DYNAMICS.

551. Meaning of Electro-dynamics.—A wire through which a current is passing, is found to be capable of producing movements in other wires also conveying currents. The theory of these movements, or more generally, of *the mechanical actions of currents upon one another*, constitutes a distinct branch of electrical science, and is called *electro-dynamics*. It stands in very close relation to electro-magnetism; and if the laws of either of the two sciences are given, those of the other may be deduced as consequences.

The science of electro-dynamics was founded by Ampère. Figs. 473, 474 represent an arrangement which he devised for rendering a conductor movable without interruption of the current conveyed by it.

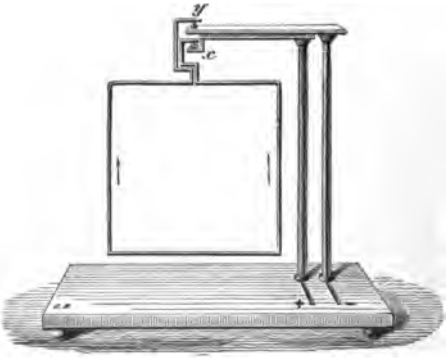

Fig. 473.—Ampère's Stand.

A wire is bent into the form of a nearly complete rectangle, and its two ends terminate in points, one above the other, so arranged that a vertical through the centre of gravity passes through them both. Accordingly, if either or both of these points be supported, the wire can turn freely about this vertical as axis. The points dip into two small metallic cups x y containing mercury, and the weight is usually borne by the upper point alone, which touches the bottom of its cup. The cups are attached to two horizontal arms of metal, supported on metallic pillars, which can be connected with the two terminals of a battery. The wire thus forms part of the circuit, the current being down one side of the rectangle and up the other. Instead of the rectangular the circular form may be employed, as in Fig. 475.

If a magnet be placed beneath, as in Fig. 474, the wire frame will set its plane perpendicular to the length of the magnet, the relative position assumed being the same as if the wire frame were fixed, and the magnet freely suspended, if we neglect the disturbing effect of the earth's magnetism.

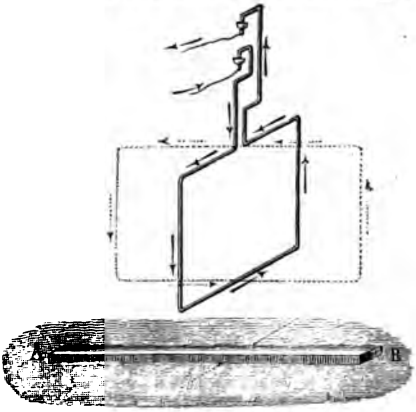

Fig. 474.—Action of Magnet on Movable Circuit.

552. Mutual Forces between Conductors conveying Currents.—The following elementary laws, regarding the mutual forces exerted between conductors through which currents are passing, were established by Ampère. For brevity of expression, it is usual to speak, in this sense, of the *mutual forces between currents*, or of the *mutual mechanical action of currents.*

I. *Successive portions of the same rectilinear current repel one another.*[1]

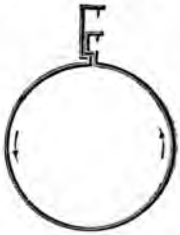

Fig. 475.

This is proved by the aid of two troughs of mercury separated by a partition (Fig. 476). A varnished wire is bent into such a form that two portions of it can float on the surface of the mercury in the two troughs, while connected with each other by an arc passing over the partition. The only portions without varnish are the ends. When the terminals of a battery are inserted in the mercury, opposite the ends, as shown in the figure, the circuit is completed through the wire, and repulsion is exhibited, the wire moving away to the further end of the vessel.

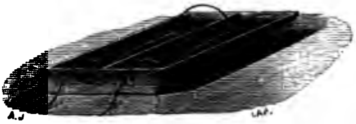

Fig. 476.—Repulsion of Successive Portions.

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[1] This first law is not universally accepted, and can scarcely be regarded as resting on the same sure foundation as the rest.

II. *Parallel currents, if in the same direction, attract, and if in the opposite direction, repel each other.*

The apparatus employed for demonstrating this twofold proposition, consists of two metallic pillars *t*, *v* (Fig. 477), which are respectively connected at their upper ends with the two cups of mercury *x*, *y*. The rectangular conductor *a b c d e* is suspended with its terminal points in these cups so as to complete the circuit between the pillars. When the current is passed, this movable conductor always places itself so that its plane coincides with that of the two pillars, and so that currents in the same direction in the pillars and in the wire are next each other, as shown in the figure.

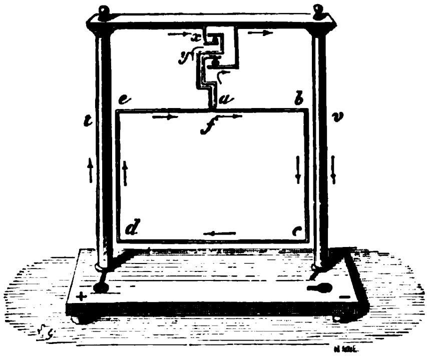

Fig. 477.—Attraction of Parallel Currents.

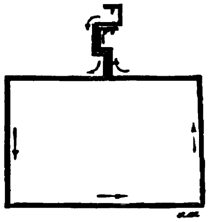

Fig. 478.—Apparatus for Repulsion.

For establishing repulsion, a slightly different form of wire is employed, which is represented in Fig. 478. When this is hung from the cups, in the position which the figure indicates, the currents in the pillars are in opposite directions to those in the neighbouring portions of the movable conductor, and the latter accordingly turns away until it is stopped by the collision of the wires above.

III. *Currents whose directions are inclined to each other at any angle, attract each other if they both flow towards the vertex of the angle,*[1] *or if they both flow from it, and repel each other if one of them flows towards the angle, and the other from it.*

A consequence of this law is that two currents, as AB, DC (Fig. 479), crossing one another near O in different planes, tend to set themselves parallel, and so that their directions shall be the same.

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[1] If the currents are not in the same plane, we must substitute *the feet of their common perpendicular* for *the vertex of the angle*, in the enunciation of this law.

r there is attraction between the portions AO and DO, and also tween the portions OB and OC; whereas there is repulsion ween AO and OC, and between B and OD. Accordingly, if the vable conductor of Fig. 477 or be traversed by a current, and ther wire carrying a current be ed horizontally at any angle un- neath its lower side, the movable ductor will turn on its point of ension till it becomes parallel to *wire* below it; and in the position able equilibrium the current in its lower side will have the same *tion* as that in the influencing wire.

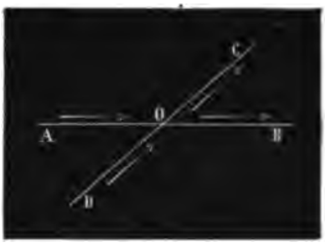

Fig. 479.—Tendency to set Parallel.

3. **Continuous Rotation produced by a Circular Current.**—Suppose ave a current flowing round a circle (Fig. 480), and also a current ing along OA, which is approximately a radius of this circle. let the current in OA be from the e towards the circumference, as indi- in the figure. Then, by law III., OA racted on one side and repelled on the , both forces combining to make OA round the circle in the opposite direc- o that in which the circular current is *g.* If the current in OA were from ference to centre, the tendency would *OA* to sweep round the circle in the lirection as the circular current.

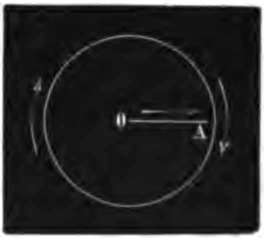

Fig. 480.—Continuous Rotation of Radial Current.

reasoning still holds if OA is in a plane parallel to that of the r current, O being a point on the axis of the circle and the of OA being not greater than the radius.

rcular current may also produce continuous rotation in a con- parallel to the axis of the circle, and movable round that Fig. 481 represents an arrangement for obtaining this effect. l of wire through which a current can be sent, is wound round per basin EF, its extremities being connected with the bind- ws *m*, *o*. From the centre of the basin rises the little pillar A, terminating above in a cup containing mercury. lar is connected with the binding-screw *n*. The basin, which cted with the binding-screw *p*, contains water mixed with a little acid to improve its conducting power, and a movable conductor BC rests, by a point, on the bottom of the cup of mercury, while its lowest portion, which consists of a light hoop, dips in the acidulated water. By connecting *m* and *n* a single circuit is obtained, of which

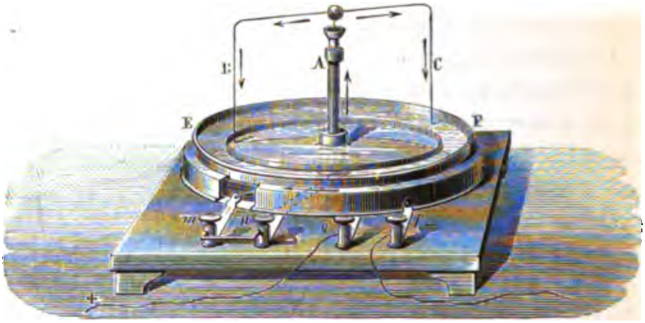

Fig. 481.—Apparatus for Continuous Rotation.

o and *p* are the terminals, so that if *o* is connected with the positive and *p* with the negative pole of a battery, the current entering at *o* first traverses the wire coil, then ascends the pillar A, returns down the sides B, C to the floating ring and liquid, and so escapes to *p*. As soon as these connections have been completed, the movable conductor commences continuous rotation in the direction opposite to that of the current in the coil.

If, instead of connecting *m* and *n*, we connect *n* and *o*, and lead the positive wire from the battery to *p* and the negative wire to *o*, the course of the current will be from *p* to the acid, thence up the sides B, C, and inwards along the top of the movable conductor to the mercury cup, then down the pillar to *n*, thence to *o*, and through the coil from *o* to *m* in the same direction as in the former experiment; but the rotation of the movable conductor will now occur in the opposite direction to that before observed, and therefore in the same direction as the current.

554. Action of an Indefinite[1] Rectilinear Current upon a Finite Current movable around one Extremity.—A finite current movable about one extremity may also be caused to rotate continuously about this extremity by the action of an indefinite rectilinear current. This is clearly indicated by Fig. 482. In the right-hand diagram, the cur-

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[1] The word *indefinite*, in this application, simply means *of great length in comparison with the distance and length of the movable current.*

t OA flowing outwards from the centre of motion O, and acted by the indefinite current MN, is first attracted into the position In this new position it is repelled by n N, and attracted by It is thus brought successively into the positions O A$''$, O A$'''$, In this last-
itioned position,
two currents being
llel and opposite,
e is repulsion; and
passing it, there
gain repulsion on
ide and attraction
ie other, till it is
d round to its
position O A. It
us kept in con-
l rotation. If the movable current flows inwards to the centre
tion O, as in the left-hand diagram, while the direction of the
nite current is the same as before, the direction of rotation will
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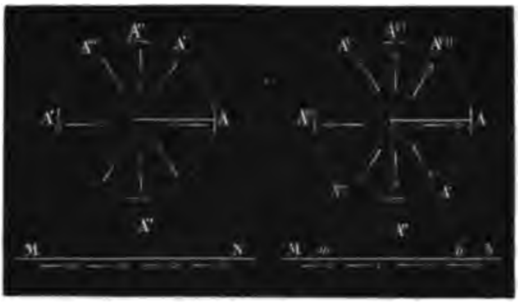

Fig. 482.—Rotation of Radial Current.

Action of an Indefinite Rectilinear Current on a Finite Current idicular to it.—Let MN, in the upper half of Fig. 483, be an *ite* rectilinear current, and
portion of another current
in the same or in any other
In the latter case let DC
common perpendicular.
if the currents have the
ns represented by the ar-
i element at p will attract
ient at m with a force
e may represent by a line
d an element at p' equal to
and situated at the same
from C on the other side,
el the element at m with
force, represented by mf. Constructing the parallelogram
the resultant force of these two elements upon m is repre-
y the diagonal mF, which is parallel to MN and in the
direction to the indefinite current. As this reasoning applies

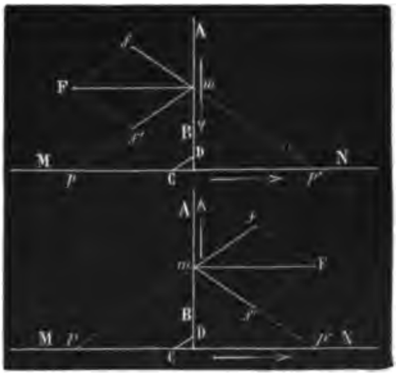

Fig. 483.—Translation Parallel to Indefinite Current.

to all the elements of both currents, it follows that the current AB will experience a force tending to give it a motion of translation parallel to MN. This motion will be opposite to the direction of the indefinite current when the direction of the finite current is towards the common perpendicular DC, as in the upper diagram, and will be in the same direction as the indefinite current when the direction of the finite current is from the common perpendicular, as in the lower diagram.

556. Action upon a Rectangular Current movable about an Axis Perpendicular to an Indefinite Current.—It follows from the preceding section that if a finite current AB (Fig. 484), perpendicular to an indefinite current, is movable round an axis O O′ parallel to itself, the plane AB O O′ will place itself parallel to the indefinite current, and AB will place itself in advance or in rear of the axis according as the current in AB is from or towards the indefinite current.

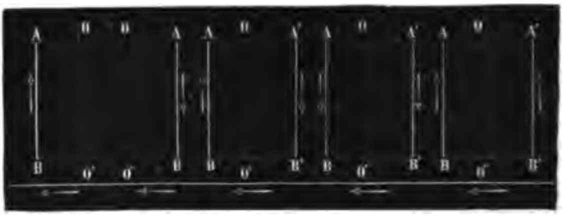

Fig. 484.—Position assumed by Perpendicular Current.

If a pair of parallel and opposite currents BA, A′B′, rigidly connected together, and movable round the axis O O′ lying between them, are submitted to the action of the indefinite current, the forces upon them will conspire to place the system in the position indicated in the figure. If the two currents A B, A′B′ are both in the same direction, their tendencies to revolve round the axis O O′ will counteract each other.

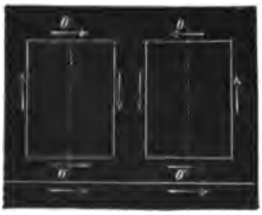

Fig. 485.—Position assumed by Rectangular Current.

557. Action upon a Rectangular Current movable round an Axis Perpendicular to an Indefinite Current.—If a rectangular current (Fig. 485) is movable round an axis *oo′* perpendicular to the direction of the indefinite rectilineal current, we have just seen that the action upon the two sides of the rectangle which are perpendicular to the latter tends to place the system so that its plane shall be parallel to the indefinite current, and that the side which carries the receding current shall be in advance of the other. The action upon the near side of the rectangle contributes to produce the same effect, since this side tends to set itself parallel to the influencing current, and so that the directions of the two shall be the same.

The action upon the further side of the rectangle tends to produce an opposite effect; but, in consequence of the greater distance, this action is feebler than that upon the near side. The system accordingly tends to take the position of stable equilibrium represented in the right-hand half of the figure. The diagram on the left hand represents a position of unstable equilibrium.

What is here proved for a rectangular current, is true for any closed plane circuit movable round an axis of symmetry perpendicular to an indefinite rectilinear current; that is to say, any such circuit tends to place itself so that the current in the near side of it is in the same direction as the indefinite current.

The results of § 556 can be verified experimentally by the aid of the apparatus represented in Fig. 486. CC, DD are two cups (shown in section) surrounding the metallic pillar AB at its upper and lower ends, and containing a conducting liquid. The lower cup is insulated from the pillar, and connected with the binding-screw *g*. The liquid in the upper cup CC is connected with the upper end of the pillar by the bent arm *dm*. *o*K is a light horizontal rod supported on a point at B, and carrying a counterpoise K at one end, while the other carries a wire *mnop*, whose two ends *nm* and *op* descend vertically into the two cups, the middle portion of the wire being wrapped tightly round the rod. The binding-screw *f* is connected with the lower end of the pillar. If a current enters at *f* and leaves at *g*, its direction in the long vertical wire *op* will be descending; and it will be ascending, if the connections are reversed. By sending a current at the same time through a long horizontal wire in the neighbourhood of the system, movements will be obtained in accordance with the foregoing conclusions.

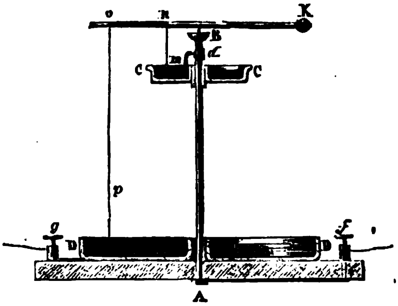

Fig. 486.—Position assumed by Vertical Current.

558. Sinuous Currents.—A sinuous current exhibits the same action as a rectilinear current, provided that they nowhere deviate far from each other. This principle can be exemplified by bringing near to a movable conductor (Fig. 487) another conductor consisting of a wire doubled back upon itself, having one of its portions straight, and the other sinuous, but very near the first. A current sent through this double wire traverses the straight and the sinuous portions in opposite directions, and it will be found that their joint effect upon the movable conductor is inappreciable.

This principle holds not only for rectilinear currents but for currents of any form, and is very extensively employed in the analytical investigations of electro-dynamics. In computing the action exercised by or upon a conductor of any form, it is generally convenient to substitute for the conductor itself an imaginary conductor, nearly coincident with it, and consisting of a succession of short straight portions at right angles to one another (Fig. 488).

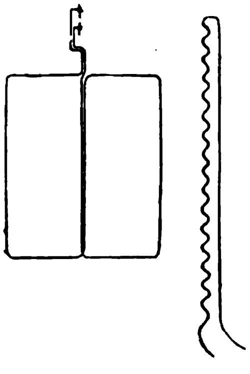

Fig. 487. Sinuous Currents.

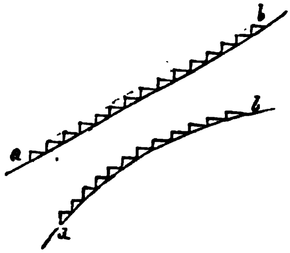

Fig. 488.

559. Mutual Action of Two Elements of Currents.—Ampère based his analytical investigations on the assumption that the action exercised by an element (*i.e.* a very short portion) of one current upon an element of another, consists of a single force directed along the joining line. This assumption conducted him to a formula for the amount of this force, which has been found to give true results in every case capable of being tested by experiment. Nevertheless, it is by no means certain that either Ampère's formula or his fundamental assumption is true. Other assumptions have been made, leading to other formulæ in contradiction to that of Ampère, which also give true results in every case capable of being experimentally tested.

The fact is that experiments can only be performed with complete circuits, and the contradictions which subsist between the different assumptions, in the case of the several parts of a circuit, vanish when the circuit is considered as a whole. All the formulæ, however, agree in making the mutual force or forces between two elements vary inversely as the square of their distance, and directly as the products of the currents which pass through them. Professor Clerk Maxwell[1] discards all assumptions as to mutual actions between elements at a distance, and employs the principle that a circuit conveying a current always tends to move in such a manner as to increase the number of magnetic force-tubes (in the sense of § 445 H) which pass through it. The work done in any displacement is measured by the number of tubes thus added; but tubes which cross the circuit in the opposite direction to those due to the current in the circuit are to be regarded as negative.

We have seen (§ 531 A, B) that the lines of magnetic force due to a current are circles surrounding it; and also that, when a line of magnetic force cuts a current, the latter experiences a force tending to move it at right angles to the plane of itself and the line of force. In the case of two parallel currents, each is cut at right angles by the lines of magnetic force due to the other; the direction of the force experienced by either current is therefore directly to or from the other current; and the criterion of § 531 B will be found to indicate attraction when the directions of the currents are the same, and repulsion when they are opposite.

In Fig. 480 the lines of magnetic force cut OA in a direction perpendicular to the plane of the diagram, OA accordingly experiences a force perpendicular to its own length in the plane of the diagram; and the same remarks apply to AB in Fig. 484. All the experimental facts above detailed are in fact thus explicable. In the experiment of Fig. 476, where the application is scarcely so obvious as in the other cases, the observed motion may be deduced from the direction in which the bridge or arc connecting the two side-wires is cut by the lines of force.[2]

560. Action of the Earth on Currents.—In virtue of terrestrial magnetism, movable circuits, when left to themselves, take up definite positions having well-marked relations to the lines of terrestrial magnetic force. For example, in the apparatus of Fig. 486, the vertical wire *o p* will place itself to the west or east (magnetic) of the pillar A B, according as the current in *o p* is ascending or descending. This effect is due to the horizontal component of terrestrial magnetism.

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[1] Maxwell "On Faraday's Lines of Force." Camb. Trans. 1858, p. 50.

[2] Some further remarks on the forces experienced by currents in magnetic fields will be found in Chap. lii.

In the apparatus of Fig. 481, if the current be sent only through the movable portion, continuous rotation will be produced, which will be with or against the hands of a watch according as the current in the top wires is inwards or outwards. This effect is due to the vertical component of the earth's magnetism, acting on the currents in the horizontal wires. Vertical lines of magnetic force falling on a horizontal current give the latter a tendency to move perpendicular to its own length in a horizontal plane.

561. Solenoids.—If we suspend from Ampère's stand (Fig. 473) a plane circuit, whether rectangular or circular, it will place itself perpendicular to the magnetic meridian, in such a manner that the current in its lower side is from east to west; or, in other words, so that the ascending current is in its western and the descending current in its eastern side; this effect being due to the action of the horizontal component of terrestrial magnetism upon the ascending and descending parts of the current. If, then, we have a number of such circuits, rigidly connected together at right angles to a common axis, and with their currents all circulating the same way, their common axis will tend to place itself in the magnetic meridian, like the axis of a magnet. Such a system was called by Ampère a solenoid (σωλὴν, a tube), and was realized by him in the following manner.

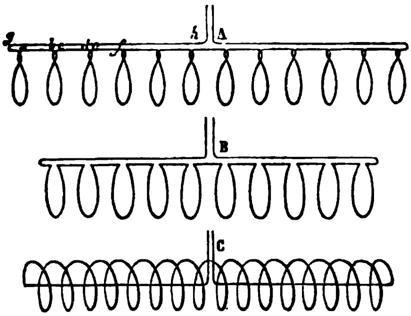


Fig. 491.—Solenoids.

Imagine a wire bent into such a shape as to consist of a number of rings united to each other by straight portions. It will differ from a theoretical solenoid only by having currents in these straight portions; but if the two ends of the wire be carried back till they nearly meet in the middle of the length, as shown at A and B (Fig. 491), the currents in these returning portions, being opposite to those in the other straight portions, will destroy their effect, and the resultant electro-dynamic action of the system will be simply due to the currents in the rings. The same effect is more conveniently obtained by substituting for the rings and intermediate straight portions, a helix, which, by the principles of sinuous currents, is equivalent to them. Each spire of the helix represents a circle perpendicular to the axis, together with a straight portion parallel to the axis and equal to the distance between two spires. The effect of all the straight portions is exactly destroyed by the wires which return from the ends of the helix and meet in the middle. This arrangement, which is represented at C, is that which is universally adopted, the returning wires being sometimes in the axis, and sometimes on the outside of the helix.

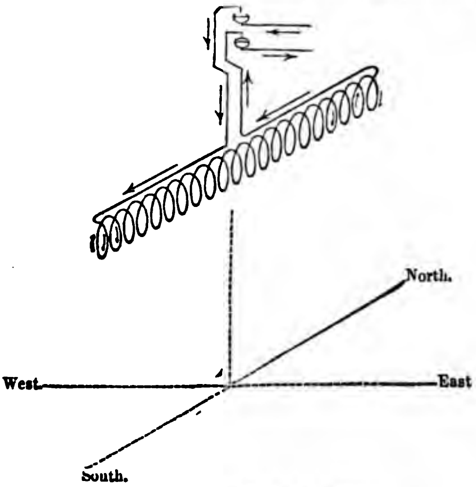


Fig. 492.—Orientation of Solenoid.

If a solenoid, thus constructed, be suspended on an Ampère's stand, as in Fig. 492, and a current sent through it, it will immediately place its axis parallel to a declination needle. It may accordingly be said to have poles. In Fig. 493, A represents the austral or north-seeking, B the boreal or south-seeking pole of the solenoid; that is to say, the direction of the current is against or with the hands of a watch according as the austral or boreal pole is presented to the observer. The same difference is illustrated by Fig. 492.

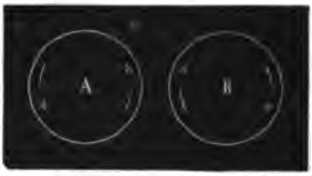


Fig. 493.—Poles of Solenoid.

562. Dip of Solenoid.—If a solenoid could be balanced so as to be perfectly free to move about its centre of gravity, it would place its axis parallel to the dipping-needle. The experiment would be scarcely practicable with a solenoid properly so called, on account of its weight; but it can be performed with a single plane circuit, such as that shown in Fig. 494. If such a circuit is nicely balanced about an axis through its centre of gravity, and placed so that it can turn freely in the plane of the magnetic meridian, the passing of a current through it will cause it to set its plane perpendicular to the direction of a dipping-needle. This effect is due to the action of terrestrial magnetism on the upper and lower sides of the rectangle. The plane of the rectangle is represented in the figure as coinciding with the direction of dip. In this position the action of terrestrial magnetism urges the upper side backwards, and the lower side forwards, and stable equilibrium will be attained when the rectangle has turned through 90°.

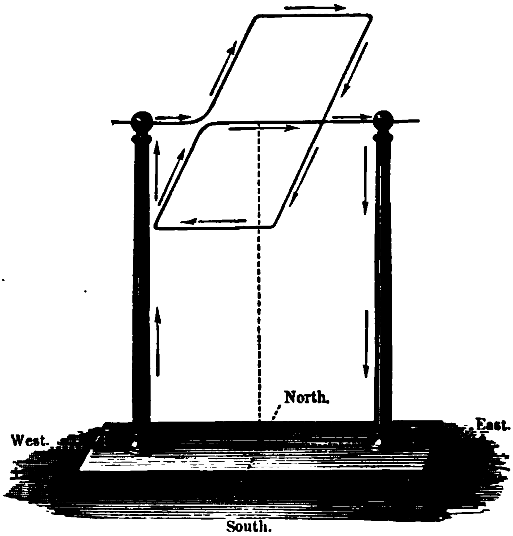


Fig. 494.—Dip of Element of Solenoid.

563. Mutual Actions of Solenoids.—Solenoids behave like magnets not only as regards the forces which they experience from terrestrial magnetism, but also as regards the actions which they exert upon one another. The similar poles of two solenoids repel, and the unlike poles attract each other, as we may easily prove by suspending one solenoid from an Ampère's stand and bringing another near it.

The reason of these attractions and repulsions is illustrated by Fig. 495. If two austral poles are placed opposite each other, as in the upper part of the figure, the currents are circulating round them in opposite directions, and, by the laws of parallel currents, should therefore repel each other; whereas if two dissimilar poles be placed face to face, the currents which circulate round them are in the same direction, and attraction should therefore ensue.

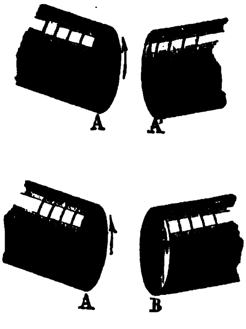


Fig. 495.—Mutual Action of Solenoids.

Lastly, if one pole of an ordinary magnet be brought near one pole of a suspended solenoid, as in Fig. 496, repulsion or attraction will be exhibited according as the poles in question are similar or dissimilar. In the position represented in the figure, this action is mainly due to the action of the boreal pole of the magnet upon the descending currents in the near side of the solenoid. This action consists in a force to the left hand, nearly parallel to the axis of the solenoid, which tends to make the solenoid rotate about its supports, and thus to bring the end A of the solenoid into contact with the end B of the magnet.

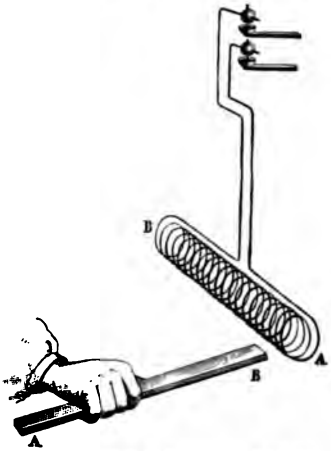
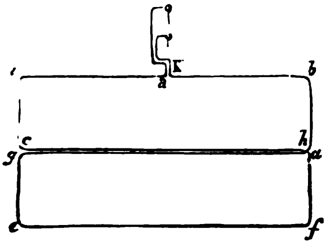


Fig. 496.—Action of Magnet on Solenoid.

It may be shown, by the aid of Ampère's formula for the mutual force between two elements, that the mutual action of two solenoids is equivalent to four forces, directed along lines joining the poles of the solenoids, and varying inversely as the squares of the distances between the poles; the forces between similar poles being repulsive, and the other two attractive. The analogy between solenoids and magnets is thus complete.



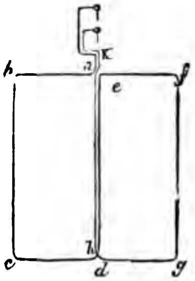
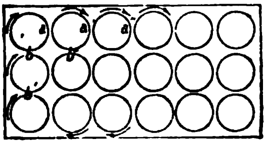


Fig. 497. Astatic Circuits. Fig. 498.

564. Astatic Circuits.—When it is desired to eliminate the influence of terrestrial magnetism in electro-dynamic experiments, an astatic circuit may be employed as the movable conductor. Two such circuits are represented in the accompanying figures (Figs. 497, 498). In each of them the current in one half of the circuit circulates with, and in the other against the hands of a watch, thus producing equal and opposite tendencies to orientation, which destroy one another.

565. Ampère's Theory of Magnetism.—In accordance with the preceding facts, Ampère propounded the hypothesis that what is called magnetism consists in the existence of electric currents circulating round the particles of magnetic bodies. In iron or steel, when unmagnetized, according to this theory, the currents around different particles have different directions; but when it is magnetized, the directions of all are the same. Fig. 499 represents an ideal section of a magnetized bar at right angles to the direction of its magnetization. On the neighbouring faces of any two particles, the currents are in opposite directions, hence, by the laws of sinuous currents, there is a mutual destruction of force through the whole interior, and the resultant effect is the same as if there were currents circulating round the exterior of the magnet, as represented in Fig. 500.



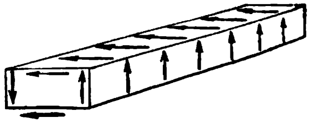


Fig. 499. Amperian Currents in Magnet. Fig. 500.

Magnetization by influence depends, according to this theory, on the tendency of currents to set themselves parallel and in similar directions; and if the substance magnetized possesses coercive force, the direction thus impressed on its currents persists after the influence is removed. In soft iron, on the contrary, they resume their former irregularity.

Ampère's theory of magnetism is in complete accordance with all known facts. But it admits of question whether it is simpler to deduce the laws of magnetism and electro-magnetism from those of electro-dynamics; or to adopt the reverse order, and deduce the laws of electro-dynamics from those of electro-magnetism.

566. Rotation of a Magnet on its Axis.—The following experiment is due to Ampère. A magnet, loaded with platinum at its lower end, floats upright in mercury contained in a glass vessel (Fig. 501). A cavity is hollowed out in the top of the magnet. This contains mercury, in which a point dips. On connecting one of the terminals of a battery with this point, and the other with the outer edge of the mercury in the vessel, the magnet is seen to rotate on its axis. If the north-seeking pole is uppermost, and the positive pole of the battery is connected with the point, the direction of rotation is N.E.S.W.

The Amperian explanation of this phenomenon is, that it is due

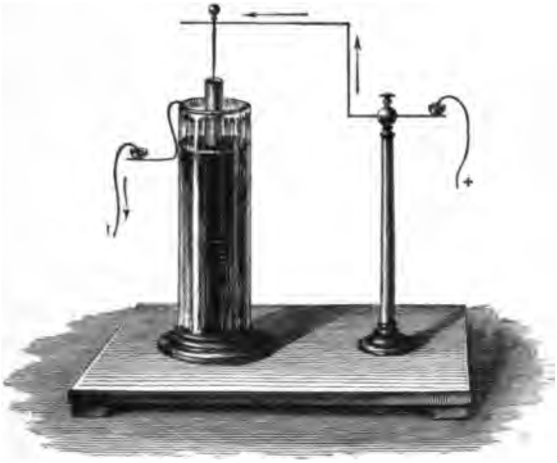


Fig. 501.—Rotation of Magnet.

he action between the outward-flowing current in the mercury
the Amperian currents which circulate round the magnet. The
er, as represented by the arrows nC, Cm in Fig. 502, are opposite
watch-hands. The outward-flowing
ent in CD attracts the current in Cm,
e they are both directed away from
angular point C, and repels the cur-
in n C. Hence the magnet is made
otate in the direction m C n, opposite
at of the Amperian current.
e experiment is sometimes varied by
ng the point dip in the mercury in
essel, the magnet being allowed to
freely near it, and a metallic ring
immersed at the outer edge of the
ry, to which the current flows out in all directions from the
As soon as the circuit is completed, the magnet begins to
ve round the point. The rotation will be in the same direction

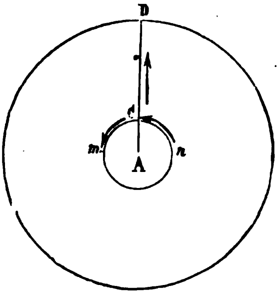


Fig. 502.—Explanation of Rotations.

as in the other form of the experiment; that is to say, if the current flows outwards from the point to the edge of the vessel, the direction of rotation will be opposite to that of the Amperian currents in the magnet. This is easily explained by the laws of parallel currents, for the current in O C (Fig. 503) attracts the Amperian current at *m*, and the current in O D repels the current at *n*. The magnet will therefore move from O D to O C, and will revolve round O in the direction N.E.S.W.

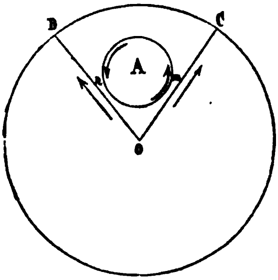


Fig. 503.—Explanation of Rotations.

567. Magnetization by Currents.—Ampère's theory of magnetism leads naturally to the conclusion that a bar of iron or steel may be magnetized by means of a current. Arago was the first to establish this fact, but without a clear apprehension of the conditions necessary for success, or of the criterion for determining which will be the austral, and which the boreal pole. Ampère conceived the idea of introducing the needle to be magnetized into the axis of a solenoid, and the result confirmed his prediction that the poles of the needle would be turned the same way as those of the magnetizing helix. This is what must happen if the currents in the helix force the Amperian currents in the bar into parallelism with themselves, so that all rotate the same way. The action, in fact, is precisely analogous to that represented in Fig. 477.

It is to be remarked that, in this process of magnetization, the portions of the currents parallel to the axis of the helix produce no effect. The wire through which the current is to be sent may be wound like thread upon a reel, returning alternately from end to end and all the convolutions will contribute to magnetize the bar the same way, although it is evident that the helices are in this case alternately right-handed and left-handed. The north-seeking and south-seeking poles may be in all cases distinguished by the rule that the direction in which the current circulates in the coil is against watch-hands as seen from the former and with watch-hands as seen from the latter; or it may be remembered by the rule, that if I identify my own body in imagination with a portion of the wire, and suppose the current to enter at my feet, while my face is towards the needle, the north-seeking pole will be to my left. In 1 and 2 (Fig. 504) *a* will be the austral (or north-seeking), and *b* the boreal pole of the inclosed needle, when the current in the helix has the direction indicated by the arrows.

If the direction of winding is changed, in the manner represented

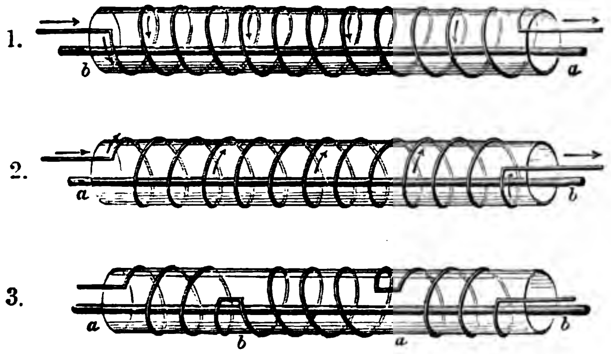


Fig. 504.

1. Right-handed Helix. 2. Left-handed Helix. 3. Arrangement for Consequent Points.

in 3, so that, as seen from one end, the direction in which the current circulates is in one part with and in another against the hands of a watch, consequent points (§ 503) will be formed at the points of change. Thus, if the current enters at the left-hand end of the coil, the points *a a* will be austral, and the points *b b* boreal poles.

568. Electro-magnets.—Arago was the first to observe the effect of a current in magnetizing soft iron. On plunging in iron-filings a wire through which a very strong current was passing, he observed that the filings clung to the wire, that they placed their length tangentially to it, and that they fell off when the current ceased to pass. Each filing was evidently, in this experiment, a little magnet placing itself at right angles to the current. A cylindrical bar of iron can be powerfully magnetized by wrapping round it a coil of insulated wire and sending a current through this coil. Stout copper wire is generally employed for this purpose. Such an arrangement is called an *electro-magnet.*

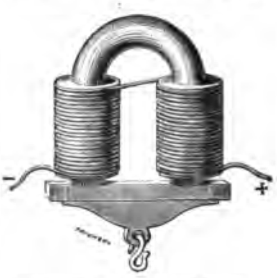


Fig. 505.—Horse-shoe Electro-magnet.

The bar has often the horse-shoe form, as in Fig. 505, and in this case the central part is usually left bare. The direction of winding on the ends must be such that, if the bar were straightened out, the current would circulate in the same direction round every part. This is clearly shown in the figure. Electro-magnets have been constructed capable of sustaining a load of many tons.

Besides the enormous power that can be given them, electro-magnets have the advantage of being readily made or unmade instantaneously, by completing or interrupting the circuit to which the coil belongs. This principle has received very numerous and varied applications, some of which will be mentioned in later chapters.

569. Residual Magnetism.—When the current round an electro-magnet is interrupted, the destruction of the magnetization is not complete. The small remaining magnetization is called *remanent* or *residual* magnetism. It is frequently sufficiently powerful to retain the armatures in contact with the magnet, and thus necessitates the employment of *opposing springs*, if instantaneous separation is desired. The mere act of separation suffices to destroy the greater part of the residual magnetism.

Fig. 507 represents an electro-magnet EE′, furnished with an opposing spring *g*. The armature A, with its lever *t*, turns about the axis VV′. The opposing spring *g* has one end fixed at K, and the other attached to the end of the lever *t*. It therefore tends to remove the armature from the magnet. *c* and *d* are two points whose distance can be regulated, and which serve to limit the movements of the armature.

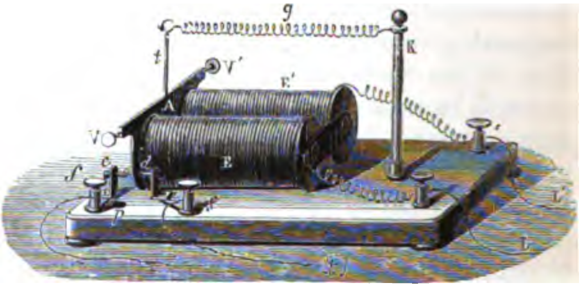


Fig. 507.—Electro-magnet with Opposing Spring.

CHAPTER XLIX.

HEATING EFFECTS OF CURRENTS.

570. Heating of Wires.—The heating of a wire by the passage of a current may conveniently be exhibited by the aid of the apparatus represented in Fig. 508. Two uprights mounted on a stand are furnished, at different heights, with pairs of insulated binding-screws *a a′*, *b b′*, *c c′*, having wires stretched between them. A current can thus be sent through any one of the wires, by connecting the terminals of a battery with the binding-screws at its extremities. When this is done with a battery of suitable power, the wire is first seen to droop in consequence of expansion, then to redden, and finally to melt, becoming inflamed if the metal is sufficiently combustible.

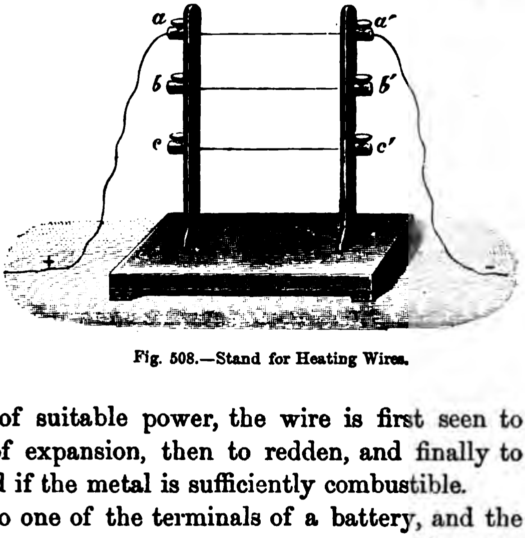


Fig. 508.—Stand for Heating Wires.

If a file is attached to one of the terminals of a battery, and the other terminal is drawn along the file, a rapid succession of sparks will be obtained; and if the battery be sufficiently powerful, globules of incandescent metal will be scattered about with brilliant effect.

571. Joule's Law.—The energy of a current is jointly proportional to the quantity of electricity that passes and the electro-motive force that drives it. As the numerical measure of a current is the quantity of electricity which passes in unit time, it follows that the energy of a current C lasting for a time t, is ECt, E denoting the electro-motive force. But again, by Ohm's law, E is equal to the product of the current C and the whole resistance R. The expression for the energy therefore becomes

$$C^2 R t, \qquad (1)$$

and this energy is all transformed into heat in the circuit, unless the current is called upon to perform some other kind of work in addition to overcoming the resistance of the circuit. It has accordingly been found, first by Joule, and afterwards by Lenz, Becquerel, and others, that the formula $C^2 R t$ represents the quantity of heat generated by a current under ordinary circumstances. The experiments have usually been conducted by passing a current through a spiral of wire immersed in water or alcohol, and observing the elevation of temperature of the liquid.

This law of Joule's, like that of Ohm, may be applied to any part of a circuit, as well as to the circuit considered as a whole; that is to say, if the circuit consists of parts whose resistances are $r_1, r_2, \ldots,$ the quantities of heat generated in them are respectively $C^2 r_1 t$, $C^2 r_2 t, \ldots,$ and are therefore proportional to the resistances $r_1, r_2 \ldots$ of the respective parts, since C and t are necessarily the same for all.

572. Relation of Heat in Circuit to Chemical Action in Battery.— The energy of a current, and consequently the heat developed in the circuit, is the exact equivalent of the potential energy of chemical affinity which runs down in the cells of the battery. This fact, first verified approximately by Joule, has been more accurately confirmed by the experiments of Favre, who introduced into the muffle of his mercurial calorimeter, already described and figured in § 351, a small voltaic cell with its poles connected by a fine wire. He found that the consumption of 33 grammes of zinc in the cell corresponded to a generation of heat amounting to 18,796 gramme-degrees. But the chemical action in the cell is complex. The 33 grammes of zinc unite with 8 grammes of oxygen, and in so doing generate 42,451 gramme-degrees. The combination of these 41 grammes of oxide of zinc with 40 grammes of sulphuric acid, produces 10,456 gramme-degrees, making in all 52,907. But an equivalent of water undergoes decomposition, and this *absorbs* 34,463, which must be subtracted from the above sum, leaving 18,444 gramme-degrees as the balance of heat generated in the whole complex action. The heat actually observed in the experiment, agreed almost precisely with this calculated amount.

573. Distribution of Heat in Different Parts of Circuit.—These experiments also served to verify the application of Joule's law to each part of the circuit considered separately. By introducing the cell into the muffle whilst a spiral of fine wire connecting the poles was outside, and then introducing the spiral while the cell was outside, Favre was able to measure separately the heat generated in the cell and in the spiral, and these were found to be proportional to their resistances.

If wires of different diameter or of different electrical conductivity form parts of the same circuit, so as to be traversed by the same current, the bad conductors will become more heated than the good, and the fine wires more than the coarse. All parts of the length of a uniform wire will be uniformly heated. The specific resistance of platinum is ten times greater than that of copper; hence ten times as much heat will be generated in a platinum as in a copper wire by a given current, if the diameters of the two wires be the same.

The *elevation of temperature* is greater in a fine than in a coarse wire, not only because of its greater resistance, which leads to the development of a greater quantity of heat in it, but also on account of its smaller capacity for heat, and its smaller surface. When the current is passed for so short a time that the heat emitted may be neglected, the elevation of temperature varies directly as the resistance per unit length, and inversely as the capacity per unit length. Each of these quantities varies directly as the section of the wire, and hence the elevation of temperature is inversely as the square of the section, or as the fourth power of the diameter.

On the other hand, if the current be continued till the permanent temperature is attained, capacity ceases to have any influence, and the heat emitted in unit time must be equal to the heat received. If x denote the elevation of temperature, the heat emitted is approximately $2\pi r B x$ by Newton's law (§ 307), B being a constant. The heat received is $\frac{A}{\pi r^2}$, A being another constant. By equating these two expressions, we find that $r^3 x$ is equal to a constant, and hence x varies inversely as r^3, that is, the elevation of temperature is inversely as the cube of the diameter.

To obtain the most rapid production of heat in the circuit considered as a whole, we must reduce the resistance to a minimum; for the heat produced in unit time is E C, which, by Ohm's law, is the same as $\frac{E^2}{R}$, and therefore varies inversely as R the total resistance.

574. Mechanical Work done by Current.—Favre's experiments also furnished a confirmation of the fact, that when a current is called upon to perform mechanical work, the amount of heat generated in the circuit is diminished by the equivalent of this work. He inclosed a battery of five cells in the muffle of one calorimeter, and an electro-magnet in another calorimeter, the connections between the coil of the electro-magnet and the poles of the battery being made by short thick wires whose resistance could be neglected. The electro-magnet attracted an armature, and thus raised a weight by means of external pulleys.

It was found that when the armature was fixed, so that no mechanical work could be performed, the heat developed was the precise equivalent of the chemical action which took place in the battery; but when the electro-magnet was allowed to raise the weight, the amount of heat indicated by the calorimeters was sensibly less. The difference was measured, and compared with the work done in raising the weight. The comparison indicated 444 kilogrammetres of work for each kilogramme-degree of heat that disappeared, a result which agrees sufficiently well with the established value of Joule's equivalent (425 kilogrammetres).

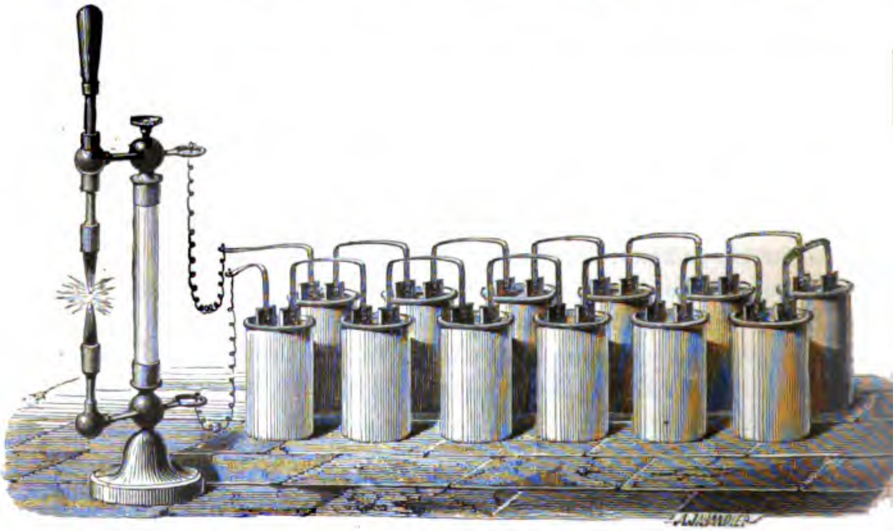


Fig. 509.—Electric Light.

575. Electric Light.—When two pointed pieces of a conducting kind of carbon, such as that which is deposited in the retorts at gasworks, are connected with the poles of a powerful battery, as in Fig. 509, a brilliant light is obtained by bringing them together so as to allow discharge to take place between them. This discharge, when once obtained, will not be interrupted by separating the points to some distance,—greater in proportion to the electro-motive force of the battery; and the interval will be occupied by a luminous arch (known as the *voltaic arc*) of intense brightness and excessively high temperature. This brilliant experiment was first performed by Sir Humphrey Davy, at the commencement of the present century, with a battery of 3000 cells. The light appears to be mainly due to the incandescence of particles of carbon which traverse the space between the points.

This transport of particles can be rendered visible to a large number of spectators by throwing an image of the heated points on a screen with the aid of a lens. Fig. 510 represents the image thus obtained, the natural size of the carbons being indicated by the sketch at the right hand. On watching the image for some time, incandescent particles will be observed traversing the length of the arc, sometimes in one direction and sometimes in the other, the prevailing direction being, however, that of the positive current. This circumstance, which appears to be connected with the higher temperature of the positive terminal, explains the difference between the forms assumed by the two carbons. The point of the positive carbon becomes concave, while the negative carbon remains pointed and wears away less rapidly. This difference is more precisely marked when the experiment is performed *in vacuo;* a kind of cone then grows up on the negative carbon, while a conical cavity is formed in the positive carbon. These phenomena are less clearly exhibited in air, on account of the combustion occasioned by the presence of oxygen.

The voltaic arc exceeds in temperature as well as in brightness all other artificial sources of heat. Despretz succeeded by its means in fusing and even volatilizing many substances which had previously proved refractory. Carbon itself was softened and bent, welded, and apparently reduced to vapour, which was condensed, in the form of black crystalline powder, on the walls of the containing vessel.

The voltaic arc must be regarded as an instance of conduction rather than of disruptive discharge, the air being rendered a conductor by the high temperature to which it is raised. Hence it is that, although discharge does not commence between the points till they have been brought close together, it is not interrupted by subsequently removing them to a considerable distance.

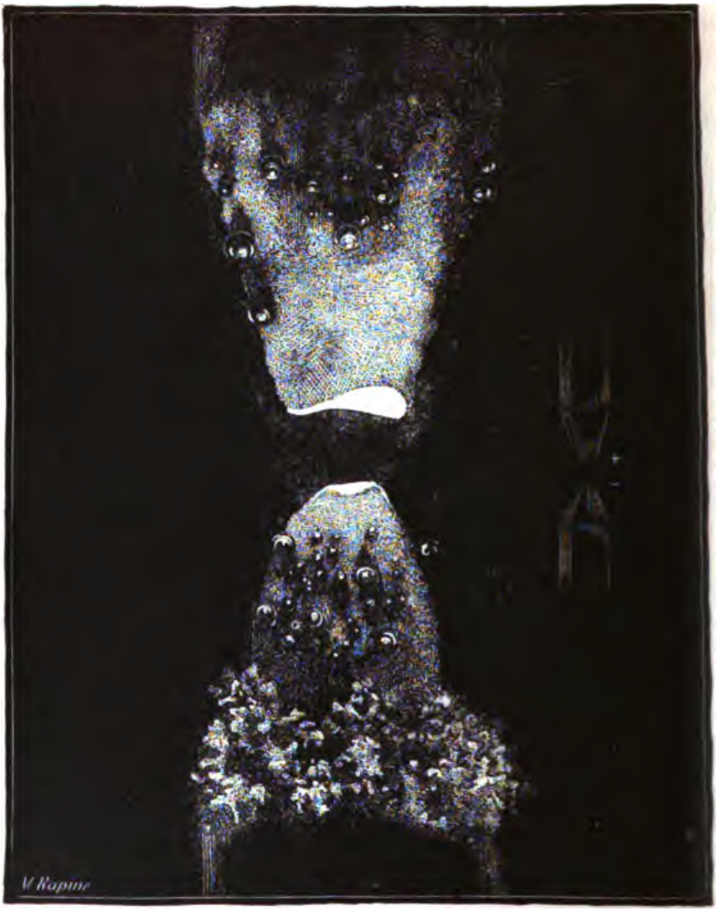


Fig. 510.—Image of the Carbon Points.

The voltaic arc is acted on by a magnet, according to the same laws as any other current. M. Quet, by employing a very powerful electro-magnet, with its poles at equal distances on opposite sides of the line joining the points, repelled the arc laterally to such an extent that it resembled a blowpipe flame (Fig. 511).

576. Light of the Voltaic Arc.—The light of the voltaic arc has a dazzling brilliancy, and attempts were long ago made to utilize it

The failures of these attempts were due not so much to its greater costliness in comparison with ordinary sources of illumination, as to the difficulty of using it effectively. Its brilliancy is painfully and even dangerously intense, being liable to injure the eyes and produce headaches. Its small size detracts from its illuminating power —*it dazzles rather than illuminates*— and it cannot be produced on a sufficiently small scale for ordinary purposes of convenience. There is no mean between the absence of light and a light of overpowering intensity.

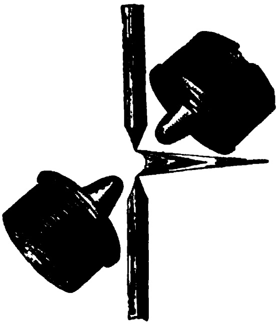


Fig. 511.—Action of Magnet on Voltaic Arc.

There is, however, one application in which these peculiarities of the electric light are positive advantages, penetration being the essential requisite; we mean the lighting-up of lighthouses. Here the office of the light is not to render other objects visible, but to be itself seen; and in this respect, in hazy weather, the electric light is found decidedly superior to oil-lamps.

The electric light is also extensively used for throwing images on a screen in lecture-illustrations, and for producing various luminous effects in theatrical exhibitions. It has also been successfully employed for enabling labourers to carry on their work at night.

As the carbons undergo waste by combustion, it is necessary to employ some means for keeping them at a nearly constant distance, so as to give a steady light. Several different regulators have been employed for this purpose, all of them depending on the principle that the strength of the current diminishes as the distance, and consequently the resistance, increases. We will briefly describe two, those of Duboscq and Foucault.

577. Duboscq's Regulator.—In Duboscq's apparatus (Fig. 512) there is a train of wheel-work, driven by a main-spring contained within the barrel P, the motion being moderated by means of the revolving fans *g*. The two racks S and T are driven by two wheels attached to the barrel, one of them (the driver of T) having double the radius of the other. One rack thus rises, and the other falls, but the rising rack T moves twice as fast as the other. The rack T is that which carries the positive carbon *c*; the negative carbon *c'* is fixed to the piece T', which travels with the rack S. It has been found by experience that the positive carbon wears away twice as fast as the negative. Hence the adoption of this arrangement, which causes the positive carbon to move double the distance of the other. If the current were generated, not by a battery, but by a magneto-electric machine, such as we shall describe in a later chapter, each carbon would be alternately positive and negative, and it would be necessary to make their velocities equal.

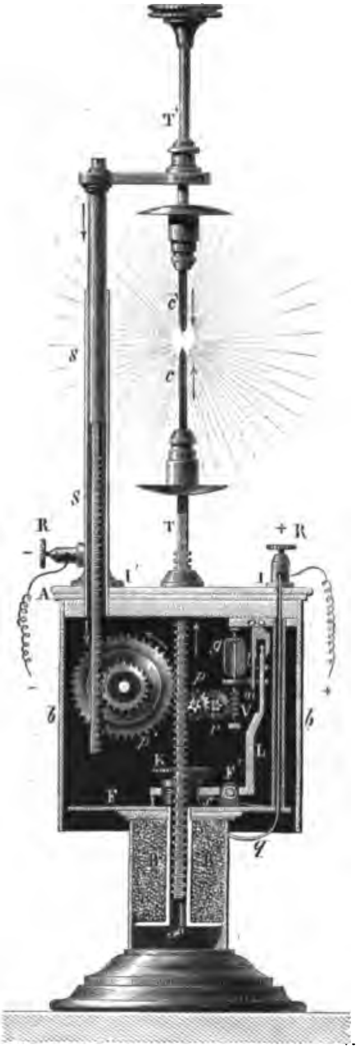


Fig. 512.—Duboscq's Regulator.

The current from the battery enters the apparatus by the binding-screw R, traverses the coil of the electro-magnet B B, whence it passes through the rack T to the positive carbon *c*. From the negative carbon *c'* it travels to the rack S, and escapes by the binding-screw R'. The soft-iron core[1] of the electro-magnet attracts an armature K, with a force which depends on the strength of the current. The armature is attached to one arm of the bent lever L, which turns about a horizontal axis at F', and an opposing spring *s* resists the attraction of the electro-magnet. The upper end of the bent lever governs the movements of a shorter lever *l* which turns about an axis at *a*. This short lever is armed at its lower end with a tooth or pallet *m*, whose office is to stop the movement of a toothed wheel, attached to the axis of the revolving fans.

When the current is passing in full strength, the electro-magnet holds down the armature, thus causing the pallet to lock the teeth

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[1] The *core* of an electro-magnet is the soft iron in its interior, which becomes magnetised by the passage of the current.

f the wheel and hinder the machinery from moving; but as the
arbons burn away, the resist-
nce increases, the current di-
iinishes, and the strength of
ıe electro-magnet falls off, until
ıe opposing spring is able to
rerpower it and raise the ar-
ature. This unlocks the pallet
om the wheel, and the racks
e accordingly driven forward,
us bringing the carbons nearer
gether, and increasing the cur-
ıt until the electro-magnet
quires sufficient power to pre-
il over the opposing spring
l lock the wheel-work again.
A small lever is provided, for
pping or starting the motion
hand. The armature can
be screwed up or down, so
o regulate its minimum dis-
e from the electro-magnet
rding to the battery power
loyed. The mechanism is
sed in a metallic box, one
of which can be removed
ı it is desired to obtain ac-
to the interior.

8. **Foucault's Regulator.**—
ault's latest form of regu-
differs from Duboscq's in
g two systems of wheel-
one for bringing the car-
nearer together, and the
for moving them further
Fig. 513 represents the
tus, with the omission of
intermediate wheels. L'
arrel driven by a spring
d within it, and driving several intermediate wheels which

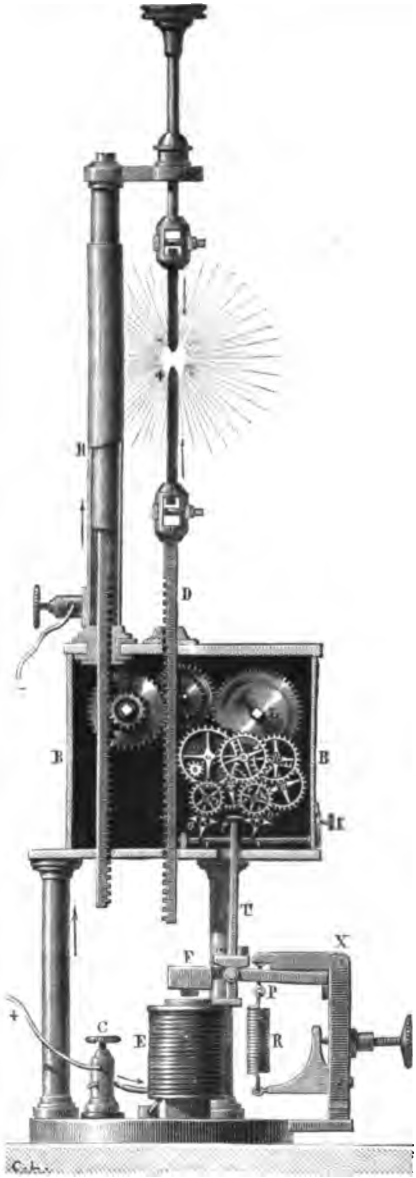

Fig. 513.—Foucault's Regulator.

transmit its motion to the fly *o*. L is the second barrel, driven by a stronger spring, and driving in like manner the fly *o'*. The racks which carry the carbons work with toothed wheels attached to the barrel L', the wheel for the positive carbon having double the diameter of the other, as in Duboscq's arrangement above described. The current enters at the binding-screw C, traverses the coil of the electro-magnet E, and passes through the wheel-work to the rack D, which carries the positive carbon. From the positive carbon it passes through the voltaic arc to the negative carbon, and thence, through the support H, to the binding-screw connected with the negative pole of the battery.

When the armature F descends towards the magnet, the other arm of the lever F P is raised, and this movement is resisted by the spiral spring R, which, however, is not attached to the lever in question, but to the end of another lever pressing on its upper side and movable about the point X. The lower side of this lever is curved, so that its point of contact with the first lever changes, giving the spring greater or less leverage according to the strength of the current. In virtue of this arrangement, which is due to Robert Houdin, the armature, instead of being placed in one or the other of two positions, as in the ordinary forms of apparatus, has its position accurately regulated according to the strength of the current. The anchor T*t* is rigidly connected with the lever F P, and follows its oscillations. If the current becomes too weak, the head *t* moves to the right, stops the fly *o'* and releases *o*, which accordingly revolves, and the carbons are moved forward. If the current becomes too strong, *o* is stopped, *o'* is released, and the carbons are drawn back. When the anchor T*t* is exactly vertical, both flies are arrested, and the carbons remain stationary. The curvature of the lever on which the spring acts being very slight, the oscillations of the armature and anchor are small, and very slight changes in the strength of the current and brilliancy of the light are immediately corrected. The details of the mechanism contain some ingenious devices, which our limits do not permit us to explain.

578 A. Peltier Effect.—When a current is sent through a heterogeneous circuit, a peculiar thermal effect occurs at each junction. We have seen, in Chap. xlv., that the heating of a junction in such a circuit tends to produce a current in a definite direction. It was discovered by Peltier that if a current be sent through the junction *in that direction*, the junction will be *cooled* by it, and that it will be warmed by a current sent in the opposite direction.

If, for example, a current from a battery is sent through an ordinary thermo-pile, the junctions at one end will rise in temperature, and those at the other end will be depressed. If the battery be then removed, and a galvanometer substituted, a current in the opposite direction to the former will be indicated by the galvanometer, until equality of temperature has been restored.

The *Peltier effect*, as it is called, is superadded to the general warming due to the overcoming of resistance in the circuit, so that he actual temperature attained by a junction depends on both auses combined.

CHAPTER L.

ELECTRO-MOTORS—TELEGRAPHS.

579. Electro-magnetic Engines.—Electro-magnetic engines are driven by means of the temporary magnetization of soft iron under the influence of a current. This magnetization can be destroyed or reversed with great rapidity; and it is thus possible to produce alternate movements of an armature, which can be readily transformed into other movements by ordinary mechanism. Since 1834, when Jacobi of St. Petersburg constructed the first engine of this kind, many other inventors have tried their powers in the same direction; but none of these attempts have been commercially successful, and the idea of employing such engines for any useful purpose is now almost abandoned.

The chief reason for this want of success is the greater cost of the material consumed as compared with the fuel of other engines. A pound of zinc costs about fifty times as much as a pound of coal, and if the full equivalent in the form of work could be obtained, both for the coal burned in a furnace and for the zinc consumed in a battery, a pound of coal would yield four times as much work as a pound of zinc. Hence, if the "efficiency"[1] of a heat-engine and of an electric engine be the same, the cost of performing a given quantity of work will be 200 times greater for the electric engine than for the other. It appears, however, that, as regards efficiency, the electro-magnetic engine may have an advantage of about 4 to 1. This would make its work only 50 times as expensive as that of a steam-engine.

Again, inasmuch as magnetic attractions decrease very rapidly with increase of distance, it is necessary for efficient working that the travel of the driving parts should be very small. This is inconvenient from a mechanical point of view.

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[1] That is to say, the ratio of the energy utilized to the whole energy expended. This is the "efficiency of an engine" in the broadest sense. The "efficiency" of Chap. xxxii. is sometimes called, by way of distinction, the "efficiency of the working fluid."

Lastly, as we shall see in a later chapter, the movement of conductors in a strong magnetic field causes induced currents which strongly oppose the motion.

We shall proceed to describe two of the most successful electromagnetic engines which have yet been constructed.

580. Bourbouze's Engine.—In the engine of M. Bourbouze (Fig. 514) the armatures have a reciprocating motion. There are two helices, having soft-iron cores in their interior for the lower half of their length. Two soft-iron rods or plungers travel up and down in the space above the cores, and are jointed to a beam which, by means of a connecting-rod and crank, turns a fly-wheel. The positive pole of a battery is permanently connected with a sliding piece of metal which travels to and fro horizontally, so as to be connected with a terminal of the left-hand or of the right-hand coil, according as it is on the left or the right of its middle position. The upper terminals of both coils are permanently connected with the negative pole of the battery. The reciprocating movement of the slider is produced by an eccentric on the axis of the fly-wheel, like the movement of the slide-valve in a steam-engine.

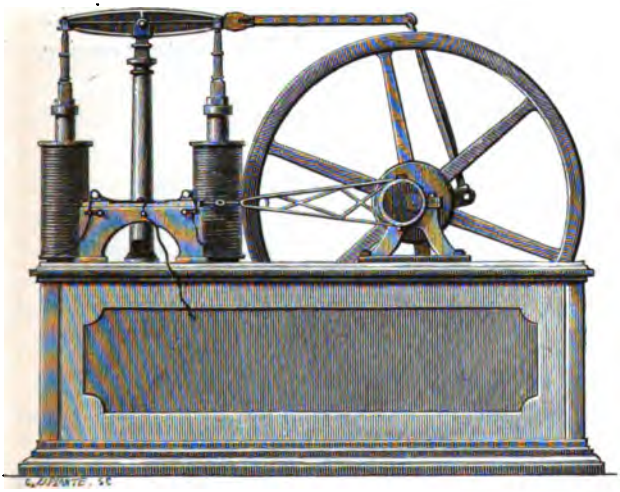

Fig. 514.—Bourbouze's Engine.

In the position represented in the figure, the eccentric and slider are nearly at the extremity of their range to the left. The left plunger is then at the middle of its down-stroke. When it reaches the bottom, the sliding-piece will be in the centre of its travel, and the left-hand coil will just have been disconnected. Immediately afterwards, the right-hand coil will be brought into circuit, its plunger being at the summit of its path; and it will continue in circuit till the plunger is nearly at the bottom. The electro-magnets are thus made and unmade whenever the eccentric passes its highest and lowest positions. On account of the shortness of the stroke, the beam is prolonged to a considerable distance before attaching to it the connecting-rod which drives the crank.

581. Froment's Engine.—Froment's is a rotatory engine. It may

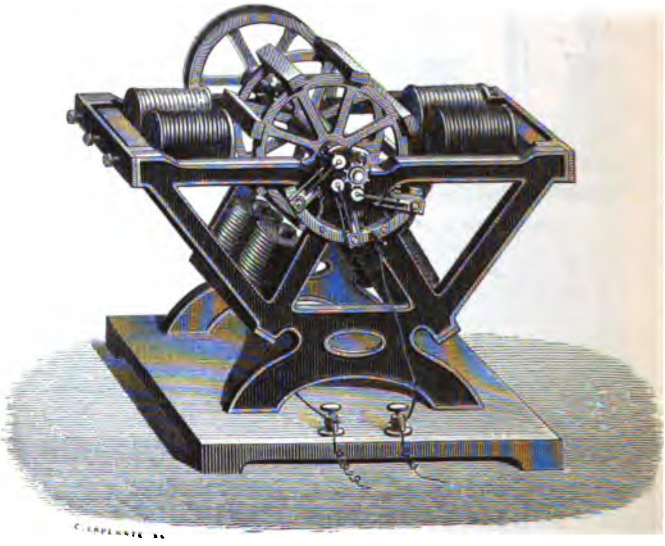

Fig. 515.—Froment's Engine.

be described as consisting of a wheel, with eight armatures of soft iron attached to its circumference at intervals of 45°, rotating under the action of four electro-magnets fixed to a cast-iron frame at intervals of 60°. Each magnet is "made" when an armature comes within 15° or 20° of it, and "unmade" as the armature is passing it.

The making and breaking of the circuits is effected by means of three distributors, one of which is shown on an enlarged scale in Fig. 516. R is an eight-toothed wheel, fixed to the axis on which the armatures revolve, and turning with them. Each tooth, as it passes the roller r, pushes it away, and brings the studs m' m into contact. As long as they remain in contact, the current circulates through the coil with which the distributor is connected. The distributors are screwed into a metallic arc, which is constantly connected with one pole of the battery. One of them serves for the two opposite horizontal magnets, which are made and unmade together. The two lower magnets have one distributor apiece. Matters are so arranged that the current is not cut off from one coil till just after it has commenced to flow in the next. This precaution prevents, or at least mitigates, the induction-spark which (for reasons to be hereafter explained) generally occurs in breaking circuit, and which has the mischievous effect of oxidizing the contacts, and thus, after a time, deranging the movements.

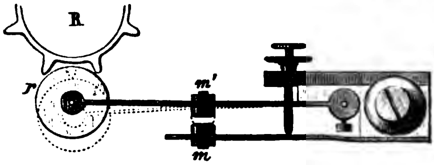

Fig. 516.—Distributor.

582. **Electric Telegraph: History.**—The discovery that electricity could be transmitted instantaneously to great distances, at once suggested the idea of employing it for signalling. Bishop Watson, already referred to in § 466, performed several experiments of this kind in the neighbourhood of London, the most remarkable being the transmission of the discharge of a Leyden-jar through 10,600 feet of wire suspended between wooden poles at Shooter's Hill. This was in 1747. A plan for an alphabetical telegraph to be worked by electricity is minutely described in the *Scot's Magazine* for 1753, but appears to have been never experimentally realized. Lesage, in 1774, erected at Geneva a telegraph line, consisting of twenty-four wires connected with the same number of pith-ball electroscopes, each representing a letter. Reusser, in Germany, proposed, in the same year, to replace the electroscopes by spangled panes exhibiting the letters themselves. The difficulty of managing frictional electricity was, however, sufficient to prevent these and other schemes founded on its employment from yielding any useful results. Volta's discoveries, by supplying electricity of a kind more easily retained on the conducting wires, afforded much greater facilities for transmitting signals to a distance.

Several suggestions were made for receiving-apparatus to exhibit the effects of the currents transmitted from a voltaic battery. Sömmering of Munich in 1811 proposed a telegraph, in which the signals were given by the decomposition of water in thirty-five vessels, each connected with a separate telegraph wire. Ampère, in 1820, proposed to utilize Œrsted's discovery, by employing twenty-four needles, to be deflected by currents sent through the same number of wires; and Baron Schilling exhibited in Russia, in 1832, a telegraphic model in which the signals appear to have been given by the deflections of a single needle.[1]

Weber and Gauss carried out this plan in 1833, by leading two wires from the observatory of Göttingen to the Physical Cabinet, a distance of about 9000 feet. The signals consisted in small deflections of a bar-magnet, suspended horizontally with a mirror attached, on the plan since adopted in Thomson's mirror galvanometer.

At their request the subject was earnestly taken up by Professor Steinheil of Munich, whose inventions contributed more perhaps than those of any other single individual to render electric telegraphs commercially practicable. He was the first to ascertain that earth-connections might be made to supersede the use of a return wire. He also invented a convenient telegraphic alphabet, in which, as in most of the codes since employed, the different letters of the alphabet are represented by different combinations of two elementary signals. Two needles were employed, one or the other of which was deflected according as a positive or a negative current was sent, the deflections being always to the same side. Sometimes the needles were merely observed by eye, sometimes they were made to strike two bells, and sometimes to produce dots, by means of capillary tubes charged with ink, on an advancing strip of paper, thus leaving a permanent record on the strip in the shape of two rows of dots. His currents were magneto-electric, like those of Weber and Gauss.

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[1] The contributions of Mr. (now Sir Francis) Ronalds to the art of telegraphy must not be altogether overlooked. According to an able notice in *Nature*, Nov. 23, 1871. "Sir Francis, before 1823, sent intelligible messages through more than eight miles of wire insulated and suspended in the air. His elementary signal was the divergence of the pith-balls of a Canton's electrometer produced by the communication of a statical charge to the wire. He used synchronous rotation of lettered dials at each end of the line, and charged the wire at the sending end whenever the letter to be indicated passed an opening provided in a cover; the electrometer at the far end then diverged, and thus informed the receiver of the message which letter was designated by the sender. The dials never stopped, and any slight want of synchronism was corrected by moving the cover. Hughes' printing instrument is the fully-developed form of this rudimentary instrument. A gas pistol was used to draw attention, just as now a bell is rung. The primary idea of reverse currents is to be found where Sir Francis suggests that the wire when charged with positive electricity should discharge not to earth but into a battery negatively charged. Equally interesting is the discussion on what we now call lateral induction, then known as compensation. The author clearly saw that in the underground wires which he suggests as substitutes for aerial lines, this induction would be or might be a cause of retardation."

The attraction of an electro-magnet on a movable armature furnishes another means of signalling. This was the foundation of Morse's telegraphic system, and was employed by Wheatstone for ringing a bell to call attention before transmitting a message.

About the year 1837 electric telegraphs were first established as commercial speculations in three different countries. Steinheil's system was carried out at Munich, Morse's in America, and Wheatstone and Cooke's in England. The first telegraphs ever constructed for commercial use were laid down by Wheatstone and Cooke, on the London and Birmingham and Great Western Railways. The wires, which were buried in the earth, were five in number, each acting on a separate needle; but the expensiveness of this plan soon led to its being given up. The single-needle and double-needle telegraphs of the same inventors have been much more extensively used, the former requiring only one wire, and the latter two.

Wheatstone (now Sir Charles Wheatstone) has since contributed several important inventions to the art of telegraphy, some of which ve shall have occasion to mention in later sections.

583. Batteries.—All the public telegraphs in this country have ow for many years been worked by voltaic currents; the magneto-lectric system, which was tried on some lines, having been found involve a needless expenditure of labour.

According to Mr. Culley,[1] engineer-in-chief to the post-office, the attery which has been adopted by the authorities of that depart-ent is a modified Daniell's, consisting of a teak trough, divided to cells by plates of glass or slate, and well coated with marine ue, each cell being divided into two by a slab of porous porcelain. e zinc plates measure 4 inches × 2, and the copper plates, which very thin, are 4 inches square. The zinc hangs at the upper part its cell, which is filled with dilute solution of sulphate of zinc. e copper cell is filled with a saturated solution of sulphate of per, and crystals of this salt are placed at the bottom. The enditure in sulphate of copper is about a pound and a half for h cell per annum.

84. Wires.—The wires for land telegraphs are commonly of what alled galvanized iron, that is, iron coated with zinc, supported on s by means of glass or porcelain insulators, so contrived that some part of the porcelain surface is sheltered from rain, and insulates the wires from the posts even in wet weather. Wires thus suspended are called *air-lines.*

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[1] *Handbook of Practical Telegraphy*, edition 1871, p. 19.

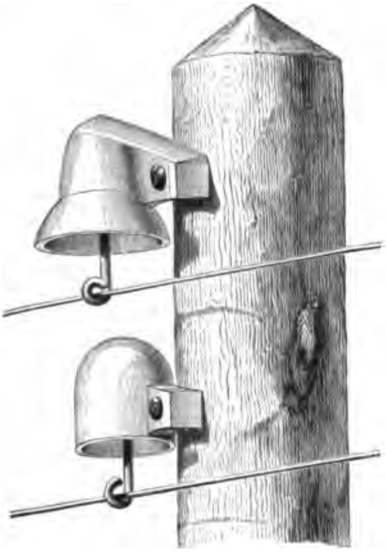

Fig. 517.—Insulators.

Underground wires are, however, sometimes employed. They are insulated by a coating of gutta-percha, and are usually laid in pipes, an arrangement which admits of their being repaired or renewed without opening the ground except at the drawing-in boxes. There is less leakage of electricity from subterranean than from air lines, but their cost is greater, and they are less suited for rapid signalling, on account of the retardation caused by the inductive action between the wire and the conducting earth, which is similar to that between the two coatings of a Leyden jar.

The early inventors of electric telegraphs supposed that a current could not be sent from one station to another without a return wire to complete the circuit. Steinheil, while conducting experiments on a railway, with the view of ascertaining whether the rails could be employed as lines of telegraph, made the discovery that the earth would serve instead of a return wire, and with the advantage of diminished resistance; the earth, in fact, behaving like a return wire of infinitely great cross-section, and therefore of no resistance.

We are not, however, to suppose that the current really returns from the receiving to the transmitting station through the earth. The duty actually performed by the earth consists in draining off the opposite electricities which would otherwise accumulate in the terminals. It keeps the two terminals at the same potential; and as long as this condition is fulfilled, the current will have the same strength as if the terminals were in actual contact.

584 A. Single-needle Telegraph.—One of the best known telegraphs in this country, though little or not at all employed elsewhere, is the single-needle instrument of Wheatstone and Cooke, represented in Figs. 517 A, B, the former showing its external appearance, and the latter its internal arrangements as seen from behind. The needle, which is visible in front, is one of an astatic pair, its fellow being in the centre of the coil C C. When the handle H hangs straight down, the instrument is in the position for receiving signals from another station. The current from the line-wire enters at L, and, after traversing the coil and deflecting the needle, escapes through the earth-wire E, having taken in its course the two tall contact-springs *t′ t.*

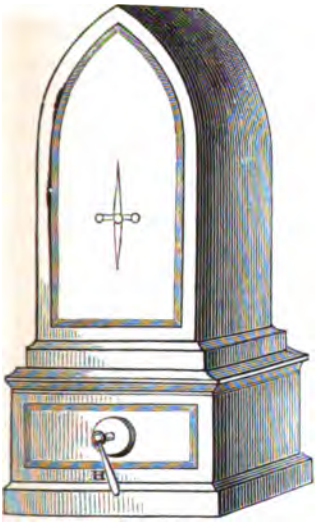

Fig. 517 A.—Single-needle Instrument.

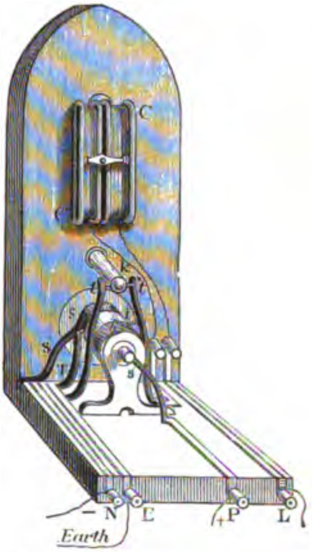

Fig. 517 B.—Internal Arrangements.

To send a current to another station, the handle H is moved to one side, and the current sent will be positive or negative according to the side to which the handle is moved. The handle turns the cylindrical arbor *a b*, which is divided electrically into two parts by an insulator in the middle of its length. Each of these parts has a pin projecting from it, one pin being above, and the other below. These are vertical when the handle is vertical, and are then doing no duty: but when the handle is put to one side, the upper pin (which is attached to *b*) makes contact with one of the tall springs *t t′*, at the same time pushing it away from the metallic rest *k*, and thus putting it out of connection with the other tall spring; while the lower pin (which is attached to *a*) makes contact with one of two short springs

T T′, only one of which is shown in the figure. There is permanent connection between *a* and the negative pole of the battery through the spring *s*, and between *b* and the positive pole through the spring *s′*. In the position represented in the figure, *a* serves to connect the negative pole of the battery with the earth, and *b* serves to connect the positive pole with the spring *t′*, down which the current passes from the point of contact of the pin, and then through the coil to the line-wire at L. The needle of the sending station is thus deflected to the same side as that of the receiving station.

If the handle were moved to the other side, *b* would serve to connect the positive pole with the earth, and *a* would establish connection between the negative pole and the coil, which is itself connected with the line-wire.

Since the telegraphs of this country came into the hands of the post-office, the alphabet devised by Wheatstone and Cooke has been given up, and the Morse alphabet, which we give in a later section, adopted in its place. In the Morse alphabet, which is now the telegraphic alphabet of all nations, the shortest signs are allotted to those letters which occur most frequently. This was not the case with the old needle-alphabet, which was rather planned with the view of assisting the memory; and experience has shown that such assistance is quite unnecessary. The needle instrument is also, to a great extent, being superseded by Morse's instrument.

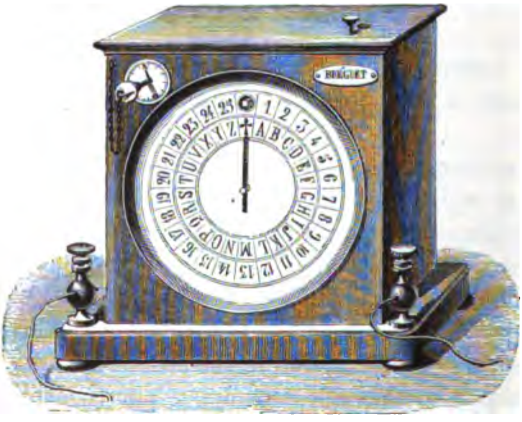

Fig. 518.—Breguet's Indicator.

585. Dial Telegraphs.—Telegraphs in which the ordinary letters of the alphabet are ranged round the circumference of a dial, and are pointed at by a revolving hand, are specially convenient for those who are not professional telegraphists. They are constructed on the principle of step-by-step motion, the hand being advanced by successive steps, each representing one current sent or stopped.

One of the simplest instruments of this class is Breguet's, which is extensively used on the French railways. Fig. 518 represents the exterior of the receiving instrument. The dial is inscribed with the 25 letters of the French alphabet and a cross, making 26 signals in all. The hand (as in other step-by-step telegraphs) advances only in one direction, which is the same as that of the hands of a clock, stopping before each letter which is to be indicated, and pointing to the cross at the end of each word. Fig. 519 shows the mechanism by which the motion is produced. A is the armature of an electro-magnet, the magnet itself being removed in the figure, to allow the other parts to be better seen. The two dotted circles traced on the armature represent vertical sections of the two coils, which rest on the bottom of the box, and have their axes horizontal. If introduced, they would nearly conceal the armature from view. The armature turns about a horizontal axis V V′, and is attached to an opposing spring which draws it back from the magnet. The tension of this spring can be regulated by means of a lever acted on by a key outside the box. When a current is sent, the armature is attracted to the magnet; when the current ceases, the spring draws it back; and it thus moves continually to and fro during the transmission of a message. An upright arm *l* is attached to the armature, and carries a horizontal arm *c*, which lies between the two prongs of a fork *d*, represented on a larger scale in Fig. 520. This fork vibrates about a horizontal axis *a b*, to which is attached the vertical pallet *i*. This

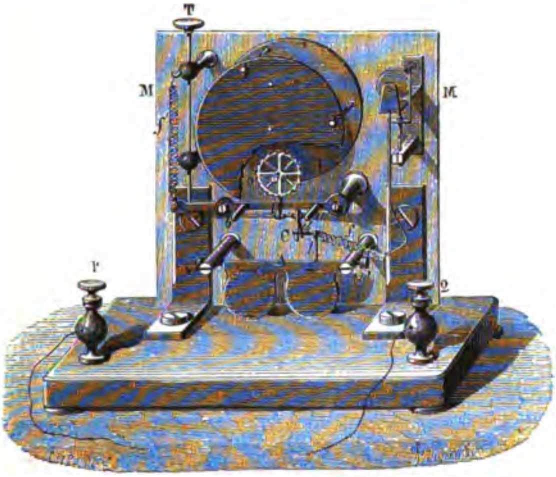

Fig. 519.—Mechanism of Indicator.

pallet acts upon an escapement wheel O, toothed in a peculiar way, the thickness of the teeth being only half the thickness of the wheel, and the teeth on one half of the thickness being opposite the spaces on the other half. The total number of teeth is 26, thirteen on each half of the thickness.

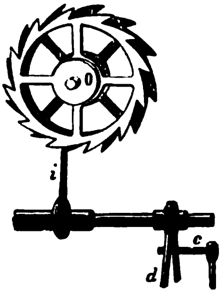

Fig. 520.—Escapement.

When no current is passing, the pallet *i* is engaged with one of the teeth on the remote side, as represented in Fig. 520. When a current passes, the armature is attracted, and the pallet is moved over to the near side, thus releasing the tooth with which it was previously engaged, and becoming engaged with the next tooth on the near side of the wheel. The wheel, which is urged by a clock-movement, thus advances $\frac{1}{26}$ of a revolution; and the hand on the dial, being attached to the wheel, moves forward one letter. When the current ceases, the pallet moves back to the remote side, and the hand is advanced another letter. If the hand is initially at the cross, it will be advanced to any required letter by so arranging matters that the number of currents *plus* the number of interruptions shall be equal to the number denoting the place of the letter in the alphabet. To effect this arrangement is the office of the sending instrument.

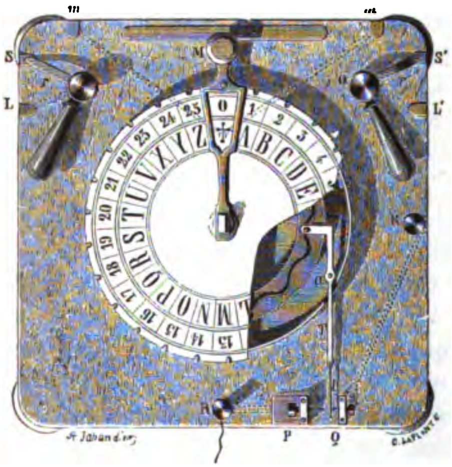

Fig. 521.—Breguet's Manipulator.

586. Sending Instrument.—This is represented in Fig. 521. There is a dial inscribed with 25 letters and a cross, like that of the receiving instrument, and an arm which can be carried round the dial by a handle M. There are 26 notches cut in the edge of the dial, in which a pin attached to the movable arm catches; and the arm is allowed sufficient play to and from the face of the dial to admit of this pin being easily released or inserted. When the pin is in one of the notches, the instrument is in position for transmitting the corresponding letter. The action is as follows:—

A toothed or rather undulated wheel is fixed on the same axis as the revolving arm, and turns with it. There are 13 projections and 13 hollows on its circumference, a few of which are shown in the figure where the face is cut away. A bent lever T, movable about an axis at *a*, bears at one end against the circumference of the undulated wheel, while its other end plays between two points P, Q, and is in contact with one or other of these points whenever its upper end bears against a hollow or a projection. P is in connection with a battery, and Q with the earth, the undulated wheel being in connection with the line-wire. The movement of the handle thus produces the requisite number of currents and interruptions.

587. Alarum.—Besides the sending and receiving apparatus above described, each station has an *alarum*, which is employed to call attention before sending a despatch. There are several different kinds. Fig. 522 represents the *vibrating alarum*, which is one of the simplest. It contains an electro-magnet *e*, with an armature *f* fixed to the end of an elastic plate. When no current is passing through the coil, the armature is held back by the elasticity of this plate, so as to press against a contact-spring *g* connected with the binding-screw *m*. The terminals of the coil are at the binding-screws *p*, *p'*, the former of which is in connection with the armature, and the latter with the earth. As long as the armature presses against the spring *g*, there is communication between the two binding-screws *m* and *p'* through the coil; but the passing of a current produces attraction of the armature, which draws it away from *g* and interrupts the current. The electro-magnet is thus demagnetized, and the armature springs back against *g*, so as to allow a fresh current to pass. The armature is thus kept in continual vibration; and a hammer K, which it carries above, produces repeated strokes on a bell T.

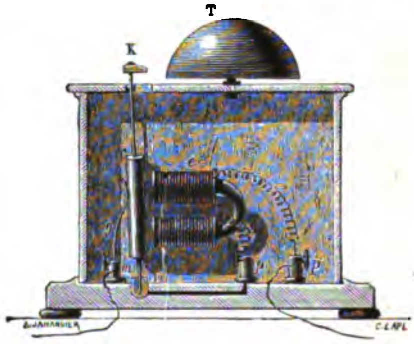

Fig. 522.—Vibrating Alarum.

587A. Wheatstone's Universal Telegraph.—The first step-by-step telegraph was invented by Wheatstone; and the most perfect instrument of the class is probably his "Universal Telegraph," which is now in such general use in this country for connecting places of business. The currents employed are magneto-electric, and are alternately positive and negative. They produce successive reversals of polarity in an electro-magnet, which acts upon a light steel magnet—a kind of astatic needle—and causes it to rotate through a large angle first in one direction, and then in the opposite. Each of these rotations causes a ratchet-wheel to advance one tooth, and this causes the pointer to advance one letter. At the same time, the turning of the handle by which the currents are generated, causes the pointer of the sending instrument to advance one letter for each current sent, so that the pointers at the two stations indicate the same letter. The same dial which serves for sending, also serves for receiving. It is surrounded by a number of keys or buttons, one against each letter. When any letter is to be sent, its key is depressed, the operator continuing all the time to turn the handle for generating currents. Previous to putting down a key, these currents complete their circuit within the instrument; but when a key is down, every current generated travels along the line to the receiving station, until the pointers have been advanced step by step to the corresponding letter. As soon as this has been reached, the currents are again confined to the sending instrument; and the pointers will make no further advance till another key is put down.[1]

588. Morse's Telegraph.—Morse's apparatus, first tried in America about 1837, is now perhaps the most extensively used of all.

His receiving instrument, or *indicator*, in its primitive simplicity, consists (Fig. 523) of an electro-magnet, a lever movable about an axis, carrying a soft-iron armature at one end, and a pencil at the other, and a strip of paper which is drawn past the pencil by a pair of rollers.

As the pencil soon became blunt, and was uncertain in its marking, a point, which scratched the paper, was substituted. This has now to a great extent been superseded by an ink-writer, which requires the exertion of less force, and at the same time leaves a more visible trace.

589. Receiving Instrument.—Fig. 524 represents Morse's indicator as modified by Digney. A train of clock-work, not shown in the

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[1] For the details of the mechanism, reference may be made to Wheatstone's Patent, No. 1239 year 1858. A condensed account will be found in *Sabine on the Electric Telegraph*, pp. 82–84.

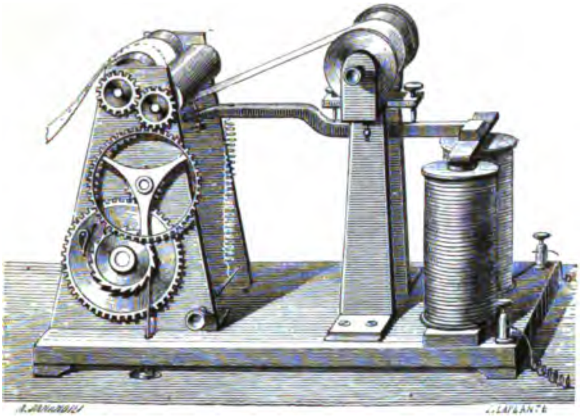


Fig. 523.—Morse's Telegraph.

figure, drives one of a pair of rollers *nm*, which draw forward a strip of paper *pp* forming part of a long roll K. The same train turns the

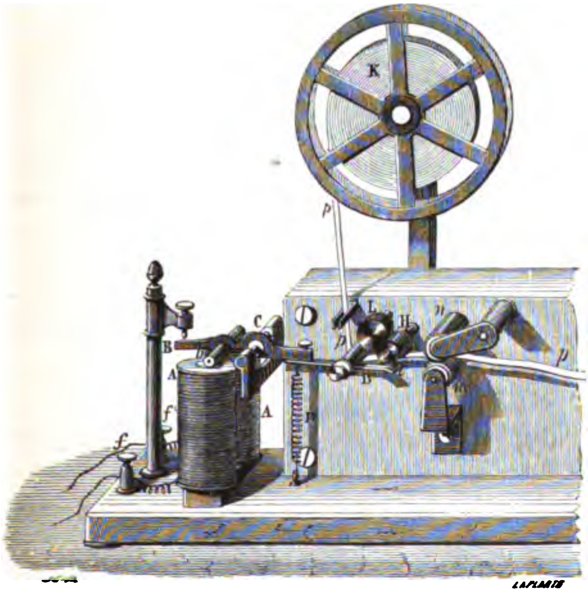


Fig. 524.—Modified Form.

nting-cylinder H, the surface of which is kept constantly charged
h a thick greasy ink by rolling-contact with the ink-pad L. The

armature B B′ of the electro-magnet A is mounted on an axis at C, and carries a style at its extremity just beneath the printing-cylinder. When a current passes, the armature is attracted, and the style presses the paper against the printing-cylinder, causing a line to be printed on it, the length of which depends on the duration of the current, as the paper continues to advance without interruption. The lines actually employed are of two lengths, one being made as short as possible (-), and called a *dot*, the other being about three times as long (—) and called a *dash*. The opposing spring D restores the armature to its original position the moment the current ceases.

590. Key for Transmitting.—Morse's key (Fig. 525) is simply a brass lever, mounted on a hinge at A, and pressed up by the spring *f*. When the operator puts down the key, by pressing on the button K with his finger, the projections *c d* are brought into contact, and a current passes from the battery-wire P to the line-wire L. When the key is up, the projections *a b* are in contact, and currents arriving by the line-wire pass by the wire R to the indicator or the relay. By keeping the key down for a longer or shorter time, a dash or a dot is produced at the station to which the signal is sent. The dash and dot are combined in different ways to indicate the different letters, as shown in the following scheme, which is now generally adopted both in Europe and America:—

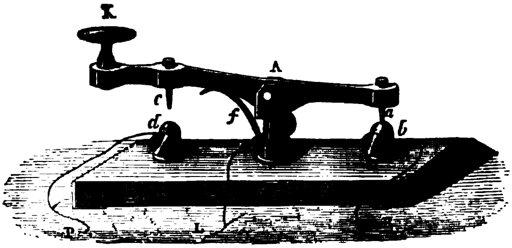


Fig. 525.—Morse's Key.

Morse's Alphabet.

A ·—	J ·———	T —	1 ·————
Ä ·—·—	K —·—	U ··—	2 ··———
B —···	L ·—··	Ü ··——	3 ···——
C —·—·	M ——	V ···—	4 ····—
D —··	N —·	W ·——	5 ·····
E ·	O ———	X —··—	6 —····
É ··—··	Ö ———·	Y —·——	7 ——···
F ··—·	P ·——·	Z ——··	8 ———··
G ——·	Q ——·—	Ch ————	9 ————·
H ····	R ·—·		0 —————
I ··	S ···	Understood ···—·	

A space about equal to the length of a dash is left between two letters, and a space of about twice this length between two words.

In needle-telegraphs, the dot is represented by a deflection to the left, and the dash by a deflection to the right.

Fig. 526 represents Morse's indicator in connection with what is called a *relay;* that is to say, an apparatus which, on receiving a feeble current from a distance, sends on a much stronger current from a battery on the spot. The key B being up, a current arriving by the line-wire passes through the key from *c* to *a*, thence through another wire to the coil of the electro-magnet belonging to the relay, nd through this coil to earth. The electro-magnet of the relay ttracts an armature, the contact of which with the magnet com-letes the circuit of the local battery, in which circuit the coil belong-ng to the indicator is included. The armature of the indicator is hus compelled to follow the movements of the armature of the relay.

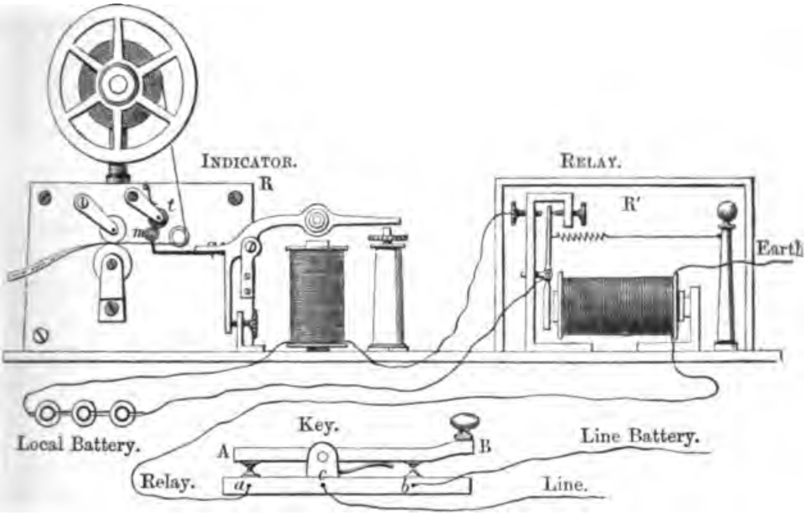


Fig. 526.—Morse's Apparatus, with Relay.

Relays are used when the currents which arrive are too much feebled to give clear indications by direct action. They are also equently introduced at intermediate points in long lines which uld not otherwise be worked through from end to end. The alogy of this use to change of horses on a long journey is the origin the name. Relays are also frequently used in connection with arums when these are large and powerful.

591. Hughes' Printing Telegraph.—The employment of Morse's alphabet requires on the average about three currents to be sent per letter. The extension of telegraphic service has stimulated the industry of inventors to devise means for obtaining more rapid transmission. Hughes, about 1859, invented a system which requires only one current to be sent for each letter, and which, accordingly, sends messages in about a third of the time required by Morse's method. Hughes' machine also prints its messages in Roman characters on a strip of paper. These advantages are, however, obtained at the expense of extreme complexity in the apparatus employed. It is only fit for the use of skilled hands; but it is extensively employed on important lines of telegraph. We will proceed to indicate the fundamental arrangements of this marvellous piece of ingenuity.

Fig. 527 is a general view of the machine. It is propelled by powerful clock-work, with a driving-weight of about 120 lbs., and with a regulator consisting of a vibrating spring *l* acting upon a 'scape-wheel. A travelling weight on the spring can be moved towards either end to regulate the quickness of the vibrations. The clock-work drives three shafts or axes: (1.) the type-shaft, so called because it carries at its extremity the type-wheel T, which has the letters of the alphabet engraved in relief on its circumference at equal distances, except that a blank space occurs at one place instead of a letter; (2.) the printing-shaft, which turns much faster than the type-shaft, making sometimes 700 revolutions per minute, and carrying the fly-wheel V. These two axes are horizontal, and are separately represented in Fig. 528; (3.) a vertical shaft *a*, having the same velocity as the type-wheel, which drives it by means of bevel-wheels.

This vertical shaft consists of two metallic portions, insulated from each other by an ivory connecting-piece. In the position represented in Fig. 528, these two metallic parts are electrically connected by means of the screw V, but they will be disconnected by raising the movable piece *v*.

The revolving arm composed of the pieces *v′v* is called the *chariot*. It revolves with the vertical shaft, and travels over a disc D pierced with as many holes as there are letters on the type-wheel, these holes being ranged in a circle round the base of the shaft, and at such a distance from the shaft that the extremity of the chariot passes exactly over them. In these holes are the upper ends of a set of pins *g*, which are raised by putting down a set of keys BN resem-

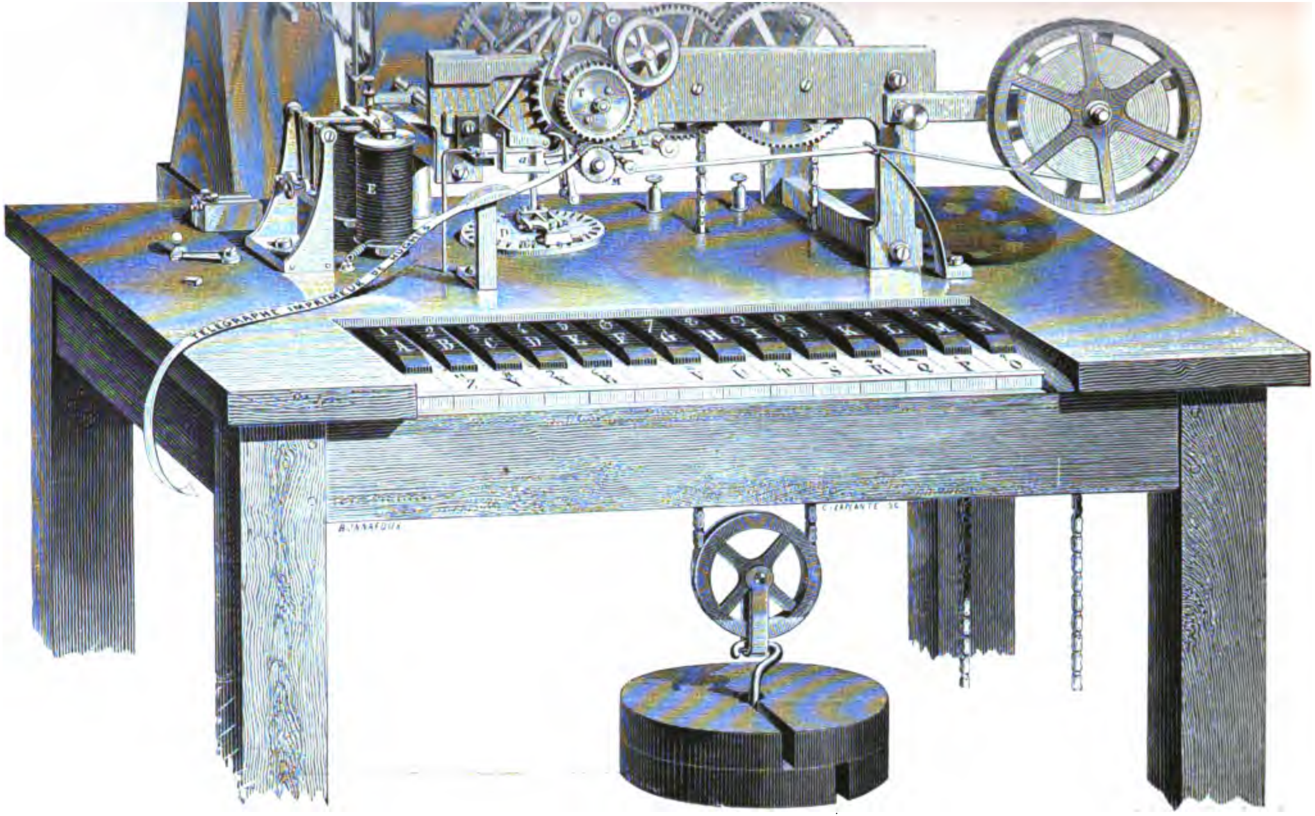


Fig. 527.—Hughes' Printing Telegraph.

bling those of a piano. When the chariot passes over a pin which is thus raised, the piece *v* is lifted away from *v'*, and the current from the battery, which previously passed from the pin through *v* and *v'* to the earth, is now cut off from *v'*, and passes through *v* to the electro-magnet, and thence to the line-wire.

This is the process for sending signals. We will now explain how a current thus sent causes a letter to be printed by the type-wheels at both the sending and receiving stations, the sending and receiving instruments being precisely alike.

The current traverses the coils of an electro-magnet E (Fig. 527), beneath which is a permanent steel horse-shoe magnet, having its poles in contact with the soft-iron cores of the electro-magnet. When no current is passing, the influence of the steel renders these cores temporary magnets, and enables them to hold the movable armature *p* against the force of an opposing spring. The current is in such a direction that it tends to reverse the magnetism induced by the steel. It is not necessary, however, that it should be strong enough to produce an actual reversal, but merely that it should weaken the induced magnetism of the cores sufficiently to enable the

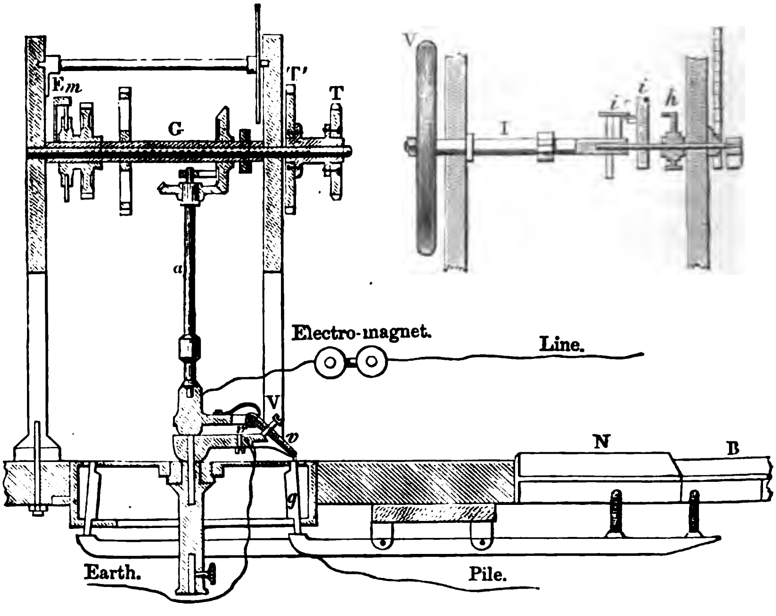


Fig. 528.—Type-shaft and Printing-shaft.

opposing spring to overpower them. This is one of the most original parts of Hughes' apparatus, and is a main cause of its extreme sensibility.

The printing-shaft consists of two portions, one of which I (Fig. 528) carries the fly-wheel V, and turns uniformly under the action of the clock movement; the other, which is next the front of the machine, remains at rest when no current is passing; but when the armature of the magnet rises, the two parts of the shaft become locked together by means of the ratchet-wheel and click *i i'*.

The portion of the shaft which is thus turned every time a current passes, carries a very acute cam or tooth *p* (Fig. 529), which suddenly raises the lever *a b*, movable about an axis at one end T, and, by so doing, raises the paper against the type-wheel, and prints the letter. In order thus to print a letter from the rim of a wheel which continues turning, very rapid movement is necessary. This is secured by making the opposing spring which moves the armature very powerful, and the cam *p* very acute. The same movement of the lever which produces the impression, raises the arm J U, which carries a spring *r* with a click at its extremity. This click, in its ascent, glides over the teeth of the ratchet-wheel E; but locks into the teeth and turns the wheel in its descent, and by so doing, advances the paper through the distance corresponding to one letter. The spacing of the words is obtained by the help of the blank on the type-wheel.

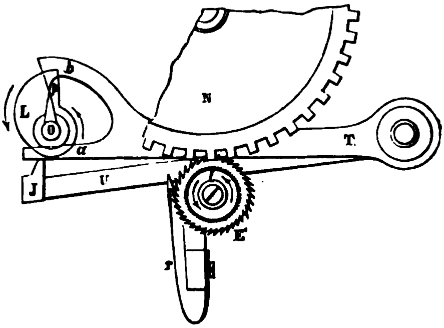


Fig. 529.—Mechanism for Printing.

The type-wheel should admit of easy adjustment to restore it to agreement with the chariot when accidental derangement may have occurred. For this purpose, the shaft G is made hollow, its internal and external portions being merely locked together by the click *m*, which is held in its place by a permanent current in either direction. On pressing down the button Q (Fig. 527), the click *m* is raised by the piece E, so as to leave the type-wheel free, and a pin is provided which catches in a notch corresponding to the blank on the type-wheel. The adjustment can also be made by hand.

Lastly, the shaft I carries a third cam, which, at each revolution

bling those of a piano. Whe toothed wheel T', set on the
thus raised, the piece *v* is li ches it a little forward or back-
the battery, which previo e driving gear. Small discrepan-
to the earth, is now cu e type-wheel and chariot are thus
electro-magnet, and the s printed. This contrivance serves to
This is the process t from gaining or losing on the sending
a current thus sent ansmission of a message. The type-wheel
at both the sending ent must be adjusted before the message
instruments bein the two instruments start at the same letter.
The current t al **Telegraph.**—Suppose a metallic cylinder,
beneath whic ted with the earth, to be revolving, carrying
poles in cor urface a strip of paper freshly impregnated with
When no c ssium. Also suppose a very light steel point per-
cores ter ected with the line-wire, and resting in contact with
armatur Every time that a current arrives by the line-wire,
in such ation will take place at the point of contact, and the paper
by th int will be discoloured by the formation of prussian blue.
principle of Bain's electro-chemical telegraph, which leaves
d in the shape of dots and dashes of prussian blue. The
atus for sending signals is the same as in Morse's system. The
paper must not be too wet, or the record will be blurred; neither
must it be too dry, for then no record will be obtained.

533. **Autographic Telegraph.**—An autographic telegraph is one which produces at the receiving station a fac-simile of the original despatch. The best known instruments of this class are those of Bonelli and Caselli. We shall describe the latter.

At the sending station a sheet of metallized paper, with the despatch written upon it in a greasy kind of ink, is laid upon a cylindric surface M (Fig. 530). At the receiving station there is a

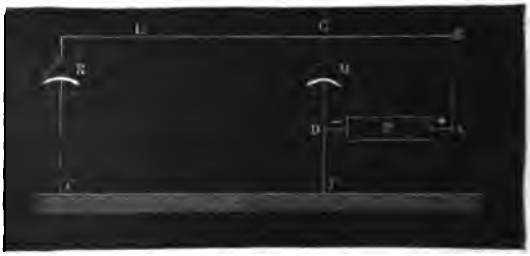


Fig. 530.—Principle of Caselli's Telegraph.

R, on which a sheet of Bain's chemical paper riven by pendulums which oscillate with exact over the surfaces of the two sheets, describing se parallel lines at a uniform distance apart, both

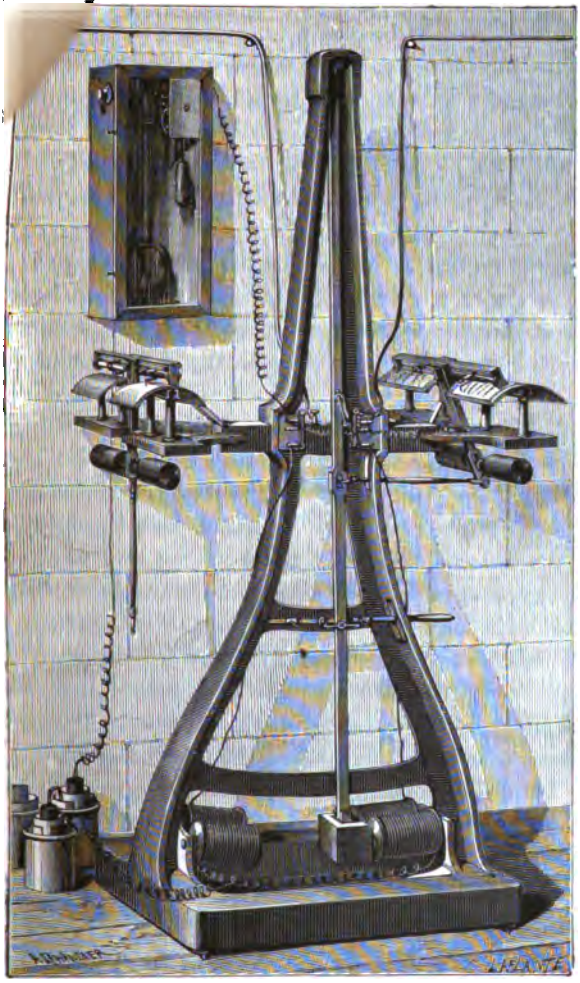


Fig. 531.—Caselli's Telegraph.

s being in permanent connection with the line-wire. The cur- is furnished by the battery P at the sending station. When the is on a conducting portion of the paper M, the current takes urse of least resistance ABCD, no sensible portion of it going to the other station. On the other hand, when the style is on the non-conducting ink in which the despatch is written, the circuit ABCD is broken, and the current travels through the line-wire. At this moment the style on the sheet R is in exactly the same position as that on the sheet M, by reason of the synchronism of the pendulums, and a blue line will be produced which will be the exact reproduction of the broken line of the despatch traversed by the style. Accordingly, when the style of M has described a series of lines close together and covering the sheet, R will be covered with a series of points or lines forming a copy of the despatch. The tracing point is carried by a lever turning about an axis near its lower end. To this lower end is attached a connecting-rod, jointed at its other end to the pendulum (Fig. 531). While the pendulum swings in one direction, the style traces a line in one direction on the sheet. At the end of this stroke, an action occurs which, besides advancing the style, raises it, so that it does not touch the sheet during the return stroke.

The synchronism of the pendulums at the two stations, which is absolutely necessary for correct working, is obtained by means of two clocks which are separately regulated to a given rate, the clock-pendulums making two vibrations for one of the telegraphic pendulum. The bob of the latter consists of a mass of iron, and vibrates between two electro-magnets, which are made and unmade according to the position of the clock-pendulum, as the latter makes and breaks the circuit of a local battery. The mass of iron is thus alternately attracted by each of the two magnets as it comes near them, and is prevented from gaining or losing on the clock.

It is evident that the Caselli telegraph may be applied to copy not only letters but a design of any kind; hence the name of *pantelegraph* which has been given it. Fig. 532 represents a copy thus obtained upon Bain's paper. Fig. 533 represents a copy obtained at the same time upon a sheet of tin-foil, such as is usually placed beneath the paper. The current decomposes the moisture of the paper, and the hydrogen thus liberated reduces the oxide of tin, of which a small quantity is always present on the surface. If the foil be then treated with a mixture of nitric and pyrogallic acid, the traces are developed and come out black.

The Caselli system has been used for some years on the telegraphs around Havre and Lyons, but has not realized the hopes of its promoters, its despatches being often illegible.

Instead of a series of parallel lines, the styles may be made to trace the successive convolutions of a fine helix, the two sheets being bent

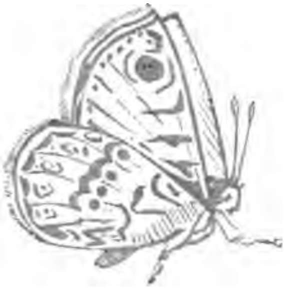


Fig. 532.—Fac-simile of Despatch.

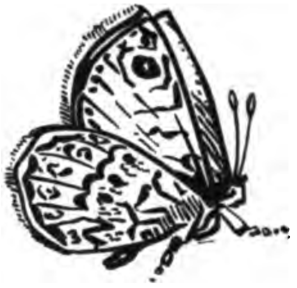


Fig. 533.—Copy on Tinfoil.

round two cylinders, which revolve in equal times, and also advance longitudinally.

595. **Submarine Telegraphs.**—The first submarine telegraph cable ras laid between Dover and Calais in 1850; but, being nsufficiently protected against the friction of the rocks, ; only lasted a few hours. The two Atlantic cables hich were laid in 1866 appear to be still in perfect rder.

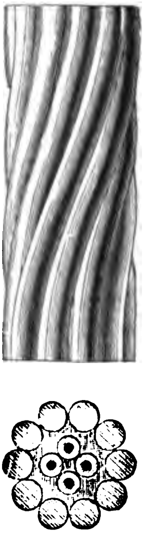


Fig. 534. Submarine Cable.

Submarine cables are now usually constructed by nbedding a certain number of straight copper wires gutta-percha (Fig. 534), which insulates them from ch other; this is surrounded with tarred hemp, and veral strands of iron wire are wound outside of all. le copper wires in the interior are the conductors the transmission of the signals; the gutta-percha for insulation; the hemp and iron are for protecn.

The Atlantic cables contain a central conductor, sisting of seven copper wires, twisted together and ered with three layers of gutta-percha, forming gether a cylinder $\frac{3}{8}$ of an inch in diameter. This covered with a layer consisting of five strands hemp, served with a composition consisting of 5 parts of Stockn tar, 5 of pitch, 1 of linseed-oil, and 1 of bees'-wax. Lastly, the le is covered by 18 strands of charcoal iron, each strand consistof seven wires $\frac{7}{10}$ of a millimetre in diameter. On leaving the machine which put on the wire covering, the cable was passed through a cauldron contain a mixture of pitch, tar, and linseed-oil. The difficulty of obtaining sufficiently good insulation has thus been completely surmounted.

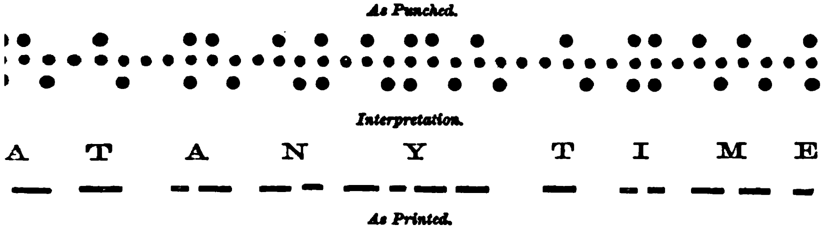
A second difficulty attaching to submarine telegraphy depends upon the inductive action of the surrounding water, or of the iron sheath. This action, which is found quite sensible in subterranean lines of no great length, becomes of immense importance in long submarine cables. The cable forms one enormous condenser, the central conductor representing the inner coating, and the sea-water, or iron sheath, the outer coating of a Leyden jar. In the Atlantic cables, the retardation of the signals due to this cause is so considerable that it would be barely possible to obtain a speed one-fifth of that usually attained on land-lines, if the same modes of sending and receiving signals were employed. The electrical capacity of the cable is in fact so enormous, that a long time is required to give it a full charge from a battery, or to discharge it again. The signals accordingly lose all their sharpness, and run into one another, unless special precautions are taken. After sending a current from one pole of the battery, the cable must be discharged, either by putting it to earth, or, still better, by connecting it for an instant with the other pole of the battery. The residual effects of the first current are thus quickly destroyed, and the line is left free for a second signal.

As the first effect received through such a cable is very slight, a very sensitive receiving instrument is necessary for quick working. Thomson's mirror galvanometer (§ 536A) is the instrument which has been hitherto employed, the signals being read off by an attendant who watches the movements of the spot of light, dots and dashes being represented by deflections in opposite directions. A self-recording instrument by the same inventor is now coming into use, in which the signals are written with ink discharged from a very light glass siphon, the siphon being moved by a very light coil of fine copper wire, suspended by a silk fibre between the poles of a very powerful permanent magnet. The coil turns in one direction or the other according as the current transmitted is positive or negative, thus producing opposite sinuosities in the ink record which is traced upon an advancing strip of paper. The regular flow of the ink is assisted by electrical attraction, on the principle of the bucket or watering-pot described in § 445; but with this difference, that it is not the ink but the paper that is electrified. An electrical machine of peculiar and novel construction, bearing some resemblance to the replenisher of § 469 E, is employed for this purpose.

595 A. Wheatstone's Automatic System.—Another very effective contrivance for increasing the speed of signalling, is Wheatstone's automatic apparatus, which is being very extensively adopted by the authorities of the postal telegraphs. The first step towards sending a message by this system consists in punching the message in a ribbon of stiff paper. The punching is done by a special instrument, the operator having merely to put down three keys, one of which represents *dot*, another *dash*, and the third *blank*. The holes punched are in three rows. Those in the middle row are equidistant, and are intended to perform the office of the teeth of a rack in guiding the paper uniformly forwards. Those in the two outside rows contain the message, a dot being represented by a pair of holes exactly opposite each other (:) one in each row, and a dash by two holes ranged obliquely (˙.).

The punched strips are then put through the transmitting instrument, and, by regulating the movements of two pins, cause the transmission of the currents necessary for printing the message at the receiving station. From 60 to 100 words are thus transmitted per minute and automatically printed.

The following is a specimen of three consecutive words of a telegraphic message, as it appears on the punched strip at the sending station, and on the printed strip at the receiving station:—



The speed thus attained is three or four times greater than that f ordinary writing. The practical limit to speed, in lines of con-derable length, arises not so much from the difficulty of making uicker movements, as from the blending together of successive gnals in travelling a great distance, especially if part of the distance under ground or under water. This evil is partly remedied by aking each signal consist, not of a single current, but of two; thus a dot will be produced by an instantaneous current, *immediately* succeeded by another of opposite sign; a dash by an equally short current followed *at a longer interval* by an opposite one. In this way, though a greater number of currents are required for each word, a greater number of words can be distinctly signalled in a given time; and, by sending three properly adjusted currents for each signal, a still greater speed of distinct transmission is possible. The transmitting instrument of Wheatstone's automatic system does in fact send three currents for each dot or dash.

595 B. Electrically-controlled Clocks.—Various schemes have been proposed for utilizing electricity in connection with the driving and government of clocks. In some of them, electricity is employed either to wind up the driving-weight, or to fulfil the office of a driving-weight by its own action, a pendulum being employed as the regulator, as in ordinary clocks. In others, electricity both drives and regulates the clock (or even a considerable number of clocks), by means of currents which keep time with the movements of a standard clock, electricity having thus to do the work both of driving and regulating the dependent clocks.

But the system which has given the best practical results is that of Mr. R. L. Jones, in which the dependent clocks are complete clocks, able to go of themselves, and keep moderately good time, without the aid of electricity. The duty devolving on the electric currents is merely to supply the small amount of accelerating or retarding action necessary to prevent the dependent clocks from gaining or losing on the standard clock by whose movements the currents are timed.

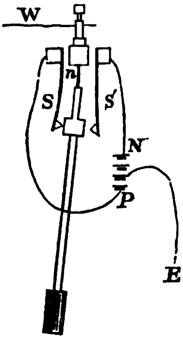


Fig. 534 A.—Controlling Pendulum.

The arrangements for attaining this end are shown in the annexed figures 534 A, 534 B, which represent the pendulums of the controlling and controlled clocks respectively. These pendulums are supposed to be almost precisely of the same length, so that they would nearly synchronize if disconnected.

The controlling pendulum, in its movement to either side, comes in contact with one or the other of two weak springs SS′, which are connected with the poles of a battery PN, having one of its middle plates connected with the earth, so as to keep its poles at potentials differing from that of the earth in opposite directions. In the position represented in the figure, a current is being sent from the positive pole P into the wire W. When the pendulum swings over to the other side, a negative current will be sent.

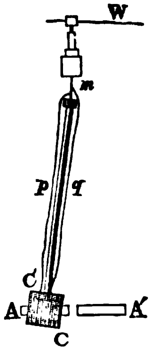


Fig. 534 B.—Controlled Pendulum.

The bob CC of the controlled pendulum (Fig. 534 B) is a hollow cylinder of soft iron encircled by a coil, whose ends are connected through two suspending springs at *m* with the wire W and the earth respectively. The consequence of this arrangement is that, whenever a current arrives by the wire W, the bob becomes an electro-magnet.

Two steel magnets AA′ are fixed, with their poles turned opposite ways, in such a position that the hollow bob of the pendulum always encircles one or both of them. Suppose, in the figure, that the poles AA′ which are turned outwards, are the two austral poles, so that the two boreal poles are facing each other. Then matters are to be so arranged that, in the position represented, the pendulum being near the left extremity of its swing, the right-hand end of the coil is a boreal pole, and magnetic force urges the pendulum to the left. When the pendulum is near the right extremity of its swing, the current is in the opposite direction, and consequently the boreal pole of the coil is its left-hand end. The pendulum will thus experience magnetic force urging it to the right. If the pendulum tends to gain upon the standard, its return from the extremities of its swing is hus opposed for a longer time than its outward movement is aided; nd if it tends to lose, the assistance to its motion lasts longer than he opposition. Its tendency to deviate from the standard clock ther way is thus checked, and the correcting action is greater as he deviation from coincidence is greater. The controlling power hus obtained is so great, that even if the electrical connections are terrupted during several consecutive beats, the accumulated errors ll be completely wiped off after the connections are restored.

of this axis, engages with a very coarse-toothed wheel T′, set on the same axis as the type-wheel, and pushes it a little forward or backward without detaching it from the driving gear. Small discrepancies between the velocities of the type-wheel and chariot are thus corrected as often as a letter is printed. This contrivance serves to keep the receiving instrument from gaining or losing on the sending instrument during the transmission of a message. The type-wheel of the receiving instrument must be adjusted before the message begins, so as to make the two instruments start at the same letter.

592. Electro-chemical Telegraph.—Suppose a metallic cylinder, permanently connected with the earth, to be revolving, carrying with it on its surface a strip of paper freshly impregnated with cyanide of potassium. Also suppose a very light steel point permanently connected with the line-wire, and resting in contact with the paper. Every time that a current arrives by the line-wire, chemical action will take place at the point of contact, and the paper at this point will be discoloured by the formation of prussian blue. This is the principle of Bain's electro-chemical telegraph, which leaves a record in the shape of dots and dashes of prussian blue. The apparatus for sending signals is the same as in Morse's system. The paper must not be too wet, or the record will be blurred; neither must it be too dry, for then no record will be obtained.

593. Autographic Telegraph.—An autographic telegraph is one which produces at the receiving station a fac-simile of the original despatch. The best known instruments of this class are those of Bonelli and Caselli. We shall describe the latter.

At the sending station a sheet of metallized paper, with the despatch written upon it in a greasy kind of ink, is laid upon a cylindric surface M (Fig. 530). At the receiving station there is a

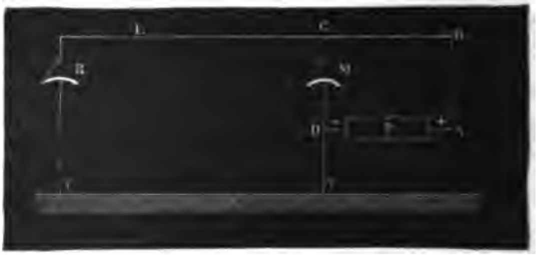


Fig. 530.—Principle of Caselli's Telegraph.

similar cylindric surface R, on which a sheet of Bain's chemical paper is laid. Two styles, driven by pendulums which oscillate with exact synchronism, move over the surfaces of the two sheets, describing upon them very close parallel lines at a uniform distance apart, both

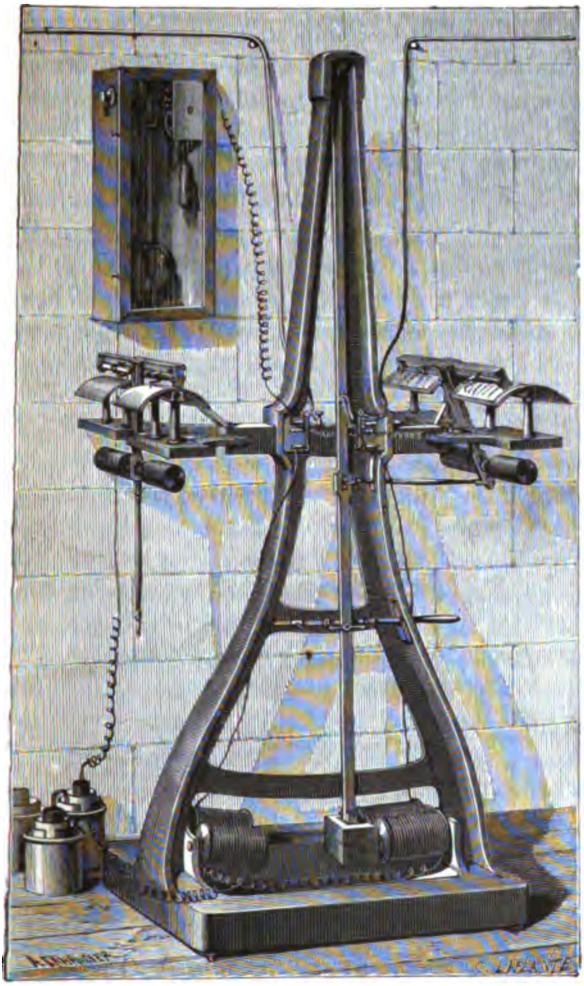


Fig. 531.—Caselli's Telegraph.

s being in permanent connection with the line-wire. The cur- s furnished by the battery P at the sending station. When the is on a conducting portion of the paper M, the current takes urse of least resistance ABCD, no sensible portion of it going position. Thus if Fig. 536 represent a line of particles traversed by the current from left to right, there will be a continual stream of hydrogen-particles along this line from left to right, and a stream of oxygen[1] particles from right to left. The hydrogen of molecule 1 will combine with the oxygen of molecule 2 to form a new molecule 1′; the hydrogen of molecule 2 will combine with the oxygen of molecule 3 to form a new molecule 2′, and so on. The oxygen of molecule 1 is given off at the left-hand extremity, which we suppose to be the point of contact with one of the strips of platinum, and the hydrogen of molecule 6 at the other strip. The molecules 1′, 2′, 3′ . . . are then in their turn decomposed to form a new set. In actual cases, the number of molecules, instead of being only 6 as represented in the figure, is of course many millions.

598. Electrolysis of Binary Compounds.—When a compound formed by the union of a metal with some other elementary substance is submitted to electrolysis, the metal always comes to the negative pole. It was in this way that several of the metals were first obtained from their oxides by Sir Humphrey Davy. Potassium, for example, was obtained by placing a piece of potash on a platinum disk connected with the negative pole of a battery of 250 cells, and then applying a platinum wire connected with the positive pole to its upper surface. The potash, which had been allowed to contract a little moisture from the atmosphere, in order to give it sufficient conducting power, soon began to fuse at the points of contact of the electrodes. A violent effervescence occurred at the upper or positive electrode; while at the lower surface small globules appeared resembling quicksilver, some of which instantly burst into flame, while others merely became tarnished and afterwards coated over with a white film.

The earthy oxides, such as magnesia and alumina, are more difficult of reduction than the alkalies potash and soda, and have never yet been electrolyzed. The metals magnesium and aluminium have, however, been obtained by the electrolysis of their chlorides. Chloride of magnesium, for example, is melted by heating it to redness in a porcelain crucible, the upper part of which is divided into two compartments by a porous partition. Pieces of carbon are employed as the terminals of the battery, and are inserted one in each of these compartments, the piece which is to serve as the negative electrode being notched like the edge of a saw, with its teeth pointing downwards ready to intercept the metal in its upward course—the metal being specifically lighter than its chloride. It was by the electric current that Deville in 1854 succeeded in preparing aluminium, which has exhibited such unexpected and interesting properties.

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[1] According to modern theories, the anion is not oxygen, as here stated, but a compound of oxygen and sulphur. (See § 600.) The explanation as it stands in the text represents the views held by Faraday and his contemporaries.

For the electrolysis of binary compounds soluble in water, their solutions are frequently employed, but these should in general be *highly* concentrated.

599. Electrolysis of Salts.—When a salt of any of the less inflammable metals is submitted to electrolysis, a continual deposition of the metal is observed on the negative electrode; while, at the positive electrode, oxygen is disengaged, and acid set free. These effects occur, for example, if platinum electrodes are plunged in a solution of sulphate of copper. If an oxydizable metal is employed as the positive electrode, the oxygen will combine with it instead of being given off. If the metal employed be copper itself, the oxide of copper formed will combine with the acid, and a quantity of sulphate of copper will be formed exactly equal to that which undergoes decomposition. The solution thus remains constantly in the same state as regards saturation, and the copper deposited on the negative electrode is exactly compensated by that dissolved off the positive electrode.

When a salt of one of the alkaline metals is electrolyzed, the appearances presented are different from those which we have just been describing; and they for a long time received an erroneous interpretation. Let the tube represented in Fig. 537 be charged with solution of sulphate of soda coloured with syrup of violets. If the current be hen passed, after the lapse of some time, red tinge will be observed in the liquid round the positive electrode, and a green nge around the negative. This shows the presence of free acid the positive and alkali at the negative pole. Oxygen is also und to be evolved at the positive and hydrogen at the negative le. The interpretation for a long time given to these results was, at, in the electrolysis of a salt, the acid went to the positive and e base to the negative pole. In the case of a metallic salt, such as

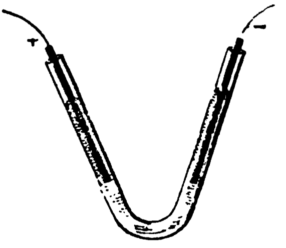

Fig. 537.—Decomposition of Salts.

sulphate of copper, it was supposed that the oxide of copper which forms the base, underwent a further electrolysis, resulting in the appearance of the metal at the negative and of the oxygen at the positive pole. This complicated hypothesis of two successive electrolyses is entirely gratuitous. It was in fact simply framed to suit the chemical theories which regarded a salt as the result of the union of an acid and a base. It is now believed that the electrolysis of sulphate of soda is single, that it consists in resolving the salt into sodium and an unstable compound of sulphur and oxygen, SO_4, to which the name of sulphion has been given. The sodium unites with the oxygen of the water at the negative pole, thus forming soda and liberating hydrogen; and the sulphion unites with the hydrogen of the water at the positive pole, forming sulphuric acid and liberating oxygen. These chemical actions immediately consequent upon electrolysis are called *secondary actions.*

600. Electrolysis of Water.—What we have just said of salts, applies equally to the oxygen acids. Thus in the electrolysis of sulphuric acid (SO_3, H_2O) the hydrogen, which is a kind of gaseous metal, goes to the negative pole, while the substance SO_4 goes to the positive pole, and there unites with the hydrogen of the water to form the primitive compound, setting oxygen free.

It is probable that what is called the electrolysis of water is really an indirect action of this kind; that, in fact, it is not the water, but the acid contained in it that is electrolyzed, decomposition and recomposition of acid being in continual progress.

Be this as it may, voltameters are frequently employed for the measurement of currents, from which use indeed they derive their name. This mode of measuring a current is due to Faraday. The quantity of electricity which passes is measured by the quantity of gas evolved; and this is best determined by measuring the hydrogen, in the first place because of its greater volume, but still more because it is less liable than oxygen to be absorbed by water. It is important that the temperature at which the operation is conducted should not be too low; for if it be under 20° C., the water may become so strongly impregnated with oxygen as to be able to take up some of the hydrogen which is separated at the negative pole, and reduce it again to the form of water.

Voltameters have the disadvantage, as compared with galvanometers, of introducing opposing electro-motive force into the circuit as well as a large amount of resistance.

601. Definite Laws of Electrolysis.—The following principles were completely established by Faraday's researches:—

1. The quantity (*i.e.* mass) of a given electrolyte decomposed by a current is simply proportional to the quantity of electricity which passes through it,—in other words, is jointly proportional to the *strength* of the current and the time that it lasts; or, the rate of decomposition of a given electrolyte is simply proportional to the *strength* of current, and is independent of all other circumstances.

2. The quantities (masses) of different electrolytes decomposed by the same quantity of electricity are directly as their chemical equivalents.[1]

These laws can be extended to the cells of the battery themselves, if we pay proper attention to the signs of the quantities involved. The essential difference between a cell of the battery and a decomposing cell included in the circuit is, that the former contributes positive, and the latter negative electro-motive force to the circuit; and if one of the cells of the battery be reversed, so that the current travels through it not from zinc to copper as usual, but from copper to zinc, it immediately becomes a decomposing cell with electromotive force opposing that of the circuit. The amount of chemical *combination* that takes place in a battery cell (or the excess of combination over decomposition, if both are going on, as in a Daniell's cell, where sulphate of copper is decomposed) is chemically equivalent to the decomposition that occurs in any one decomposing cell in the same circuit. This is on the supposition that no local action (§ 521) is allowed to take place in the battery. Keeping Grotthus' hypothesis in view, we may therefore assert that the total chemical action is the same in amount for all sections of the current, whether hese sections are taken in battery cells or in decomposing cells; but is *opposite* in sign, according as the sections are made across cells which assist, or which oppose the current by their electro-motive orce.

If a current generated by a battery consisting of several cells ranged in a series, is passed through a succession of decomposing lls, one containing acidulated water, another chloride of lead, other protochloride of tin in a state of fusion, and another a concentrated solution of nitrate of silver, then for 65 parts by weight of zinc dissolved in any one cell of the battery, 2 parts of hydrogen will be evolved from the acidulated water, and 207 parts of lead, 118 of tin, and 216 of silver, from the other electrolytes respectively; these numbers being proportional to the chemical equivalents of zinc, hydrogen, lead, tin, and silver.

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[1] This law applies directly, and without exception, to the decompositions effected by the ct action of the current. It is not always (though usually) applicable to the final ults of electrolysis, when secondary actions come into play. In certain cases, for mple, the final result will be exactly double what the law would give.

If several cells containing the same electrolyte are placed in different parts of the circuit, so as to be traversed in succession by the same current, the same amount of decomposition will be effected in them all; and this amount for each cell will be the full equivalent of the action in each cell of the battery.

In order to effect a given amount of decomposition in a given cell, with the smallest possible consumption of zinc in the battery, the number of battery cells employed should be the smallest that will suffice to effect the operation at all; in other words, the resultant electro-motive force in circuit should barely exceed zero. This is obvious from considering that the quantity of electricity required to effect the given operation is irrespective of the number of cells of the battery, and is absolutely constant. The quantity of zinc dissolved *in each cell* is therefore constant also, and hence the whole zinc dissolved is proportional to the number of cells. If we employ more cells than are necessary, we shall effect the required operation more quickly, but at the expense of an extra consumption of zinc.

Time may be saved by increasing the number both of the decomposing cells and of the battery cells. By doubling them both, we shall double the electro-motive force and also the resistance, so that the current will be unaltered. The chemical action in each cell will therefore be the same as before; and as the number of decomposing cells is doubled, a double quantity of the given electrolyte is decomposed.

602. Polarization of Electrodes.—When electrodes have been doing duty for some time in the decomposition of an electrolyte, if we detach them from the battery, plunge them in a conducting liquid, and connect them externally by a wire, we shall find that a current is circulating in the opposite direction to the original current.

Suppose, for example, that the Bunsen battery M (Fig. 538) has been employed for electrolyzing sulphate of potash by means of the electrodes A B, A being the positive, and B the negative electrode, the current flows through the decomposing cell from A to B. Now let the battery be removed, and the electrodes connected externally by the wire N. A current will now pass through the liquid *from B to A*, completing its circuit through the wire.

The origin of this current can be explained in the following way. During the process of decomposition, potash collects on the electrode

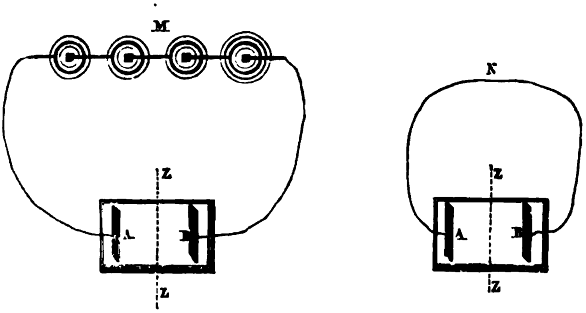

Fig. 538.—Polarization of Electrodes.

B, and sulphuric acid on the electrode A. When the connecting wire is substituted for the battery, the two substances which have been forced to separate begin to unite again; and as their tendency to do so produced an opposing electro-motive force while the direct current was passing, this tendency is now manifested in the actual production of a reverse current and reverse transport of elements. It is not necessary that the deposition of the two substances on the plates should have been brought about by electrolysis. A similar result will be obtained if the plates be coated with the two substances in any other way

Grove's *gas-battery* is constructed by immersing two platinum plates in a vessel of acidulated water, the upper halves of the plates being surrounded, one by oxygen, and the other by hydrogen, inclosed in inverted tubes. External communication must be made etween the plates by means of wires sealed into the upper ends of he tubes, and a current will pass through the circuit thus completed; r, if there are more cells than one, the hydrogen plate of each cell ust be connected with the oxygen plate of the next, and the first nd last plates must also be connected. In each cell, union of the vo gases will gradually take place through the acidulated water, nd the direction of the current will be opposite to that which would store the gases to their places if the cell were used as a voltameter; at is to say, it will be through the liquid from the plate in hydrogen the plate in oxygen. The plates employed for this purpose are usually covered with a deposit of finely-divided platinum, as the great extent of surface thus obtained conduces to rapid action.

Ritter's *secondary pile* consists of a number of discs all of the same metal separated by pieces of moistened cloth. If its two extremities be connected for a few seconds with the poles of a battery, the pile will be found to have acquired the power of producing, for a short time, a current opposite to that of the battery.

603. Feeble Currents through Electrolytes.—Liquids capable of undergoing electrolysis never conduct electricity without being electrolyzed. Some exceptions were at one time supposed to exist in cases of very feeble currents; but experiments in vacuo have shown that these are explained by the re-absorption by the liquid of the gases evolved. Under the exhausted receiver of an air-pump the gases are actually given off. There appears to be no real exception to the rule that electricity, in traversing an electrolyzable liquid, always produces its full equivalent of decomposition.

604. Electro-metallurgy.—The applications of electrolysis to the arts are numerous and important. They are of two kinds. In one, the electrolytic deposit is intended as a permanent covering, and should adhere perfectly so as to form one mass with the body which it covers. In the other, the adhesion is temporary, and must not be too close, the object being merely to obtain an exact copy of the original form. Electro-plating belongs to the former class; electrotype to the latter.

605. Electro-gilding and Electro-plating.—The deposition of a coating of gold or silver on the surface of a less precious metal is merely an example of the electrolysis of a salt, as described in § 599. The metal in solution is always deposited on the negative electrode, hence we have merely to make the negative electrode consist of the article which we wish to coat. The only points to be decided practically relate to the means of making the deposit solid and firmly adherent. These ends have been completely attained by the methods patented about 1840 by Elkington in England and Ruolz in France.

The solutions are always alkaline, and usually consist of the cyanide or chloride of the metal, dissolved in an alkaline cyanide.

To prepare the *gold-bath*, 50 grammes of fine gold are dissolved in aqua regia; and the solution is evaporated till it has the consistence of syrup. Water is then added, together with 50 grammes of cyanide of potassium, and the mixture is boiled. The quantities named give about 50 litres of solution.

The negative electrode consists of the article to be gilded. The positive electrode is a plate of fine gold, which constitutes a soluble electrode, and serves to keep the solution at a constant strength. In order that the gilding may be well done, the bath must be maintained, during the operation, at a temperature of from 60° to 70° Centigrade.

Fig. 539 represents a form of apparatus which is very frequently employed. The poles of the battery are connected with two metallic rods resting on the top of the cistern which contains the bath. The articles to be gilded are hung from the negative rod. From the positive rod is hung a plate of gold, whose size should be proportional to the total surface of the articles which form the negative electrode.

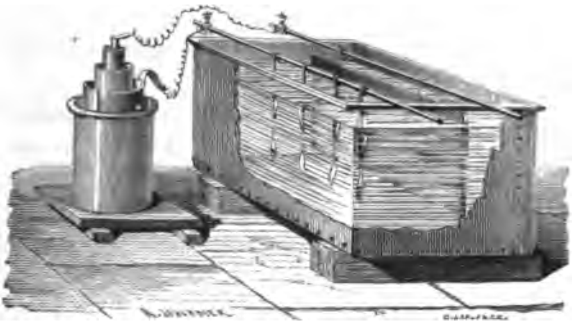

Fig. 539.—Apparatus for Electro-gilding.

The *silver bath* is a solution containing 2 parts of cyanide of silver, 10 of cyanide of potassium, and 250 of water. The operation of plating is the same as that of gilding, except that the apparatus is usually on a larger scale, and that the temperature may be lower.

In both cases the surfaces to be coated must be thoroughly leansed from grease. For this purpose they are subjected to the rocesses of pickling and dipping, which we cannot stay to describe.

Other bodies, as well as metals, can be coated, if their surfaces are irst covered with some conducting material. Baskets, fruits, leaves, c., have thus been gilded or silvered.

Similar processes are employed for depositing other metals, of hich copper is the most frequent example.

606. Electrotype.—Electrotyping consists in obtaining copper casts fac-similes of medals, engraved plates, &c., by means of electrolytic position. The first successful attempts in this direction were made about 1839 by Jacobi at St. Petersburg and Spencer in England. The art is now very extensively practised.

If a fac-simile of a medal is required, a cast is first taken of it, either in fusible alloy, plaster of Paris, or gutta-percha softened by heating to 100° C., this last material being the most frequently employed. The fusible alloy is a conductor; the other materials are not, and their surfaces are therefore rendered conducting by rubbing them over with plumbago. The mould thus prepared is made to serve as the negative electrode in a bath of sulphate of copper, a copper plate being used as the positive electrode. When the current passes, copper is deposited on the surface of the mould, forming a thin sheet, which, when detached, is a fac-simile of one side of the original medal. A similar process can be applied to the other side, and thus a complete copy can be obtained.

In operations of this kind, the bath itself is often made to serve as the battery. Fig. 540 represents such an arrangement.

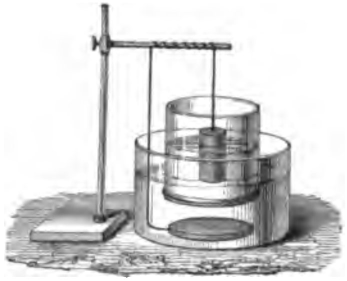

Fig. 540.—Bath and Battery in one.

In the interior of a vessel containing a saturated solution of sulphate of copper, a second vessel is supported, consisting either of porous earthenware or of a glass cylinder closed below by a membrane. In this second vessel is placed acidulated water, with a cylinder of zinc suspended in it. The mould is placed in the outer vessel under the bottom of the porous cylinder, and is connected with the zinc by a stout wire which completes the circuit. The arrangement is evidently equivalent to a Daniell's cell. The current passes through the liquids from the zinc to the mould, electrolyzing the solution of sulphate of copper; and as the metal travels with the current, it is deposited on the surface of the mould. The strength of the solution is kept up by suspending in it crystals of sulphate of copper contained in a vessel pierced with holes.

607. Applications of Electrotype.—One of the commonest applications of electrotype is to the production of copies of wood engravings. The original blocks, as they leave the hand of the engraver, could not yield a large number of impressions without being materially injured by wear. When many impressions are required, they are not taken directly from the wood, but from an electrotype taken in copper from a gutta-percha mould. The process of deposition is continued only for twenty-four hours, and the plate of copper thus obtained is very thin. It is strengthened by filling up its back with melted type-metal. Such plates will afford about 80,000 impressions, and it is from them that nearly all the illustrations in popular works are printed. Postage stamps, which must be exactly alike in order to prevent counterfeits, are also printed from electrotypes; and, on account of the great number of impressions required, the electrotypes themselves need frequent renewal; but the operations necessary for this purpose do not sensibly injure the original.

Copperplate engravings and even daguerreotypes can be very accurately reproduced in copper. No preparation of the surface is necessary, as the thin film of oxide which is present is quite sufficient to prevent the deposit from adhering too closely.

Gasaliers are usually of cast-iron coated with copper by electrolysis. The copper is not, however, deposited on the surface of the iron, as the contact of the two metals would greatly promote the oxidation of the iron, if any of it were accidentally exposed to the air. The iron is first painted over with red-lead, which, when dry, is covered with a very thin layer of plumbago to render it conducting; and it is on this that the copper is deposited.

CHAPTER LII.

INDUCTION OF CURRENTS.

608. Induced Currents.—Induced currents may be described as currents produced in conductors by the influence of neighbouring currents or magnets. Their discovery by Faraday in 1831 constitutes an epoch in the history of electrical science. We shall first describe some modes of producing them; and then state their general laws.

609. Currents induced by Commencement and Cessation of Currents.—Let two coils be wound upon the same frame B, one of them, called the secondary coil, having its ends connected with the binding-screws of the galvanometer G, while the ends of the other, which is called the primary coil, dip in two cups of mercury g g' connected with the two plates of the voltaic element P. As long as the current is passing steadily in the primary coil, the needle of the galvanometer remains undeflected; but if the current be stopped, by lifting a wire out of one of the mercury cups, the needle is immediately deflected, indicating the existence of a current in the same direction as that which

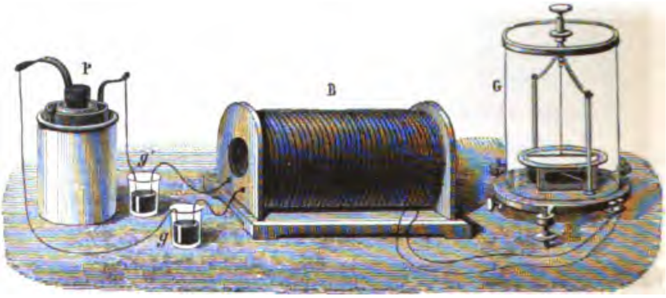

Fig. 541.—Current induced by Commencement or Cessation.

was previously circulating in the primary coil. This effect is very transitory. The needle appears to receive a sudden impulse which immediately passes away. If the current be then re-established, there is a deviation to the other side, indicating a current in the opposite direction to that in the primary coil; and this deviation, like that which occurred before, is merely the effect of an instantaneous impulse, the needle making a few oscillations from side to side, and then remaining steadily at zero. This experiment, which is substantially the same as that by which Faraday first made the discovery, establishes the following proposition:—*When a current begins to flow, it induces an inverse current in a neighbouring conductor; when it ceases, it induces a direct current; and both the currents thus induced are merely instantaneous.*

610. Currents induced by Variations of Strength of Primary Current.—Employing the same apparatus, let us, while the primary current

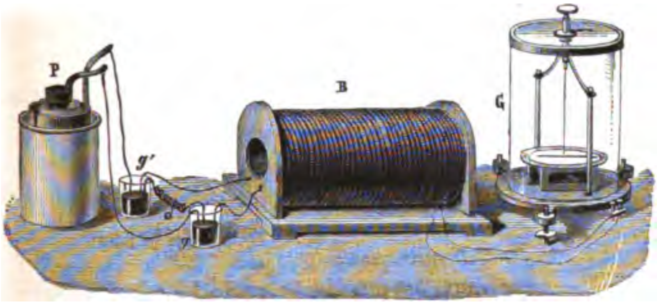

Fig. 542.—Current induced by Change of Strength.

is passing, connect the two mercury cups by the wire *d* (Fig. 542), thus dividing the circuit (§ 547), and causing a great diminution of the current in the primary coil. At the instant of making this connection, the needle of the galvanometer is affected, moving in the same direction as if the primary current were stopped; and on lifting he connecting wire out of one of the cups, so as to produce a sudden ncrease in the current in the primary coil, the needle moves in the pposite direction. *When a current receives a sudden increase, this roduces an inverse current in a neighbouring conductor; and when is suddenly decreased, a direct current is induced.*

611. Currents induced by Variations of Distance.—Currents may so be induced by change of distance between the primary and condary conductors. Let the secondary coil, for example, be hollow, as in Fig. 543, and let the primary coil, with the current passing in it, be suddenly introduced into its interior. The galvanometer will indicate the production of an inverse current in the secondary coil.

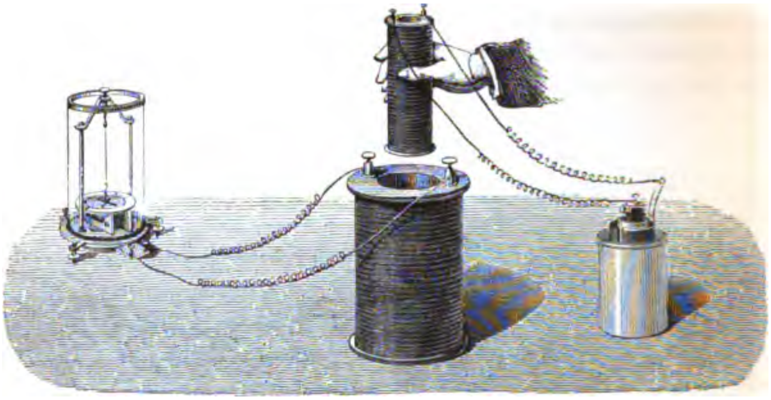

Fig. 543.—Current induced by Change of Distance.

When the needle has come to rest, let the primary coil be withdrawn, and a direct current will be indicated by the galvanometer. These currents differ from those previously mentioned in being less sudden. They last as long as the relative motion of the two coils continues. *When a conductor conveying a current approaches or is approached by a neighbouring conductor, an inverse current is induced in the latter; and when one of these conductors moves away from the other, a direct current is induced.*

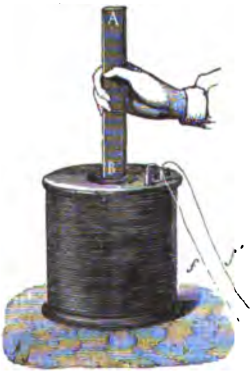

Fig. 544.—Current induced by Motion of Magnet.

612. Magneto-electric Induction.—As a current may be regarded as a magnet (§ 531 A), and a magnet may be regarded as a system of currents (§ 565), induction can be effected by a magnet as well as by a coil.

Let a hollow coil be connected with a galvanometer, and a magnet held over it, as in Fig. 544. As long as the magnet remains stationary, no current is indicated; but when one pole of the magnet is thrust into the interior of the coil, the needle is deflected by an impulse which lasts only as long as the motion of the magnet. If the magnet is allowed to remain at rest in this position, the needle, as soon as it has time to recover from its oscillations, stands at zero; but on withdrawing the magnet, another current will be indicated in the opposite direction to the former.

Currents may also be induced, with even more striking effect, by moving one pole of a magnet towards or from one end of a soft-iron bar previously placed in the interior of the coil (Fig. 545). These currents are due to the magnetism produced and destroyed in the soft iron. *When the intensity of magnetization of a piece of iron or steel undergoes changes, currents are induced in neighbouring conductors.* The directions of these currents can be inferred from the preceding rules by supposing a solenoid to be substituted for the magnet.

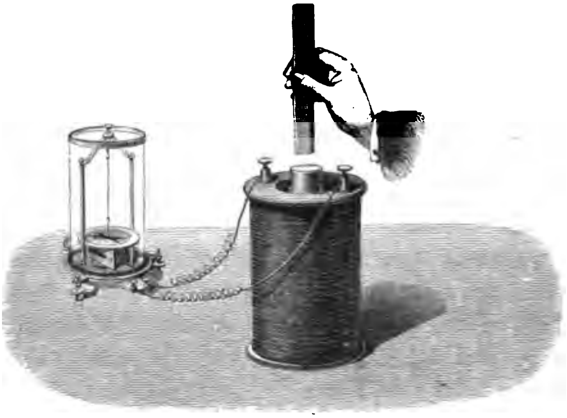

Fig. 545.—Current induced by Magnetization of Soft Iron.

613. Lenz's Law.—*The currents induced by the relative movement either of two circuits or of a circuit and a magnet are always in such directions as to produce mechanical forces tending to oppose the movement.* For example, when two parallel wires, through one of which a current is passing, are made to approach, an opposite current is induced in the other; and opposite currents by their mutual repulsion resist approach. This general law as to the direction of nduced currents was first distinctly enunciated by Lenz, a Russian philosopher.

***613 A.* Direction of Induced Currents specified by Reference to Lines f Magnetic Force.**—We have already mentioned, in connection with he mutual forces between magnets and currents (§ 531 B), that a wire conveying a current experiences force perpendicular to its length, and at the same time perpendicular to the lines of magnetic force, when placed in a magnetic field. We have seen that, if the current is from foot to head, and the lines of force (for an austral pole) run from front to back, the force experienced by the wire is a force to the right. Motion of the wire to the right will diminish this force by diminishing the current, motion to the left will increase it by increasing the current, and the amount of increase or diminution is quite independent of the original amount of current. Let the direction of the lines of magnetic force for an austral pole be called *from front to back;* then the motion of a conductor *to the right* generates a current in it *from head to foot,* and motion in the opposite direction generates an opposite current. We shall have frequent occasion to recur to this criterion of direction, which applies to every case of induced currents.

As the generation of currents by induction depends not on absolute but on relative motion, namely the relative motion of the conductor and the lines of magnetic force, the criterion of direction will take the following form when the conductor is supposed to be stationary, and the lines of force to move.[1] Let the direction of the lines of magnetic force for an austral pole be called *from front to back,* then *if the lines of force move so as to cut through the conductor from right to left, a current will be induced in the conductor from head to foot.*

If the conductor forms part of a closed circuit, we shall have a continuous current flowing through it as long as the motion lasts. If the circuit is open, there will merely be an incipient current, which, if its direction be from head to foot, will reduce the end of the conductor which we are regarding as its foot to a higher electrical potential than the other, and this difference of potential will be maintained as long as the motion lasts.

613 B. Quantitative Statements.—In order to state the quantitative laws of induced currents in the simplest and most general manner, we must employ the conception of tubes of force as explained in § 445 G (but they will now be tubes not of electrical but of magnetic force), and we must suppose them to be arranged in the equable manner described in § 445 H. That is to say, we must suppose the whole field cut up into tubes of force in such a manner that, if a cross-section (an equipotential surface as regards magnetic potential) be made in any part of the field, the number of tubes per unit of sectional area is equal to the intensity in that part of the field. It is more usual to speak of *number of lines of force* than of *number of tubes*, the convention being that each tube contains one line; but the counting of tubes rather than lines has the advantage of naturally allowing fractional parts to be reckoned, and not suggesting the idea of discontinuity.

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[1] It may be noted that when a bar-magnet is rotated on its axis, it induces no current in a neighbouring wire, inasmuch as its lines of force cut such a wire once positively and once negatively.

The tubes of force due to a magnet are to be regarded as rigidly attached to the magnet, and carried with it in all its movements, whether of translation or rotation. They undergo no change of size or form unless the magnet itself undergoes changes in its magnetization.

These conceptions being premised, the quantitative laws of induced currents can be stated with great simplicity and complete generality.

1. When a conductor is moved in a magnetic field, the ELECTROMOTIVE FORCE generated by the motion is equal to the number of tubes which the conductor cuts through per unit time.

2. If the conductor forms part of a closed circuit, the CURRENT generated in the circuit is the quotient of the number of tubes cut through per unit time, by the resistance of the circuit; and, lastly,

3. The whole QUANTITY of electricity conveyed by the current is the quotient of the number of tubes cut through, by the resistance of the circuit. The quantity of electricity conveyed by a current of brief duration is measured by observing the swing of a galvanometer needle. It is proportional to the greatest deviation of the needle from zero, provided that this deviation is small, and that the duration of the current is less than that of the swing. When experiments on induced currents are made under these conditions, it is found that the deviation of the needle is not sensibly increased by moving the inducing magnet or coil more rapidly, as long as the ground moved over is the same.

The dependence of the quantity of electricity induced upon the number of tubes cut through, was discovered by Faraday, who estabished it experimentally by moving a loop of wire in various ways n the vicinity of a magnet. The three foregoing laws were all, in act, substantially established by the series of researches in which hese experiments occur.[1]

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[1] *Researches*, vol. iii. series xxviii.

In counting the tubes cut through, it is necessary to attend to the direction of the current due to the cutting of each tube. Those tubes which are so cut as to give currents in one direction round the circuit (when tested by the criterion of § 613 A) must have one sign given them, and those which give currents in the opposite direction must be reckoned as of the opposite sign. It is in every case the algebraic sum that is to be taken; and if a tube is cut once positively and once negatively, it may be left out of the reckoning.

613 C. Relation of Induced Current to Work done.—The direction of the force experienced by any straight portion of a circuit conveying a current in a uniform field, is perpendicular at once to its own length and to the lines of force. If L be its length, a its inclination to the lines of force, C the current flowing through it, and I the intensity of the field, the magnitude of the force will be C I L sin a, and the work done in any movement of translation will be the product of this force by that component of the distance moved which lies in the direction of the force. All this may be more concisely expressed by saying that the work done by magnetic force is the product of C by the number of tubes of force which the wire cuts through in its motion, any tubes which are cut in a direction opposed to that of the force which the wire experiences being counted negatively. *When a closed circuit conveying a current is moved in any magnetic field, the work done upon it by magnetic force is the product of the strength of current by the algebraic number of tubes cut through.*

Comparing this law with the first of the three laws given in the preceding section, we see that the work done per unit time is the product of the actual current and the induced electro-motive force. The original current is *increased* or diminished by the motion, according as work is done *against* or by the magnetic forces of the field.

This result can be shown to be in harmony with Joule's law (§ 571), according to which the energy-value of a current, for each unit of time that it lasts, is the product of the current by the electromotive force producing it. For, if C and E denote the actual amounts of current and electro-motive force, and $C_0 E_0$ the values which these elements would have if there were no motion, the energy required from without to produce the motion is, by the law we are now stating, $C(E-E_0)$, and the energy represented by the additional consumption of zinc in the battery is $(C-C_0)E_0$, since, with a given number of cells, the zinc consumed is simply proportional to the current. The sum of these two expressions is $CE - C_0E_0$; which, by Joule's law, represents the increase in current energy.

When there is no current in the circuit except the induced current, work must always be done against the forces of the field to an amount precisely equal to CE, the energy-value of the current.

613 D. Movement of Lines of Force with Change of Magnetization.—As long as a piece of iron or steel remains unchanged in its magnetization, its tubes of force are to be conceived of as a rigid system rigidly connected with it. When the intensity of magnetization is increased, new tubes are added and the old ones are crushed together. The new tubes are to be regarded as coming into existence at the magnetic axis of the magnet, and pushing the old ones further away from the axis. When the intensity of magnetization falls off, a reverse motion occurs, and the axis absorbs those tubes which lie next it.

Similar remarks apply to changes of strength in a current. The lines of magnetic force due to a current in a wire are circles, and the tubes of force are rings, having the wire for their common axis. When the current receives an increase of strength, the new rings must all be conceived of as starting from the wire, and pushing out the old rings before them, and on the diminution or cessation of the current a reverse movement occurs.

When a current suddenly commences in a wire, or a piece of soft iron is suddenly magnetized, a neighbouring wire is cut through by as many tubes of force, and subjected to the same inductive influence, as if it were suddenly moved up from a great distance into its actual position. The experimental results described in § 609–612 are thus only particular cases of the general principles of § 613 A, B.

613 E. Motion in Uniform Field.—If we define a uniform magnetic field as a field of uniform intensity, it can be shown to follow, as a mathematical consequence, that the equipotential surfaces must be parallel planes, and the lines of force parallel straight lines. The tubes of force will, of course, be of uniform section, and the number of tubes per unit of cross-section will be equal to I, the intensity of the field. The electro-motive force generated by the motion of a straight wire of length L in such a field, with a velocity of translation V, being equal to the number of tubes cut through in unit time, will be LVI, if the length of the wire and the direction of motion are perpendicular to each other and to the lines of force. For any other position of the wire, and for any other direction of motion, the number of tubes cut through will evidently be less. If the length of the wire is parallel to the lines of force, no tubes will be cut through, whatever be the direction of motion; and if the direction of motion be parallel to the lines of force, no tubes will be cut through, whatever be the position of the wire. In these two cases, then, there is no generation of electro-motive force tending to produce a current along the wire.

Terrestrial magnetism furnishes us with an example of a uniform field, so long as we confine our attention to a space of moderate dimensions, such as the interior of a room.

613F. Unit of Resistance.—Units of *length*, *mass*, and *time*, having been selected, unit *force* is defined as that which, acting on unit mass for unit time, generates unit velocity.

A magnetic pole of unit strength, or a unit *pole*, is defined as that which attracts or repels an equal pole at unit distance with unit force.

Unit *intensity of field* is defined as the intensity at a place where a unit pole experiences unit force.

A unit *current*, or a current of unit strength, is one which, for each unit of its length, affects a unit pole at unit distance with unit force. In passing through a circular coil of unit radius and length l, the force which it exerts on a unit pole at the centre is l.

Unit *electro-motive force* is the electro-motive force existing in a circuit in which unit current does unit work in each unit of time; and unit *resistance* is the resistance of a circuit in which unit electro-motive force would produce unit current.

The course of the above investigation shows that the units of length, mass, and time are sufficient to determine all the other units mentioned. It can further be shown[1] that the unit of resistance is independent of the unit of mass, and depends only on the units of length and time, being directly as the unit of length, and inversely as the unit of time—a property which is also characteristic of the unit of velocity. Hence a resistance, like a velocity, can be adequately expressed in *metres per second*. The unit of resistance now commonly employed is the *ohm*, which is defined as *ten million metres per second*. The resistance of an ordinary Daniell's cell is about half an ohm. The resistance of a mile of submarine telegraph-cable is from 4 to 12 ohms.

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[1] See appendix at the end of this chapter.

614. Induction by means of Terrestrial Magnetism.—If a wire ring, or any other form of closed circuit, receives a movement of translation in a uniform field, no current is generated, because the same number of force-tubes are cut negatively as positively. Whatever currents are generated by the motion of a closed circuit in the terrestrial magnetic field, must therefore be due solely to rotational movements. Suppose the circuit to consist of a single circle of wire, and let it be initially placed so that its plane is perpendicular to the dipping-needle, and therefore perpendicular to the lines of magnetic force. In this position, the number of force-tubes which it incloses is equal to the product of the inclosed area by the total intensity of terrestrial magnetic force, that is to $\pi r^2 \mathrm{I}$, I denoting this intensity, and r the radius of the circle. Now let the ring rotate through 180° about any diameter, so that it comes back into its original place, but facing the opposite way. During this semi-revolution, each half of the ring has cut through all the tubes which passed through the ring, and though in one sense the two halves have been cutting the tubes in opposite directions, the application of the criterion of § 613 B shows that the resulting currents are in the same direction round the circuit. The number of tubes cut through is therefore to be reckoned as $2\pi r^2 \mathrm{I}$, and the quotient of this by the time occupied in a semi-revolution is the average electro-motive force (§ 613 E). If the rotation be *uniform*, the actual electro-motive force is greatest in the middle of the semi-revolution, and is zero at its commencement and termination. During the other half-revolution the circumstances are precisely the same, except that the two halves of the ring have changed places. If we compare the currents in two positions of the ring which differ by 180°, we see that the current round the ring has the same direction in space, but opposite directions as regards the ring itself.

If, instead of a single ring of wire, we have a circular coil consisting of any number of convolutions, with its two ends united, the same principles apply. If there are n convolutions, the electro-motive force will be n times greater than with one, but as the resistance is also n times greater the strength of current is the same.

In the apparatus called *Delezenne's Circle*, a coil of wire revolves about a diameter, but the two ends of the coil, instead of being directly united, are so connected with the two ends of the axis of rotation that the circuit is completed through a galvanometer. On rotating the coil rapidly by means of a handle provided for the purpose, a current is indicated by the galvanometer, and this current is found to be strongest (for a given rate of rotation) when the axis is perpendicular to the dipping-needle. If the axis is inclined at an angle θ to the dipping-needle, the current is proportional to $\sin\theta$; and if the axis is parallel to the dipping-needle there is no current at all. For a given position of the axis, the current varies directly as the speed of rotation. When the time of a revolution is only a small fraction of the time in which the needle would oscillate, the variations of electro-motive force, and consequently of current, which take place during a revolution, have not time to manifest themselves, and the deflection of the needle is that due to the average current. It is necessary, however, that a commutator be employed to prevent the reversal of the current at each half-revolution. The proportionality of the current to $\sin\theta$ is easily inferred from the principles of the foregoing sections; for if the plane of a circle, instead of being perpendicular to the lines of force, is inclined to them at an angle θ, the number of force-tubes which it incloses will be not $\pi r^2 I$, but $\pi r^2 I \sin\theta$.

614 A. British Association Experiment.—The experiments upon which the present standards of resistance depend for their authority, were conducted by a committee of the British Association in 1862. A circular coil of wire, with its ends joined, was made to revolve rapidly, at a measured rate, about a vertical axis; and the current induced was measured by the deflection of a magnetized needle suspended, within a glass case, in the centre of the coil. The part of the earth's magnetic force which comes into play in this arrangement, is only the horizontal component, or $I\cos\delta$, δ denoting the dip; and it is worthy of remark that variations in the horizontal intensity do not alter the deflection of the needle, since they affect to the same extent the amount of the induced current, and the terrestrial couple on the needle tending to resist deflection.

All the other elements involved were determined by observation, and hence the value of R in metres per second was calculated. By comparing the resistances of other coils with that of the coil used in this experiment (a comparison easily made by ordinary methods), their values in metres per second were at once determined; and it was easy to construct a resistance-coil of ten million metres per second, or any other desired amount of resistance. Standard resistance-coils are usually made of German silver; this material being selected on account of the smallness of its temperature correction. All metals have their resistances increased by heat, and a standard coil can therefore only be correct at one particular temperature.

615. Induction of a Current on Itself: Extra Current.—If two portions of the same wire are side by side, the sudden commencement or cessation of a current in one, induces a current in the other, just as if they were portions of two unconnected circuits. An action of this kind occurs whenever a current commences or ceases in a coil, each convolution exercising an inductive influence on the rest. This action is called the *induction of a current upon itself*, and the current due to it is called an *extra current.*

The extra current on the commencement of the primary current is inverse, and merely acts as a hindrance to commencement; but the extra current on the stoppage of the primary current is direct, and is often a strongly-marked phenomenon. Hence it is that, with batteries of ordinary power, a spark is obtained on breaking, but not on making connection. The spark is particularly brilliant when a coil of many convolutions is included in the circuit, and especially if this coil incloses a core of soft iron. If an observer holds in his hands two metallic handles permanently connected with the two ends of such a coil, and if the circuit of the battery is alternately made and broken, he will receive a shock from the extra current at each interruption. If the interruptions succeed each other rapidly, the physiological effect may become very intense. Many of the machines employed for medical purposes are constructed on this plan.

Special contrivances are provided for producing a rapid succession of interruptions at regular intervals. They are called *rheotomes* or *contact-breakers.* Sometimes they consist of toothed wheels turned by hand,—sometimes of vibrating armatures moved automatically.

616. Ruhmkorff's Induction-coil.—Induced currents capable of prolucing very striking effects are furnished by the apparatus first uccessfully constructed by Ruhmkorff, and hence known as Ruhmorff's coil.

It contains two coils of wire, one of them forming part of the rcuit of a battery, and called the primary coil; while in the other, lled the secondary coil, the induced currents are generated. In the is of the coils is a bundle of stout straight wires of soft iron, with disc of the same material at each end, to which the wires are ited. Around this core is wound the primary coil, consisting of opper wire about two millimetres in diameter. The ends of this wire are shown at *f* and *f′*. The secondary coil consists of much finer wire (about a quarter of a millimetre in diameter) and of much greater length. In large instruments the primary coil may have a length of 80 metres, and the secondary a length of 150 kilometres (94 miles). Special precautions must be taken to insulate the different convolutions of the secondary coil from one another, and from the primary coil. The two ends of the secondary wire are at the binding-screws A, B, which are supported on glass pillars. It is obvious that if currents are alternately passed and stopped in the primary coil, there will be an alternate generation of currents (or at all events of electro-motive forces) in opposite directions in the secondary coil. The action of the core is similar to that of the soft-iron bar in Fig. 545, and its inductive effect is always in the same direction as that of the primary coil, for the primary coil may itself be regarded as a temporary magnet with its poles turned the same way as those of the core.

The successive makes and breaks are effected automatically in various ways. In small instruments the arrangement adopted is usually the same as that of the vibrating alarum described in § 587; but for large instruments Foucault's contact-breaker is preferred. It is represented in its place in Fig. 547.

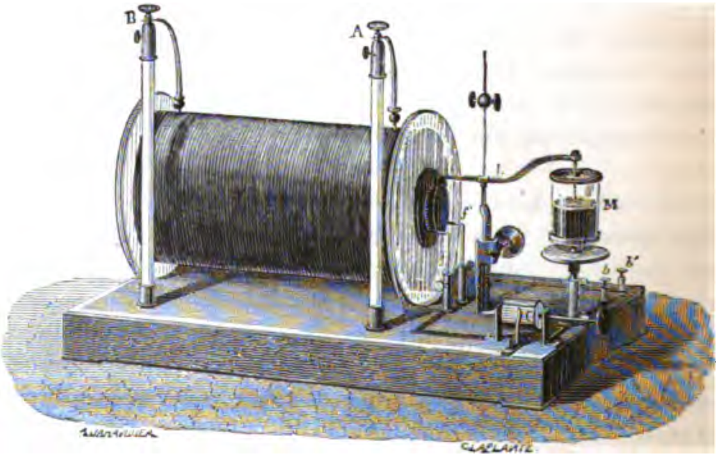

Fig. 547.—Ruhmkorff's Coil.

The wires from the battery are attached at *b* and *b′*. The curren entering for example at *b*, passes to the commutator C, and thenc through a brass bar let into the table, to the end *f* of the primar coil. Having traversed this coil, it comes out at f', and is conducted to a vertical pillar, carrying at its upper end a spring, to which the transverse lever L is attached. One end of the lever carries a point which just dips in the mercury of the vessel M, the bottom of which is metallic, and is in communication with b'. The other end of the lever carries a small armature of soft iron just above the end of the core.

When the current passes, the core becomes magnetized and attracts this armature, thus lifting the point at the other end of the lever out of the mercury and breaking circuit. The core being thus demagnetized, the elasticity of the spring releases the armature, and the point again dips in the mercury, and completes the circuit. A thin layer of absolute alcohol is usually poured on the surface of the mercury, and serves, by its eminent non-conducting power, to make the interruptions and renewals of the current more sudden.

The *commutator* C is a frequent appendage to electrical apparatus, its office being to stop the current from passing, or to make it pass in either direction at pleasure. As fitted to Ruhmkorff's coil, it has usually the form represented in end view and bird's-eye view in the two parts of Fig. 548. There is a cylinder of insulating material turning by means of metallic axle-ends on insulating supports. One of the axle-ends is connected by means of the screw g with the brass plate C on the surface of the ylinder. A similar plate C′ on the pposite side is in like manner peranently connected with the other le-end by the screw g'. These two lates CC′ leave between them a considerable portion of the insulating surface of the cylinder uncovered. In the position represented in Fig. 548, the two binding-screws A A′ are connected respectively ith the two axle-ends. If the commutator were turned (by its illed head) through 180°, these connections would be reversed; and

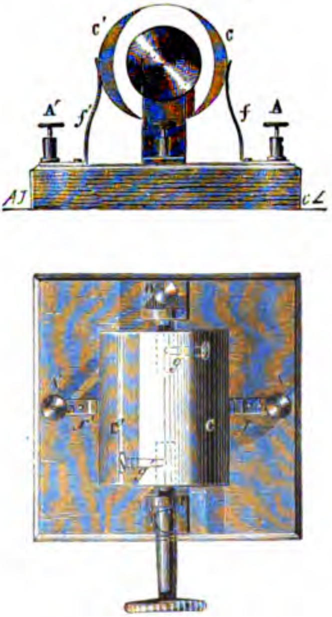

Fig. 548.—Commutator.

if it were turned through 90°, the connections would be interrupted, as the contact-springs *ff'* would bear against the uncovered portions of the insulating cylinder. The milled head is of course insulated from the axle-ends so as to protect the operator.

617. Spark from Induction Coil.—When the ends of the secondary coil are connected, currents traverse it alternately in opposite directions, as the primary circuit is made and broken. These opposite currents convey equal quantities of electricity, and if they are employed for decomposing water in a voltameter, the same proportions of oxygen and hydrogen are collected at both electrodes. If, however, the ends are disconnected, so that only disruptive discharge can occur between them, the inverse current, on account of its lower electro-motive force, is unable to overcome the intervening resistance, and only the direct current passes (that is, the current produced by breaking the primary circuit). The sparks are from an inch to about 18 inches long, according to the size and power of the apparatus, and exhibit effects comparable to those obtained by electrical machines. A Leyden battery may be charged, glass pierced, or combustible bodies inflamed.

The great electro-motive force of the induced current, which enables it to produce these striking effects, depends on the great number of convolutions of the secondary coil, and on the suddenness of the interruptions of the primary current. The quantity of electricity which passes through the secondary coil depends on the product of the number of convolutions by the number of tubes of force which cut through them (§ 613 F), and is the same whether the cessation be sudden or gradual; but the electro-motive force varies inversely as the time occupied.

The discharges from a Ruhmkorff's coil become more violent and detonating if the two electrodes are connected respectively with the two coatings of a Leyden jar or other condenser. An apparatus consisting of numerous sheets of tin-foil separated by oiled silk (alternate sheets of foil being connected) is frequently employed for this purpose, and is placed beneath the instrument so as to be out of sight.

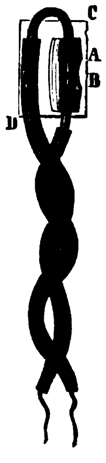

Fig. 549.
Statham's Fuse.

Induction coils are often used for firing mines, by means of Statham's fuse, which is represented in the annexed figure (Fig. 549). Two copper wires covered with gutta-percha have their ends separated by a space of a few millimetres, and inclosed in a little cylinder of gutta-percha containing sulphuret of copper. This, again, is inclosed in a cartridge, C D, which is filled up with gunpowder. The two wires are connected with the two ends of the secondary coil, and when the instrument is set in action, sparks pass between the ends A, B, heating the sulphuret of copper to redness, and exploding the powder.

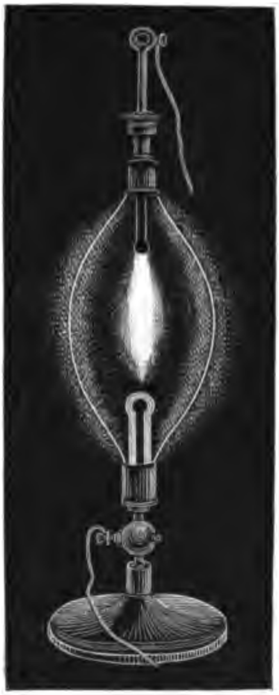

Fig. 550.—Electric Egg.

618. Discharge in Rarefied Gases.—When the ends of the secondary coil are connected with the electrodes of the electric egg (Fig. 550), which has first been exhausted as completely as possible by the air-pump, a luminous sheaf, of purple colour, is seen extending from the positive ball to within a little distance of the negative ball. The latter is surrounded by a bluish glow. The blue and purple lights are separated by a small interval of darkness. If other gases are used instead of air, the tints change, but there is always a decided difference of tint between the positive and negative extremities. By the aid of the commutator it is easy to reverse the current, and thus produce at pleasure an interchange of the appearances presented by the two terminals.

If, before exhausting, we introduce into the egg a little alcohol, turpentine, or other volatile liquid, the light presents a series of bright bands alternating with dark spaces. Plate II. Fig. 1 represents these stratifications as seen in vapour of alcohol.

The phenomenon of stratification is seen to more advantage in ong tubes than in the electric egg; and the presence of alcoholic or other vapour may be dispensed with if the exhaustion be carried ufficiently far, as in the tubes constructed by Geissler of Bonn, which contain various gases very highly rarefied, and have plainum wires sealed into their extremities to serve as electrodes. our such tubes are represented in Plate II. Certain substances, uch as uranium glass, and solution of sulphate of quinine, become uminous in the presence of the electric light, and are called *fluores* *cent.* Such substances are often introduced into Geissler's tubes, for the sake of the brilliant effects which they produce.

619. Action of Magnets on Currents in Rarefied Gases.—The luminous discharges in Geissler's tubes are, like the *voltaic arc,* veritable currents. They are capable of deflecting a magnetized needle, and are themselves acted on by magnets, as in the following experiment. A soft-iron rod (Fig. 551) is fitted in the interior of a glass vessel from which the air can be exhausted, and is coated with an insulating substance to prevent discharge between it and a metallic ring which surrounds it near its lower end. When the terminals of a battery are connected, one with this ring, and the other with the upper end of the apparatus, a luminous sheaf extends from the summit towards the wire ring, and surrounds the soft iron. If, while things are in this condition, we place beneath the apparatus one pole either of a permanent magnet or an electro-magnet, the soft-iron rod is magnetized, and the luminous streaks immediately begin to revolve round it, the direction of rotation being always in accordance with the rule of § 531 B.

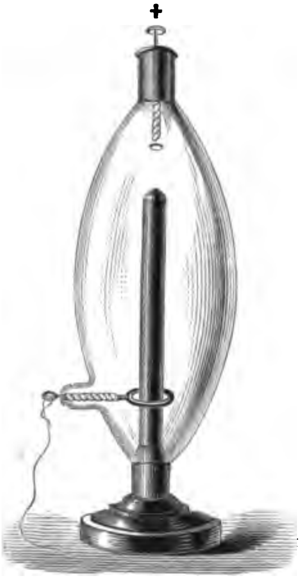

Fig. 551.—Action of Magnets on the Discharge.

620. Magneto-electric Machines.—Faraday's discovery of the induction of currents by magnets, was speedily utilized in the construction of magneto-electric machines, which, without a battery, and with no other stimulus than that afforded by the presence of a permanent magnet, enable the operator, by the expenditure of mechanical work, to obtain powerful electrical effects. The first machine of this kind was constructed in 1833 by Pixii. A magnet A was made to revolve close to a double coil B B′, in which a current was thus generated. The construction was improved by Saxton, and afterwards by Clarke, who made the magnet fixed, and caused the coil, which is much lighter, to rotate in front of it. Clarke's machine is extremely well known, being found in nearly all collections of physical apparatus.

621. Clarke's Machine.—In this machine there is a compound horse-shoe magnet fixed to a vertical support. Close in front of the magnet, near its poles, are two connected coils t, t', each containing a soft-iron core. The two cores are united by a plate of copper on the side next the magnet, and by a plate of soft iron on the remote side. The direction of winding in the two coils is the same as for an ordinary horse-shoe electro-magnet. The coils are mounted on an axis f, which passes through the support of the steel magnet, and car-

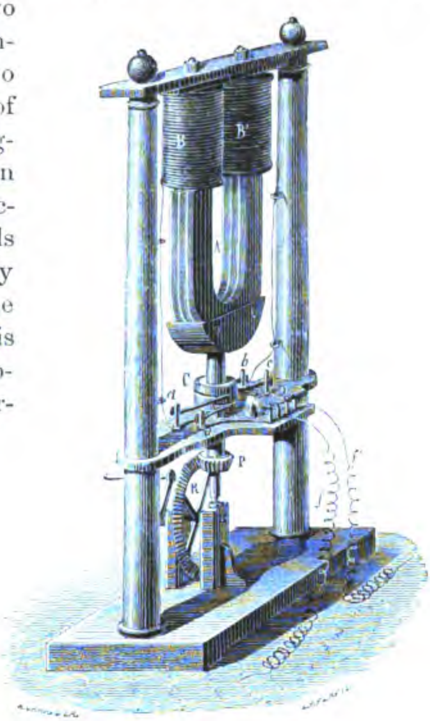

Fig. 552.—Pixii's Machine.

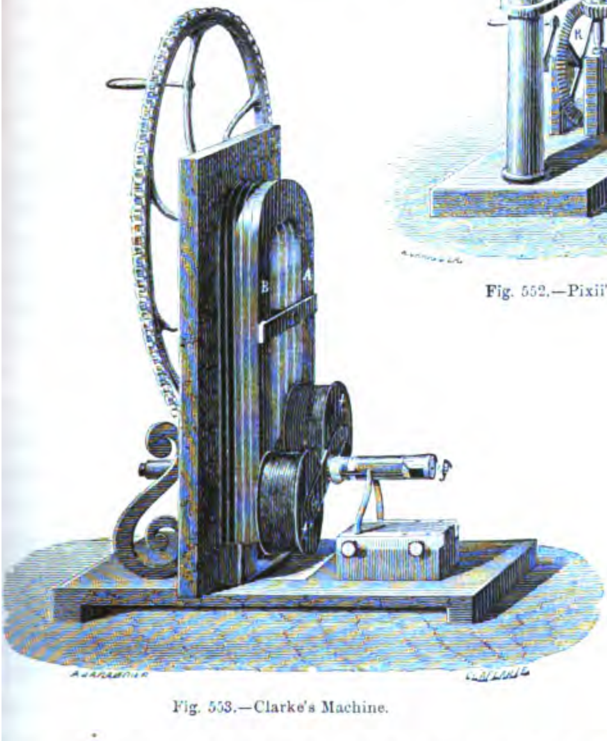

Fig. 553.—Clarke's Machine.

ies a pinion. By means of an endless chain passing over this inion, and over a large wheel to which a handle is attached, the pinion, and with it the coils, can be made to revolve rapidly. The ends of the wire which forms the two coils are connected respectively with the two metallic pieces E, E′ (Fig. 554), which are mounted on the axis, but insulated from it and from each other.

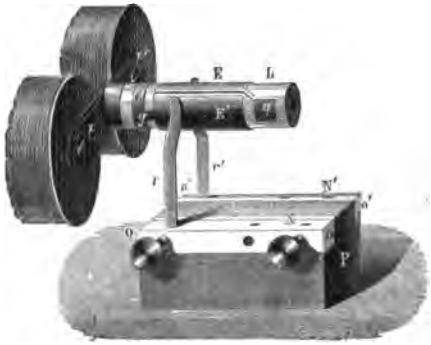

Fig. 554.—Commutator of Clarke's Machine.

Let us now examine the formation of the currents. The two iron cores, with their connecting iron plate, may be regarded as a temporary horseshoe magnet, whose poles are always of opposite name to those of the steel magnet which are respectively nearest to them. The intensity of magnetization is greatest when the soft-iron magnet is horizontal, vanishes when it is vertical, and in passing through the vertical position undergoes reversal. If we call one direction of magnetization positive and the opposite direction negative, the strongest positive magnetization corresponds to one of the two horizontal positions, and the strongest negative to the other, the two positions differing by 180°. While the magnet, then, is revolving from one horizontal position to the other, its magnetization is changing from the strongest positive to the strongest negative, and this change produces a current in one definite direction in the surrounding coil. During the next half-revolution the magnetization is again gradually reversed, and an opposite current is generated in the coil. If we examine the direction of the currents due to the cutting across of the lines of force of the permanent magnet by the convolutions of the coil, we shall find that they concur with those due to the action of the cores. The current in the coils circulates in one direction as long as the electro-magnet is moving from one horizontal position to the other, and changes its direction at the instant when the cores come opposite the poles of the steel magnet.

By the aid of the commutator represented in Fig. 554, the currents may be made to pass always in the same direction through an external circuit. *r* and *r′* are two contact-springs bearing against the two metal pieces E, E′, which are the terminals of the coil. At the instant when the current in the coil is reversed, these springs are in contact with intermediate insulating pieces which separate the metallic pieces E, E′. When the current in the coil is in one direction (say from E to E′), r is in contact with E, and r' with E′. When the current in the coil is in the opposite direction (E′ to E), r is in contact with E′, and r' with E; thus in each case r is the positive and r' the negative spring, and the current will be from r to r' in an external connecting wire. O O, O′O′, are metallic pieces insulated from each other, and connected with the springs $r\,r'$ respectively. Binding-screws are provided for attaching wires through which the current is to be passed.

With this machine water can be decomposed, wire heated to redness, or soft iron magnetized; but these effects are usually on a small scale on account of the small dimensions of the machine.

For giving shocks, two wires furnished with metallic handles are attached to the binding-screws, and a third spring is employed which puts the terminals E E′ in direct connection with each other twice in each revolution, by making contact with two plates q. When these contacts cease, the current is greatly diminished by having to pass through the body of the person holding the handles, and the extra-current thus induced gives the shock. To obtain the strongest effect, the hands should be moistened with acidulated water before grasping the handles.

622. Magneto-electric Machines for Lighthouses.—Very powerful effects can be obtained from magneto-electric machines of large size driven rapidly. Such machines were first suggested by Professor Nollet of Brussels; and they have been constructed by Holmes of London and the Compagnie l'Alliance of Paris. It is by means of these machines that the electric light is maintained in lighthouses; they have also been employed to some extent in electro-metallurgy. Fig. 555 represents the pattern adopted by the French company. It has eight rows of compound horse-shoe magnets fixed symmetrically round a cast-iron frame. They are so arranged that opposite poles always succeed each other, both in each row and in each circular set. There are seven of these circular sets, with of course six intervening spaces. Six bronze wheels, mounted on one central axis, revolve in these intervals, the axis being driven by steam-power transmitted by a pulley and belt. The speed of rotation is usually about 350 revolutions of the axis per minute. Each of the six bronze wheels carries at its circumference sixteen coils, corresponding to the number of poles in each circular set. The core of each coil is a cleft tube of soft iron, this form having been found peculiarly favourable to rapid demagnetization.

Each core has its magnetism reversed sixteen times in each revolution, by the influence of the sixteen successive pairs of poles between which it passes, and the same number of currents in alternately opposite directions are generated in the coils. The coils can be connected in different ways, according as great electro-motive force or small resistance is required. The positive ends are connected with the axis of the machine, which thus serves as the positive electrode, and a concentric cylinder, well insulated from it, is employed as the negative electrode.

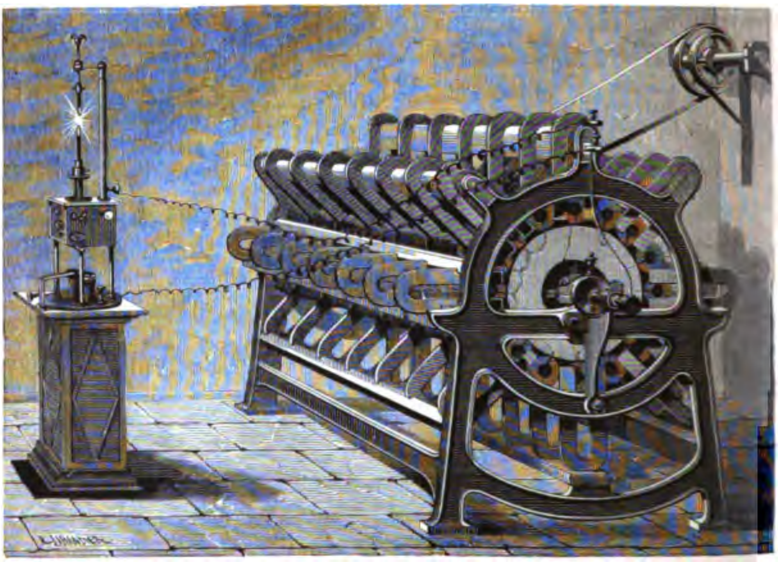

Fig. 555.—Lighthouse Machine.

When the machine is employed for the production of the electric light, the currents may be transmitted to the carbon points in alternate directions, as they are produced. For electro-metallurgical purposes they are brought into one constant direction by a commutator, as in Clarke's machine above described. The driving-power required for lighthouse purposes is about three horse-power.

623. Siemens' Armature.—An important improvement in Clarke's machine was introduced by Siemens of Berlin in 1854. It consists in the adoption of a peculiar form of electro-magnet, which is represented in Fig. 556. The iron portion is a cylinder with a very deep and wide groove cut along a pair of opposite sides, and continued round the ends. The coil is wound in this groove like thread upon a shuttle. Regarded as an electro-magnet, the poles are not the ends of the cylinder, but are the two cylindrical faces which have not been cut away. In Fig. 557, *a b* is a section of the armature with the coil wound upon it. ABMN is a socket within which the armature revolves, the portions AB being of iron, and MN of brass.

The advantage of Siemens' armature is that, on account of the small space required for its rotation, it can be kept in a region of very intense magnetic force by the use of comparatively small magnets. Its form is also eminently favourable to rapid rotation. It is placed between the opposite poles of a row of horse-shoe magnets which bestride it along the whole of its length, as shown at the *top* of Fig. 559, and is rotated by means of a driving-band passing over the pulley shown at the lower end of Fig. 556.

The polarity of the electro-magnet is reversed at each half-revolution as in Clarke's arrangement, and the alternately opposite currents generated are reduced to a common direction by a commutator nearly identical with Clarke's, and represented in Figs. 556, 558. Siemens' machines are much more powerful than Clarke's when of the same size.

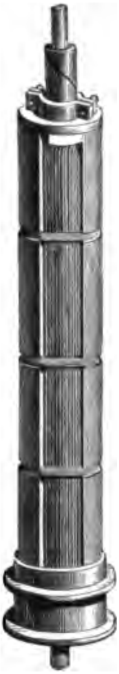

Fig. 556. Siemens' Armature.

624. Accumulation by Successive Action: Wilde's Machine.—By employing the current from a Siemens' machine to magnetize soft

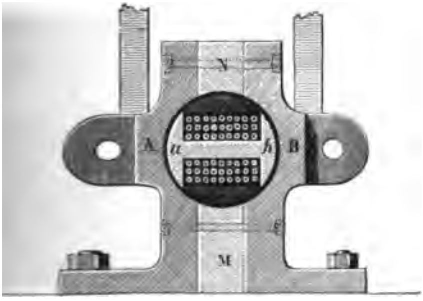

Fig. 557.—Section of Siemens' Armature.

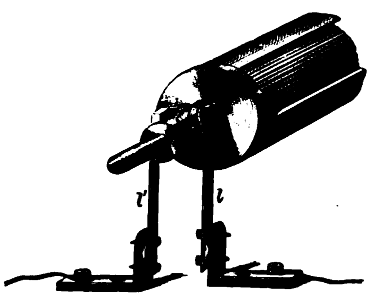

Fig. 558.—Commutator.

iron, we can obtain an electro-magnet of much greater power than the steel magnets from whose induction the current was derived. By causing a second coil to rotate between the poles of this electro-magnet, we can obtain a current of much greater power than the

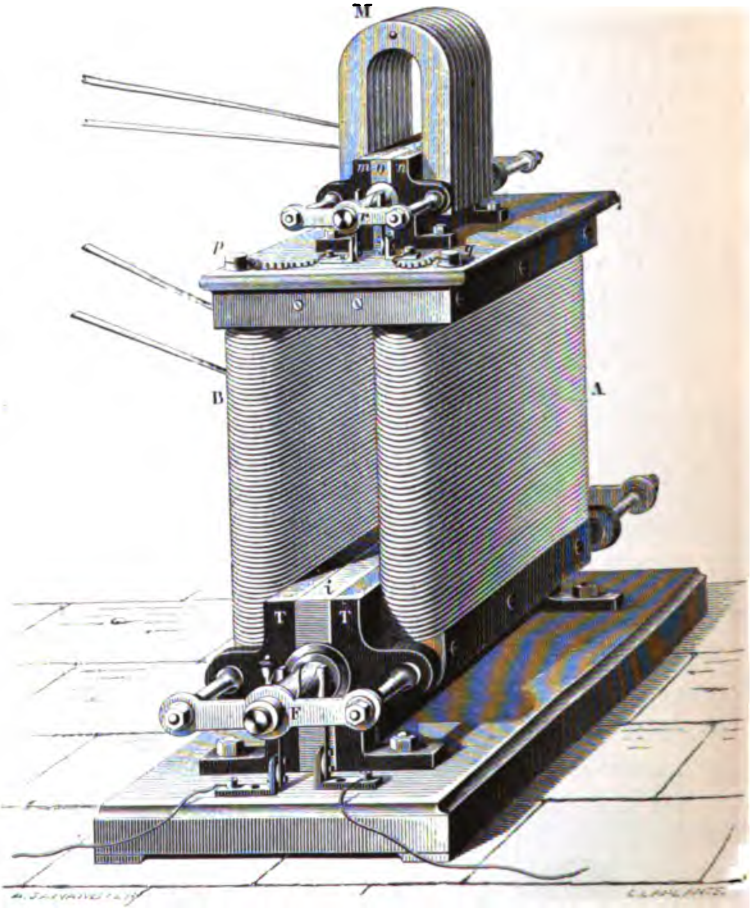

Fig. 559.—Wilde's Machine.

primary current. This is the principle of Wilde's machine, which is represented in Fig. 559. It consists of two Siemens' machines, one above the other. The upper machine derives its inductive action from a row of steel magnets M, whose poles rest on the soft-iron masses *m*, *n*, forming the sides of the socket within which a Siemens' armature *r* rotates. The currents generated in the coil, after being reduced to a uniform direction by a commutator, flow to the binding-screws p, q. These are the terminals of the coil of the large electro-magnet AB, through which accordingly the current circulates. The core of this electro-magnet consists of two large plates of iron, connected above by another iron plate, which supports the primary machine. Its lower extremities rest, like those of the primary magnets, on two iron masses T, T, separated by a mass of brass i; and a second Siemens' armature F, of large size, revolving within this system, furnishes the currents which are utilized externally.

Wilde's machine produces calorific and luminous effects of remarkable intensity; but the speed of rotation required is very great, being sometimes 1500 revolutions a minute for the large, and 2000 for the small armature. This great speed involves serious inconveniences; and the machine does not appear to have been used for lighthouses, or other practical purposes.

Wilde's principle can be carried further. The current of the second armature can be employed to animate a second electro-magnet of greater power than the first, with a third Siemens' armature revolving between its poles. This has actually been done by Wilde. By means of the current from this triple machine, driven by 15 horse-power, the electric light was maintained between two carbons as thick as a man's finger, and a bar of platinum 2 feet long and a quarter of an inch in diameter was quickly melted.

This system of accumulation could probably be carried several steps further, but always with the expenditure of a proportionately large amount of energy in driving it. In no magneto-electric machine can the electrical energy obtained exceed the mechanical energy expended in producing it.

625. Accumulation by Mutual Action: Siemens' and Wheatstone's Machine.—Siemens and Wheatstone nearly simultaneously proposed the construction of a magneto-electric machine in which the induced currents are made to circulate round the soft-iron magnet which produced them. Iron has usually some traces of permanent magnetism, especially if it has once been magnetized. This magnetism serves to induce very feeble currents in a revolving armature. These cur-ents are sent round the iron magnet, thus increasing its magnetiza-ion. This again produces a proportionate increase in the induced urrents; and thus, by a successive alternation of mutual actions, ery intense magnetization and very powerful currents are speedily btained. In the machine as exhibited by Siemens in 1867, the current was diverted into an external circuit, at regular intervals, by an automatic arrangement.

626. Ladd's Machine.—Ladd in 1867 constructed a machine based on the principle of mutual action just described; but, instead of utilizing the current by occasional interruptions, he employed a second revolving armature whose coil was in permanent connection with the external circuit.

B, B′ (Fig. 560) are two plates of iron surrounded by coils which are connected at the right-hand end so as to form but one circuit. The other ends are attached to two binding-screws connected with the ends of the coil of a Siemens' armature *a′*. The direction of winding of the two large coils BB′ is the same as for a horse-shoe magnet, so that the two poles at either end are of opposite sign. The ends of the cores are let into masses of soft iron M M, N N, between which two armatures *a a′* rotate. The coil of the armature *a* is connected with the external circuit containing, for example, two carbon points for exhibiting the electric light.

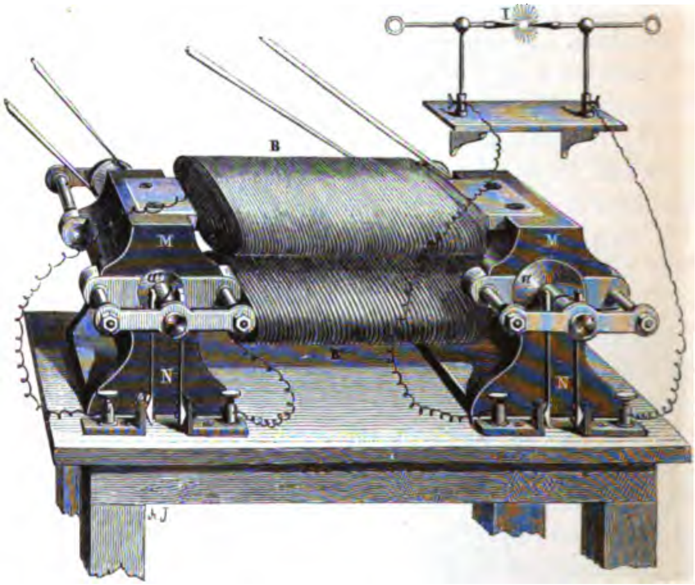

Fig. 560.—Ladd's Machine.

On the principle of mutual action, the electro-magnets B, B′, which we may suppose to have at first only a trace of magnetism, are soon raised to very intense magnetization by the rapid rotation of the armature *a'*, and as long as the rotation continues, the magnetization is maintained. The rapid rotation of the other armature *a* between the poles thus strongly excited, produces a very powerful current which can be utilized externally.

Ruhmkorff has modified the arrangement by using a single rotating armature with two coils wound upon it, one of them being connected with the electro-magnet, and the other with the external circuit.

The efficiency of machines of this description, regarded as means for the transformation of mechanical into electrical energy, is undoubtedly very considerable; nevertheless it is not perfect, a large amount of energy being wasted in generating heat. On account of the high velocity necessary for efficient working, and the small size of the apparatus in comparison with the currents obtained, the elevation of temperature is often so great as to prove a source of much annoyance.

626 A. Wheatstone's Telegraphic Currents.—In Wheatstone's Universal Telegraph, which has been partially described in a previous chapter, the magneto-electric currents which give the signals are produced by causing a small flat bar of soft iron to rotate rapidly before the poles of a steel horse-shoe magnet, which has two connected coils of wire wound upon it in the same manner as upon electro-magnets. It is in these coils that the currents are generated, the iron bar being a temporary magnet, and thus influencing the coils nearly in the same manner as if it were a permanent magnet. A current is induced in one direction as it approaches the poles, and in the opposite direction as it recedes from them, so that altogether four currents are generated in each complete revolution. On account of the lightness of the bar, it can be rotated with great rapidity.

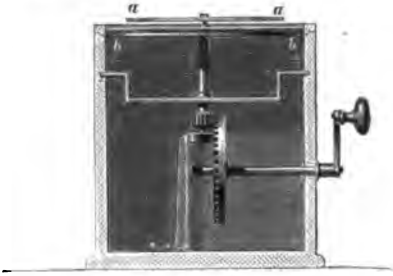

Fig. 561.—Arago's Rotations.

627. Arago's Rotations.—Faraday successfully applied his discovery of magneto-electric induction to account for a phenomenon first observed by Arago in 1824, and subsequently investigated by Babbage and Sir John Herschel. A horizontal disc of copper *b b*, placed in the interior of a box, is set in rapid rotation by turning a handle. Just over the copper disc, but above the thin plate which forms the top of the box, a magnetized needle *aa* is balanced horizontally. When the disc is made to rotate, the needle is observed to deviate from the meridian in the direction of the rotation. When the speed of rotation exceeds a certain limit, the needle is not only deflected, but carried round in continuous rotation in the same direction as the disc.

The explanation is to be found in the currents which are induced in the disc by its motion in the vicinity of the magnetized needle. The forces between these currents and the needle are (by Lenz's law) such as to urge the disc backwards; and, from the universal relation which subsists between action and reaction, they must be such as to urge the needle forwards; hence the motion. The direction of the induced current at any instant is in fact along that diameter of the disc which is directly under the needle, the circuit being completed through the lateral portions of the disc; and it is evident that a current thus flowing parallel to the needle underneath it tends to produce deflection. If the continuity of the disc is interrupted by radial slits, the observed effect is considerably weakened inasmuch as the return circuit is broken. Faraday succeeded in directly demonstrating the existence of currents in a disc rotating near a fixed magnet, by exploring its surface with the amalgamated ends of two wires connected with a galvanometer.

The experiment performed by Arago may be reversed by setting the magnet in rotation, and observing the effect produced on the disc. The latter, if delicately suspended, will be found to rotate in the same direction as the magnet. This experiment was first performed by Babbage and Herschel. Its explanation is identical with that just given. In both cases the induced rotation must be slower than that of the body turned by hand, as the existence of the induced currents depends upon the motion of the one body relative to the other.

When an iron disc is used instead of a copper one, magnetism is induced in the portions which pass under the poles of the magnet; and as this requires a sensible time for its disappearance, there is always attraction between the poles of the needle and the portions of the disc which have just moved past. The needle is thus drawn forwards by magnetic attraction, and the observed effect is similar to that obtained with the copper disc, though the cause[1] is altogether different.

627A. Copper Dampers.—Precisely similar to the above is the explanation of the utility of a copper disc in checking the vibrations of a magnetized needle under which it is fixed. As the needle swings to either side, its motion induces currents in the copper which urge the needle in the opposite direction to that in which it is moving. When it rests for an instant at the extremity of its swing, the currents cease; and as soon as it begins to return, the currents again resist its motion. A copper plate thus used is called a *damper*, and the vibrations thus resisted and destroyed are said to be *damped*. The name is applied to any other means for gradually destroying vibrations, and is probably based on the analogy between this action and the steadying action of a liquid upon a suspended body immersed in it.

The resistance which induced currents oppose to the motion producing them is well illustrated by Faraday's experiment of the *copper cube*. A cube of copper is suspended by a thread, and set spinning by twisting the thread and then allowing it to untwist. If, while spinning, it is held between the poles of a powerful magnet, like that represented in Fig. 432, it is instantly brought to rest. If the poles are brought very near together, so as to heighten the intensity of the field, and a thin sheet of copper is inserted between them and moved rapidly in its own plane, the operator feels its motion resisted by some invisible influence. The sensation has been compared to that of cutting cheese. Foucault's apparatus for the heating of a copper disc by rotating it between the poles of a magnet (§ 356), is another illustration of the same principle. In all cases where induced currents are generated, and are not called upon to perform external work, they yield their full equivalent of heat.

The advantage of employing copper in experiments of this kind arises from its superior conductivity, to which the induced currents are proportional.

628. Electro-medical Machines.—The application of electricity is often resorted to for certain nervous affections and local paralyses. Many different forms of apparatus are employed for this purpose.

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[1] That is to say, the *main cause;* for there must be induced currents in the iron as well as in the copper, though inferior in strength, on account of the inferior conductivity of the former metal.

One of the most convenient is represented in Fig. 562. Two small coils connected with each other, and furnished with a vibrating contact-breaker, are traversed by the current from a miniature battery. The coils are surrounded by hollow cylinders of copper or brass, in which induced currents are generated as often as the current in the coils is established or interrupted. This action diminishes the energy of the extra-currents on which the shock depends, and the operator can accordingly regulate its strength at pleasure by sliding the cylinders on or off.

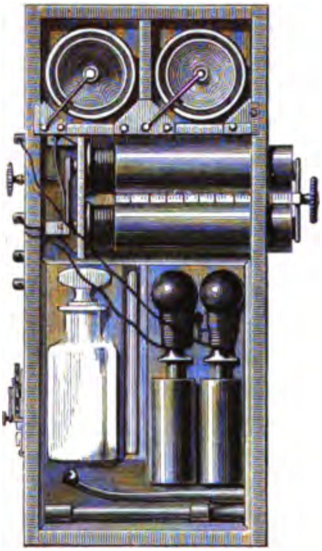

Fig. 562.—Electro-medical Machine.

628A. Caution regarding Lines of Force.—After the very extensive use which has been made in this volume of lines and tubes of force, we think it right to caution the reader against supposing that these conceptions depend upon any doubtful hypothesis. They merely serve, like meridians and parallels of latitude, to map out space in a mode convenient for the statement of physical laws.

APPENDIX.

ON ELECTRICAL AND MAGNETIC UNITS.

(1). The numerical value of a concrete quantity is its ratio to a particular unit of the same kind; the selection of this unit being always more or less arbitrary.

(2). One kind of quantity may, however, be so related to two or more others, as to admit of being specified in terms of units of these other kinds. For example, of the three kinds of quantity, called distance (or length), time, and velocity, any one is capable of being expressed in terms of the other two. Velocity can be specified (as regards amount) by stating the distance passed over in a specified time. Distance can be specified by stating the velocity required for describing it in a specified time, and time can be specified by stating the distance described with a specified velocity.

Force, distance, and work are in like manner three kinds of quantity, of which any two are just sufficient to specify the third.

(3). Calculation is greatly facilitated by employing as few original or underived units as possible. These should be of kinds admitting of easy and accurate comparison; and all other units should be derived from them by the simplest modes of derivation which are available.

(4). Velocity is proportional directly to distance described, and inversely to the time of its description; and is independent of all other elements. This is expressed, by saying that *the dimensions f velocity* are $\frac{\text{distance}}{\text{time}}$ or $\frac{\text{length}}{\text{time}}$.

Again, if we define the unit of velocity to be that with which unit listance would be described in unit time, the real magnitude of the mit of velocity will depend upon the units of length and time elected, being proportional directly to the real magnitude of the rmer, and inversely to the real magnitude of the latter. This is expressed by saying that *the dimensions of the unit of velocity* are $\frac{\text{length}}{\text{time}}$. Both forms of expression are convenient; and the ideas which they are intended to express are logically equivalent.

(5). All electrical and magnetic units can be derived from units of length, mass, and time. We shall denote length by l, mass by m, and time by t.

(6). The unit of *velocity* is the velocity with which unit length is described in unit time. Its dimensions are $\frac{l}{t}$.

(7). The unit of *acceleration* is the acceleration which gives unit increase of velocity in unit time. Its dimensions are $\frac{\text{velocity}}{\text{time}}$ or $\frac{l}{t^2}$.

(8). The unit *force* is that which acting on unit mass produces unit acceleration. Its dimensions are mass × acceleration, or $\frac{ml}{t^2}$.

(9). The unit of *work* is the work done by unit force working through unit length. Its dimensions are force × length, or $\frac{ml^2}{t^2}$.

(10). The unit of *kinetic energy* is the kinetic energy of *two* units of mass moving with unit velocity (according to the formula $\frac{1}{2}mv^2$). Its dimensions are mass × (velocity)^2, or $\frac{ml^2}{t^2}$, and are the same as the dimensions of work. It might appear simpler to make it the energy of *one* unit of mass moving with unit velocity; but if this change were made, it would be necessary either to halve the unit of work, or else to make kinetic energy double of the work which produced it. Either of these alternatives would involve greater inconvenience and complexity than the selection made above.

UNITS OF STATICAL ELECTRICITY.

(11). Let q denote *quantity* of electricity measured statically, so that the mutual repulsion of two equal quantities q at distance l, is $\frac{q^2}{l^2}$. This being equal to a force, the dimensions of q^2 must be (length)^2 × force, or $\frac{ml^3}{t^2}$, and the dimensions of q must be $\frac{m^{\frac{1}{2}}l^{\frac{3}{2}}}{t}$.

(12). Let V denote *difference of potential.* Then the work required to raise a quantity q through a difference of potential V. is qV. The dimensions of V are therefore $\frac{\text{work}}{q}$, or $\frac{ml^2}{t^2}\,\frac{t}{m^{\frac{1}{2}}l^{\frac{3}{2}}}$, or

$\frac{m^{\frac{1}{2}}l^{\frac{1}{2}}}{t}$. The dimensions of potential are of course the same as those of difference of potential.

(13). The *capacity* of a conductor is the quotient of the quantity of electricity with which it is charged, by the potential which this charge produces in the conductor. The dimensions of capacity are therefore $\frac{m^{\frac{1}{2}}l^{\frac{3}{2}}}{t} \cdot \frac{t}{m^{\frac{1}{2}}l^{\frac{1}{2}}}$, or simply l. In fact, as we have seen (§ 445 M), the capacity of a spherical conductor is equal to its radius.

MAGNETIC AND ELECTRO-MAGNETIC UNITS.

(14). Let P denote the numerical value of a *pole* (or the strength of a pole). Then, since two equal poles P at distance l repel each other with the force $\frac{P^2}{l^2}$, which must be of the dimensions $\frac{ml}{t^2}$, the dimensions of P are $\frac{m^{\frac{1}{2}}l^{\frac{3}{2}}}{t}$.

(15). Let I denote the *intensity of* a magnetic *field*. Then, a pole P in this field is acted on with a force P I. This must be of the dimensions $\frac{ml}{t^2}$. Hence, the dimensions of I are $\frac{ml}{t^2} \cdot \frac{t}{m^{\frac{1}{2}}l^{\frac{3}{2}}}$, or $\frac{m^{\frac{1}{2}}}{l^{\frac{1}{2}}t}$

(16). Let M denote the *moment* of a magnet. Since it is the product of the strength of a pole by the distance between two poles, its dimensions are $\frac{m^{\frac{1}{2}}l^{\frac{5}{2}}}{t}$.

(17). Intensity of *magnetization* is the quotient of moment by volume. Its dimensions are therefore $\frac{M}{l^3}$ or $\frac{m^{\frac{1}{2}}}{l^{\frac{1}{2}}t}$. These are the same as the dimensions of intensity of field.

(18). When a magnetic substance is placed in a magnetic field, it is magnetized by induction; and each substance has its own specific *co-efficient of magnetic induction* (constant, or nearly so, when the field is not excessively intense), which expresses the ratio of the intensity of the induced magnetization to the intensity of the field. For diamagnetic substances, this co-efficient is negative, that is to say, the induced polarity is reversed, end for end, as compared with that of a paramagnetic substance placed in the same field.

(19). The work required to move a pole P from one point to another, is the product of P by the difference of the magnetic potentials of the two points. Hence, the dimensions of *magnetic potential* are $\frac{m l^2}{t^2} \frac{t}{m^{\frac{1}{2}} l^{\frac{3}{2}}}$ or $\frac{m^{\frac{1}{2}} l^{\frac{1}{2}}}{t}$.

(20). A *current* C flowing along a circular arc, produces at the centre of the circle an intensity of field equal to C multiplied by length of arc divided by square of radius. Hence, C divided by a length is equal to a field-intensity, the dimensions of which are $\frac{m^{\frac{1}{2}}}{l^{\frac{1}{2}} t}$, and the dimensions of C are $\frac{m^{\frac{1}{2}} l^{\frac{1}{2}}}{t}$.

(21). The *quantity* Q of electricity conveyed by a current is the product of the current by the time that it lasts. Its dimensions are therefore $m^{\frac{1}{2}} l^{\frac{1}{2}}$.

(22). The work done in urging a quantity Q by an electro-motive force E is E Q, hence the dimensions of *electro-motive force* are $\frac{m l^2}{t^2} \frac{1}{m^{\frac{1}{2}} l^{\frac{1}{2}}}$ or $\frac{m^{\frac{1}{2}} l^{\frac{3}{2}}}{t^2}$; and as electro-motive force is difference of potential, these are also the dimensions of *potential*.

(23). The *capacity* of a conductor is the quotient of quantity of electricity by potential; its dimensions are therefore $\frac{t^2}{l}$.

(24). The *resistance* R of a circuit is, by Ohm's law, equal to $\frac{E}{C}$. Its dimensions are therefore $\frac{m^{\frac{1}{2}} l^{\frac{3}{2}}}{t^2} \frac{t}{m^{\frac{1}{2}} l^{\frac{1}{2}}}$ or $\frac{l}{t}$, and are the same as the dimensions of velocity.

(25). On comparing the dimensions of the same element as measured according to the two systems, it will be observed that they are not identical. The dimensions of quantity of electricity, for example, in the first system, are to its dimensions in the second, as l to t; and the dimensions of capacity are as l^2 to t^2. Accordingly, the ratio of the electro-static to the electro-magnetic unit of quantity is equal to a length divided by a time; that is to say, is equal to a velocity. From experiments in which the same quantity of electricity was measured both statically and magnetically, it appears that this ratio is in fact identical with the velocity of light. Professor Clerk Maxwell maintains that light, electricity, and magnetism are all affections of one and the same medium; that light is an electro-magnetic phenomenon, and that its laws can be deduced from those of electricity and magnetism.

Notwithstanding this difference of dimensions, two quantities of electricity which are equal when compared statically, are also equal when compared magnetically; or if one be double of the other when compared statically, it will also be double of the other when compared magnetically.

(26). An illustration from a somewhat more familiar department may assist the reader in convincing himself that it is possible for one and the same kind of quantity to have different dimensions according to the line of derivation employed. It is well known that uniform spheres attract each other with a force which is directly as the product of their masses, and inversely as the square of the distance between their centres. If this law were made to furnish the unit of force, the dimensions of force would be $\frac{m^2}{l^2}$, instead of $\frac{ml}{t^2}$, as previously found. The ambiguity depends partly on the fact that l in the one formula denotes distance between attracting centres, and in the other distance moved over. It is only when the mode of derivation is distinctly specified, or is too obvious to need specification, that the dimensions of a quantity admit of being determinately stated. As the definition of a derived unit necessarily involves a specification of the mode of its derivation, there is some advantage in speaking of the *dimensions of a unit*, rather than of the dimensions of the quantity which the unit serves to measure.

(27). Derived units are often called *absolute* units; but it seems an abuse of language to define a unit by its *relation* to other arbitrary units, and then call it *absolute*.

ACOUSTICS.

CHAPTER LIII.

PRODUCTION AND PROPAGATION OF SOUND.

629. Sound is a Vibration.—Sound, as directly known to us by the sense of hearing, is an impression of a peculiar character, very broadly distinguished from the impressions received through the rest of our senses, and admitting of great variety in its modifications. The attempt to explain the physiological actions which constitute hearing forms no part of our present design. The business of physics is rather to treat of those external actions which constitute sound, considered as an objective existence external to the ear of the percipient.

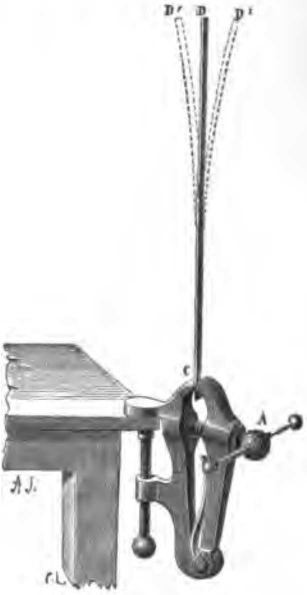

Fig. 563.—Vibration of Straight Spring.

It can be shown, by a variety of experiments, that sound is the result of vibratory movement. Suppose, for example, we fix one end C of a straight pring C D (Fig. 563) in a vice A, then raw the other end D aside into the osition D′, and let it go. In virtue of s elasticity (§ 23), the spring will return its original position; but the kinetic nergy which it acquires in returning is fficient to carry it to a nearly equal stance on the other side; and it thus wings alternately from one side to the other through distances very adually diminishing, until at last it comes to rest. Such movement is called vibratory. The motion from D′ to D″, or from D″ to , is called a *single vibration*. The two together constitute a *double* or *complete vibration;* and the time of executing a complete vibration is the *period* of vibration. The *amplitude* of vibration for any point in the spring is the distance of its middle position from one of its extreme positions. These terms have been already employed (§ 44) in connection with the movements of pendulums, to which indeed the movements of vibrating springs bear an extremely close resemblance. The property of isochronism, which approximately characterizes the vibrations of the pendulum, also belongs to the spring, the approximation being usually so close that the period may practically be regarded as altogether independent of the amplitude.

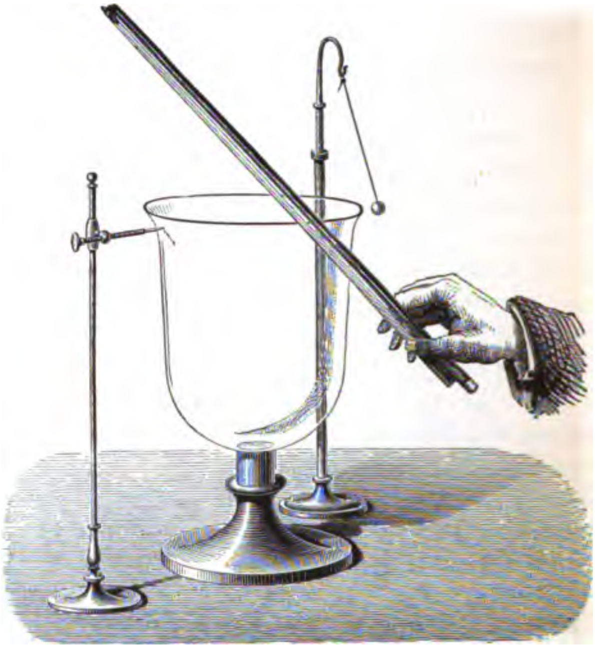

Fig. 564.—Vibration of Bell.

When the spring is long, the extent of its movements may generally be perceived by the eye. In consequence of the persistence of impressions, we see the spring in all its positions at once; and the edges of the space moved over are more conspicuous than the central parts, because the motion of the spring is slowest at its extreme positions.

As the spring is lowered in the vice, so as to shorten the vibrating portion of it, its movements become more rapid, and at the same time more limited, until, when it is very short, the eye is unable to detect any sign of motion. But where sight fails us hearing comes to our aid. As the vibrating part is shortened more and more, it emits a musical note, which continually rises in pitch; and this effect continues after the movements have become much too small to be visible.

It thus appears that a vibratory movement, if sufficiently rapid, may produce a sound. The following experiments afford additional illustration of this principle, and are samples of the evidence from which it is inferred that vibratory movement is essential to the production of sound.

Vibration of a Bell.—A point is fixed on a stand, in such a position as to be nearly in contact with a glass bell (Fig. 564). If a rosined fiddle-bow is then drawn over the edge of the bell, until a musical note is emitted, a series of taps are heard due to the striking of the bell against the point. A pith-ball, hung by a thread, is driven out by the bell, and kept in oscillation as long as the sound continues. By lightly touching the bell, we may feel that it is vibrating; and if we press strongly, the vibration and the sound will both be stopped.

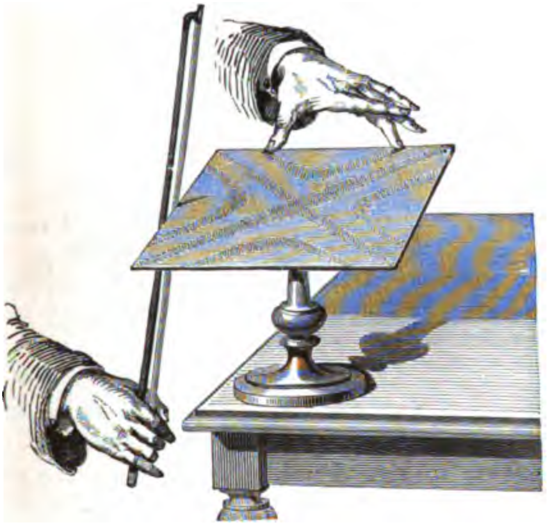

Fig. 565.—Vibration of Plate.

Vibration of a Plate.—Sand is strewn over the surface of a horizontal plate (Fig. 565), which is then made to vibrate by drawing a bow over its edge. As soon as the plate begins to sound, the sand dances, leaves certain parts bare, and collects in definite lines, which are called *nodal lines.* These are, in fact, the lines which separate portions of the plate whose movements are in opposite directions. Their position changes whenever the plate changes its note.

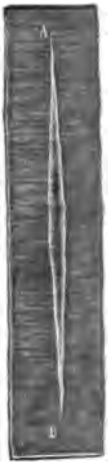

Fig. 566. Vibration of String.

The vibratory condition of the plate is also manifested by another phenomenon, opposite—so to speak—to that just described. If very fine powder, such as lycopodium, be mixed with the sand, it will not move with the sand to the nodal lines, but will form little heaps in the centre of the vibrating segments; and these heaps will be in a state of violent agitation, with more or less of gyratory movement, as long as the plate is vibrating. This phenomenon, after long baffling explanation, was shown by Faraday to be due to indraughts of air, and ascending currents, brought about by the movements of the plate. In a moderately good vacuum, the lycopodium goes with the sand to the nodal lines.

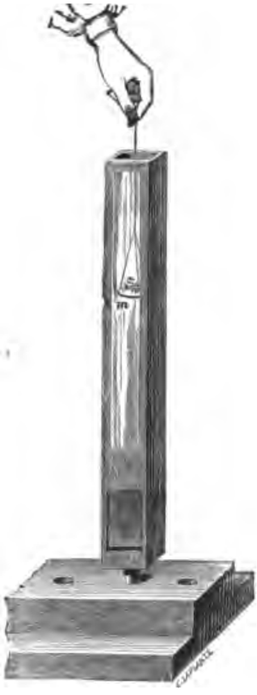

Fig. 567.—Vibration of Air.

Vibration of a String.—When a note is produced from a musical string or wire, its vibrations are often of sufficient amplitude to be detected by the eye. The string thus assumes the appearance of an elongated spindle (Fig. 566).

Vibration of the Air.—The sonorous body may sometimes be air, as in the case of organ-pipes, which we shall describe in a later chapter. It is easy to show by experiment that when a pipe *speaks*, the air within it is vibrating. Let one side of the tube be of glass, and let a small membrane *m*, stretched over a frame, be strewed with sand, and lowered into the pipe. The sand will be thrown into violent agitation, and the rattling of the grains, as they fall back on the membrane, is loud enough to be distinctly heard.

Singing Flames.—An experiment on the production of musical sound by flame, has long been known under the name of the *chemical harmonica.* An apparatus for the production of hydrogen gas (Fig. 568) is furnished with a tube, which tapers off nearly to a point at its upper end, where the gas issues and is lighted. When a tube, open at both ends, is held so as to surround the flame, a musical tone is heard, which varies with the dimensions of the tube, and often attains considerable power. The sound is due to the vibration of the air and products of combustion within the tube; and on observing the reflection of the flame in a mirror rotating about a vertical axis, it will be seen that the flame is alternately rising and falling, its successive images, as drawn out into a horizontal series by the rotation of the mirror, resembling a number of equidistant tongues of flame, with depressions between them. The experiment may also be performed with ordinary coal-gas.

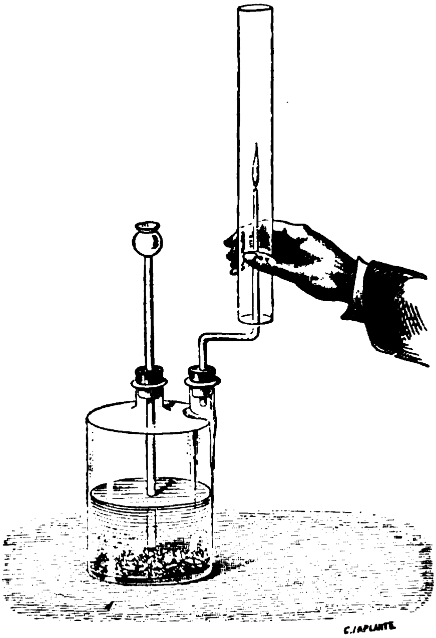

Fig. 568.—Chemical Harmonica.

Trevelyan Experiment.—A fire-shovel (Fig. 569) is heated, and balanced upon the edges of two sheets of lead fixed in a vice; it is then seen to execute a series of small oscillations—each end being alternately raised and depressed—and a sound is at the same time emitted. The oscillations are so small as to be scarcely perceptible in themselves; but they can be rendered very obvious by attaching to the shovel a small silvered mirror, on which a beam of light is directed. The reflected light can be made to form an image upon a screen, and this image is seen to be in a state of oscillation as long as the sound is heard.

The movements observed in this experiment are due to the sudden expansion of the cold lead. When the hot iron comes in contact with it, a protuberance is instantly formed by dilatation, and the iron is thrown up. It then comes in contact with another portion

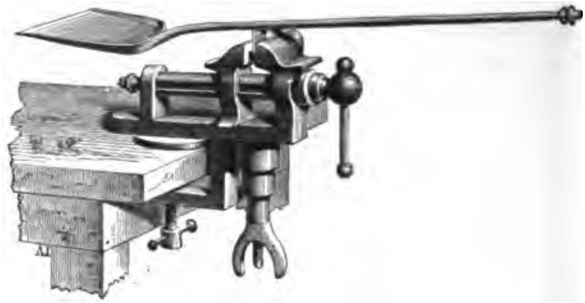

Fig. 569.—Trevelyan Experiment.

of the lead, where the same phenomenon is repeated while the first point cools. By alternate contacts and repulsions at the two points, the shovel is kept in a continual state of oscillation, and the regular succession of taps produces the sound.

The experiment is more usually performed with a special instrument invented by Trevelyan, and called a *rocker*, which, after being heated and laid upon a block of lead, rocks rapidly from side to side and yields a loud note.

630. Distinctive Character of Musical Sound.—It is not easy to draw a sharp line of demarcation between musical sound and mere noise. The name of noise is usually given to any sound which seems unsuited to the requirements of music.

This unfitness may arise from one or the other of two causes. Either,

1. The sound may be unpleasant from containing discordant elements which jar with one another, as when several consecutive keys on a piano are put down together. Or,

2. It may consist of a confused succession of sounds, the changes being so rapid that the ear is unable to identify any particular note. This kind of noise may be illustrated by sliding the finger along a violin-string, while the bow is applied.

All sounds may be resolved into combinations of elementary musical tones occurring simultaneously and in succession. Hence the study of musical sounds must necessarily form the basis of acoustics.

Every sound which is recognized as musical is characterized by what may be called smoothness, evenness, or regularity; and th physical cause of this regularity is to be found in the accurat *periodicity* of the vibratory movements which produce the sound. By *periodicity* we mean the recurrence of precisely similar states at equal intervals of time, so that the movements exactly repeat themselves; and the time which elapses between two successive recurrences of the same state is called the *period* of the movements.

Practically, musical and unmusical sounds often shade insensibly into one another. The tones of every musical instrument are accompanied by more or less of unmusical noise. The sounds of bells and drums have a sort of intermediate character; and the confused assemblage of sounds which is heard in the streets of a city blends at a distance into an agreeable hum.

631. Vehicle of Sound.—The origin of sound is always to be found in the vibratory movements of a sonorous body; but these vibratory movements cannot bring about the sensation of hearing unless there be a medium to transmit them to the auditory apparatus. This medium may be either solid, liquid, or gaseous, but it is necessary that it be elastic. A body vibrating in an absolute vacuum, or in a medium utterly destitute of elasticity, would fail to excite our sensations of hearing. This assertion is justified by the following experiments:—

1. Under the receiver of an air-pump is placed a clock-work arrangement for producing a number of strokes on a bell. It is placed on a thick cushion of felt, or other inelastic material, and the air in the receiver is exhausted as completely as possible. If the clock-work is then started by means of the handle *g*, the hammer will be seen to strike the bell, but the sound will be scarcely audible. If hydrogen be introduced into the vacuum, and the receiver be again exhausted, the sound will be much more completely extinguished, being heard with difficulty even when the ear is placed in contact with the receiver. Hence it may fairly be concluded that if the receiver could be perfectly exhausted, and a perfectly inelastic support could be found for the bell, no sound at all would be emitted.

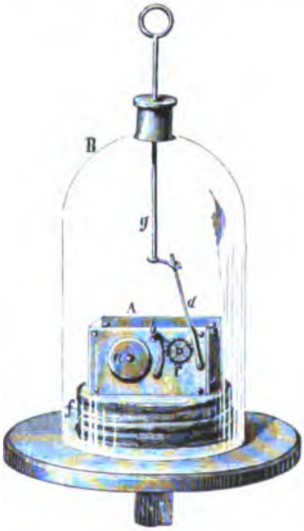

Fig. 570.—Sound in Exhausted Receiver.

2. The experiment may be varied by using a glass globe, furnished with a stop-cock, and having a little bell suspended within it by a thread. If the globe is exhausted of air, the sound of the bell will be scarcely audible. The globe may be filled with any kind of gas, or with vapour either saturated or non-saturated, and it will thus be found that all these bodies transmit sound.

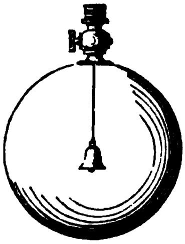

Fig. 571.
Globe with Stop-cock.

Sound is also transmitted through liquids, as may easily be proved by direct experiment. Experiment, however, is scarcely necessary for the establishment of the fact, seeing that fishes are provided with auditory apparatus, and have often an acute sense of hearing.

As to solids, some well-known facts prove that they transmit sound very perfectly. For example, light taps with the head of a pin on one end of a wooden beam, are distinctly heard by a person with his ear applied to the other end, though they cannot be heard at the same distance through air. This property is sometimes employed as a test of the soundness of a beam, for the experiment will not succeed if the intervening wood is rotten, rotten wood being very inelastic.

The *stethoscope* is an example of the transmission of sound through solids. It is a cylinder of wood, with an enlargement at each end, and a perforation in its axis. One end is pressed against the chest of the patient, while the observer applies his ear to the other. He is thus enabled to hear the sounds produced by various internal actions, such as the beating of the heart and the passage of the air through the tubes of the lungs. Even simple *auscultation,* in which the ear is applied directly to the surface of the body, implies the transmission of sound through the walls of the chest.

By applying the ear to the ground, remote sounds can often be much more distinctly heard; and it is stated that savages can in this way obtain much information respecting approaching bodies of enemies.

We are entitled then to assert that *sound, as it affects our organs of hearing, is an effect which is propagated, from a vibrating body, through an elastic and ponderable medium.*

632.[1] **Mode of Propagation of Sound.**—We will now endeavour to explain the action by which sound is propagated.

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[1] The numbering of §§ 632–638 does not correspond with the original, some transpositions having been made.

Let there be a plate *a* vibrating opposite the end of a long tube, and let us consider what happens during the passage of the plate from its most backward position *a*″, to its most advanced position *a*′. This movement of the plate may be divided in imagination into a number of successive parts, each of which is communicated to the layer of air close in front of it, which is thus compressed, and, in its endeavour to recover from this compression, reacts upon the next layer, which is thus in its turn compressed. The compression is thus passed on from layer to layer through the whole tube, much in the same way as, when a number of ivory balls are laid in a row, if the first receives an impulse which drives it against the second, each ball will strike against its successor and be brought to rest.

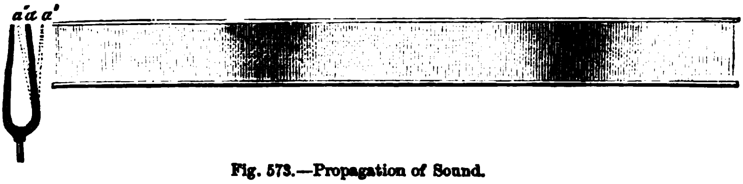

Fig. 573.—Propagation of Sound.

The compression is thus passed on from layer to layer through the tube, and is succeeded by a rarefaction corresponding to the backward movement of the plate from *a*′ to *a*″. As the plate goes on vibrating, these compressions and rarefactions continue to be propagated through the tube in alternate succession. The greatest compression in the layer immediately in front of the plate, occurs when the plate is at its middle position in its forward movement, and the greatest rarefaction occurs when it is in the same position in its backward movement. These are also the instants at which the plate is moving most rapidly.[1] When the plate is in its most advanced position, the layer of air next to it, A (Fig. 574), will be in its natural state, and another layer at A_1, half a wave-length further on, will also be in its natural state, the pulse having travelled from A to A_1, while the plate was moving from *a*″ to *a*′. At intervening points between A and A_1, the layers will have various amounts of compression corresponding to the different positions of the plate in its forward move-

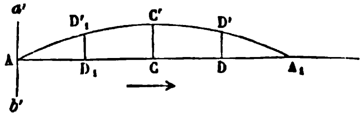

Fig. 574.—Graphical Representation.

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[1] See § 632 A, also Note A at the end of this chapter.

ment. The greatest compression is at C, a quarter of a wave-length in advance of A, having travelled over this distance while the plate was advancing from *a* to *a′*. The compressions at D and D_1 represent those which existed immediately in front of the plate when it had advanced respectively one-fourth and three-fourths of the distance from *a″* to *a′*, and the curve A C′ A_1 is the graphical representation both of condensation and velocity for all points in the air between A and A_1.

If the plate ceased vibrating, the condition of things now existing in the portion of air A A_1 would be transferred to successive portions of air in the tube, and the curve A C′ A_1 would, as it were, slide onward through the tube with the velocity of sound, which is about 1100 feet per second. But the plate, instead of remaining permanently at *a′*, executes a backward movement, and produces rarefactions and retrograde velocities, which are propagated onwards in the same manner as the condensations and forward velocities. A complete wave of the undulation is accordingly represented by the curve A E′ A_1 C′ A_2 (Fig. 575), the portions of the curve below the line of abscissas being intended to represent rarefactions and retrograde velocities. If we suppose the vibrating plate to be rigidly connected with a piston which works air-tight in the tube, the velocities of the particles of air in the different points of a wave-length will be identical with the velocities of the piston at the different parts of its motion.

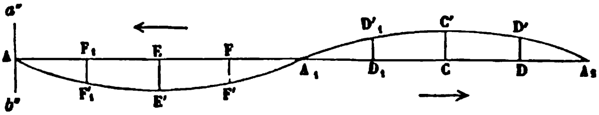


Fig. 575.—Graphical Representation of Complete Wave.

The wave-length A A_2 is the distance that the pulse has travelled while the vibrating plate was moving from its most backward to its most advanced position, and back again. During this time, which is called the *period* of the vibrations, each particle of air goes through its complete cycle of changes, both as regards motion and density. The period of vibration of any particle is thus identical with that of the vibrating plate, and is the same as the time occupied by the waves in travelling a wave-length. Thus, if the plate be one leg of a common A tuning-fork, making 435 complete vibrations per second, the period will be $\frac{1}{435}$th of a second, and the undulation will travel in this time a distance of $\frac{1120}{448}$ feet, or 2 feet 6 inches, which is therefore the wave-length in air for this note. If the plate continues to vibrate in a uniform manner, there will be a continual series of equal and similar waves running along the tube with the velocity of sound. Such a succession of waves constitutes an undulation. Each wave consists of a condensed portion, and a rarefied portion, which are distinguished from each other in Fig. 573 by different tints, the dark shading being intended to represent condensation.

632A. Nature of Undulations.—The possibility of condensations and rarefactions being propagated continually in one direction, while each particle of air simply moves backwards and forwards about its original position, is illustrated by Fig. 575A, which represents, in an exaggerated form, the successive phases of an undulation propagated through 7 particles A B C D E F *a* originally equidistant, the distance from the first to the last being one wave-length of the undulation. The diagram is composed of thirteen horizontal rows, the first and last being precisely alike. The successive rows represent the positions of the particles at successive times, the interval of time from each row to the next being $\frac{1}{12}$th of the period of the undulation.

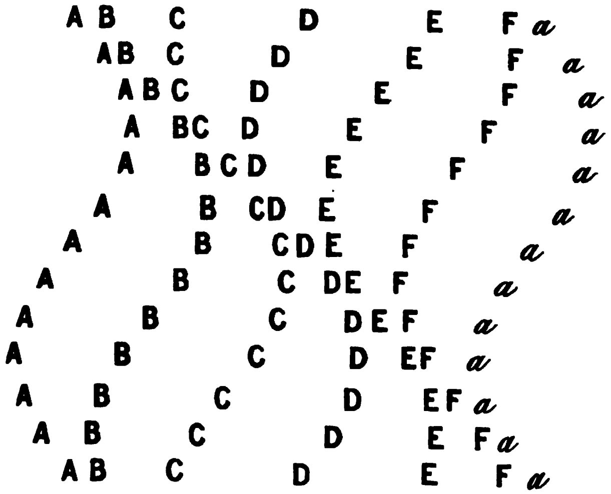


Fig. 575A.—Longitudinal Vibration.

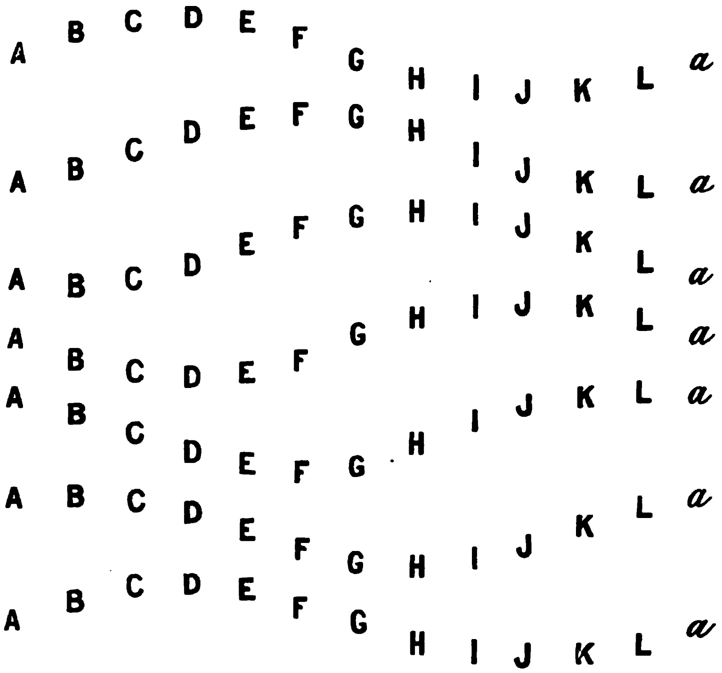
In the first row A and *a* are centres of condensation, and D is a centre of rarefaction. In the third row B is a centre of condensation, and E a centre of rarefaction. In the fifth row the condensation and rarefaction have advanced by one more letter, and so on through the whole series, the initial state of things being reproduced when each of these centres has advanced through a wave-length, so that the thirteenth row is merely a repetition of the first.

The velocities of the particles can be estimated by the comparison of successive rows. It is thus seen that the greatest forward velocity is at the centres of condensation, and the greatest backward velocity at the centres of rarefaction. Each particle has its greatest velocities, and greatest condensation and rarefaction, in passing through its mean position, and comes for an instant to rest in its positions of greatest displacement, which are also positions of mean density.

The distance between A and *a* remains invariable, being always a wave-length, and these two particles are always in the same phase. Any other two particles represented in the diagram are always in different phases, and the phases of A and D, or B and E, or C and F, are always opposite; for example, when A is moving forwards with the maximum velocity, D is moving backwards with the same velocity.

The vibrations of the particles, in an undulation of this kind, are called *longitudinal;* and it is by such vibrations that sound is propagated through air. Fig. 575 B illustrates the manner in which an undulation may be propagated by means of *transverse* vibrations, that is to say, by vibrations executed in a direction perpendicular to that in which the undulation advances. Thirteen particles A B C D E F G H I J K L *a* are represented in the positions which they occupy at successive times, whose interval is one-sixth of a period. At the instant first considered, D and J are the particles which are furthest displaced. At the end of the first interval, the wave has advanced two letters, so that F and L are now the furthest displaced. At the end of the next interval, the wave has advanced two letters further, and so on, the state of things at the end of the six intervals, or of one complete period, being the same as at the beginning, so that the seventh line is merely a repetition of the first. Some examples of this kind of wave-motion will be mentioned in later chapters.

633. Propagation in an Open Space.—When a sonorous disturbance occurs in the midst of an open body of air, the undulations to which it gives rise run out in all directions from the source. If the disturbance is symmetrical about a centre, the waves will be spherical; but this case is exceptional. A disturbance usually produces condensation on one side, at the same instant that it produces rarefaction on another. This is the case, for example, with a vibrating plate, since, when it is moving towards one side, it is moving away from the other. These inequalities which exist in the neighbourhood of the sonorous body, have, however, a tendency to become less marked, and ultimately to disappear, as the distance is increased. Fig. 576 represents a diametral section of a series of spherical waves. Their mode of propagation has some analogy to that of the circular waves produced on water by dropping a stone into it; but the particles which form the waves of water are elevated and depressed; whereas those which form sonorous waves merely advance and retreat, their lines of motion being always coincident with the directions along which the sound travels. In both cases it is important to remark that *the undulation does not involve a movement of transference.* Thus, when the surface of a liquid is traversed by waves, bodies floating on it rise and fall, but are not carried onward. This property is characteristic of undulations generally. *An undu-*


A B C D E F G H I J K L a
A B C D E F G H I J K L a
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A B C D E F G H I J K L a

Fig. 575 B.—Transverse Vibration.

lation may be defined as a system of movements in which the several particles move to and fro, or round and round, about definite points, in such a manner as to produce the continued onward transmission of a condition, or series of conditions.

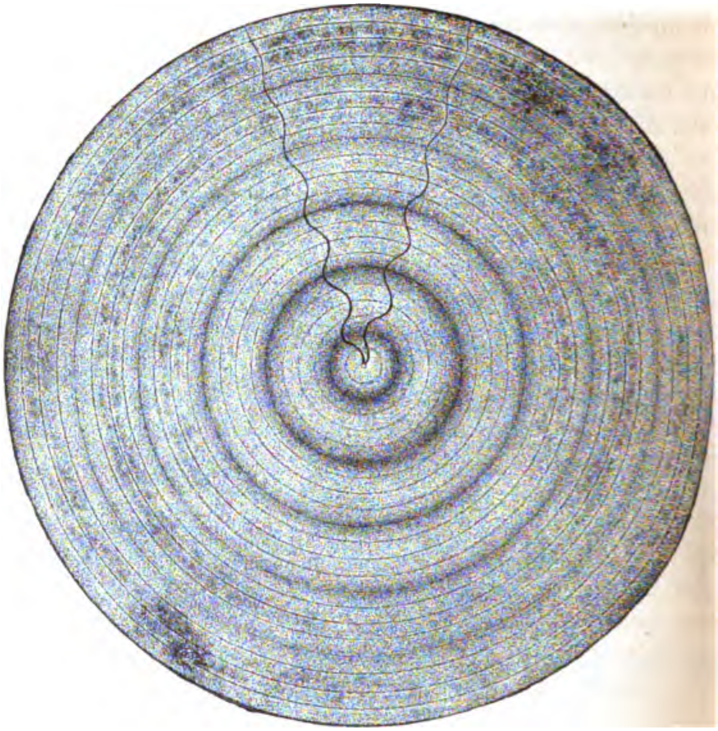

Fig. 576.—Propagation in Open Space.

There is one important difference between the propagation of sound in a uniform tube and in an open space. In the former case, the layers of air corresponding to successive wave-lengths are of equal mass, and their movements are precisely alike, except in so far as they are interfered with by friction. Hence sound is transmitted through tubes to great distances with but little loss of intensity, especially if the tubes are large.[1]

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[1] Regnault, in his experiments on the velocity of sound, found that in a conduit ·108 of a metre in diameter, the report of a pistol charged with a gramme of powder ceased to be heard at the distance of 1150 metres. In a conduit of ·3^{m}, the distance was 3810^{m}. In the great conduit of the St. Michel sewer, of 1^{m}·10, the sound was made by successive reflections to traverse a distance of 10,000 metres without becoming inaudible.—*D.*

In an open space, each successive layer has to impart its own condition to a larger layer; hence there is a continual diminution of amplitude in the vibrations as the distance from the source increases. This involves a continual decrease of loudness. An undulation involves the onward transference of energy; and the amount of energy which traverses, in unit time, any closed surface described about the source, must be equal to the energy which the source emits in unit time. Hence, by the reasoning which we employed in the case of radiant heat (§ 308), it follows that the intensity of sonorous energy diminishes according to the law of inverse squares.

The energy of a particle executing simple vibrations in obedience to forces of elasticity, varies as the square of the amplitude of its excursions; for, if the amplitude be doubled, the distance worked through, and the mean working force, are both doubled, and thus the work which the elastic forces do during the movement from either extreme position to the centre is quadrupled. This work is equal to the energy of the particle in any part of its course. At the extreme positions it is all in the shape of potential energy; in the middle position it is all in the shape of kinetic energy; and at intermediate points it is partly in one of these forms, and partly in the other.

If we sum the potential energies of all the particles which constitute one wave, and also sum their kinetic energies, we shall find the two sums to be equal.[1]

633 A. Dissipation of Sonorous Energy.—The reasoning by which we have endeavoured to establish the law of inverse squares, assumes that onward propagation involves no loss of sonorous energy. This assumption is not rigorously true, inasmuch as vibration implies friction, and friction implies the generation of heat, at the expense of the energy which produces the vibrations. Sonorous energy must therefore diminish with distance somewhat more rapidly than according to the law of inverse squares. All sound, in becoming extinct, becomes converted into heat.

This conversion is greatly promoted by defect of homogeneity in the medium of propagation. In a fog, or a snow-storm, the liquid or solid particles present in the air produce innumerable reflections, in each of which a little sonorous energy is converted into heat.

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[1] In the case of one of the particles, the potential energy at distance y from the position of equilibrium is half the product of force by distance, and may be denoted by $\frac{\mu}{2} \cdot y^2$; the kinetic energy is $\frac{\mu}{2}(a^2-y^2)$, a being the amplitude. The former of these quantities may be written $\frac{\mu}{2} a^2 \cos^2 \theta$, and the latter will be $\frac{\mu}{2} a^2 \sin^2 \theta$. In dealing with the series of particles which form one wave, θ is equicrescent from particle to particle, and its limiting values differ by an entire circumference. Under these conditions, it is obvious that the mean values of $\cos^2 \theta$ and $\sin^2 \theta$ are equal, and that each of them is equal to $\frac{1}{2}$.

634. Velocity of Sound in Air.—The propagation of sound through an elastic medium is not instantaneous, but occupies a very sensible time in traversing a moderate distance. For example, the flash of a gun at the distance of a few hundred yards is seen some time before the report is heard. The interval between the two impressions may be regarded as representing the time required for the propagation of the sound across the intervening distance, for the time occupied by the propagation of light across so small a distance is inappreciable.

It is by experiments of this kind that the velocity of sound in air has been most accurately determined. Among the best determinations may be mentioned that of Lacaille, and other members of a commission appointed by the French Academy in 1738; that of Arago, Bouvard, and other members of the Bureau de Longitudes in 1822; and that of Moll, Vanbeek, and Kuytenbrouwer in Holland, in the same year. All these determinations were obtained by firing cannon at two stations, several miles distant from each other, and noting, at each station, the interval between seeing the flash and hearing the sound of the guns fired at the other. If guns were fired only at one station, the determination would be vitiated by the effect of wind blowing either with or against the sound. The error from this cause is nearly eliminated by firing the guns alternately at the two stations, and still more completely by firing them simultaneously. This last plan was adopted by the Dutch observers, the distance of the two stations in their case being about nine miles. Regnault has quite recently repeated the investigation, taking advantage of the important aid afforded by modern electrical methods for registering the times of observed phenomena. All the most careful determinations agree very closely among themselves, and show that the velocity of sound through air at 0° C. is about 332 metres, or 1090 feet per second.[1] The velocity increases with the temperature, being proportional to the *square root of* what we have called in § 219 A *the absolute temperature.* If t denote the ordinary Centigrade temperature, and α the coefficient of expansion ·00366, the velocity of sound through air at any temperature is given by the formula

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[1] A recent determination by Mr. Stone at the Cape of Good Hope is worthy of note as being based on the comparison of observations made through the sense of hearing alone. It had previously been suggested that the two senses of sight and hearing, which are concerned in observing the flash and report of a cannon, might not be equally prompt in receiving impressions (Airy on *Sound*, p. 131). Mr. Stone accordingly placed two observers—one near a cannon, and the other at about three miles distance; each of whom, on hearing the report, gave a signal through an electric telegraph. The result obtained was in precise agreement with that stated in the text.

$$332\sqrt{1+\alpha t} \text{ in metres per second, or}$$
$$1090\sqrt{1+\alpha t} \text{ in feet per second.}$$

The actual velocity of sound from place to place on the earth's surface is found by compounding this velocity with the velocity of the wind.

There is some reason, both from theory and experiment, for believing that very loud sounds travel rather faster than sounds of moderate intensity.

635. Theoretical Computation of Velocity.—By applying the principles of dynamics to the propagation of undulations,[1] it is computed that the velocity of sound through air must be given by the formula

$$v = \sqrt{\frac{E}{D}}, \qquad (1)$$

D denoting the density of the air, and E its elasticity, as measured by the quotient of pressure applied by compression produced.

Let P denote the pressure of the air in units of force per unit of area; then, if the temperature be kept constant during compression, a small additional pressure p will, by Boyle's law, produce a compression equal to $\frac{p}{P}$, and the value of E, being the quotient of p by this quantity, will be simply P.

On the other hand, if no heat is allowed either to enter or escape, the temperature of the air will be raised by compression, and additional resistance will thus be encountered. In this case the compression ($\frac{v}{V}$ in § 347 A) will be $\frac{p}{P(1+\beta)}$, $1+\beta$ denoting the ratio of the two specific heats, which for air and simple gases is about 1·41; and the value of E will be $P(1+\beta)$.

It thus appears that the velocity of sound in air cannot be less than $\sqrt{\frac{P}{D}}$ nor greater than $\sqrt{1{\cdot}41\frac{P}{D}}$. Its actual velocity, as determined by observation, is nearly identical with the latter of these limiting values. It is probable that the compressions and extensions which the particles of air undergo in transmitting sound are of too brief duration to allow of any sensible transference of heat from particle to particle.

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[1] See Note B at the end of this chapter.

The following is the actual process of calculation for perfectly dry air at 0°C., the centimetre, gramme, and second being taken as the units of length, mass, and time.

The density of dry air at 0°, under the pressure of 1033 grammes per square centimetre, at Paris, is ·001293 of a gramme per cubic centimetre. But the gravitating force of a gramme at Paris is 981 (§ 38). The density ·001293 therefore corresponds to a pressure of 1033×981 units of force per unit of area; and the expression for the velocity in centimetres per second is

$$v=\sqrt{1{\cdot}41\frac{P}{D}}=\sqrt{1{\cdot}41\ \frac{1033\times 981}{{\cdot}001293}}=33240 \text{ nearly};$$

that is, 332·4 metres per second, or 1093 feet per second.

635A. Effects of Pressure, Temperature, and Moisture.—The velocity of sound is independent of the height of the barometer, since changes of this element (at constant temperature) affect P and D in the same direction, and to the same extent.

For a given density, if P_0 denote the pressure at 0°, and a the coefficient of expansion of air, the pressure at t° Centigrade is $P_0\ (1+at)$, the value of a being about $\frac{1}{273}$.

Hence, if the velocity at 0° be 1090 feet per second, the velocity at t° will be $1090\sqrt{1+\frac{t}{273}}$. At the temperature 50° F. or 10° C., which is approximately the mean annual temperature of this country, the value of this expression is about 1110, and at 86° F. or 30° C. it is about 1148. The increase of velocity is thus about a foot per second for each degree Fahrenheit.

The humidity of air has some influence on the velocity of sound, inasmuch as aqueous vapour is lighter than air, but the effect is comparatively trifling, at least in temperate climates. At the temperature 50° F., air saturated with moisture is less dense than dry air by about 1 part in 220, and the consequent increase of velocity cannot be greater than about 1 part in 440, which will be between 2 and 3 feet per second. The increase should, in fact, be somewhat less than this, inasmuch as the value of $1+\beta$ (the ratio of the two specific heats) appears to be only 1·31 for aqueous vapour.[1]

635B. Newton's Theory, and Laplace's Modification.—The earliest theoretical investigation of the velocity of sound was that given by Newton in the *Principia* (book 2, section 8). It proceeds on the tacit assumption that no changes of temperature are produced by the compressions and extensions which enter into the constitution of a sonorous undulation; and the result obtained by Newton is equivalent to the formula

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[1] Rankine on the *Steam Engine*, p. 320.

$$v=\sqrt{\frac{P}{D}};$$

or since (§ 111 A) $\frac{P}{D}=g\,H$, where H denotes the *height of a homogeneous atmosphere,* and the velocity acquired in falling through any height s is $\sqrt{2gs}$, the velocity of sound in air is, according to Newton, the same as *the velocity which would be acquired by falling in vacuo through half the height of a homogeneous atmosphere.* This in fact, is the form in which Newton states his result.[1]

Newton himself was quite aware that the value thus computed theoretically was too small, and he throws out a conjecture as to the cause of the discrepancy; but the true cause was first pointed out by Laplace, as depending upon increase of temperature produced by compression, and decrease of temperature produced by expansion.

635c. Velocity in Gases generally.—The same principles which apply to air apply to gases generally; and since for all simple gases the ratio of the two specific heats is 1·41, the velocity of sound in any simple gas is $\sqrt{1{\cdot}41\,\frac{P}{D}}$, D denoting its absolute density at the pressure P. Comparing two gases at the same pressure, we see that the velocities of sound in them will be inversely as the square roots of their absolute densities; and this will be true whether the temperatures of the two gases are the same or different.

636. Velocity of Sound in Liquids.—The velocity of sound in water was measured by Colladon, in 1826, at the Lake of Geneva. Two boats were moored at a distance of 13,500 metres (between 8 and 9 miles). One of them carried a bell, weighing about 140 lbs., immersed in the lake. Its hammer was moved by an external lever, so arranged as to ignite a small quantity of gunpowder at the instant of striking the bell. An observer in the other boat was enabled to hear the sound by applying his ear to the extremity of a trumpet-shaped tube (Fig. 572), having its lower end covered with a membrane and facing towards the direction from which the sound proceeded. By noting the interval between seeing the flash and hearing the sound, the velocity with which the sound travelled through the water was determined. The velocity thus computed was 1435 metres per second, and the temperature of the water was 8°·1 C.

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[1] Newton's investigation relates only to *simple* waves; but if these have all the same velocity (as Newton shows), this must also be the velocity of the complex wave which they compose. Hence the restriction is only apparent.

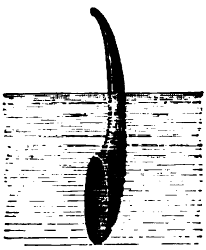

Fig. 572.

Formula (1) of § 633 holds for liquids as well as for gases, and is easily applied to the case of water if we neglect the changes of temperature produced by compression and extension. We have stated in § 22 (Part I.) that the compressibility of water, as determined by the most recent experiments, is ·0000457 per atmosphere, at the temperature of 15° C. The value of E in terms of the units employed in § 635 is therefore $\frac{1033 \times 981}{\cdot 0000457}$, and D, the mass of a cubic centimetre of water expressed in grammes, is unity. We have therefore

$$v = \sqrt{\frac{E}{D}} = \sqrt{\frac{1033 \times 981}{\cdot 0000457}} = 148920 \text{ nearly,}$$

which, reduced to metres per second, is 1489·2.

This computation applies to water at 15°, which is 7° warmer than the water of the lake. As the elasticity of water is known to increase with its temperature, the difference between the two results is in the right direction. The agreement is sufficiently close to show that the increase of elasticity from the instantaneous changes of temperature produced by sonorous undulations is insignificant in the case of water.[1]

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[1] Sir W. Thomson has investigated, on thermo-dynamic principles, the additional pressure required to produce a given diminution of volume, when the heat of compression is not allowed to escape. He computes that the elasticity of a fluid under these circumstances is to its elasticity at constant temperature as $1 + \frac{E \alpha^2 T}{J C}$ to 1, E denoting the elasticity at constant temperature, α the coefficient of expansion of the fluid per degree Centigrade, T the absolute temperature of the fluid, or the common Centigrade temperature increased by 273°, C the thermal capacity of unit volume of the liquid, and J Joule's equivalent for a unit of heat. If E be expressed in absolute units of force per unit of area, J must be expressed in absolute units of work, and will be 42400 × 981 if the centimetre, the gramme, and the second be the units of length, mass, and time.

For water at 15° C. the coefficient of expansion α is about ·00015, T is 288, and C is unity. The value of $\frac{E \alpha^2 T}{J C}$ will be found to be about ·003, so that the heat of compression and cold of expansion increase the effective elasticity by 3 parts in a thousand, and therefore increase the velocity of sound by 1½ part in a thousand.

The same formula applies to solids if we make E denote Young's modulus, and α the coefficient of linear expansion. For iron it gives, according to Sir W. Thomson (*Proc. R. S. E.* 1865–6), an increase of about ⅓ per cent. in Young's modulus, and therefore of ⅙ per cent. in the velocity of sound.

Wertheim has measured the velocity of sound in some liquids by an indirect method, which will be explained in a later chapter. He finds it to be 1160 metres per second in ether and alcohol, and 1900 in a solution of chloride of calcium.

637. Velocity of Sound in Solids.—The velocity of sound in cast-iron was determined by Biot and Martin by means of a connected series of water-pipes, forming a conduit of a total length of 951 metres. One end of the conduit was struck with a hammer, and an observer at the other end heard two sounds, the first transmitted by the metal, and the second by the air, the interval between them being 2·5 seconds. Now the time required for travelling this distance through air, at the temperature of the experiment (11° C.), is 2·8 seconds. The time of transmission through the metal was therefore ·3 of a second, which is at the rate of 3170 metres per second. It is, however, to be remarked, that the transmitting body was not a continuous mass of iron, but a series of 376 pipes, connected together by collars of lead and tarred cloth, which must have considerably delayed the transmission of the sound. But in spite of this, the velocity is about nine times greater than in air.

Wertheim, by the indirect methods above alluded to, measured the velocity of sound in a number of solids, with the following results, the velocity in air being taken as the unit of velocity:—

Lead,	3·974 to 4·120
Tin,	7·338 to 7·480
Gold,	5·603 to 6·424
Silver,	7·903 to 8·057
Zinc,	9·863 to 11·009
Copper,	11·167
Platinum,	7·823 to 8·467
Steel,	14·361 to 15·108
Iron,	15·108
Brass,	10·224
Glass,	14·956 to 16·759
Flint Glass,	11·890 to 12·220
Oak,	9·902 to 12·02
Fir,	12·49 to 17·26

638. Theoretical Computation.—The formula $\sqrt{\frac{E}{D}}$ serves for solids as well as for liquids and gases; but as solids can be subjected to many different kinds of strain, whereas liquids and gases can be subjected to only one, we may have different values of E, and different velocities of transmission of pulses for the same solid. This is true even in the case of a solid whose properties are alike in all directions (called an *isotropic* solid); but the great majority of solids are very far from fulfilling this condition, and transmit sound more rapidly in some directions than in others.

When the sound is propagated by alternate compressions and extensions running along a substance which is not prevented from extending and contracting laterally, the elasticity E becomes identical[1] with Young's modulus (§ 23). On the other hand, if uniform spherical waves of alternate compression and extension spread outwards, symmetrically, from a point in the centre of an infinite solid, lateral extension and contraction will be prevented by the symmetry of the action. The effective elasticity is, in this case, greater than Young's modulus, and the velocity of sound will be increased accordingly.

By the table on p. 29 the value of Young's modulus for copper is 12,558 kilogrammes per square millimetre, or 1,255,800,000 grammes per square centimetre, and by the table on p. 89 the density of copper in grammes per cubic centimetre is 8·8. Hence, for the velocity of sound through a copper rod, in centimetres per second, we have

$$v = \sqrt{\frac{E}{D}} = \sqrt{\frac{1255800000 \times 981}{8 \cdot 8}} = 374150 \text{ nearly,}$$

or 3741·5 metres per second.

This is about 11·2 times the velocity in air.

639. Reflection of Sound.—When sonorous waves meet a fixed obstacle they are reflected, and the two sets of waves—one direct, and the other reflected—are propagated just as if they came from two separate sources. If the reflecting surface is plane, waves diverging from any centre O in front of it are reflected so as to diverge

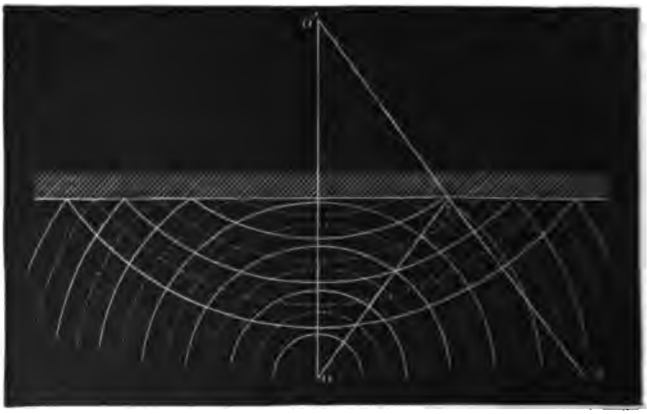

Fig. 577.—Reflection of Sound.

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[1] Subject to the small correction mentioned in the foot-note to § 636.

from a centre O′ symmetrically situated behind it, and an ear at any point M in front hears the reflected sound as if it came from O′.

The direction from which a sound appears to the hearer to proceed is determined by the direction along which the sonorous pulses are propagated, and is always normal to the waves. A normal to a set of sound-waves may therefore conveniently be called a *ray* of sound. O I is a direct ray, and I M the corresponding reflected ray; and it is obvious, from the symmetrical position of the points O O′, that these two rays are equally inclined to the surface, or *the angles of incidence and reflection are equal.*

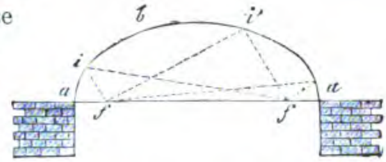

Fig. 578.—Reflection from Elliptic Roof.

640. Illustrations of Reflection of Sound.—The reflection of sonorous waves explains some well-known phenomena. If *a b a* be an elliptic dome or arch, a sound emitted from either of the foci *f f* will be reflected from the elliptic surface in such a direction as to pass through the other focus. A sound emitted from either focus may thus be distinctly heard at the other, even when quite inaudible at nearer points. This is a consequence

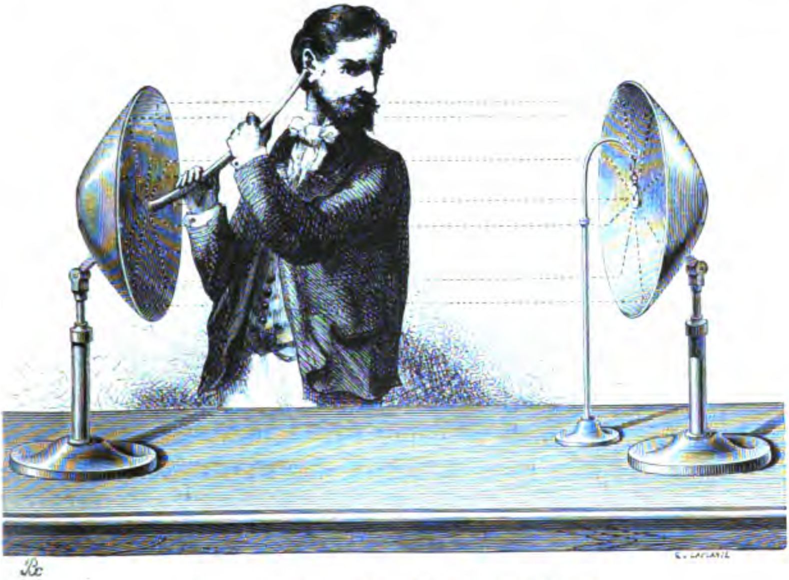
Fig. 579.—Reflection of Sound from Conjugate Mirrors.

of the property, that lines drawn to any point on an ellipse from the two foci are equally inclined to the curve.

The experiment of the conjugate mirrors (§ 311) is also applicable to sound. Let a watch be hung in the focus of one of them (Fig. 579), and let a person hold his ear at the focus of the other; or still better, to avoid intercepting the sound before it falls on the second mirror, let him employ an ear-trumpet, holding its open end at the focus. He will distinctly hear the ticking, even when the mirrors are many yards apart.[1]

641. Echo.—Echo is the most familiar instance of the reflection of sound. In order to hear the echo of one's own voice, there must be a distant body capable of reflecting sound directly back, and the number of syllables that an echo will repeat is proportional to the distance of this obstacle. Reckoning $\frac{1}{5}$ of a second as the time of pronouncing a syllable, the space traversed by sound in this time is about 200 feet, and an obstacle must be at half this distance in order that it may be able to send back a single syllable. The sounds reflected to the speaker have travelled first over the distance O A (Fig. 580) from him to the reflecting body, and then back from A to O. Supposing five syllables to be pronounced in a second, and taking the velocity of sound as 1100 feet per second, a distance of

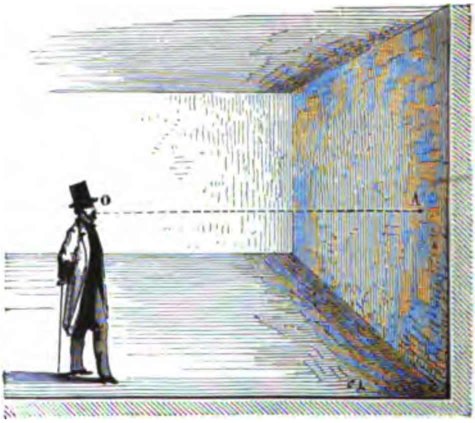

Fig. 580.—Echo.

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[1] Sondhaus has shown that sound, like light, is capable of being *refracted*. A spherical balloon of collodion, filled with carbonic acid gas, acts as a sound-lens. If a watch be hung at some distance from it on one side, an ear held at the conjugate focus on the other side will hear the ticking.

550 feet from the speaker to the reflecting body would enable the speaker to complete the fifth syllable before the return of the first; this is at the rate of 110 feet per syllable. At distances less than about 100 feet there is not time for the distinct reflection of a single syllable; but the reflected sound mingles with the voice of the speaker. This is particularly observable under vaulted roofs.

Multiple echoes are not uncommon. They are due, in some cases, to independent reflections from obstacles at different distances; in others, to reflections of reflections. A position exactly midway between two parallel walls, at a sufficient distance apart, is favourable for the observance of this latter phenomenon. One of the most frequently cited instances of multiple echoes is that of the old palace of Simonetta, near Milan, which forms three sides of a quadrangle. According to Kircher, it repeats forty times.

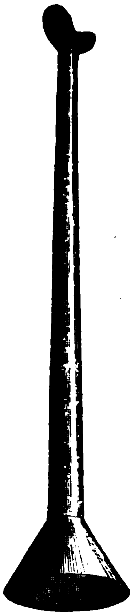
Fig. 581. Speaking-trumpet.

642. Speaking and Hearing Trumpets.—The complete explanation of the action of these instruments presents considerable difficulty. The speaking-trumpet (Fig. 581) consists of a long tube (sometimes 6 feet long), slightly tapering towards the speaker, furnished at this end with a hollow mouth-piece, which nearly fits the lips, and at the other with a funnel-shaped enlargement, called the *bell*, opening out to a width of about a foot. It is much used at sea, and is found very effectual in making the voice heard at a distance. The explanation usually given of its action is, that the slightly conical form of the long tube produces a series of reflections in directions more and more nearly parallel to the axis; but this explanation fails to account for the utility of the *bell*, which experience has shown to be considerable.

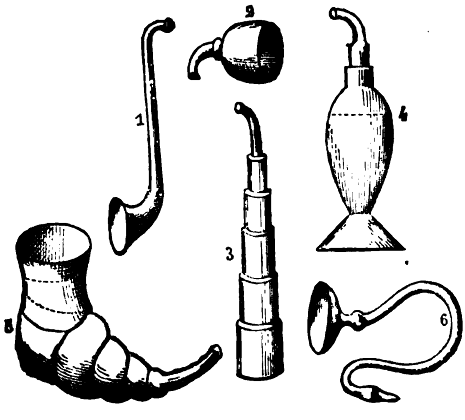

Fig. 582.—Ear-trumpets.

Ear-trumpets have various forms, as represented in Fig. 582;

having little in common, except that the external opening or *bell* is much larger than the end which is introduced into the ear. Membranes of gold-beaters' skin are sometimes stretched across their interior, in the positions indicated by the dotted lines in Nos. 4 and 5. No. 6 consists simply of a bell with such a membrane stretched across its outer end, while its inner end communicates with the ear by an indian-rubber tube with an ivory end-piece. These light membranes are peculiarly susceptible of impression from aerial vibrations. In Regnault's experiments above cited, it was found that membranes were affected at distances greater than those at which sound was heard.

643. Interference of Sonorous Undulations.—When two systems of waves are traversing the same matter, the actual motion of each particle of the matter is the resultant of the motions due to each system separately. When these component motions are in the same direction the resultant is their sum; when they are in opposite directions it is their difference; and if they are equal, as well as opposite, it is zero. Very remarkable phenomena are thus produced when the two undulations have the same, or nearly the same wave-length; and the action which occurs in this case is called *interference*.

When two sonorous undulations of exactly equal wave length and amplitude are traversing the same matter in the same direction, their phases must either be the same, or must everywhere differ by the same amount. If they are the same, the amplitude of vibration for each particle will be double of that due to either undulation separately. If they are opposite—in other words, if one undulation be half a wave-length in advance of the other—the motions which they would separately produce in any particle are equal and opposite, and the particle will accordingly remain at rest. Two sounds will thus, by their conjoint action, produce silence.

In order that the extinction of sound may be complete, the rarefied portions of each set of waves must be the *exact* counterparts of the condensed portions of the other set, a condition which can only be approximately attained in practice.

The following experiment, due to M. Desains, affords a very direct illustration of the principle of interference. The bottom of a wooden box is pierced with an opening, in which a powerful whistle fits. The top of the box has two larger openings symmetrically placed with respect to the lower one. The inside of the box is lined with felt, to prevent the vibrations from being communicated to the box, and to weaken internal reflection. When the whistle is sounded, if a membrane, with sand strewn on it, is held in various positions in the vertical plane which bisects, at right angles, the line joining the two openings, the sand will be agitated, and will arrange itself in nodal lines. But if it is carried out of this plane, positions will be found, at equal distances on both sides of it, at which the agitation is scarcely perceptible. If, when the membrane is in one of these positions, we close one of the two openings, the sand is again agitated, clearly showing that the previous absence of agitation was due to the interference of the undulations proceeding from the two orifices.

In this experiment the proof is presented to the eye. In the following experiment, which is due to M. Lissajous, it is presented to the ear. A circular plate, supported like the plate in Fig. 565, is made to vibrate in sectors separated by radial nodes. The number of sectors will always be even, and adjacent sectors will vibrate in opposite directions. Let a disk of card-board of the same size be divided into the same number of sectors, and let alternate sectors be cut away, leaving only enough near the centre to hold the remaining sectors together. If the card be now held just over the vibrating disk, in such a manner that the sectors of the one are exactly over sectors of the other, a great increase of loudness will be observed, consequent on the suppression of the sound from alternate sectors; but if the card-board disk be turned through the width of half a sector, the effect no longer occurs. If the card is made to rotate rapidly in a continuous manner, the alternations of loudness will form a series of beats.

It is for a similar reason that, when a large bell is vibrating, a person in its centre hears the sound as only moderately loud, while within a short distance of some portions of the edge the loudness is intolerable.

644. Interference of Direct and Reflected Waves. Nodes and Antinodes.—Interference may also occur between undulations travelling in opposite directions; for example, between a direct and a reflected system. When waves proceeding along a tube meet a rigid obstacle, forming a cross section of the tube, they are reflected directly back again, the motion of any particle close to the obstacle being compounded of that due to the direct wave, and an equal and opposite motion due to the reflected wave. The reflected waves are in fact the images (with reference to the obstacle regarded as a plane mirror) of the waves which would exist in the prolongation of the tube if the obstacle were withdrawn. At the distance of half a wave-length from the obstacle the motions due to the direct and reflected waves will accordingly be equal and opposite, so that the particles situated at this distance will be permanently at rest; and the same is true at the distance of any number of half wave-lengths from the obstacle. The air in the tube will thus be divided into a number of vibrating segments separated by nodal planes or cross sections of no vibration arranged at distances of half a wave-length apart. One of these nodes is at the obstacle itself. At the centres of the vibrating segments—that is to say, at the distance of a quarter wave-length *plus* any number of half wave-lengths from the obstacle or from any node—the velocities due to the direct and reflected waves will be equal and in the same direction, and the amplitude of vibration will accordingly be double of that due to the direct wave alone. These are the sections of greatest disturbance as regards change of place. We shall call them *antinodes*. On the other hand, it is to be remembered that motion *with* the direct wave is motion *against* the reflected waves, and *vice versa*, so that (§ 632) at points where the velocities due to both have the same absolute direction they correspond to condensation in the case of one of these undulations, and to rarefaction in the case of the other. Accordingly, these sections of maximum movement are the places of no change of density; and on the other hand, the nodes are the places where the changes of density are greatest. If the reflected undulation is feebler than the direct one, as will be the case, for example, if the obstacle is only imperfectly rigid, the destruction of motion at the nodes and of change of density at the antinodes will not be complete; the former will merely be places of minimum motion, and the latter of minimum change of density.

Direct experiments in verification of these principles, a wall being the reflecting body, were conducted by Savart, and also by Seebeck, the latter of whom employed a testing apparatus called the acoustic pendulum. It consists essentially of a small membrane stretched in a frame, from the top of which hangs a very light pendulum, with its bob resting against the centre of the membrane. In the middle portions of the vibrating segments the membrane, moving with the air on its two faces, throws back the pendulum, while it remains nearly free from vibration at the nodes.

Regnault made extensive use of the acoustic pendulum in his experiments on the velocity of sound. The pendulum, when thrown back by the membrane, completed an electric circuit, and thus effected a record of the instant when the sound arrived.

644A. Beats Produced by Interference.—When two notes, which are not quite in unison, are sounded together, a peculiar palpitating effect is produced;—we hear a series of bursts of sound, with intervals of comparative silence between them. The bursts of sound are called *beats*, and the notes are said to *beat* together. If we have the power of tuning one of the notes, we shall find that as they are brought more nearly into unison, the beats become slower, and that, as the departure from unison is increased, the beats become more rapid, till they degenerate first into a rattle, and then into a discord. The effect is most striking with deep notes.

These beats are completely explained by the principle of interference. As the wave-lengths of the two notes are slightly different, while the velocity of propagation is the same, the two systems of waves will, in some portions of their course, agree in phase, and thus strengthen each other; while in other parts they will be opposite in phase, and will thus destroy each other. Let one of the notes, for example, have 100 vibrations per second, and the other 101. Then, if we start from an instant when the maxima of condensation from the two sources reach the ear together, the next such conjunction will occur exactly a second later. During the interval one of the systems of waves has been gradually falling behind the other, till, at the end of the second, the loss has amounted to one wave-length. At the middle of the second it will have amounted to half a wave-length, and the two sounds will destroy each other. We shall thus have one beat and one extinction in each second, as a consequence of the fact that the higher note has made one vibration more than the lower. In general, the frequency of beats is the difference of the frequencies of vibration of the beating notes.

NOTE A. § 632.

That the particles which are moving forward are in a state of compression, may be shown in the following way:—Consider an imaginary cross section travelling forward through the tube with the same velocity as the undulation. Call this velocity v, and the velocity of any particle of air u. Also let the density of any particle be denoted by ρ. Then u and ρ remain constant for the imaginary moving section, and the mass of air which it traverses in its motion per unit time is $(v-u)\rho$. As there is no permanent transfer of air in either direction through the tube, the mass thus traversed must be the same as if the air were at rest at its natural density. Hence the value of $(v-u)\rho$ is the same for all cross sections; whence it follows, that where u is greatest ρ must be greatest, and where u is negative ρ is less than the natural density.

If ρ_0 denote the natural density, we have $(v-u)\rho=v\rho_0$, whence $\frac{u}{v}=\frac{\rho-\rho_0}{\rho}$; that is to say, *the ratio of the velocity of a particle to the velocity of the undulation is equal to the condensation existing at the particle.* If u is negative—that is to say, if the velocity be retrograde—its ratio to v is a measure of the rarefaction.

From this principle we may easily derive a formula for the velocity of sound, bearing in mind that u is always very small in comparison with v, and that consequently the ratio of ρ to ρ_0 is very nearly unity.

For, consider a thin lamina of air, whose natural thickness is δx, and let $\delta\rho$ and δp be the differences between the densities and pressures respectively on its two faces. Then the equation above investigated leads to the condition $\frac{\delta u}{\delta\rho}=\frac{v}{\rho}$. But the time which the moving section occupies in traversing the lamina is approximately $\frac{\delta x}{v}$, and in this time the velocity of the lamina changes by the amount δu. The force producing this change of velocity is δp, or $1{\cdot}41\frac{p}{\rho}\delta\rho$, and must be equal to the quotient of change of momentum by time, that is to $\rho\,\delta x\,.\,\delta u\div\frac{\delta x}{v}$ or to $\rho v\,\delta u$. Hence $\frac{\delta u}{\delta\rho}=1{\cdot}41\frac{p}{\rho^2 v}$. Equating this to the other expression for $\frac{\delta u}{\delta\rho}$ we have

$$\frac{v}{\rho}=1{\cdot}41\frac{p}{\rho^2 v},\quad v^2=1{\cdot}41\frac{p}{\rho}.$$

This investigation is due to Professor Rankine, *Phil. Trans.* 1869.

NOTE B. § 635.

The following is the usual investigation of the velocity of transmission of sound through a uniform tube filled with air, friction being neglected: Let x denote the original distance of a particle of air from the section of the tube at which the sound originates, and $x+y$ its distance at time t, so that y is the displacement of the particle from the position of equilibrium. Then a particle which was originally at distance $x+\delta x$ will at time t be at the distance $x+\delta x+y+\delta y$; and the thickness of the intervening lamina, which was originally δx, is now $\delta x+\delta y$. Its compression is therefore $-\frac{\delta y}{\delta x}$ or ultimately $-\frac{dy}{dx}$, and if P denote the original pressure, the increase of pressure is $-1{\cdot}41\,\mathrm{P}\frac{dy}{dx}$. The excess of pressure behind a lamina δx above the pressure in front is $\frac{d}{dx}\left(1{\cdot}41\,\mathrm{P}\frac{dy}{dx}\right)\delta x$, or $1{\cdot}41\,\mathrm{P}\frac{d^2y}{dx^2}\delta x$; and if D denote the original density of the air, the acceleration of the lamina will be the quotient of this expression by $\mathrm{D}.\delta x$. But this acceleration is $\frac{d^2y}{dt^2}$. Hence we have the equation

$$\frac{d^2y}{dt^2}=1{\cdot}41\frac{\mathrm{P}}{\mathrm{D}}\frac{d^2y}{dx^2};$$

the integral of which is

$$y=\mathrm{F}(x-vt)+f(x+vt);$$

where v denotes $\sqrt{1{\cdot}41\frac{\mathrm{P}}{\mathrm{D}}}$, and F, f denote any functions whatever.

The term $F(x-vt)$ represents a wave, of the form $y=F(x)$, travelling forwards with velocity v; for it has the same value for $t_1+\delta t$ and $x_1+v.\delta t$ as for t_1 and x_1. The term $f(x+vt)$ represents a wave, of the form $y=f(x)$, travelling backwards with the same velocity.

In order to adapt this investigation, as well as that given in Note A, to the propagation of longitudinal vibrations through any elastic material, whether solid, liquid, or gaseous, we have merely to introduce E in the place of 1·41 P, E denoting the coefficient of elasticity of the substance, as defined by the condition that a compression $\frac{dy}{dx}$ is produced by a force (per unit area) of $E\frac{dy}{dx}$.

CHAPTER LIV.

NUMERICAL EVALUATION OF SOUND.

645. Qualities of Musical Sound.—Musical tones differ one from another in respect of three qualities;—loudness, pitch, and character.

Loudness.—The loudness of a sound considered subjectively is the intensity of the sensation with which it affects the organs of hearing. Regarded objectively, it depends, in the case of sounds of the same pitch and character, upon the energy of the aerial vibrations in the neighbourhood of the ear, and is proportional to the square of the amplitude.

Our auditory apparatus is, however, so constructed as to be more susceptible of impression by sounds of high than of low pitch. A bass note must have much greater energy of vibration than a treble note, in order to strike the ear as equally loud. The intensity of sonorous vibration at a point in the air is therefore not an absolute measure of the intensity of the sensation which will be received by an ear placed at the point.

The word loud is also frequently applied to a source of sound, as when we say a loud voice, the reference being to the loudness as heard at a given distance from the source. The diminution of loudness with increase of distance according to the law of inverse squares is essentially connected with the proportionality of loudness to square of amplitude.

Pitch.—Pitch is the quality in respect of which an acute sound differs from a grave one; for example, a treble note from a bass note. All persons are capable of appreciating differences of pitch to some extent, and the power of forming accurate judgments of pitch constitutes what is called a *musical ear.*

Physically, pitch depends solely on *frequency of vibration,* that is to say, on the number of vibrations executed per unit time. In ordinary circumstances this frequency is the same for the source of sound, the medium of transmission, and the drum of the ear of the person hearing; and in general the transmission of vibrations from one body or medium to another produces no change in their frequency. The second is universally employed as the unit of time in treating of sonorous vibrations; so that *frequency* means *number of vibrations per second.* Increase of frequency corresponds to elevation of pitch.

Period and *frequency* are reciprocals. For example, if the period of each vibration is $\frac{1}{100}$ of a second, the number of vibrations per second is 100. Period therefore is an absolute measure of pitch, and the longer the period the lower is the pitch.

The wave-length of a note in any medium is the distance which sound travels in that medium during the period corresponding to the note. Hence wave-length may be taken as a measure of pitch, provided the medium be given; but, in passing from one medium to another, wave-length varies directly as the velocity of sound. The wave-length of a given note in air depends upon the temperature of the air, and is shortened in transmission from the heated air of a concert-room to the colder air outside, while the pitch undergoes no change.

If we compare a series of notes rising one above another by what musicians regard as equal differences of pitch, their frequencies will not be equidifferent, but will form an increasing geometrical progression, and their periods (and wave-lengths in a given medium) will form a decreasing geometrical progression.

Character.—Musical sounds may, however, be alike as regards pitch and loudness, and may yet be easily distinguishable. We speak of the *quality* of a singer's voice, and the *tone* of a musical instrument; and we characterize the one or the other as rich, sweet, or mellow, on the one hand; or as poor, harsh, nasal, &c., on the other. These epithets are descriptive of what musicians call *timbre*—a French word literally signifying *stamp.* German writers on acoustics denote the same quality by a term signifying *sound-tint.* It might equally well be called *sound-flavour.* We adopt *character* as the best English designation.

Physically considered, as wave-length and wave-amplitude fall under the two previous heads, *character* must depend upon the only remaining point in which aerial waves can differ—namely their *form,* meaning by this term the law according to which the velocities and densities change from point to point of a wave. This subject will be more fully treated in Chapter lvi. Every musical sound is more or less mingled with non-musical noises, such as puffing, scraping, twanging, hissing, rattling, &c. These are not comprehended under *timbre* or *character* in the usage of the best writers on acoustics. The gradations of loudness which characterize the commencement, progress, and cessation of a note, and upon which musical effect often greatly depends, are likewise excluded from this designation. In distinguishing the sounds of different musical instruments, we are often guided as much by these gradations and extraneous accompaniments as by the character of the musical tones themselves.

646. Musical Intervals.—When two notes are heard, either simultaneously or in succession, the ear experiences an impression of a special kind, involving a perception of the relation existing between them as regards difference of pitch. This impression is often recognized as identical where absolute pitch is very different, and we express this identity of impression by saying that the *musical interval* is the same.

Each musical interval, thus recognized by the ear as constituting a particular relation between two notes, is found to correspond to a particular *ratio* between their frequencies of vibration. Thus the *octave*, which of all intervals is that which is most easily recognized by the ear, is the relation between two notes whose *frequencies* are as 1 to 2, the upper note making twice as many vibrations as the lower in any given time.

It is the musician's business so to combine sounds as to awaken emotions of the peculiar kind which are associated with works of art. In attaining this end he employs various resources, but musical intervals occupy the foremost place. It is upon the judicious employment of these that successful composition mainly depends.

647. Gamut.—The *gamut* or *diatonic scale* is a series of eight notes having certain definite relations to one another as regards frequency of vibration. The first and last of the eight are at an interval of an octave from each other, and are called by the same name: and by taking in like manner the octaves of the other notes of the series, we obtain a repetition of the gamut both upwards and downwards, which may be continued over as many octaves as we please.

The notes of the gamut are usually called by the names

Do	Re	Mi	Fa	Sol	La	Si	Do_2

and their vibration-frequencies are proportional to the numbers

1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

or, clearing fractions, to

24	27	30	32	36	40	45	48

The intervals from Do to each of the others in order are called a *second*, a *major third*, a *fourth*, a *fifth*, a *sixth*, a *seventh*, and an *octave* respectively. The interval from La to Do_2 is called a *minor third*, and is evidently represented by the ratio $\frac{6}{5}$.

The interval from Do to Re, from Fa to Sol, or from La to Si, is represented by the ratio $\frac{9}{8}$, and is called a *major tone*. The interval from Re to Mi, or from Sol to La, is represented by the ratio $\frac{10}{9}$, and is called a *minor tone*. The interval from Mi to Fa, or from Si to Do_2, is represented by the ratio $\frac{16}{15}$, and is called a *limma*. As the square of $\frac{16}{15}$ is a little greater than $\frac{10}{9}$, a limma is rather more than half a major tone.

The intervals between the successive notes of the gamut are accordingly represented by the following ratios[1]:—

Do Re Mi Fa Sol La Si Do_2

$\frac{9}{8}$ $\frac{10}{9}$ $\frac{16}{15}$ $\frac{9}{8}$ $\frac{10}{9}$ $\frac{9}{8}$ $\frac{16}{15}$

Do (with all its octaves) is called the *key-note*, or simply the *key*, of the piece of music, and may have any pitch whatever. In order to obtain perfect harmony, the above ratios should be accurately maintained whatever the key-note may be.

648. Tempered Gamut.—A great variety of keys are employed in music, and it is a practical impossibility, at all events in the case of instruments like the piano and organ, which have only a definite set of notes, to maintain these ratios strictly for the whole range of possible key-notes. Compromise of some kind becomes necessary, and different systems of compromise are called different *temperaments* or different *modes of temperament*. The temperament which is most in favour in the present day is the simplest possible, and is called *equal temperament*, because it favours no key above another, but makes the tempered gamut exactly the same for all. It ignores the difference between major and minor tones, and makes the limma exactly half of either. The interval from Do to Do_2 is thus divided into 5 tones and 2 semitones, a tone being $\frac{1}{6}$ of an octave, and a semitone $\frac{1}{12}$ of an octave. The ratio of frequencies corresponding to a tone will therefore be the sixth root of 2, and for a semitone it will be the 12th root of 2.

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[1] The logarithmic differences, which are accurately proportional to the intervals, are approximately as under, omitting superfluous zeros.

Do Re Mi Fa Sol La Si Do

51 46 28 51 46 51 28

The difference between the natural and the tempered gamut for the key of C is shown by the following table, which gives the number of complete vibrations per second for each note of the middle octave of an ordinary piano:—

	Tempered Gamut.	Natural Gamut.		Tempered Gamut.	Natural Gamut.
C . .	258·7	258·7	G . .	387·6	388·0
D . .	290·3	291·0	A . .	435·0	431·1
E . .	325·9	323·4	B . .	488·2	485·0
F . .	345·3	344·9	C . .	517·3	517·3

The absolute pitch here adopted is that of the Paris Conservatoire, and is fixed by the rule that A (the middle A of a piano, or the A string of a violin) is to have 435 complete vibrations per second in the tempered gamut. This is rather lower than the concert-pitch which has prevailed in this country in recent years, but is probably not so low as that which prevailed in the time of Handel. It will be noted that the number of vibrations corresponding to C is approximately equal to a power of 2 (256 or 512). Any power of 2 accordingly expresses (to the same degree of approximation) the number of vibrations corresponding to one of the octaves of C.

The Stuttgard congress (1834) recommended 528 vibrations per second for C, and the C tuning-forks sold under the sanction of the Society of Arts are guaranteed to have this pitch. By multiplying the numbers 24, 27 . . . 48, in § 647, by 11, we shall obtain the frequencies of vibration for the natural gamut in C corresponding to this standard. What is generally called *concert-pitch* gives C about 538. The C of the Italian Opera is 546. Handel's C is said to have been 499$\frac{1}{2}$.

649. Limits of Pitch employed in Music.—The deepest note regularly employed in music is the C of 32 vibrations per second, which is emitted by the longest pipe (the 16-foot pipe) of large organs. Its wave-length in air at a temperature at which the velocity of sound is 1120 feet per second, is $\frac{1120}{32}=35$ feet. The highest note employed seldom exceeds A, the third octave of the A above defined. Its number of vibrations per second is $435 \times 2^3 = 3480$, and its wave-length in air is about 4 inches. Above this limit it is difficult to appreciate pitch, but notes of at least ten times this number of vibrations are audible.

The average compass of the human voice is about two octaves. The deep F of a bass-singer has 87, and the upper G of the treble 775 vibrations per second. Voices which exceed either of these limits are regarded as deep or high, though a few voices exceed them to the extent of nearly an octave.

650. **Minor Scale and Pythagorean Scale.**—The interval expressed by the ratio $\frac{25}{24}$ is called a *diesis*, and the interval $\frac{81}{80}$ is called a *comma*. The former is the difference between a limma and a minor tone, the latter is the difference between a major and a major tone. The signs $\sharp$ and $\flat$ (sharp and flat) appended to a note indicate, when strictly understood, that it is to be raised or lowered by the amount of a diesis. The major scale or gamut, as above given, is modified in the following way to obtain the minor scale:—

Do Re Mi$\flat$ Fa Sol La$\flat$ Si$\flat$ Do$_2$
$\frac{9}{8}$ $\frac{16}{15}$ $\frac{10}{9}$ $\frac{9}{8}$ $\frac{16}{15}$ $\frac{9}{8}$ $\frac{10}{9}$

the numbers in the second line being the ratios which represent the intervals between the successive notes.

It is worthy of note that Pythagoras, who was the first to attempt the numerical evaluation of musical intervals, laid down a scheme of values slightly different from that which is now generally adopted. According to him, the intervals between the successive notes of the major scale are as follows:—

Do Re Mi Fa Sol La Si Do
$\frac{9}{8}$ $\frac{9}{8}$ $\frac{256}{243}$ $\frac{9}{8}$ $\frac{9}{8}$ $\frac{9}{8}$ $\frac{256}{243}$

This scheme agrees exactly with the common system as regards the values of the fourth, fifth, and octave, and makes the values of the major third, the sixth, and the seventh each greater by a comma, while the small interval from *mi* to *fa*, or from *si* to *do*, is diminished by a comma. In the ordinary system, the prime numbers which enter the ratios are 2, 3, and 5; in the Pythagorean system they are only 2 and 3; hence the interval between any two notes of the Pythagorean scale can be expressed as the sum or difference of a certain number of octaves and fifths. In tuning a violin by making the intervals between the strings true fifths, the Pythagorean scheme is virtually employed.

651. Methods of Counting Vibrations. Siren.—The instrument which is chiefly employed for counting the number of vibrations corresponding to a given note, is called the *siren*, and was devised by Cagniard de Latour. It is represented in Figs. 583, 584, the former being a front, and the latter a back view.

There is a small wind-chest, nearly cylindrical, having its top pierced with fifteen holes, disposed at equal distances round the circumference of a circle. Just over this, and nearly touching it, is a movable circular plate, pierced with the same number of holes similarly arranged, and so mounted that it can rotate very freely about its centre, carrying with it the vertical axis to which it is attached. This rotation is effected by the action of the wind, which enters the wind-chest from below, and escapes through the holes. The form of the holes is shown by the section in Fig. 584. They do not pass perpendicularly through the plates, but slope contrary ways, so that the air when forced through the holes in the lower plate impinges *upon one side* of the holes in the upper plate, and thus blows it round in a definite direction. The instrument is driven by means of the bellows shown in Fig. 595 (§ 664). As the rotation of one plate upon the other causes the holes to be alternately opened and closed, the wind escapes in successive puffs, whose frequency

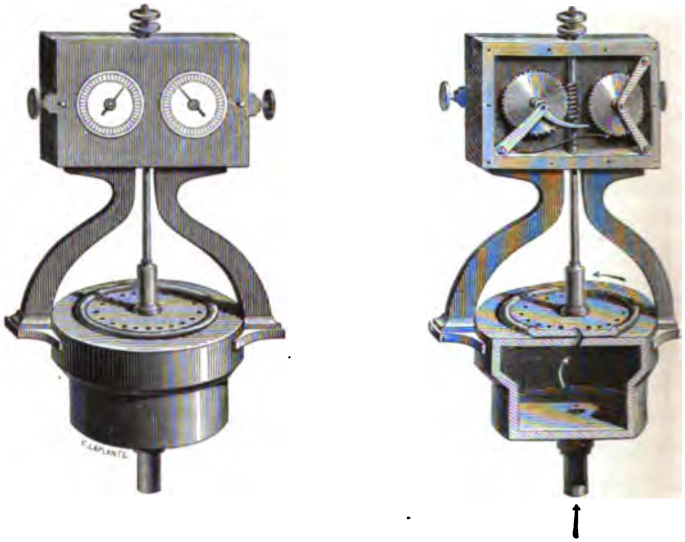
Fig. 583. Siren. Fig. 584.

depends upon the rate of rotation. Hence a note is emitted which rises in pitch as the rotation becomes more rapid.

The siren will sound under water, if water is forced through it instead of air; and it was from this circumstance that it derived its name.

In each revolution, the fifteen holes in the upper plate come opposite to those in the lower plate 15 times, and allow the compressed air in the wind-chest to escape; while in the intervening positions its escape is almost entirely prevented. Each revolution thus gives rise to 15 vibrations; and in order to know the number of vibrations corresponding to the note emitted, it is only necessary to have a means of counting the revolutions.

This is furnished by a counter, which is represented in Fig. 584. The revolving axis carries an endless screw, driving a wheel of 100 teeth, whose axis carries a hand traversing a dial marked with 100 divisions. Each revolution of the perforated plate causes this hand to advance one division. A second toothed-wheel is driven intermittently by the first, advancing suddenly one tooth whenever the hand belonging to the first wheel passes the zero of its scale. This second wheel also carries a hand traversing a second dial; and at each of the sudden movements just described this hand advances one division. Each division accordingly indicates 100 revolutions of the perforated plate, or 1500 vibrations. By pushing in one of the two buttons which are shown, one on each side of the box containing the toothed-wheels, we can instantaneously connect or disconnect the endless screw and the first toothed-wheel.

In order to determine the number of vibrations corresponding to any given sound which we have the power of maintaining steadily, we fix the siren on the bellows, the screw and wheel being disconnected, and drive the siren until the note which it emits is judged to be in unison with the given note. We then, either by regulating the pressure of the wind, or by employing the finger to press with more or less friction against the revolving axis, contrive to keep the note of the siren constant for a measured interval of time, which we observe by a watch. At the commencement of the interval we suddenly connect the screw and toothed-wheel, and at its termination we suddenly disconnect them, having taken care to keep the siren in unison with the given sound during the interval. As the hands do not advance on the dials when the screw is out of connection with the wheels, the readings before and after the measured interval of time can be taken at leisure. Each reading consists of four figures, indicating the number of revolutions from the zero position, units and tens being read off on the first dial, and hundreds and thousands on the second. The difference of the two readings is the number of revolutions made in the measured interval, and when multiplied by 15 gives the number of vibrations in the interval, whence the number of vibrations per second is computed by division.

652. Graphic Method.—In the hands of a skilful operator, with a good musical ear, the siren is capable of yielding very accurate determinations, especially if, by adding or subtracting the number of beats, correction be made for any slight difference of pitch between the siren and the note under investigation.

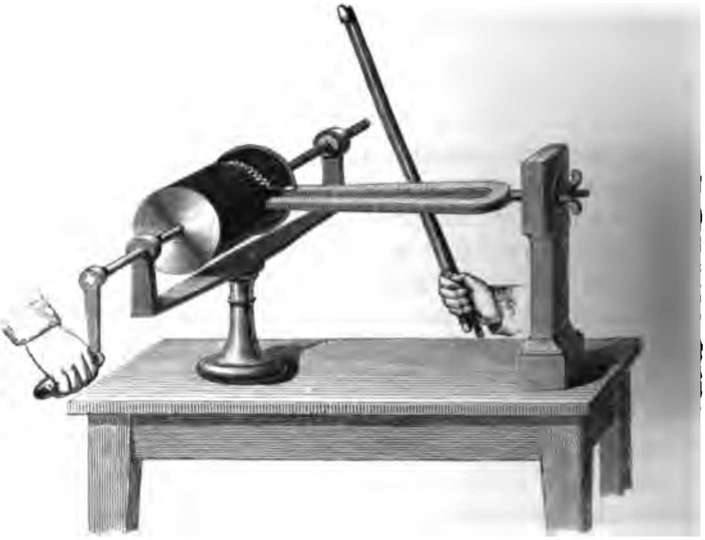

Fig. 585.—Vibroscope.

The vibrations of a tuning-fork can be counted, without the aid of the siren, by a graphical method, which does not call for any exercise of musical judgment, but simply involves the performance of a mechanical operation.

The tuning-fork is fixed in a horizontal position, as shown in Fig. 585, and has a light style, which may be of brass wire, quill, or bristle, attached to one of its prongs by wax. To receive the trace, a piece of smoked paper is gummed round a cylinder, which can be turned by a handle, a screw cut on the axis causing it at the same time to travel endwise. The cylinder is placed so that the style barely touches the blackened surface. The fork is then made to vibrate by bowing it, and the cylinder is turned. The result is a wavy line traced on the blackened surface, and the number of wave-forms (each including a pair of bends in opposite directions) corresponds to the number of vibrations. If the experiment lasts for a measured interval of time, we have only to count these wave-forms, and divide by the number of seconds, in order to obtain the number of vibrations per second for the note of the tuning-fork. By plunging the paper in ether, the trace will be fixed, so that the paper may be laid aside and the vibrations counted at leisure. The apparatus is called the *vibroscope*, and was invented by Duhamel.

M. Léon Scott has invented an instrument called the *phonautograph*, which is adapted to the graphical representation of sounds in general. The style, which is very light, is attached to a membrane stretched across the smaller end of what may be called a large ear-trumpet. The membrane is agitated by the aerial waves proceeding from any source of sound, and the style leaves a record of these agitations on a blackened cylinder, as in Duhamel's apparatus. Fig. 586 represents the traces thus obtained from the sound of a tuning-fork in three different modes of vibration.

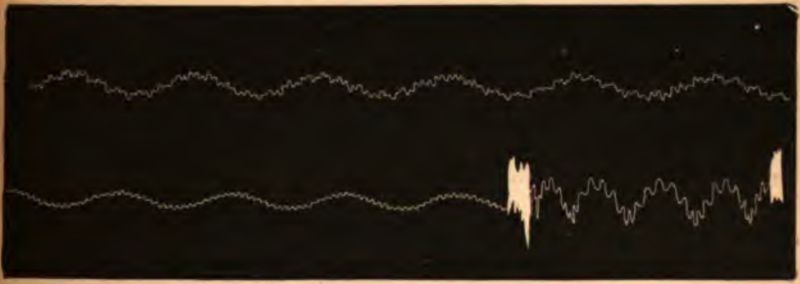

Fig. 586.—Traces by Phonautograph.

653. Tonometer.—When we have determined the frequency of vibration for a particular tuning-fork, that of another fork, nearly in unison with it, can be deduced by making the two forks vibrate simultaneously, and counting the beats which they produce.

Scheibler's *tonometer*, which is constructed by Koenig of Paris, consists of a set of 65 tuning-forks, such that any two consecutive forks make 4 beats per second, and consequently differ in pitch by

4 vibrations per second. The lowest of the series makes 256 vibrations, and the highest 512, thus completing an octave. Any note within this range can have its vibration-frequency at once determined, with great accuracy, by making it sound simultaneously with the fork next above or below it, and counting beats.

With the aid of this instrument, a piano can be tuned with certainty to any desired system of temperament, by first tuning the notes which come within the compass of the tonometer, and then proceeding by octaves.

In the ordinary methods of tuning pianos and organs, temperament is to a great extent a matter of chance; and a tuner cannot attain the same temperament in two successive attempts.

653 A. Pitch modified by Relative Motion.—We have stated in § 645 that, in ordinary circumstances, the frequency of vibration in the source of sound, is the same as in the ear of the listener, and in the intervening medium. This identity, however, does not hold if the source of sound and the ear of the listener are approaching or receding from each other. Approach of either to the other produces increased frequency of the pulses on the ear, and consequent elevation of pitch in the sound as heard; while recession has an opposite effect. Let n be the number of vibrations performed in a second by the source of the sound, v the velocity of sound in the medium, and a the relative velocity of approach. Then the number of waves which reach the ear of the listener in a second, will be n *plus* the number of waves which cover a length a, that is (since n waves cover a length v), will be $n+\frac{a}{v}n$ or $\frac{v+a}{v}n$.

If the motion considered be simply that of the listener, the medium and the source being at rest relatively to each other, the case is extremely clear, being analogous to that of a ship sailing in such a direction as to meet the waves of the sea. If, on the other hand, the listener is at rest relatively to the medium, while the source is advancing through it, this advance shortens the wave-lengths in the medium, in the ratio $\frac{v}{v+a}$, without altering the velocity of propagation. The frequency of vibration of the particles of the medium, being equal to velocity divided by wave-length, is accordingly increased in the ratio $\frac{v+a}{v}$. It may be remarked that, in this case, there is a corresponding *diminution* of frequency of vibration in that part of the medium which lies behind the source. Different portions of the medium will in fact vibrate with all frequencies intermediate between $\frac{v+a}{v}n$ and $\frac{v-a}{v}n$.

If the listener and the source are at rest relatively to each other, but the medium is flowing from him to the source with velocity a, the particles of the medium which flow past him will vibrate with the frequency $\frac{v+a}{v}n$, but the waves will pass him with only the natural frequency n.

Careful observation of the sound of a railway whistle, as an express train dashes past a station, has confirmed the fact that the sound as heard by a person standing at the station is higher while the train is approaching than when it is receding. A speed of about 40 miles an hour will sharpen the note by a semitone in approaching, and flatten it by the same amount in receding, the natural pitch being heard at the instant of passing.[1]

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[1] The best observations of this kind were those of Buys Ballot, in which trumpeters, with their instruments previously tuned to unison, were stationed, one on the locomotive, and others at three stations beside the line of railway. Each trumpeter was accompanied by musicians, charged with the duty of estimating the difference of pitch between the note of his trumpet and those of the others, as heard before and after passing.

CHAPTER LV.

MODES OF VIBRATION.

654. Longitudinal and Transverse Vibrations of Solids.—Sonorous vibrations are manifestations of elasticity. When the particles of a solid body are displaced from their natural positions relative to one another by the application of external force, they tend to return, in virtue of the elasticity of the body. When the external force is removed, they spring back to their natural position, pass it in virtue of the velocity acquired in the return, and execute isochronous vibrations about it until they gradually come to rest. The isochronism of the vibrations is proved by the constancy of pitch of the sound emitted; and from the isochronism we can infer, by the aid of mathematical reasoning, that the restoring force increases directly as the displacement of the parts of the body from their natural relative position (§§ 53 A, B, C).

The same body is, in general, susceptible of many different modes of vibration, which may be excited by applying forces to it in different ways. The most important of these are comprehended under the two heads of *longitudinal* and *transverse* vibrations.

In the former the particles of the body move to and fro in the direction along which the pulses travel, which is always regarded as the longitudinal direction, and the deformations produced consist in alternate compressions and extensions. In the latter the particles move to and fro in directions transverse to that in which the pulses travel, and the deformation consists in bending. To produce longitudinal vibrations, we must apply force in the longitudinal direction. To produce transverse vibration, we must apply force transversely.

655. Transverse Vibrations of Strings.—To the transverse vibrations of strings, instrumental music is indebted for some of its most precious resources. In the violin, violoncello, &c., the strings are set in vibration by drawing a bow across them. The part of the bow which acts on the strings consists of hairs tightly stretched and rubbed with rosin. The bow adheres to the string, and draws it aside till the reaction becomes too great for the adhesion to overcome. As the bow continues to be drawn on, slipping takes place, and the mere fact of slipping diminishes the adhesion. The string accordingly springs back suddenly through a finite distance. In rebounding, it is again caught by the bow, and the same action is repeated. In the harp and guitar, the strings are plucked with the finger, and then left to vibrate freely. In the piano the wires are struck with little hammers faced with leather. The pitch of the sound emitted in these various cases depends only on the string itself, and is the same whichever mode of excitation be employed.

656. Laws of the Transverse Vibrations of Strings.—It can be shown by an investigation closely analogous to that which gives the velocity of sound in air, that the velocity with which transverse vibrations travel along a perfectly flexible string is given by the formula

$$v=\sqrt{\frac{t}{m}}; \qquad (1)$$

t denoting the tension of the string, and m the mass of unit length of it. If m be expressed in grammes per centimetre of length, t should be expressed in terms of the unit of force which is derived from the gramme, centimetre, and second (§ 42), and the value obtained for v will be in centimetres per second. The sudden disturbance of any point in the string, causes two pulses to start from this point, and run along the string in opposite directions. Each of these, on arriving at the end of the free portion of the string, is reflected from the solid support to which the string is attached, and at the same time undergoes reversal as to side. It runs back, thus reversed, to the other end of the free portion, and there again undergoes reflection and reversal. When it next arrives at the origin of the disturbance it has travelled over just twice the length of the string; and as this is true of both the pulses, they must both arrive at this point together. At the instant of their meeting, things are in the same condition as when the pulses were originated, and the movements just described will again take place. The period of a complete vibration of the string is therefore the time required for a pulse to travel over twice its length; that is,

$$\frac{1}{n}=\frac{2l}{v}=2l\sqrt{\frac{m}{t}},$$

$$\text{or } n=\frac{1}{2l}\sqrt{\frac{t}{m}}; \qquad (2)$$

l denoting the length of the string between its points of attachment, and n the number of vibrations per second.

This formula involves the following laws:—

1. When the length of the vibrating portion of the string is altered, without change of tension, the frequency of vibration varies inversely as the length.

2. If the tension be altered, without change of length in the vibrating portion, the frequency of vibration varies as the square root of the tension.

3. Strings of the same length and tension have frequencies of vibration which are inversely as the square roots of their masses (or weights).

4. Strings of the same length and density, but of different thicknesses, will vibrate in the same time, if they are stretched with forces proportional to their sectional areas.

All these laws are illustrated (qualitatively, if not quantitatively) by the strings of a violin.

The first is illustrated by the fingering, the pitch being raised as the portion of string between the finger and the bridge is shortened.

The second is illustrated by the mode of tuning, which consists in tightening the string if its pitch is to be raised, or slackening the string if it is to be lowered.

The third law is illustrated by the construction of the bass string, which is wrapped round with metal wire, for the purpose of adding to its mass, and thus attaining slow vibration without undue slackness. The tension of this string is in fact greater than that of the string next it, though the latter vibrates more rapidly in the ratio of 3 to 2.

The fourth law is indirectly illustrated by the sizes of the first three strings. The treble string is the smallest, and is nevertheless stretched with much greater force than any of the others. The third string is the thickest, and is stretched with less force than any of the others. The increased thickness is necessary in order to give sufficient power in spite of the slackness of the string.

657. Experimental Illustration: Sonometer.—For the quantitative illustration of these laws, the instrument called the sonometer, represented in Fig. 587, is commonly employed. It consists essentially of a string or wire stretched over a sounding-box by means of a weight. One end of the string is secured to a fixed point at one end of the sounding-box. The other end passes over a pulley, and carries weights which can be altered at pleasure. Near the two ends of the box are two fixed bridges, over which the cord passes. There is also a movable bridge, which can be employed for altering the length of the vibrating portion.

To verify the law of lengths, the whole length between the fixed bridges is made to vibrate, either by plucking or bowing; the movable bridge is then introduced exactly in the middle, and one of the halves is made to vibrate; the note thus obtained will be found to be the upper octave of the first. The frequency of vibration is therefore doubled. By making two-thirds of the whole length vibrate, a note will be obtained which will be recognized as the fifth of the fundamental note, its vibration-frequency being therefore greater in the ratio $\frac{3}{2}$. To obtain the notes of the gamut, we commence with the string as a whole, and then employ portions of its length represented by the fractions $\frac{8}{9}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{8}{15}$, $\frac{1}{2}$.

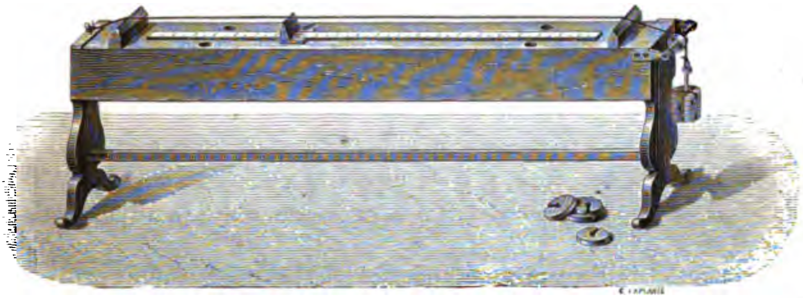

Fig. 587.—Sonometer.

To verify the law independently of all knowledge of musical intervals, a light style may be attached to the cord, and caused to trace its vibrations on the vibroscope. This mode of proof is also more general, inasmuch as it can be applied to ratios which do not correspond to any recognized musical interval.

To verify the law of tensions, we must change the weight. It will be found that, to produce a rise of an octave in pitch, the weight must be increased fourfold.

To verify the third and fourth laws, two strings must be employed, their masses having first been determined by weighing them.

If the strings are thick, and especially if they are thick steel wires, their flexural rigidity has a sensible effect in making the vibrations quicker than they would be if the tension acted alone.

658. **Harmonics.**—Any person of ordinary musical ear may easily, by a little exercise of attention, detect in any note of a piano the presence of its upper octave, and of another note a fifth higher than this; these being the notes which correspond to frequencies of vibration double and triple that of the fundamental note. A highly trained ear can detect the presence of other notes, corresponding to still higher multiples of the fundamental frequency of vibration. Such notes are called *harmonics.*

When the vibration-frequency of one note is an exact multiple of that of another note, the former note is called a harmonic of the latter. The notes of all stringed instruments contain numerous harmonics blended with the fundamental tones. Bells and vibrating plates have higher tones mingled with the fundamental tone; but these higher tones are not harmonics in the sense in which we use the word.

A violin string sometimes fails to yield its fundamental note, and gives the octave or some other harmonic instead. This result can be brought about at pleasure, by lightly touching the string at a properly-selected point in its length, while the bow is applied in the usual way. If touched at the middle point of its length, it gives the octave. If touched at one-third of its length from either end, it gives the fifth above the octave. The law is, that if touched at $\frac{1}{n}$ of its length[1] from either end, it yields the harmonic whose vibration-frequency is n times that of the fundamental tone. The string in these cases divides itself into a number of equal vibrating-segments, as shown in Fig. 589.

The division into segments is often distinctly *visible* when the string of a sonometer is strongly bowed, and its existence can be verified, when less evident, by putting paper riders on different parts of the string. These (as shown in the figure) will be thrown off by the vibrations of the string, unless they are placed accurately at the nodal points, in which case they will retain their seats. If two strings tuned to unison are stretched on the same sonometer, the vibration of the one induces similar vibrations in the other; and the experiment of the riders may be varied, in a very instructive way, by bowing one string, and placing the riders on the other. This is an instance of a general principle of great importance—that a vibrating body communicates its vibrations to other bodies which are capable of vibrating in unison with it. The propagation of a sound may indeed be regarded as one grand vibration in unison; but, besides the general waves of *propagation*, there are waves of *reinforcement*, due to the synchronous vibrations of limited portions of the transmitting medium. This is the principle of resonance.

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[1] Or at $\frac{m}{n}$ of its length, if m be prime to n.

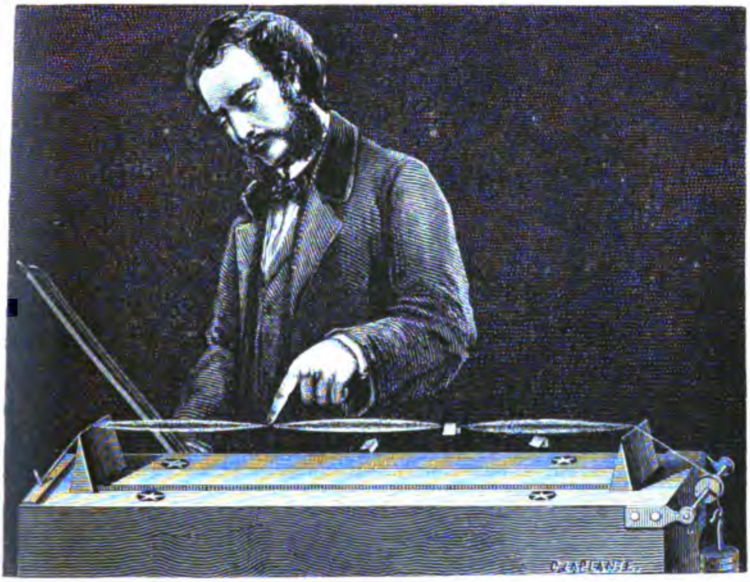

Fig. 589.—Production of a Harmonic.

658 A. Resonance.—By applying to a pendulum originally at rest a series of very feeble impulses, at intervals precisely equal to its natural time of vibration, we shall cause it to swing through an arc of considerable magnitude.

The same principle applies to a body capable of executing vibrations under the influence of its own elasticity. A series of impulses keeping time with its own natural period may set it in powerful vibration, though any one of them singly would have no appreciable effect.

Some bodies, such as strings and confined portions of air, have definite periods in which they can vibrate freely when once started;

and when a note corresponding to one of these periods is sounded in their neighbourhood, they readily take it up and emit a note of the same pitch themselves.

Other bodies, especially thin pieces of dry straight-grained deal, such as are employed for the faces of violins and the sounding-boards of pianos, are capable of vibrating, more or less freely, in any period lying between certain wide limits. They are accordingly set in vibration by all the notes of their respective instruments; and by the large surface with which they act upon the air, they contribute in a very high degree to increase the sonorous effect. All stringed instruments are constructed on this principle; and their quality mainly depends on the greater or less readiness with which they respond to the vibrations of the strings.

All such methods of reinforcing a sound must be included under *resonance;* but the word is often more particularly applied to the reinforcement produced by masses of air.

659. Longitudinal Vibrations of Strings.—Strings or wires may also be made to vibrate *longitudinally,* by rubbing them, in the direction of their length, with a bow or a piece of chamois leather covered with rosin. The sounds thus obtained are of much higher pitch than those produced by transverse vibration.

In the case of the fundamental note, each of the two halves A C, C B (Fig. 590), is alternately extended and compressed, one being extended while the other is compressed. At the middle point C there is no extension or compression, but there is greater amplitude of movement than at any other point. The amplitudes diminish in passing from C towards either end, and vanish at the ends, which are therefore nodes. The extensions and compressions, on the other hand, increase as we travel from the middle towards either end, and obtain their greatest values at the ends.

Fig. 590.—Longitudinal Vibration. First Tone.

But the string may also divide itself into any number of separately-vibrating segments, just as in the case of transverse vibrations. Fig. 591 represents the motions which occur when there are three such segments, separated by two nodes D, E. The upper portion of the figure is true for one-half of the period of vibration, and the lower portion for the remaining half.

The frequency of vibration, for longitudinal as well as for transverse vibrations, varies inversely as the length of the vibrating string, or segment of string. We shall return to this subject in § 670.

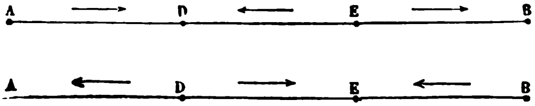

Fig. 591.—Longitudinal Vibration. Third Tone.

660. Stringed Instruments.—Only the transversal vibrations of strings are employed in music. In the violin and violoncello there are four strings, each being tuned a fifth above the next below it; and intermediate notes are obtained by fingering, the portion of string between the finger and the bridge being the only part that is free to vibrate. The bridge and sounding-post serve to transmit the vibrations of the strings to the body of the instrument. In the piano there is also a bridge, which is attached to the sounding-board, and communicates to it the vibrations of the wires.

661. Transversal Vibrations of Rigid Bodies: Rods, Plates, Bells.—We shall not enter into detail respecting the laws of the transverse vibrations of rigid bodies. The relations of their overtones to their fundamental tones are usually of an extremely complex character, and this fact is closely connected with the unmusical or only semi-musical character of the sounds emitted.

When one face of the body is horizontal, the division into separate vibrating segments can be rendered visible by a method devised by Chladni, namely, by strewing sand on this face. During the vibration, the sand, as it is tossed about, works its way to certain definite lines, where it comes nearly to rest. These nodal lines must be regarded as the intersections of internal nodal surfaces with the surface on which the sand is strewed, each nodal surface being the boundary between parts of the body which have opposite motions.

The figures composed by these nodal lines are often very beautiful, and quite startling in the suddenness of their production. Chladni and Savart published the forms of a great number. A complete theoretical explanation of them would probably transcend the powers of the greatest mathematicians.

Bells and bell-glasses vibrate in segments, which are never less than four in number, and are separated by nodal lines meeting in the middle of the crown. They are well shown by putting water in a bell-glass, and bowing its edge. The surface of the water will immediately be covered with ripples, one set of ripples proceeding from each of the vibrating segments. The division into any possible number of segments may be effected by pressing the glass with the fingers in the places where a pair of consecutive nodes ought to be formed, while the bow is applied to the middle of one of the segments. The greater the number of segments the higher will be the note emitted.

662. Tuning-fork. — Steel rods, on account of their comparative freedom from change, are well suited for standards of pitch. The tuning-fork, which is especially used for this purpose, consists essentially of a steel rod bent double, and attached to a handle of the same material at its centre. Besides the fundamental tone, it is capable of yielding two or three overtones, which are very much higher in pitch; but these are never used for musical purposes. If the fork is held by the handle while vibrating, its motion continues for a long time, but the sound emitted is too faint to be heard except by holding the ear near it. When the handle is pressed against a table, the latter acts as a sounding-board, and communicates the vibrations to the air, but it also causes the fork to come much more speedily to rest. For the purposes of the lecture-room the fork is often mounted on a sounding-box (Fig. 592), which should be separated from the table by two pieces of india-rubber tubing. The box can then vibrate freely in unison with the fork, and the sound is both loud and lasting. The vibrations are usually excited either by bowing the fork or by drawing a piece of wood between its prongs.

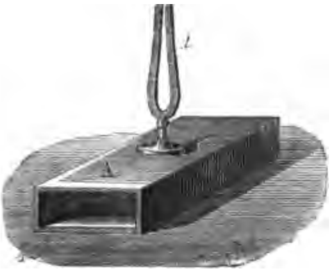

Fig. 592.—Fork on Sounding-box.

The pitch of a tuning-fork varies slightly with temperature, becoming lower as the temperature rises. This effect is due in some trifling degree to expansion, but much more to the diminution of elastic force.

663. Law of Linear Dimensions.—The following law is of very wide application, being applicable alike to solid, liquid, and gaseous bodies: —*When two bodies differing in size, but in other respects similar and similarly circumstanced, vibrate in the same mode, their vibration-periods are directly as their linear dimensions.* Their vibration-frequencies are consequently in the inverse ratio of their linear dimensions.

In applying the law to the transverse vibrations of strings, it is to be understood that the stretching force per unit of sectional area is constant. In this case the velocity of a pulse (§ 656) is constant, and the period of vibration, being the time required for a pulse to travel over twice the length of the string, is therefore directly as the length.

664. Organ-pipes.—In organs, and wind-instruments generally, the sonorous body is a column of air confined in a tube. To set this air in vibration some kind of mouth-piece must be employed. That which is most extensively used in organs is called the *flute mouth-piece*,[1] and is represented, in conjunction with the pipe to which it is attached, in Figs. 593, 594. It closely resembles the mouth-piece of

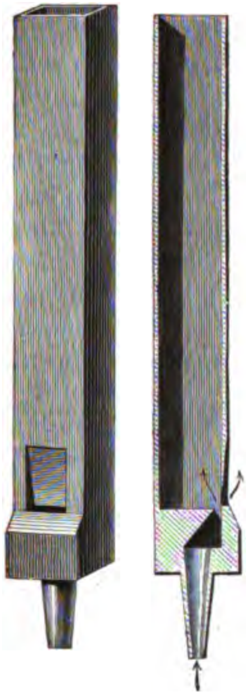

Fig. 593.—Block Pipe.

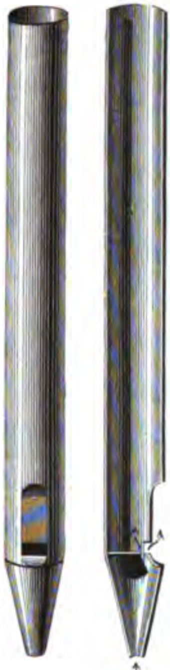

Fig. 594.—Flue Pipe.

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[1] This is not the trade name. English organ-builders have no generic name for this mouth-piece.

an ordinary whistle. The air from the bellows arrives through the conical tube at the lower end, and before entering the main body of the pipe has to pass through a narrow slit, in issuing from which it impinges on the thin end of a wedge placed directly opposite, called the lip. This lip is itself capable of vibrating in unison with any note lying within a wide range, and the note which is actually emitted is determined by the resonance of the column of air in the pipe. Fig. 593 represents a wooden, and Fig. 594 a metal organ-pipe, both of them being furnished with flute mouthpieces. The two arrows in the sections are intended to suggest the two courses which the wind may take as it issues from the slit.

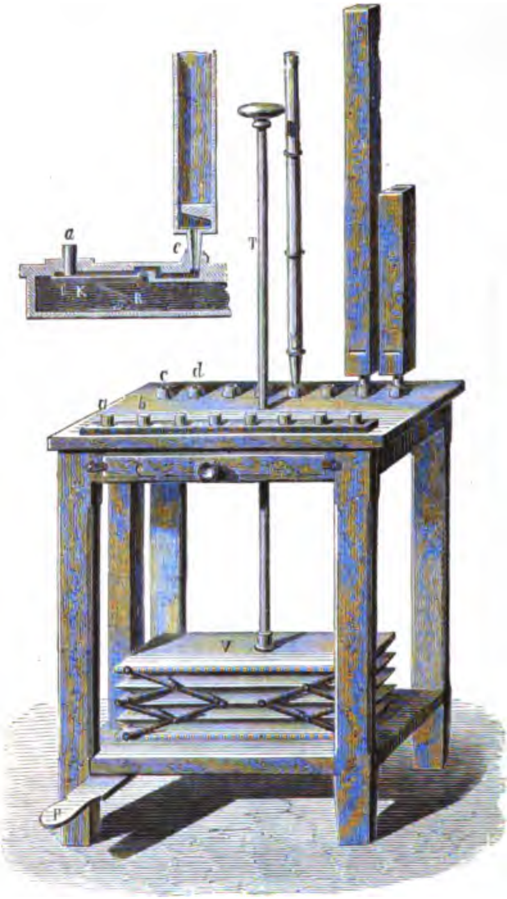

Fig. 595.—Experimental Organ.

The arrangements for admitting the wind to the pipes by putting down the keys are shown in Fig. 595. The bellows V are worked by the treadle P. The force of the blast can be increased by weighting the top of the bellows, or by pressing on the rod T. The air passes up from the bellows, through a large tube shown at one end, into a reservoir C, called the wind-chest. In the top of the wind-chest there are numerous openings *c*, *d*, &c., in which the tubes are to be fixed. The sectional drawing in the upper part of the figure shows the internal communications. A plate K, pressed up by a spring R, cuts off the tube *c* from the wind-chest, until the pin *a* is depressed. The putting down of this pin lowers the plate, and admits the wind. This description only applies to the experimental organs which are constructed for lecture illustration. In real organs the pressure of the wind in the bellows is constant; and as this pressure would be too great for most of the pipes, the several apertures of admission are partially plugged, to diminish the force of the blast.

665. The Air is the Sonorous Body.—It is easily shown that the sound emitted by an organ-pipe depends, mainly at least, on the dimensions of the inclosed column of air, and not on the thickness or material of the pipe itself. For let three pipes, one of wood, one of copper, and the other of thick card, all of the same internal dimensions, be fixed on the wind-chest. On making them speak, it will be found that the three sounds have exactly the same pitch, and but slight difference in character. If, however, the sides of the tube are *excessively* thin, their yielding has a sensible influence, and the pitch of the sound is modified.

666. Law of Linear Dimensions.—The law of linear dimensions, stated in a previous section (§ 663) as applying to the vibrations of similar solid bodies, applies to gases also. Let two box-shaped pipes (Fig. 596) of precisely similar form, and having their linear dimensions in the ratio of 2 : 1, be fixed on the wind-chest; it will be found, on making them speak, that the note of the small one is an octave higher than the other;—showing double frequency of vibration.

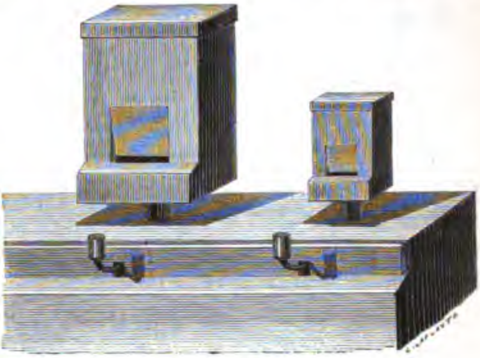

Fig. 596.—Law of Linear Dimensions.

667. Bernoulli's Laws.—The law just stated applies to the comparison of similar tubes of any shape whatever. When the length of a tube is a large multiple of its diameter, the note emitted is sensibly independent of the diameter, and depends on the length alone. The relations between the fundamental note of such a tube and its overtones were discovered by Daniel Bernoulli, and are as follows:—

I. *Overtones of Open Pipes.*—Let the pipe B (Fig. 597), which is open at the upper end, be fixed on the wind-chest; let the corresponding key be put down, and the wind gradually turned on, by means of the cock below the mouth-piece. The first note heard will be feeble and deep; it is the fundamental note of the pipe. As the wind is gradually turned full on, and increasing pressure afterwards applied to the bellows, a series of notes will be heard, each higher than its predecessor. These are the overtones of the pipe. They are the harmonics of the fundamental note; that is to say, if 1 denote the frequency of vibration for the fundamental tone, the frequencies of vibration for the overtones will be approximately 2, 3, 4, 5 . . . respectively.

II. *Overtones of Stopped Pipes.*—If the same experiment be tried with the pipe A, which is closed at its upper end; the overtones will form the series of odd harmonics of the fundamental note, all the even harmonics being absent: in other words, the frequencies of vibration of the fundamental tone and overtones will be approximately represented by the series of odd numbers 1, 3, 5, 7 . . .

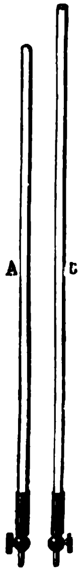

Fig. 597.
Tubes for Overtones.

It will also be found, that if both pipes are of the same length, the fundamental note of the stopped pipe is an octave lower than that of the open pipe.

668. Mode of Production of Overtones.—In the production of the overtones, the column of air in a pipe divides itself into vibrating segments, separated by nodal cross-sections. At equal distances on opposite sides of a node, the particles of air have always equal and opposite velocities, so that the air at the node is always subjected to equal forces in opposite directions, and thus remains unmoved by their action. The portion of air constituting a vibrating segment, sways alternately in opposite directions, and as the movements in two consecutive segments are opposite, two consecutive nodes are always in opposite conditions as regards compression and extension. The middle of a vibrating segment is the place where the amplitude of vibration is greatest, and the variation of density least. It may be called an *antinode.* The distance from one node to the next is half a wave-length, and the distance from a node to an antinode is a quarter of a wave-length. Both ends of an open pipe, and the end next the mouth-piece of a stopped pipe, are antinodes, being preserved from changes of density by their free communication with the external air. At the closed end of a stopped pipe there must always be a node.

The swaying to and fro of the internodal portions of air between fixed nodal planes, is an example of *stationary undulation;* and the vibration of a musical string is another example. A stationary undulation may always be analyzed into two component undulations equal and similar to one another, and travelling in opposite directions, their common wave-length being double of the distance from node to node. These undulations are constantly undergoing reflection from the ends of the pipe or string, and, in the case of pipes, the reflection is opposite in kind according as it takes place from a closed or an open end. In the former case a condensation propagated towards the end is reflected as a condensation, the forward-moving particles being compelled to recoil by the resistance which they there encounter; and a rarefaction is, in like manner, reflected as a rarefaction. On the other hand, when a condensation arrives at an open end, the sudden opportunity for expansion which is afforded causes an outward movement in excess of that which would suffice for equilibrium of pressure, and a rarefaction is thus produced which is propagated back through the tube. A condensation is thus reflected as a rarefaction; and a rarefaction is, in like manner, reflected as a condensation.

The period of vibration of the fundamental note of a stopped pipe is the time required for propagating a pulse through four times the length of the pipe. For let a condensation be suddenly produced at the lower end by the action of the vibrating lip. It will be propagated to the closed end and reflected back, thus travelling over twice the length of the pipe. On arriving at the aperture where the lip is situated, it is reflected as a rarefaction. This rarefaction travels to the closed end and back, as the condensation did before it, and is then reflected from the aperture as a condensation. Things are now in their initial condition, and one complete vibration has been performed. The period of the movements of the lip is determined by the arrival of these alternate condensations and rarefactions; and the lip, in its turn, serves to divert a portion of the energy of the blast, and employ it in maintaining the energy of the vibrating column.

The wave-length of the fundamental note of a stopped pipe is thus four times the length of the pipe.

In an open pipe, a condensation, starting from the mouth-piece, is reflected from the other end as a rarefaction. This rarefaction, on reaching the mouth-piece, is reflected as a condensation; and things are thus in their initial state after the length of the pipe has been traversed twice. The period of vibration of the fundamental note is accordingly the time of travelling over twice the length of the pipe; and its wave-length is twice the length of the pipe. In every case of longitudinal vibration, if the reflection is alike at both ends, the wave-length of the fundamental tone is twice the distance between the ends.

669. Explanation of Bernoulli's Laws.—In investigating the theoretical relations between the fundamental tone and overtones for a pipe of either kind, it is convenient to bear in mind that the distance from an open end to the nearest node is a quarter of a wave-length of the note emitted.

In the case of the open pipe the first or fundamental tone requires one node, which is at the middle of the length. The second tone requires two nodes, with half a wave-length between them, while each of them is a quarter of a wave-length from the nearest end. A quarter wave-length has thus only half the length which it had for the fundamental tone, and the frequency of vibration is therefore doubled.

The third tone requires three nodes, and the distance from either end to the nearest node is $\frac{1}{6}$ of the length of the pipe, instead of $\frac{1}{4}$ the length as in the case of the first tone. The wave-length is thus divided by 3, and the frequency of vibration is increased threefold We can evidently account in this way for the production of the complete series of harmonics of the fundamental note.

In the case of the stopped pipe, the mouth-piece is always distant a quarter wave-length from the nearest node, and this must be distant an even number of quarter wave-lengths from the stopped end, which is itself a node.

For the fundamental tone, a quarter wave-length is the whole length of the pipe.

For the second tone, there is one node besides that at the closed end, and its distance from the open end is $\frac{1}{3}$ of the length of the pipe.

For the third tone, there are two nodes besides that at the closed end. The distance from the open end to the nearest node is therefore $\frac{1}{5}$ of the length of the pipe.

The wave-lengths of the successive tones, beginning with the fundamental, are therefore as 1, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$ . . . , and their vibration-frequencies are as 1, 3, 5, 7 . . .

Also, since the wave-length of the fundamental tone is four times the length of the pipe if stopped, and only twice its length if open, it is obvious that the wave-length is halved, and the frequency of vibration doubled, by unstopping the pipe.

No change of pitch, or only very slight change, will be produced by inserting a solid partition at a node, or by putting an antinode in free communication with the external air. These principles can be illustrated by means of the jointed pipe represented in Fig. 598.

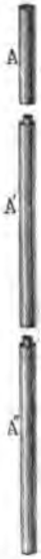

Fig. 598. Jointed Pipe.

670. **Application to Rods and Strings.**—The same laws which apply to open organ-pipes, also apply to the longitudinal vibrations of rods free at both ends, and to both the longitudinal and transverse vibrations of strings. In all these cases the overtones form the complete series of harmonics of the first or fundamental tone, and the period of vibration for this first tone is the time occupied by a pulse in travelling over twice the length of the given rod or string. In the case of longitudinal vibrations the velocity of a pulse is $\sqrt{\frac{M}{D}}$, M denoting the value of Young's modulus for the rod or string, and D its density. This is identical with the velocity of sound through the rod or string, and is independent of its tension. In the case of transverse pulses in a string (regarded as perfectly flexible), the formula for the velocity of transmission, (1) § 656, may be written $\sqrt{\frac{F}{D}}$, F denoting the stretching force per unit of sectional area. The ratio of the latter velocity to the former is $\sqrt{\frac{F}{M}}$, which is always a small fraction, since $\frac{F}{M}$ express the fraction of itself by which the string is lengthened by the force F.

If a rod, free at both ends, is made to vibrate longitudinally, its nodes and antinodes will be distributed exactly in the same way as those of an open organ-pipe. The experiment can be performed by holding the rod at a node, and rubbing it with rosined chamois leather.

671. **Application to Measurement of Velocity in Gases.**—Let v denote the velocity of sound in a particular gas, in feet per second, λ the wave-length of a particular note in this gas in feet, and n the frequency of vibration for this note, that is the number of vibrations per second which produce it. Then λ is the distance travelled in $\frac{1}{n}$

of a second, and the distance travelled in a second is

$$v = n\lambda,$$

For the same note, n is constant for all media whatever, and v varies directly as λ. The velocities of sound in two gases may thus be compared by observing the lengths of vibrating columns of the two gases which give the same note; or if columns of equal length be employed, the velocities will be directly as the frequencies of vibration, which are determined by observing the pitch of the notes emitted.

By these methods, Dulong, and more recently Wertheim, have determined the velocity of sound in several different gases. The following are Wertheim's results, in metres per second, the gases being supposed to be at 0°C.

Air, ...	331	Carbonic acid, . .	262
Oxygen, ...	317	Nitrous oxide, . .	262
Hydrogen, ...	1269	Olefiant gas, . . .	314
Carbonic oxide, . . .	337		

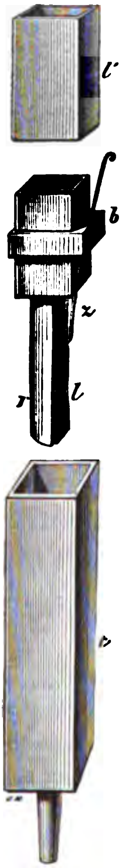

Fig. 599.—Reed Pipe.

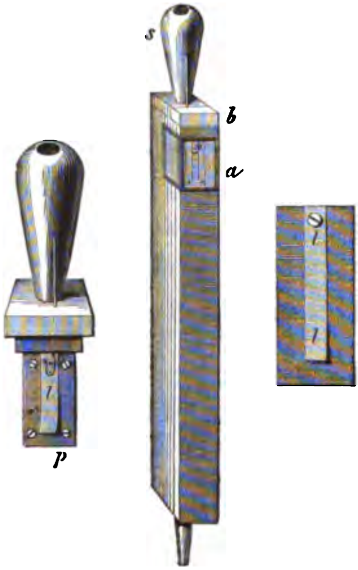

Fig. 600.—Free Reed.

The same principle is applicable to liquids and solids; and it was by means of the longitudinal vibrations of rods that the velocities given in § 637 were ascertained.

672. Reed-pipes.—Instead of the flute mouth-piece above described, organ-pipes are often furnished with what is called a *reed*. A reed contains an elastic plate *l* (Figs. 599, 600) called the *tongue*, which, by its vibrations, alternately opens and closes or nearly closes an aperture through which the wind passes. In Fig. 599, the air from the bellows enters first the lower part t of the pipe, and thence (when permitted by the tongue) passes through the channel[1] r into the upper part t'. The stiff wire z, movable with considerable friction through the hole b, limits the vibrating portion of the tongue, and is employed for tuning. Reed-pipes are often terminated above by a trumpet-shaped expansion.

Fig. 599 represents a *striking reed*, so called because the tongue closes the orifice by striking its edges. The sound thus produced is somewhat harsh. In the *free reed* (Fig. 600), the tongue passes through the orifice without striking its edges, and the tone produced is much smoother.

The striking reed is generally preferred in organs, its peculiar character rendering it very effective by way of contrast. It is always used for the *trumpet* stop. Reed-pipes can be very strongly blown without breaking into overtones.

Elevation of temperature sharpens pipes with flute mouth-pieces, and flattens reed-pipes. The sharpening is due to the increased velocity of sound in hot air. The flattening is due to the diminished elasticity of the metal tongue. It is thus proved that the pitch of a reed-pipe is not always that due to the free vibration of the inclosed air, but may be modified by the action of the tongue.

673. Wind-instruments.—In all wind-instruments, the sound is originated by one of the two methods just described. With the flute-pipe must be classed the flute, the flageolet, and the Pandean-pipes. The clarionet, hautboy, and bassoon have reed mouth-pieces, the vibrating tongue being a piece of reed or cane. In the bugle, trumpet, and French-horn, which are mere tubes without keys, the lips of the performer act as the reed-tongue, and the notes produced are approximately the natural overtones. These, when of high order, are so near together, that a gamut can be formed by properly selecting from among them.

The fingering of the flute and clarionet, has the effect sometimes of altering the effective length of the vibrating column of air, and sometimes of determining the production of overtones. In the trombone and cornet-à-piston, the length of the vibrating column of air is altered. The harmonium, accordion, and concertina are reed instruments, the reeds employed being always of the free kind.

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[1] The piece r, which is approximately a half-cylinder, is called the *reed* by organ-builders.

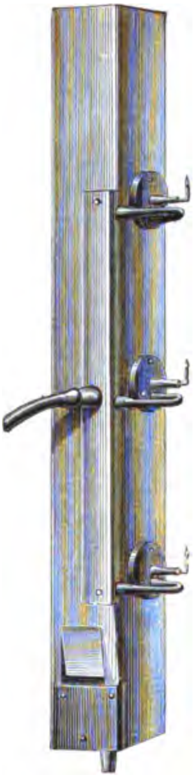

Fig. 601.—Manometric Flames.

674. Manometric Flames.—Koenig, of Paris, constructs several forms of apparatus, in which the variations of pressure produced by vibrations of air in a pipe are rendered evident to the eye by their effect upon flames. One of these is represented in Fig. 601. Three small gas-burners are fixed at definite points in the side of a pipe, as represented in the figure. When the pipe gives its second tone, the central flame is at an antinode and remains unaffected, while the other two, being at nodes, are agitated or blown out. When it gives its first tone, the central flame, which is now at a node, is more powerfully affected than the others. The gas which supplies these burners is separated from the air in the pipe only by a thin membrane. When the pipe is made to speak, the flame at the node is violently agitated, in consequence of the changes of pressure on the back of the membrane, while those at the ventral points are scarcely affected. The agitation of the flame is a true vibration; and, when examined by the aid of a revolving mirror, presents the appearance of tongues of flame alternating with nearly dark spaces. If two pipes, one an octave higher than the other, are connected with the same gas flame, or with two gas flames which can be viewed in the same mirror, the tongues of flame corresponding to the upper octave are seen to be twice as numerous as the others.

CHAPTER LVI.

ANALYSIS OF VIBRATIONS. CONSTITUTION OF SOUNDS.

675. Optical Examination of Sonorous Vibrations.—Sound is a special sensation belonging to the sense of hearing; but the vibrations which are its physical cause often manifest themselves to other senses. For instance, we can often feel the tremors of a sonorous body by touching it; we see the movements of the sand on a vibrating plate, the curve traced by the style of a vibroscope, &c. The aid which one sense can thus furnish in what seems the peculiar province of another is extremely interesting. M. Lissajous has devised a very beautiful optical method of examining sonorous vibrations, which we will briefly describe.

676. Lissajous' Experiment.—Suppose we introduce into a dark room (Fig. 602) a beam of solar rays, which, after passing through a lens L, is reflected, first, from a small mirror fixed on one of the branches of a tuning-fork D, and then from a second mirror M, which throws it on a screen E; we can thus, by proper adjustments, form upon the screen a sharp and bright image of the sun, which will appear as a small spot of light. As long as the apparatus remains at rest, we shall not observe any movement of the image; but if the tuning-fork vibrates, the image will move rapidly up and down along the line I, I′, producing, in consequence of the persistence of impressions, the appearance of a vertical line of light. If the tuning-fork remains at rest, but the mirror M is rotated through a small angle about a vertical axis, the image will move horizontally. Consequently, if both these motions take place simultaneously, the spot of light will trace out on the screen a sinuous line, as represented in the figure, each S-shaped portion corresponding to one vibration of the tuning-fork.

Now, let the mirror M be replaced by a small mirror attached to a second tuning-fork, which vibrates in a horizontal plane, as in Fig. 603. If this fork vibrates alone, the image will move to and fro horizontally, presenting the appearance of a horizontal line of

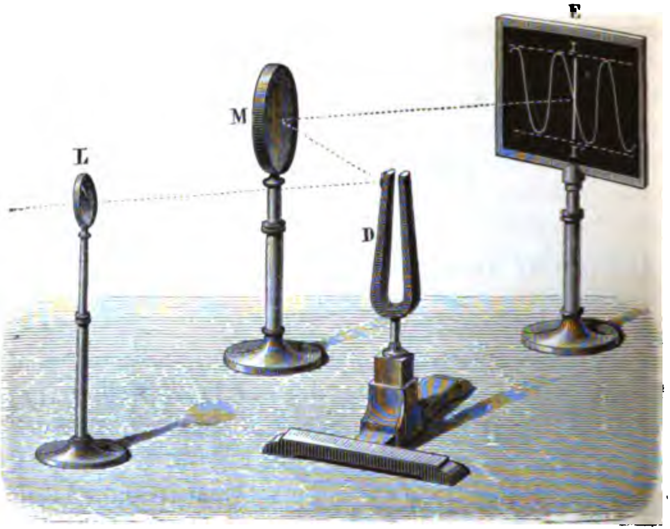

Fig. 602.—Principle of Lissajous' Experiment.

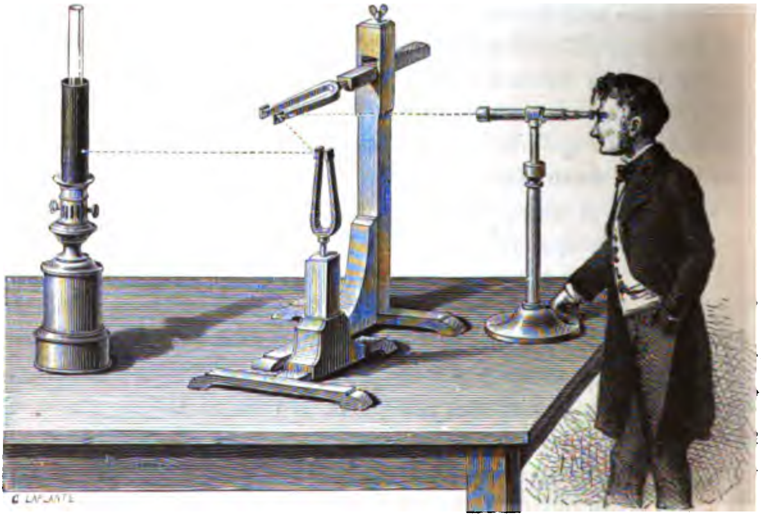

Fig. 603.—Lissajous' Experiment.

light, which gradually shortens as the vibrations die away. If both forks vibrate simultaneously, the spot of light will rise and fall according to the movements of the first fork, and will travel left and right according to the movements of the second fork. The curve actually described, as the resultant of those two component motions, is often extremely beautiful. Some varieties of it are represented in Fig. 604.

Instead of throwing the curves on a screen, we may see them by looking into the second mirror, either with a telescope, as in Fig. 603,

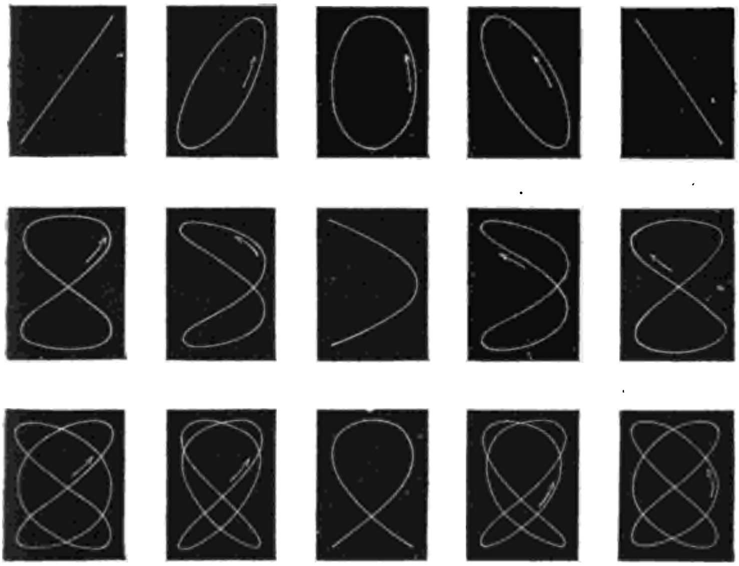

Fig. 604.—Lissajous' Figures, Unison, Octave, and Fifth.

or with the naked eye. In this form of the experiment, a lamp surrounded by an opaque cylinder, pierced with a small hole just opposite the flame, as represented in the figure, is a very convenient source of light.

The movement of the image depends almost entirely on the angular movements of the mirrors, not on their movements of translation; but the distinction is of no importance, for, in the case of such small movements, the linear and angular changes may be regarded as strictly proportional.

Either fork vibrating alone, would cause the image to execute the particular kind of movement which we have described in § 53 A, under the designation of *simple vibration* or *simple harmonic motion;* so that the movement actually executed will be the resultant of two simple harmonic motions in directions perpendicular to each other.

Suppose the two forks to be in unison. Then the two simple harmonic motions will have the same period, and the path described will always be some kind of ellipse,[1] the circle and straight line being included as particular cases. It will be a straight line if both forks pass through their positions of equilibrium at the same instant. In order that it may be a circle, the amplitudes of the two simple harmonic motions must be equal, and one fork must be in a position of maximum displacement when the other is in the position of equilibrium.

If the unison were rigorous, the curve once obtained would remain unchanged, except in so far as its breadth and height became reduced by the dying away of the vibrations. But this perfect unison is never attained in practice, and the eye detects changes depending on differences of pitch too minute to be perceived by the ear. These changes are illustrated by the upper row of forms in Fig. 604, commencing, say, with the sloping straight line at the left hand, which gradually opens out into an ellipse, and afterwards contracts into a straight line, sloping the opposite way. It then retraces its steps, moving in opposition to the arrows in the figure, and goes through the same changes again.

If the interval between the two forks is an octave, we shall obtain the curves represented in the second row;[2] if the interval is a fifth, we shall obtain the curves in the lowest row. In each case the order of the changes will be understood by proceeding from left to right.

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[1] Employing horizontal and vertical co-ordinates, and denoting the amplitudes by a and b, we have, in the case of unison, $\frac{x}{a} = \sin\theta$, $\frac{y}{b} = \sin\,(\theta+\beta)$, where β denotes the difference of phase, and θ is an angle varying directly as the time. Eliminating θ, we obtain the equation to an ellipse, whose form and dimensions depend upon the given quantities, a, b, β.

[2] The middle curve in this row is a parabola, and corresponds to the elimination of θ between the equations $\frac{x}{a} = \cos 2\,\theta$, $\frac{y}{a} = \cos\theta$. The coefficient 2 indicates the double frequency of horizontal as compared with vertical vibrations.

The general equations to Lissajous' figures are $\frac{x}{a} = \sin m\,\theta$, $\frac{y}{b} = \sin\,(n\,\theta+\beta)$, where m and n are proportional to the frequencies of horizontal and vertical vibrations. The gradual changes from one figure to another depend on the gradual change of β, and all the figures can be inscribed in a rectangle, whose length and breadth are $2a$ and $2b$.

and then back again; but the curves obtained in returning will be inverted.

677. Optical Tuning.—By the aid of these principles, tuning-forks can be compared with a standard fork with much greater precision than would be attainable by ear. Fig. 605 represents a convenient arrangement for this purpose. A lens *f* is attached to one of the prongs of a standard fork, which vibrates in a horizontal plane; and above it is fixed an eye-piece *g*, the combination of the two being equivalent to a microscope. The fork to be compared is placed upright beneath, and vibrates in a vertical plane, the end of one prong being in the focus of the microscope. A bright point *m*, produced by making a little scratch on the end of the prong with a diamond, is observed through the microscope, and is illuminated, if necessary, by converging a beam of light upon it through the lens *c*. When the forks are set vibrating, the bright point is seen as a luminous ellipse, whose permanence of form is a test of the closeness of the unison. The ellipse will go through a complete cycle of changes in the time required for one fork to gain a complete vibration on the other.

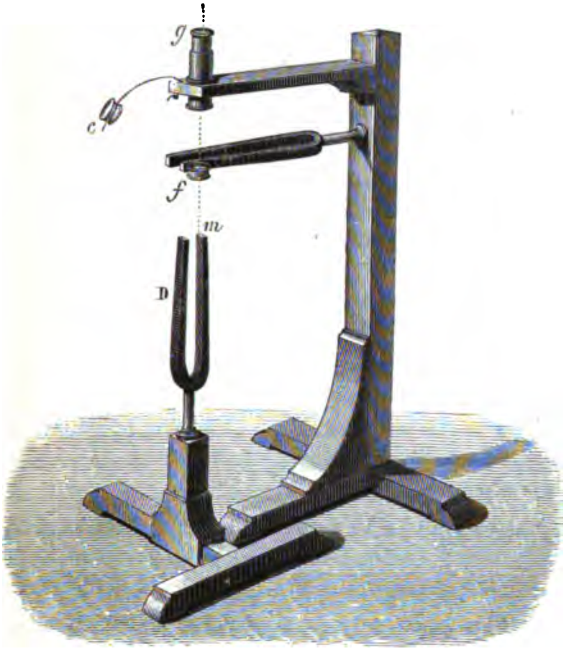

Fig. 605.—Optical Comparison of Tuning-forks.

677 A. Other Modes of producing Lissajous' Figures.—An arrangement devised in 1844 by Professor Blackburn, of Glasgow, then a student at Cambridge, affords a very easy mode of obtaining, by a slow motion, the same series of curves which, in the above arrangements, are obtained by a motion too quick for the eye to follow. A cord A B C (Fig. 605 A) is fastened at A and C, leaving more or less slack, according to the curves which it is desired to obtain; and to any intermediate point B of the cord another string is tied, carrying at its lower end a heavy body D to serve as pendulum-bob.

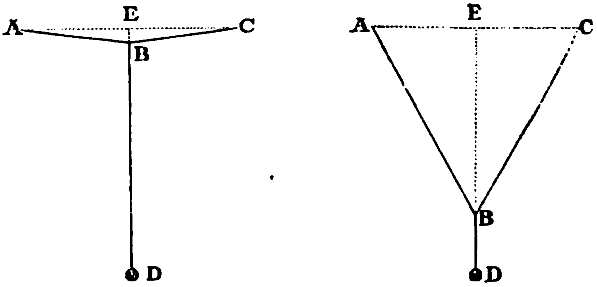

Fig. 605 A.—Blackburn's Pendulum.

If, when the system is in equilibrium, the bob is drawn aside in the plane of A B C and let go, it will execute vibrations in that plane, the point B remaining stationary, so that the length of the pendulum is B D. If, on the other hand, it be drawn aside in a plane perpendicular to the plane A B C, it will vibrate in this perpendicular plane, carrying the whole of the string with it in its motion, so that the length of the pendulum is the distance of the bob from the point E, in which the straight line A C is cut by D B produced. The frequencies of vibration in the two cases will be inversely as the square roots of the pendulum-lengths B D, E D.

If the bob is drawn aside in any other direction, it will not vibrate in one plane, but will perform movements compounded of the two independent modes of vibration just described, and will thus describe curves identical with Lissajous'. If the ratio of E D to B D is nearly equal to unity, as in the left-hand figure, we shall have curves corresponding to approximate unison. If it be approximately 4, as in the right-hand figure, we shall obtain the curves of the octave. Traces of the curves can be obtained by employing for the bob a vessel containing sand, which runs out through a funnel-shaped opening at the bottom.

The curves can also be exhibited by fixing a straight elastic rod at one end, and causing the other end to vibrate transversely. This was the earliest known method of obtaining them. If the flexural rigidity of the rod is precisely the same for all transverse directions, the vibrations will be executed in one plane; but if there be any inequality in this respect, there will be two mutually perpendicular directions possessing the same properties as the two principal directions of vibration in Blackburn's pendulum. A small bright metal knob is usually fixed on the vibrating extremity to render its path visible.

678. Character.—*Character* or *timbre,* which we have already defined in § 645, must of necessity depend on the *form* of the vibration of the aerial particles by which sound is transmitted, the word *form* being used in the metaphorical sense there explained, for in the literal sense the form is always a straight line. When the changes of density are represented by ordinates of a curve, as in Fig. 575, the form of this curve is what is meant by the form of vibration.

The subject of *timbre* has been very thoroughly investigated in recent years by Helmholtz; and the results at which he has arrived are now generally accepted as correct.

The first essential of a musical note is, that the aerial movements which constitute it shall be strictly *periodic;* that is to say, that each vibration shall be exactly like its successor, or at all events, that, if there be any deviation from strict periodicity, it shall be so gradual as not to produce sensible dissimilarity between several consecutive vibrations of the same particle.

There is scarcely any proposition more important in its application to modern physical investigations than the following mathematical theorem, which was discovered by Fourier:—*Any periodic vibration executed in one line can be definitely resolved into simple vibrations, of which one has the same frequency as the given vibration, and the others have frequencies* 2, 3, 4, 5 . . . *times as great,* no fractional multiples being admissible. The theorem may be briefly expressed by saying that *every periodic vibration consists of a fundamental simple vibration and its harmonics.*

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[1] Mr. Hubert Airy has obtained very beautiful traces by attaching a glass pen to the bob. See *Nature,* Aug. 17 and Sept. 7, 1871.

We cannot but associate this mathematical law with the experimental fact, that a trained ear can detect the presence of harmonics in all but the very simplest musical notes. The analysis which Fourier's theorem indicates, appears to be actually performed by the auditory apparatus.

The *constitution* of a periodic vibration may be said to be known if we know the ratios of the amplitudes of the simple vibrations which compose it; and in like manner the constitution of a sound may be said to be known if we know the relative intensities of the different elementary tones which compose it.

Helmholtz has shown that the *character* of a musical note depends upon its *constitution* as thus defined; and that, while change of intensity in any of the components produces a modification of character, change of phase has no influence upon it whatever. Change of phase does however affect the form of the resultant vibration. Thus certain changes of form are admissible without change of character.

The harmonics which are present in a note, usually find their origin in the vibrations of the musical instrument itself. In the case of stringed instruments, for example, along with the vibration of the string as a whole, a number of segmental vibrations are simultaneously going on. Fig. 588 represents curves obtained by the composition of the fundamental mode of vibration with another an octave higher. The broken lines indicate the forms which the string would assume if yielding only its fundamental note.[1] The continuous lines in the first and third figures are forms which a string may assume in its two positions of greatest displacement, when yielding the octave along with the fundamental, the time required for the string to pass from one of these positions to the other being the same as the time in which each of its two segments moves across and back again. The second and fourth figures must in like manner be taken together, as representing a pair of extreme positions. The number of harmonics thus yielded by a pianoforte wire is usually some four or five; and a still larger number are yielded by the strings of a violin.

The notes emitted from wide organ-pipes with flute mouth-pieces are very deficient in harmonics. This defect is remedied by combining with each of the larger pipes a series of smaller pipes,[1] each yielding one of its harmonics. An ordinary listener hears only one note, of the same pitch as the fundamental, but much richer in character than that which the fundamental pipe yields alone. A trained ear can recognize the individual harmonics in this case as in any other.

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[1] The form of a string vibrating so as to give only one tone (whether fundamental or harmonic) is a curve of sines, all its ordinates increasing or diminishing in the same proportion, as the string moves.

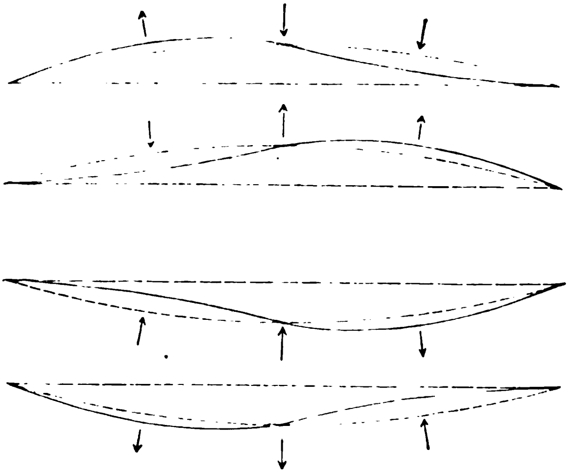

Fig. 588.—String giving first Two Tones.

It is important to remark, that though the presence of harmonic subdivisions in a vibrating body necessarily produces harmonics in the sound emitted, the converse cannot be asserted. Simple vibrations, executed by a vibrating body, produce vibrations *of the same frequency* as their own, in any medium to which they are transmitted, but not necessarily *simple* vibrations. If they produce compound vibrations, these, as we have seen (§ 678), must consist of a fundamental simple vibration and its harmonics.

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[1] The stops called *open diapason* and *stop diapason* (consisting respectively of open and stopped pipes), give the fundamental tone, almost free from harmonics. The stop absurdly called *principal* gives the second tone, that is the octave above the fundamental. The stops called *twelfth* and *fifteenth* give the third and fourth tones, which are a twelfth (octave + fifth), and a fifteenth (double octave) above the fundamental. The fifth and sixth tones are included in the stop called *mixture*.

As many of our readers will be unacquainted with the structure of organs, it may be desirable to state that an organ contains a number of complete instruments, each consisting of several octaves of pipes. Each of these complete instruments is called a *stop*, and is brought into use at the pleasure of the organist by pulling out a slide, by means of a knob-handle, on which the name of the stop is marked. To throw it out of use, he pushes in the slide. A large number of stops are often in use at once.

679. Helmholtz's Resonators.—Helmholtz derived material aid in his researches from an instrument devised by himself, and called a *resonator* or *resonance globe* (Fig. 606). It is a hollow globe of thin brass, with an opening at each end, the larger one serving for the admission of sound, while the smaller one is introduced into the ear. The inclosed mass of air has, like the column of air in an organ-pipe, a particular fundamental note of its own, depending upon its size; and whenever a note of this particular pitch is sounded in its neighbourhood, the inclosed air takes it up and intensifies it by resonance.

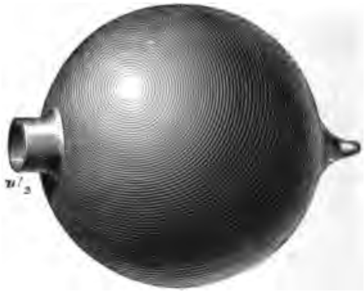

Fig. 606.—Resonator.

In order to test the presence or absence of a particular harmonic in a given musical tone, a resonator, in unison with this harmonic, is applied to the ear, and if the resonator speaks it is known that the harmonic is present. These instruments are commonly constructed so as to form a series, whose notes correspond to the bass C of a man's voice, and its successive harmonics as far as the 10th or 12th.

Koenig has applied the principle of manometric flames to enable a large number of persons to witness the analysis of sounds by resonators. A series of 6 resonators, whose notes have frequencies proportional to 1, 2, 3, 4, 5, 6, are fixed on a stand (Fig. 607), and their smaller ends, instead of being applied to the ear, are connected each with a separate manometric capsule, which acts on a gas jet. When the mirrors are turned, it is easy to see which of the flames vibrate while a sonorous body is passed in front of the resonators.

A simple tone, unaccompanied by harmonics, is dull and uninteresting, and, if of low pitch, is very destitute of penetrating quality.

Sounds composed of the first six elementary tones in fair proportion, are rich and sweet.

The higher harmonics, if sufficiently subdued, may also be present without sensible detriment to sweetness, and are useful as contributing to expression. When too loud, they render a sound harsh and grating; an effect which is easily explained by the discordant combinations which they form one with another; the 8th and 9th tones, for example, are at the same interval as the notes *Do* and *Re*.

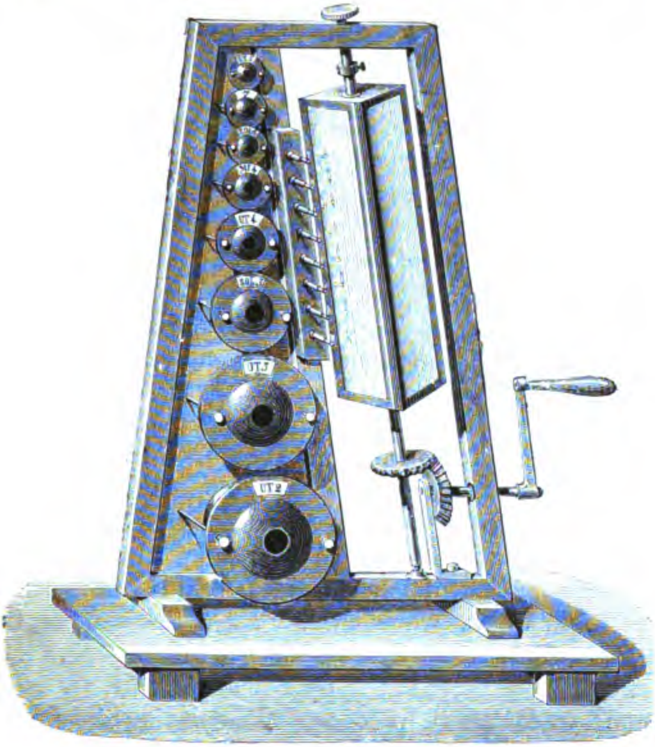

Fig. 607.—Analysis by Manometric Flames.

680. Vowel Sounds.—The human voice is extremely rich in harmonics, as may be proved by applying the series of resonators to the ear while the fundamental note is sung. The origin of the tones of the voice is in the vocal chords, which, when in use, form a diaphragm with a slit along its middle. The edges of this slit vibrate when air is forced through, and, by alternately opening and closing the passage, perform the part of a reed. The cavity of the mouth serves as a resonance chamber, and reinforces particular notes depending on the position of the organs of speech. It is by this resonance that the various vowel sounds are produced. The deepest pitch belongs to the vowel sound which is expressed in English by *oo* (as in *moon*), and the highest to *ee* (as in *screech*).

Willis in 1828[1] succeeded in producing the principal vowel sounds by a single reed fitted to various lengths of tube. Wheatstone, a few years later, made some advances in theory,[2] and constructed a machine by which nearly all articulate sounds could be imitated.

The best determinations of the particular notes which are reinforced in the case of the several vowel sounds, have been made by Helmholtz, who employed several methods, but chiefly the two following:—

1. Holding resonators to the ear, while a particular vowel sound was loudly sung.

2. Holding vibrating tuning-forks in front of the mouth when in the proper position for pronouncing a given vowel; and observing which of them had their sounds reinforced by resonance.[3]

Helmholtz has verified his determinations synthetically. He employs a set of tuning-forks which are kept in vibration by the alternate making and unmaking of electro-magnets, the circuit being made and broken by the vibrations of one large fork of 64 vibrations per second. The notes of the other forks are the successive harmonics of this fundamental note. Each fork is accompanied by a resonance-tube, which, when open, renders the note of the fork audible at a distance; and by means of a set of keys, like those of a piano, any of these tubes can be opened at pleasure. The different vowel-sounds can thus be produced by employing the proper combinations.

The same apparatus served for establishing the principle (§ 678), that the character of a musical sound depends only on *constitution*, irrespective of change of phase.

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[1] *Cambridge Transactions*, vol. iii.

[2] *London and Westminster Review*, October, 1837.

[3] According to Koenig (*Comptes Rendus*, 1870) the notes of strongest resonance for the vowels *u*, *o*, *a*, *e*, *i*, as pronounced in North Germany, are the five successive octaves of B flat, commencing with that which corresponds to the space above the top line of the base clef. Willis, Helmholtz, and Koenig all agree as regards the note of the vowel *o*, which is very nearly that of a common A tuning-fork. They are also agreed respecting the note of *a* (as in *father*), which is an octave higher.

CHAPTER LVIᴬ.

CONSONANCE, DISSONANCE, AND RESULTANT TONES.

680A. Concord and Discord.—Every one not utterly destitute of musical ear is familiar with the fact that certain notes, when sounded together, produce a pleasing effect by their combination, while certain others produce an unpleasing effect. The combination of two or more notes, when agreeable, is called *concord* or *consonance;* when disagreeable, *discord* or *dissonance.* The distinction is found to depend almost entirely on difference of pitch, that is, on relative frequency of vibration; so that the epithets consonant and dissonant can with propriety be applied to intervals.

The following intervals are consonant: unison (1 : 1), octave (1 : 2), octave + fifth (1 : 3), double octave (1 : 4), fifth (2 : 3), fourth (3 : 4).

The major third (4 : 5) and major sixth (3 : 5), together with the minor third (5 : 6) and minor sixth (5 : 8), are less perfect in their consonance.

The second and the seventh, whether major or minor, are dissonant intervals, whatever system of temperament be employed, as are also an indefinite number of other intervals not recognized in music.

Besides the difference as regards pleasing or unpleasing effect, it is to be remarked that consonant intervals can be identified by ear with much greater accuracy than those which are dissonant. Musical instruments are generally tuned by octaves and fifths, because very slight errors of excess or defect in these intervals are easily detected by the ear. To tune a piano by the mere comparison of successive notes would be beyond the power of the most skilful musician. A sharply marked interval is always a consonant interval.

680 B. Jarring Effect of Dissonance.—According to the theory propounded by Helmholtz, the unpleasant effect of a dissonant interval consists essentially in the production of beats. These have a jarring effect upon the auditory apparatus, which becomes increasingly disagreeable as the beats increase in frequency up to about 33 per second, and becomes gradually less disagreeable as the frequency is still further increased. The sensation produced by beats is comparable to that which the eye experiences from the *bobbing* of a gas flame in a room lighted by it; and the frequency which entails the maximum of annoyance is in this case much smaller, on account of the greater persistence of visible impressions. The annoyance must evidently cease when the succession becomes so rapid as to produce the effect of a continuous impression.

We have already (§ 644 A) described a mode of producing beats with any degree of frequency at pleasure; and this experiment is one of the main foundations on which Helmholtz bases his view.

680 C. Beats of Harmonics.—The beats in the experiment above alluded to, are produced by the imperfect unison of two notes, and indicate the number of vibrations gained by one note upon the other. Their existence is easily and completely explained by the considerations adduced in § 644 A. But it is well known to musicians, and easily established by experiment, that beats are also produced between notes whose interval is approximately an octave, a fifth, or some other consonance; and that, in these cases also, the beats become more rapid as the interval becomes more faulty.

These beats are ascribed by Helmholtz to the common harmonic of the two fundamental notes. For example, in the case of the fifth (2 : 3), the third tone of the lower note would be identical with the second tone of the upper, if the interval were exact; and the beats which occur are due to the imperfect unison consequent on the deviation from exact truth. All beats are thus explained as due to imperfect *unison*.

This explanation is not merely conjectural, but is established by the following proofs:—

1. When an arrangement is employed by which the fifth is made false by a known amount, the number of beats is found to agree with the above explanation. Thus, if the interval is made to correspond to the ratio 200 : 301, it is observed that there are 2 beats to every 200 vibrations of the lower note. Now the harmonics which

are in approximate unison are represented by 600 and 602, and the difference of these is 2.

2. When the resonator corresponding to this common harmonic is held to the ear, it responds to the beats, showing that this harmonic is undergoing variations of strength; but when a resonator corresponding to either of the fundamental notes is employed, it does not respond to the beats, but indicates steady continuance of its appropriate note.

3. By a careful exercise of attention, a person with a good ear can hear, without any artificial aids, that it is the common harmonic which undergoes variations of intensity, and that the fundamental notes continue steady.

680D. Beating Notes must be Near Together.—In order that two simple tones may yield audible beats, it is necessary that the musical interval between them should be small; in other words, that the ratio of their frequencies of vibration should be nearly equal to unity. Two simple notes of 300 and 320 vibrations per second will yield 20 beats in a second, and will be eminently discordant, the interval between them being only a semitone (15 : 16), but simple notes of 40 and 60 vibrations per second will not give beats, the interval between them being a fifth (2 : 3). The wider the interval between two simple notes, the feebler will be their beats; and accordingly, for a given frequency of beats, the harshness of the effect increases with the nearness of the notes to each other on the musical scale.[1] By taking joint account of the number of beats and the nearness of the beating tones, Helmholtz has endeavoured to express numerically the severity of the discords resulting from the combination of the note C of 256 vibrations per second with any possible note lying within an octave on the upper side of it, a particular constitution (approximately that of the violin) being assumed for both notes. He finds a complete absence of discord for the intervals of unison, the octave, and the fifth, and very small amounts of discord for the fourth, the sixth, and the third. By far the worst discords are found for the intervals of the semitone and major seventh, and the next worst are for intervals a little greater or less than the fifth.

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[1] The explanation adopted by Helmholtz is, that a certain part of the ear, called *Corti's organ*, contains a number of elastic fibres, each of which is attuned to a particular simple tone, and is thrown into vibration when this tone, or one nearly in unison with it, is sounded. Two tones in approximate unison, when sounded together, affect several fibres in common, and cause them to beat. Tones not in approximate unison affect entirely distinct sets of fibres, and thus cannot produce interference.

680E. Imperfect Concord.—When there is a complete absence of discord between two notes, they are said to form a perfect concord. The intervals unison, fifth, octave, octave + fifth, and the interval from any note to any of its harmonics, are of this class. The third, fourth, and sixth are instances of imperfect concord. Suppose, for example, that the two notes sounded together are C of 256 and E of 320 vibrations per second, the interval between these notes being a true major third (4 : 5); and suppose each of these notes to consist of the first six simple tones.

The first six multiples of 4 are

4, 8, 12, 16, 20, 24.

The first six multiples of 5 are

5, 10, 15, 20, 25, 30.

In searching for elements of discord, we select (one from each line) two multiples differing by unity.

Those which satisfy this condition are

4 and 5; 16 and 15; 24 and 25.

But the first pair (4 and 5) may be neglected, because their ratio differs too much from unity. Discordance will result from each of the two remaining pairs; that is to say, the 4th element of the lower of our two given notes is in discordance with the 3d element of the upper; and the 6th element of the lower is in discordance with the 5th element of the higher. To find the frequencies of the beats, we must multiply all these numbers by 64, since 256 is 4 times 64, and 320 is 5 times 64. Instead of a difference of 1, we shall then find a difference of 64, that is to say, the number of beats per second is 64 in the case of each of the two discordant combinations which we have been considering.

680F. Resultant Tones.—Under certain conditions it is found that two notes, when sounded together, produce by their combination other notes, which are not found as constituents of either. They are called *resultant tones*, and are of two kinds, *difference-tones* and *summation-tones*. A difference-tone has a frequency of vibration which is the difference of the frequencies of its components. A summation-tone has a frequency of vibration which is the sum of the frequencies of its components. As the components may either be fundamental tones or overtones, two notes which are rich in harmonics may yield, by their combination, a large number of resultant tones.

The difference-tones were observed in the last century by Sorge and Tartini, and were, until recently, attributed to beats. The frequency of beats is always the difference of the frequencies of vibration of the two elementary tones which produce them; and it was supposed that a rapid succession of beats produced a note of pitch corresponding to this frequency.

This explanation, if admitted, would furnish an exception to what otherwise appears to be the universal law, that every *elementary tone* arises from a corresponding *simple vibration.*[1] Such an exception should not be admitted without necessity; and in the present instance it is not only unnecessary, but also insufficient, inasmuch as it fails to render any account of the summation-tones.

Helmholtz has shown, by a mathematical investigation, that when two systems of simple waves agitate the same mass of air, their mutual influence must, according to the recognized laws of dynamics, give rise to two derived systems, having frequencies which are respectively the sum and the difference of the frequencies of the two primary systems. Both classes of resultant tones are thus completely accounted for.

The resultant tones—especially the summation-tones, which are fainter than the others—are only audible when the primary tones are loud; for their existence depends upon small quantities of the second order, the amplitudes of the primaries being regarded (in comparison with the wave-lengths) as small quantities of the first order.

If any further proof be required that the difference tones are not due to the coalescence of beats, it is furnished by the fact that, in certain circumstances, the beats and the difference-tones can both be heard together.

680G. Beats due to Resultant Tones.—The existence of resultant tones serves to explain, in certain cases, the production of beats between notes which are wanting in harmonics. For example, if two *simple* sounds, of 100 and 201 vibrations per second respectively, are sounded together, one beat per second will be produced between the difference-tone of 101 vibrations and the primary tone of 100 vibrations. By the beats to which they thus give rise, resultant tones exercise an influence on consonance and dissonance.

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[1] The discovery of this law is due to Ohm.

Resultant tones, when sufficiently loud, are themselves capable of performing the part of primaries, and yielding what are called *resultant tones of the second order*, by their combination with other primaries. Several higher orders of resultant tones can, under peculiarly favourable circumstances, be sometimes detected.

OPTICS.

CHAPTER LVII.

PROPAGATION OF LIGHT.

681. Light.—Light is the immediate external cause of our visual impressions. Objects, except such as are styled *self-luminous*, become invisible when brought into a dark room. The presence of something additional is necessary to render them visible, and that mysterious agent, whatever its real nature may be, we call *light*.

Light, like sound, is believed to consist in vibration; but it does not, like sound, require the presence of air or other gross matter to enable its vibrations to be propagated from the source to the percipient. When we exhaust a receiver, objects in its interior do not become less visible; and the light of the heavenly bodies is not prevented from reaching us by the highly vacuous spaces which lie between.

It seems necessary to assume the existence of a medium far more subtle than ordinary matter; a medium which pervades alike the most vacuous spaces and the interior of all bodies, whether solid, liquid, or gaseous; and which is so highly elastic, in proportion to its density, that it is capable of transmitting vibrations with a velocity enormously transcending that of sound.

This hypothetical medium is called *æther*. From the extreme facility with which bodies move about in it, we might be disposed to call it a subtle *fluid;* but the undulations which it serves to propagate are not such as can be propagated by fluids. Its elastic properties are rather those of a solid; and its waves are analogous to the pulses which travel along the wires of a piano rather than to the waves of extension and compression by which sound is propagated through air. *Luminous vibrations are transverse, while those of sound are longitudinal.*

A self-luminous body, such as a red-hot poker or the flame of a candle, is in a peculiar state of vibration. This vibration is communicated to the surrounding æther, and is thus propagated to the eye, enabling us to see the body. In the majority of cases, however, we see bodies not by their own but by reflected light; and we are enabled to recognize the various kinds of bodies by the different modifications which light undergoes in reflection from their surfaces.

As all bodies can become sonorous, so also all bodies can become self-luminous. To render them so, it is only necessary to raise them to a sufficiently high temperature, whether by the communication of heat from a furnace, or by the passage of an electric current, or by causing them to enter into chemical combination. It is to chemical combination, in the active form of combustion, that we are indebted for all the sources of light in ordinary use.

The vibrations of the æther are capable of producing other effects besides illumination. They constitute what is called radiant heat, and they are also capable of producing chemical effects, as in photography. Vibrations of high frequency, or short period, are the most active chemically. Those of low frequency or long period have usually the most powerful heating effects; while those which affect the eye with the sense of light are of moderate frequency.

682. Rectilinear Propagation of Light.—All the remarks which have been made respecting the relations between period, frequency, and wave-length, in the case of sound, are equally applicable to light, inasmuch as all kinds of luminous waves (like all kinds of sonorous waves) have the same velocity in the same medium; but this velocity is many hundreds of thousands of times greater for light than for sound, and the wave-lengths of light are at the same time very much shorter than those of sound. Frequency, being the quotient of velocity by wave-length, is accordingly about a million of millions of times greater for light than for sound. The colour of lowest pitch is deep red, its frequency being about 400 million million vibrations per second. and its wave-length in air 760 millionths of a millimetre. The colour of highest pitch is deep violet; its frequency is about 760 million million vibrations per second, and its wave-length in air 400 millionths of a millimetre. It thus appears that the range of seeing is much smaller than that of hearing, being only about one octave.

The excessive shortness of luminous as compared with sonorous waves is closely connected with the strength of the shadows cast by a light, as compared with the very moderate loss of intensity produced by interposing an obstacle in the case of sound. Sound may, for ordinary purposes, be said to be capable of turning a corner, and light to be only capable of travelling in straight lines. The latter fact may be established by such an arrangement as is represented in Fig. 608. Two screens, each pierced with a hole, are arranged so that these holes are in a line with the flame of a candle. An eye placed in this line, behind the screens, is then able to see the flame; but a slight lateral displacement, either of the eye, the candle, or either of the screens, puts the flame out of sight. It is to be noted that, in this experiment, the same medium (air) extends from the eye to the candle. We shall hereafter find that, when light has to pass from one medium to another, it is often bent out of a straight line.

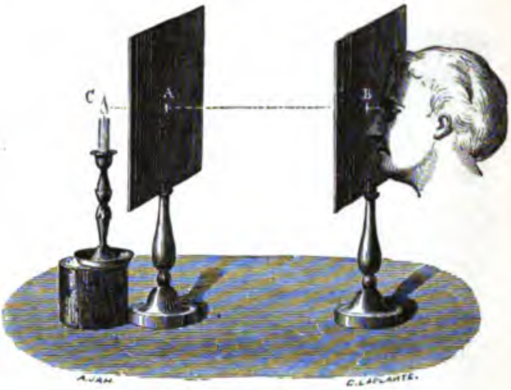

Fig. 608.—Rectilinear Propagation.

We have said that the strength of light-shadows as compared with sound-shadows is connected with the shortness of luminous waves. Theory shows that, if light is transmitted through a hole or slit, whose diameter is a very large multiple of the length of a light-wave, a strong shadow should be cast in all oblique directions; but that, if the hole or slit is so narrow that its diameter is comparable to the length of a wave, a large area not in the direct path of the beam will be illuminated. The experiment is easily performed in a dark room, by admitting sunlight through an exceedingly fine slit, and receiving it on a screen of white paper. The illuminated area will be marked with coloured bands, called diffraction-fringes; and if the slit is made narrower, these bands become wider.

On the other hand, Colladon, in his experiments on the transmission of sound through the water of the Lake of Geneva, established the presence of a very sharply defined sound-shadow in the water, behind the end of a projecting wall.

For the present we shall ignore diffraction,[1] and confine our attention to the numerous phenomena which result from the rectilinear propagation of light.

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[1] See Chap. lxiv.

683. Images produced by Small Apertures.—If a white screen is placed opposite a hole in the shutter of a room otherwise quite dark, an inverted picture of the external landscape will be formed upon it, in the natural colours. The outlines will be sharper in proportion as the hole is smaller, and distant objects will be more distinctly represented than those which are very near.

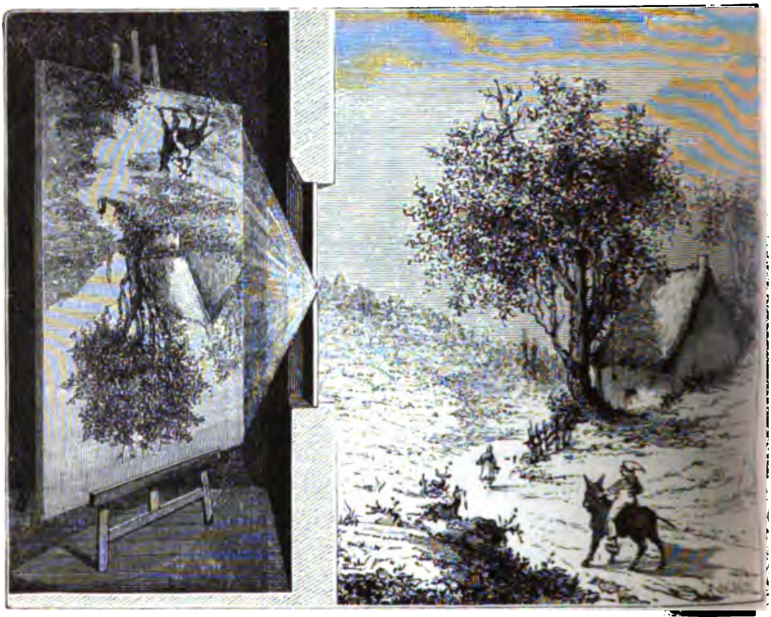

Fig. 609.—Image formed by Small Aperture.

These results are easily explained. Consider, in fact, an external object A B (Fig. 610), and let O be the hole in the shutter. The point A sends rays in all directions into space, and among them a small pencil, which, after passing through the opening O, falls upon the screen at A′. A′ receives light from no other point but A, and A sends light to no part of the screen except A′. The colour and brightness of the spot A′ will accordingly depend upon the colour and brightness of A; in other words, A′ will be the

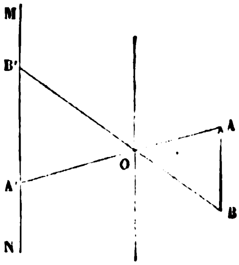

Fig. 610.—Explanation.

image of A. In like manner B′ will be the image of B, and points of the object between A and B will have their images between A′ and B′. An inverted image A′ B′ will thus be formed of the object A B.

As the image thus formed of an external point is not a point, but a spot, whose size increases with that of the opening, there must always be a little blurring of the outlines from the overlapping of the spots which represent neighbouring points; but this will be comparatively slight if the opening is very small.

An experiment, substantially the same as the above, may be performed by piercing a card with a large pin-hole, and holding it between a candle and a screen, as in Fig. 611. An inverted image of the candle will thus be formed upon the screen.

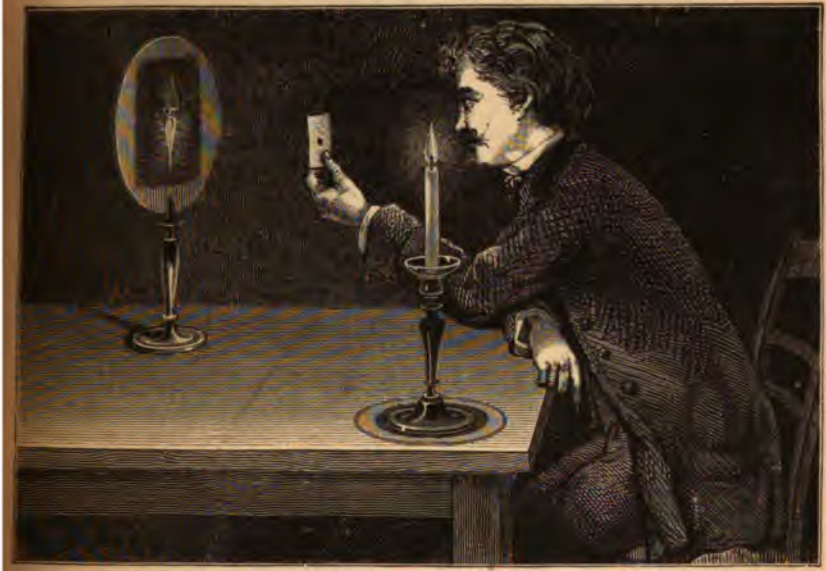

Fig. 611.—Image formed by Hole in a Card.

When the sun shines through a small hole into a room with the blinds down, the cone of rays thus admitted is easily traced by the lighting up of the particles of dust which lie in its course. The image of the sun which is formed at its lower extremity may be either circular or elliptical, according to the aspect of the surface on which it is received. Fine images of the sun are sometimes thus formed by the chinks of a venetian-blind, especially when the sun is low, and there is a white wall opposite to receive the image. In these circumstances it is sometimes possible to detect the presence of spots on the sun by examining the image.

When the sun's rays shine through the foliage of a tree, the spots of light which they form upon the ground are always round or oval, whatever may be the shape of the interstices through which they have passed, provided always that these interstices are small. When the sun is undergoing eclipse, the progress of the eclipse can be traced by watching the shape of these images, which resembles that of the uneclipsed portion of the sun's disc.

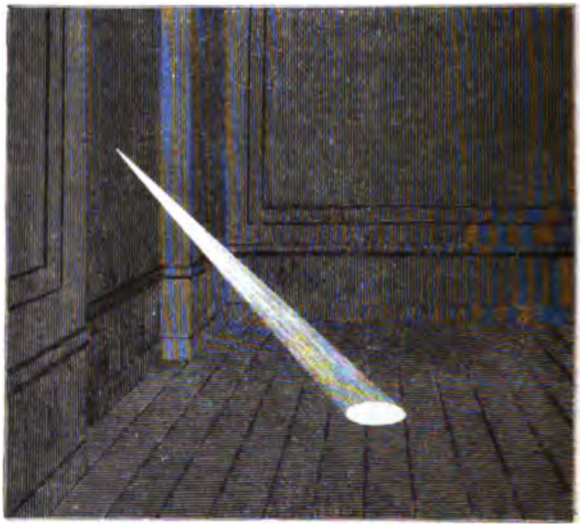

Fig. 612.—Conical Sunbeam.

684. Theory of Shadows.—The rectilinear propagation of light is the foundation of the geometry of shadows. Let the source of light be a luminous point, and let an opaque body be placed so as to intercept a portion of its rays. If we construct a conical surface touching the body all round, and having its vertex at the luminous point, it is evident that all the space within this surface on the further side of the opaque body is completely screened from the rays. The cone thus constructed is called the shadow-cone, and its intersection with any surface behind the opaque body defines the shadow cast upon that surface. In the case which we have been supposing—that of a luminous point—the shadow-cone and the shadow itself will be sharply defined.

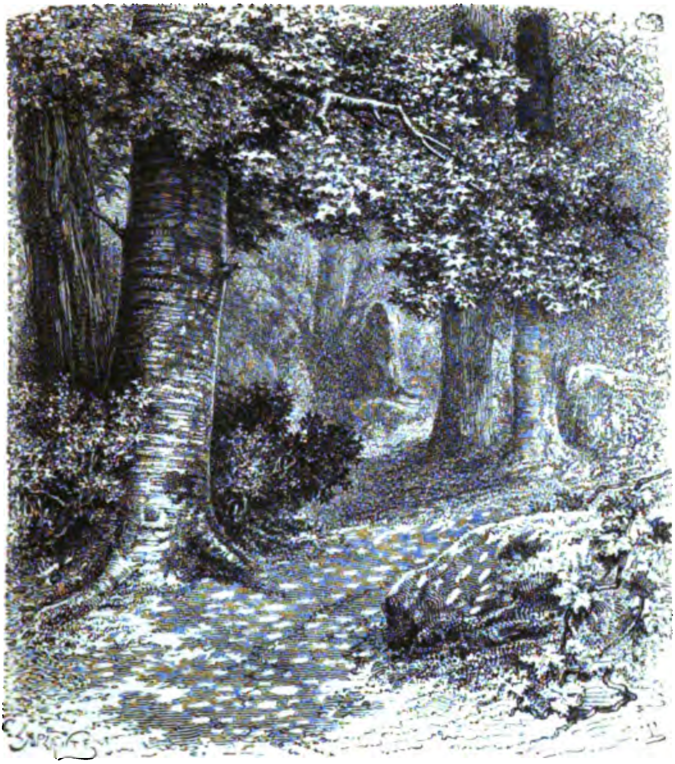

Fig. 613.—Images of Sun formed by Foliage.

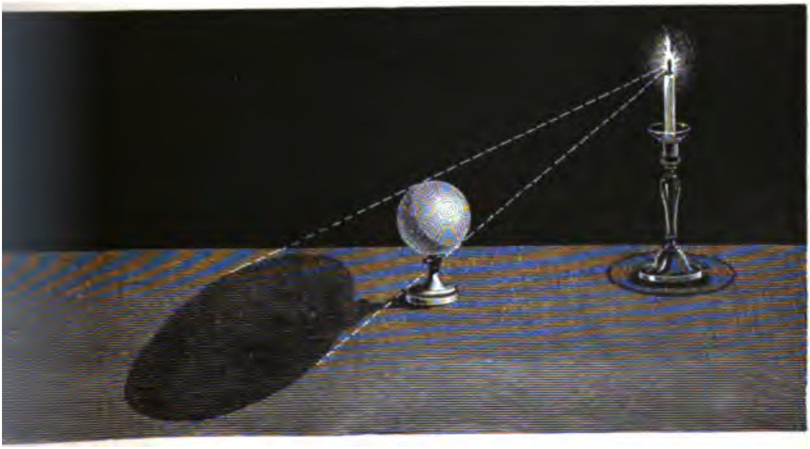

Fig. 614.—Shadow.

Actual sources of light, however, are not mere luminous points, but have finite dimensions. Hence some complication arises. Consider, in fact (Fig. 615), a luminous body situated between two opaque bodies, one of them larger, and the other smaller than itself. Conceive a cone touching the luminous body and either of the opaque bodies *externally*. This will be the cone of *total shadow*, or the cone of the *umbra*. All points lying within it are completely excluded from view of the luminous body. This cone narrows or enlarges as it recedes, according as the opaque body is smaller or larger than the luminous body. In the former case it terminates at a finite distance. In the latter case it extends to infinite distance.

Now conceive a double cone touching the luminous body and either of the opaque bodies *internally*. This cone will be wider than the cone of total shadow, and will include it. It is called the cone of *partial shadow*, or the cone of the *penumbra*. All points lying within it are excluded from the view of some portion of the luminous body, and are thus partially shaded by the opaque body. If they are near its outer boundary, they are very slightly shaded. If they are so far within it as to be near the total shadow, they are almost completely shaded. Accordingly, if the shadow of the opaque body is received upon a screen, it will not have sharply defined edges, but will show a gradual transition from the total shadow which covers a finite central area to a complete absence of shadow at the outer boundary of the penumbra. Thus neither the edges of the umbra nor those of the penumbra are sharply defined.

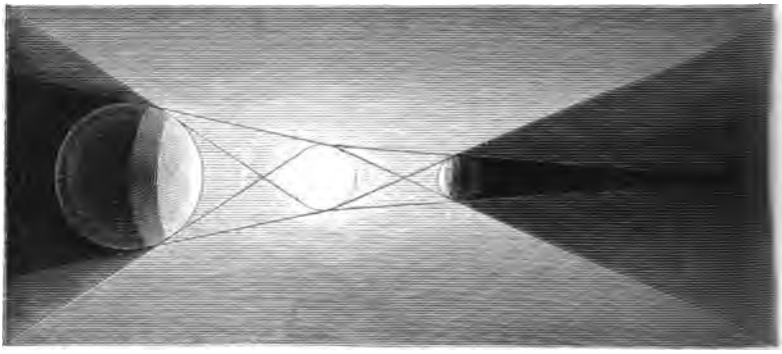

Fig. 615.—Umbra and Penumbra.

The umbra and penumbra show themselves on the surface of the opaque body itself, the line of contact of the umbral cone being further back from the source of light than the line of contact of the penumbral cone. The zone between these two lines is in partial shadow, and separates the portion of the surface which is in total shadow from the part which is not shaded at all.

685. Velocity of Light.—Luminous undulations, unlike those of sound, advance with a velocity which may fairly be styled inconceivable, being about 298 million metres per second, or 185,000 miles per second. As the circumference of the earth is only 40 million metres, light would travel seven and a half times round the earth in a second.

Hopeless as it might appear to attempt the measurement of such an enormous velocity by mere terrestrial experiments, the feat has actually been performed, and that by two distinct methods. In Fizeau's experiments the distance between the two experimental stations was about 5½ miles. In Foucault's experiments the whole apparatus was contained in one room, and the movement of light within this room served to determine the velocity.

We will first describe Fizeau's experiment.

686. Fizeau's Experiment.—Imagine a source of light placed directly in front of a plane mirror, at a great distance. The mirror will send back a reflected beam along the line of the incident beam, and an observer stationed behind the source will see its image in the mirror as a luminous point.

Now imagine a toothed-wheel, with its plane perpendicular to the path of the beam, revolving uniformly in front of the source, in such a position that its teeth pass directly between the source of light and the mirror. The incident beam will be stopped by the teeth, as they successively come up, but will pass through the spaces between them. Now the velocity of the wheel may be such that the light which has thus passed through a space shall be reflected back from the mirror just in time to meet a tooth and be stopped. In this case it will not reach the observer's eye, and the image may thus become permanently invisible to him. From the velocity of the wheel, and the number of its teeth, it will be possible to compute the time occupied by the light in travelling from the wheel to the mirror, and back again. If the velocity of the wheel is such that the light is sometimes intercepted on its return, and sometimes allowed to pass, the image will appear steadily visible, in consequence of the persistence of impressions on the retina, but with a loss of brightness proportioned to the time that the light is intercepted. The wheel employed by Fizeau had 720 teeth, the distance between the two stations was 8663 metres, and 12·6 revolutions per second produced disappearance of the image. The width of the teeth being equal to the width of the spaces, the time required to turn through the width of a tooth was $\frac{1}{2} \times \frac{1}{720} \times \frac{1}{12.6}$ of a second, that is $\frac{1}{18144}$ of a second.

In this time the light travelled a distance of $2 \times 8663 = 17326$ metres. The distance traversed by light in a second would therefore be $17{,}326 \times 18{,}144 = 314{,}262{,}944$ metres. This determination of M. Fizeau's is believed to be somewhat in excess of the truth.

A double velocity of the wheel would allow the reflected beam to pass through the space succeeding that through which the incident beam had passed; a triple velocity would again produce total eclipse, and so on. Several independent determinations of the velocity of light may thus be obtained.

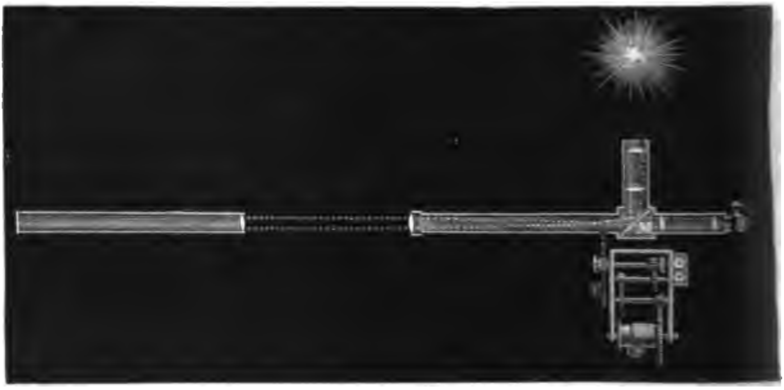

Fig. 616.—Fizeau's Experiment.

Thus far, we have merely indicated the principle of calculation. It will easily be understood that special means were necessary to prevent scattering of the light, and render the image visible at so great a distance. Fig. 616 will serve to give an idea of the apparatus actually employed.

A beam of light from a lamp, after passing through a lens, falls on a plate of unsilvered glass M, placed at an angle of 45°, by which it is reflected along the tube of a telescope; the object-glass of the telescope is so adjusted as to render the rays parallel on emergence, and in this condition they traverse the interval between the two stations. At the second station they are collected by a lens, which brings them to a focus on the surface of a plane mirror, and this mirror sends them back along the same course by which they came. A portion of the light thus sent back to the glass plate M passes through it, and is viewed by the observer through an eye-piece.

The wheel R is driven by clock-work. Figs. 617, 618, 619 respectively represent the appearance of the luminous point as seen between the teeth of the wheel when not revolving, the total eclipse produced by an appropriate speed of rotation, and the partial eclipse produced by a different speed.

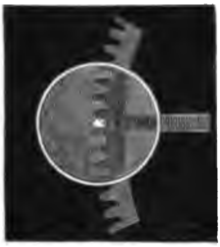
Fig. 617.—Wheel at Rest.

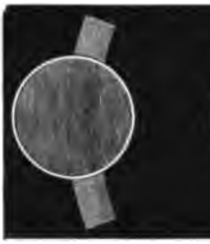
Fig. 618.—Total Eclipse.

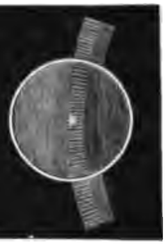
Fig. 619.—Partial Eclipse.

It was found that there was some uncertainty in the phenomenon of total eclipse, probably because the light, when not entirely intercepted, might be so much reduced in amount as to be incapable of affecting the retina.

The experiment of M. Fizeau was very remarkable as the first instance in which light was proved to occupy a finite time in traversing terrestrial distances; but the method is scarcely capable of giving accurate results. Foucault's method, which we now proceed to explain, is much better fitted for rigorous determinations.

687. Foucault's Experiment.—Foucault employed the principle of the rotating mirror, first adopted by Wheatstone in his experiments on the duration of the electric spark and the velocity of electricity (§ 437, 466). The following was the construction of his original apparatus:—

A beam of light enters a room by a square hole, which has a fine platinum wire stretched across it, to serve as a mark; it is then concentrated by an achromatic lens, and, before coming to a focus, falls upon a plane mirror, revolving about an axis in its own plane. In one part of the revolution the reflected beam is directed upon a concave mirror, whose centre of curvature is in the axis of rotation, so that the beam is reflected back to the revolving mirror, and thence back to the hole at which it first entered. Before reaching the hole, it has to traverse a sheet of glass, placed at an angle of 45°, which reflects a portion of it towards the observer's eye; and the image which it forms (an image of the platinum wire) is viewed through a powerful eye-piece. The image is only formed during a small part of each revolution; but when 30 turns are made per second, the appearance presented, in consequence of the persistence of impressions, is that of a permanent image occupying a fixed position. When the speed is considerably greater, the mirror turns through a sensible angle while the light is travelling from it to the concave mirror and back again, and a sensible displacement of the image is accordingly observed. The actual speed of rotation was from 700 to 800 revolutions per second.[1]

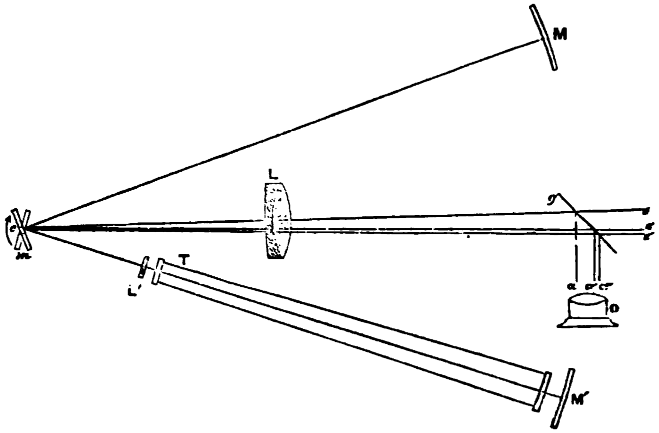

Fig. 619 A.—Foucault's Experiment.

On interposing a tube filled with water between the two mirrors, it was found that the displacement was increased, showing that a longer time was occupied in traversing the water than in traversing the same length of air.

This result, as we shall have occasion to point out later, is very important as confirming the undulatory theory and disproving the emission theory of light.

In Fig. 619 A, *a* is the position of the platinum wire, L is the achromatic lens, *m* the revolving mirror, *c* the axis of revolution, M the concave mirror, a' the image of the platinum wire, displaced from a in virtue of the rotation of the mirror; α, α' images of a, a', formed by the glass plate g, and viewed through the eye-piece O.

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[1] It was found that, at this high speed, the amalgam at the back of ordinary looking-glasses was driven off by centrifugal force. The mirror actually employed was silvered in front with real silver.

M′ is a second concave mirror, at the same distance as M from the revolving mirror; T is a tube filled with water, and having plane glass ends, and L′ a lens necessary for completing the focal adjustment; a'' and α'' are the images formed by the light which has traversed the water.[1]

Foucault's experiment, as thus described, was performed in 1850, very shortly after that of Fizeau. Some important improvements have since been introduced in the method, especially as regards the measurement of the speed of rotation of the mirror, which is evidently a principal element in the calculation. The mirror is driven by means of a bellows, furnished with a special arrangement for keeping up a constant pressure of air, and driving a kind of syren, on which the mirror is mounted. Instead of making a separate determination of the speed of rotation in each experiment, M. Foucault has adopted a very rigorous method of keeping it always at one constant value, namely, 400 revolutions per second. This is less than the speed attained in the earlier experiments; but, on the other hand, the length of the path traversed by the light between its two reflections from the revolving mirror is increased, by means of successive reflections, so as to be about 20 metres, instead of 4 as in the original experiments.

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[1] The distances are such that La and Lc+cM are conjugate focal distances with respect to the lens L. An image of the wire a is thus formed at M, and an image of this image is formed at a, the mirror being supposed stationary; and this relation holds not only for the central point of the concave mirror, but for any part of it on which the light may happen to fall at the instant considered.

Let l denote the distance c M between the revolving and the fixed mirror, l' the distance c L of the revolving mirror from the centre of the lens, r the distance a L of the platinum wire from the centre of the lens, n the number of revolutions per second, V the space traversed by light in a second, t the time occupied by light in travelling from one mirror to the other and back, θ the angle turned by the mirror in this time, and δ the angle subtended at the centre of the lens by the distance $a\,a'$ between the wire and its displaced image.

Then obviously $t = \frac{2\,l}{V}$, but also $t = \frac{\theta}{2\,\pi\,n}$; hence $V = \frac{4\,\pi\,n\,l}{\theta}$.

Now the distance between the two images (corresponding to a, a' respectively) at the back of the revolving mirror is $(l+l')\,\delta$, and is also $2\,\theta\,l$ (§ 705 A). Hence $\theta = \frac{(l+l')\,\delta}{2\,l}$, and $V = \frac{8\,\pi\,n\,l^2}{(l+l')\,\delta}$. The observed distance $a\,a'$ between the two images is equal to the distance between α, α', that is to $r\,\delta$. Calling this distance d, we have finally,

$$V = \frac{8\,\pi\,n\,l^2\,r}{(l+l')\,d}$$

The constant rate of revolution is maintained by comparison with a clock. A wheel with 400 teeth, driven by the clock, makes exactly one revolution per second. A tooth and a space alternately cover the part of the field where the image of the wire-grating (which has been substituted for the single wire) is formed. The same instantaneous flashes of light from the revolving mirror which form the image, also illuminate the rim of the wheel. If the wheel advances exactly one tooth and space between consecutive flashes, its illuminated positions are undistinguishable one from another, and the wheel accordingly appears stationary. When this is the case, it is known that the mirror is making exactly 400 turns per second. A slight departure from this rate either way, makes the wheel appear to be slowly revolving either forwards or backwards, and the bellows must be regulated until the stationary appearance is presented.

By means of this admirable combination, Foucault has made what must be regarded as the best determination yet obtained of the velocity of light. The value thus found, namely, 298 million metres per second, is smaller than that which, until a few years ago, was generally received; but recent astronomical discussions have shown that the sun's distance is somewhat less than was previously supposed; and when this correction is made, the astronomical determinations of the velocity of light agree well with that of Foucault.

688. Velocity of Light deduced from Observations of the Eclipses of Jupiter's Satellites.—The fact that light occupies a sensible time in travelling over celestial distances, was first established about 1675, by Roemer, a Danish astronomer, who also made the first computation of its velocity. He was led to this discovery by comparing the observed times of the eclipses of Jupiter's first satellite, as contained in records extending over many successive years.

The four satellites of Jupiter revolve nearly in the plane of the planet's orbit, and undergo very frequent eclipse by entering the cone of total shadow cast by Jupiter. The satellites and their eclipses are easily seen, even with telescopes of very moderate power; and being visible at the same absolute time at all parts of the earth's surface at which they are visible at all, they serve as signals for comparing local time at different places, and thus for determining longitudes. The first satellite (that is, the one nearest to Jupiter), from its more rapid motion and shorter time of revolution, affords both the best and the most frequent signals. The interval of time between two successive eclipses of this satellite is about 42½ hours, but was found by Roemer to vary by a regular law according to the position of the earth with respect to Jupiter. It is longest when the earth is increasing its distance from Jupiter most rapidly, and is shortest when the earth is diminishing its distance most rapidly. Starting from the time when the earth is nearest to Jupiter, as at T, J (Fig. 620), the intervals between successive eclipses are always longer than the mean value, until the greatest distance has been attained, as at T′, J′, and the sum of the excesses amounts to 16 min. 26·6 sec. From this time until the nearest distance is again attained, as at T″, J″, the intervals are always shorter than the mean, and the sum of the defects amounts to 16 min. 26·6 sec. It is evident, then, that the eclipses are visible 16 m. 26·6 s. earlier at the nearest than at the remotest point of the earth's orbit; in other words, that this is the time required for the propagation of light across the diameter of the orbit. Taking this diameter as 183 millions of miles,[1] we have a resulting velocity of about 185,500 miles per second.

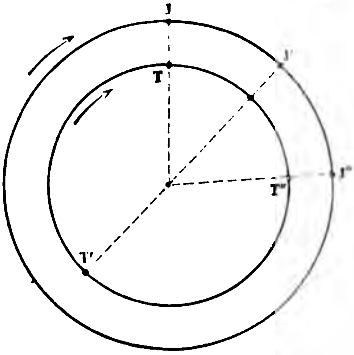

Fig. 620.—Earth and Jupiter.

688A. Velocity of Light deduced from Aberration.—About fifty years after Roemer's discovery, Bradley, the English astronomer, employed the velocity of light to explain the astronomical phenomenon called *aberration*. This consists in a regular periodic displacement of the stars as seen from the earth, the period of the displacement being a year. If the direction in which the earth is moving in its orbit at any instant be regarded as the *forward* direction, every star constantly appears on the forward side of its true place, so that, as the earth moves once round its orbit in a year, each star describes in this time a small apparent orbit about its true place.

The phenomenon is explained in the same way as the familiar fact, that a shower of rain falling vertically, seems, to a person running forwards, to be coming in his face. The relative motion of the ain-drops with respect to his body, is found by compounding the ctual velocity of the drops (whether vertical or oblique) with a velocity equal and opposite to that with which he runs. Thus if A B (Fig. 620 A) represents the velocity with which he runs, and C A the true velocity of the drops, the apparent velocity of the drops will be represented by D A. If a tube pointed along A D moves forward parallel to itself with the velocity A B, a drop entering at its upper end will pass through its whole length without wetting its sides; for while the drop is falling along D B (we suppose with uniform velocity) the tube moves along A B, so that the lower end of the tube reaches B at the same time as the rain-drop.

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[1] The sun's mean distance from the earth was, until recently, estimated at 95 millions f miles. It is now estimated at 92 or 91½ millions.

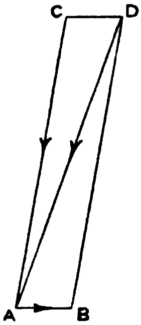

Fig. 620 A. Aberration.

In like manner, if A B is the velocity of the earth, and C A the velocity of light, a telescope must be pointed along A D to see a star which really lies in the direction of A C or B D produced. When the angle B A C is a right angle (in other words, when the star lies in a direction perpendicular to that in which the earth is moving), the angle C A D, which is called the aberration of the star, is 20″·5, and the tangent of this angle is the ratio of the velocity of the earth to the velocity of light. Hence it is found by computation that the velocity of light is about ten thousand times greater than that with which the earth moves in its orbit. The latter is easily computed, if the sun's distance is known, and is about 18½ miles per second. Hence the velocity of light is about 185,000 miles per second. It will be noted that both these astronomical methods of computing the velocity of light, depend upon the knowledge of the sun's distance from the earth, and that, if this distance is overestimated, the computed velocity of light will be too great in the same ratio.

Conversely, the velocity of light, as determined by Foucault's method, can be employed, in connection either with aberration or the eclipses of the satellites, for computing the sun's distance; and the first correct determination of the sun's distance was, in fact, that deduced by Foucault from his own results.

689. Photometry.—Photometry is the measurement of the relative amounts of light emitted by different sources. The methods employed for this purpose all consist in determinations of the relative distances at which two sources produce equal intensities of illumination. The eye would be quite incompetent to measure the ratio of two unequal illuminations; but a pretty accurate judgment can be formed as to equality or inequality of illumination, at least when the surfaces compared are similar, and the lights by which they are illuminated are of the same colour. The law of inverse squares is always made the basis of the resulting calculations; and this law may itself be verified by showing that the illumination produced by one candle at a given distance is equal to that produced by four candles at a distance twice as great.

690. Bouguer's Photometer.—Bouguer's photometer consists of a semi-transparent screen, of white tissue paper, ground glass, or thin white porcelain, divided into two parts by an opaque partition at right angles to it. The two lamps which are to be compared are

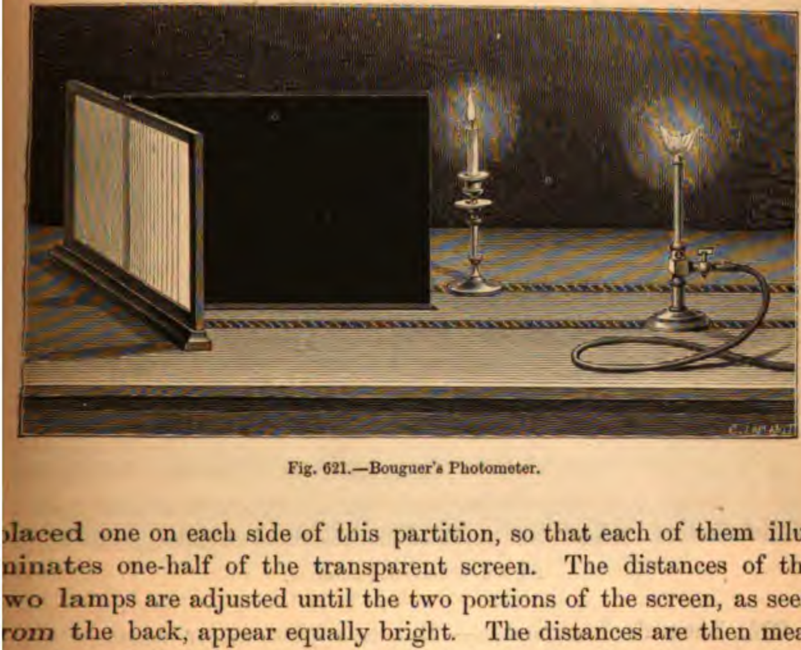

Fig. 621.—Bouguer's Photometer.

laced one on each side of this partition, so that each of them illu-
ninates one-half of the transparent screen. The distances of the
wo lamps are adjusted until the two portions of the screen, as seen
rom the back, appear equally bright. The distances are then mea-
ured, and their squares are assumed to be directly proportional to
e illuminating powers of the lamps.

691. Rumford's Photometer.—Rumford's photometer is based on
e comparison of shadows. A cylindric rod is so placed that each of
e two lamps casts a shadow of it on a screen; and the distances
e adjusted until the two shadows are equally dark. As the shadow
rown by one lamp is illuminated by the other lamp, the compari-
n of shadows is really a comparison of illuminations.

692. Foucault's Photometer.—The two photometers just described are alike in principle. In each of them the two surfaces compared are illuminated each by one only of the sources of light. In Rumford's the remainder of the screen is illuminated by both. In Bouguer's it consists merely of an intervening strip which is illuminated by neither. If the partition is movable, the effect of moving it further from the screen will be to make this dark strip narrower until it disappears altogether; and if it be advanced still further, the two illuminated portions will overlap. In Foucault's photometer there is an adjusting screw, for the purpose of advancing the partition so far that the dark strip shall just vanish. The two illuminated portions, being then exactly contiguous, can be more easily and certainly compared.

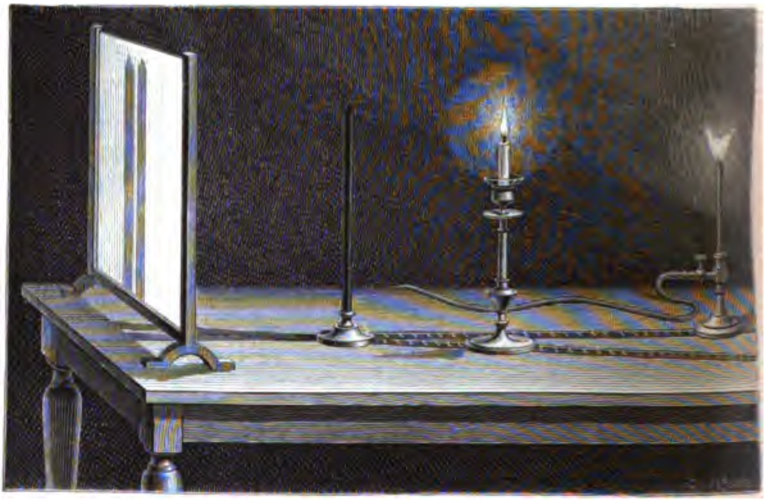

Fig. 622.—Rumford's Photometer.

693. Bunsen's Photometer.—Bunsen's photometer consists of a screen of white paper with a grease-spot in its centre. The lights to be compared are placed on opposite sides of this screen, and their distances are so adjusted that the grease-spot appears neither brighter nor darker than the rest of the paper, from whichever side it is viewed. When the distances have not been correctly adjusted, the grease-spot will appear darker than the rest of the paper when viewed from the side on which the illumination is most intense, and lighter than the rest of the paper when viewed from the other side.

CHAPTER LVIII.

REFLECTION OF LIGHT.

694. Reflection.—If a beam of the sun's rays A B (Fig. 623) be admitted through a small hole in the shutter of a dark room, and allowed to fall on a polished plane surface, it will be seen to continue its course in a different direction B C. This is an example of reflection. A B is called the incident beam, and B C the reflected beam. The angle A B D contained between an incident ray and the normal is called the angle of incidence; and the angle C B D contained between the corresponding reflected ray and the normal is called the angle of reflection. The plane A B D containing the incident ray and the normal is called the plane of incidence.

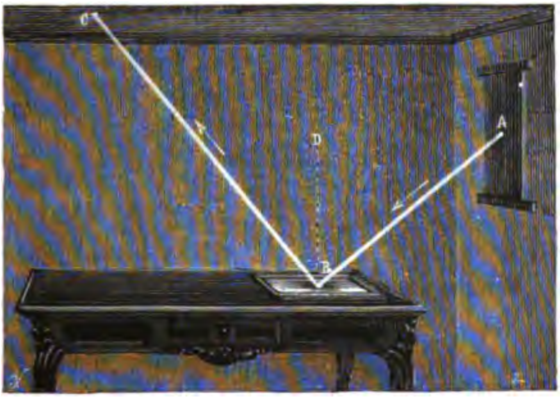

Fig. 623.—Reflection of Light.

695. Laws of Reflection.—The reflection of light from polished surfaces takes place according to the following laws:—

1. The reflected ray lies in the plane of incidence.

2. The angle of reflection is equal to the angle of incidence.

These laws may be verified by means of the apparatus represented in Fig. 624. A vertical divided circle has a small polished plate fixed at its centre, at right angles to its plane, and two tubes travelling on its circumference with their axes always directed towards the centre. The zero of the divisions is the highest point of the circle, the plate being horizontal.

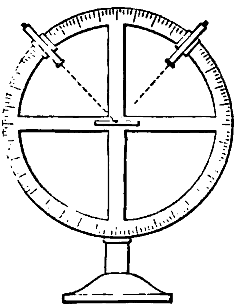
Fig. 624.—Verification of Laws of Reflection.

A source of light, such as the flame of a candle, is placed so that its rays shine through one of the tubes upon the plate at the centre. As the tubes are blackened internally, no light passes through except in a direction almost precisely parallel to the axis of the tube. The observer then looks through the other tube, and moves it along the circumference till he finds the position in which the reflected light is visible through it. On examining the graduations, it will be found that the two tubes are at the same distance from the zero point, on opposite sides. Hence the angles of incidence and reflection are equal. Moreover the plane of the circle is the plane of incidence, and this also contains the reflected rays. Both the laws are thus verified.

696. Artificial Horizon.—These laws furnish the basis of a method of observation which is frequently employed for determining the altitude of a star, and which, by the consistency of its results, furnishes a very rigorous proof of the laws.

A vertical divided circle (Fig. 625) is set in a vertical plane by proper adjustments. A telescope movable about the axis of the circle is pointed to a particular star, so that its line of collimation I′S′ passes through the apparent place of the star. Another telescope,[1] similarly mounted on the other side of the circle, is directed downwards along the line I′R towards the image of the star as seen in a trough of mercury I. Assuming the truth of the laws of reflection as above stated, the altitude of the star is half the angle between the directions of the two telescopes; for the ray S I from the star to the mercury is parallel to the line S′I′, by reason of the excessively great distance of the star; and since the rays S I, I R are equally inclined to the normal I N, which is a vertical line, the lines I′S′, I′R are also equally inclined to the vertical, or, what is the same thing, are equally inclined to a horizontal plane. A reflecting surface of mercury thus used is called a mercury horizon, or an *artificial horizon*. Observations thus made give even more accurate results than those in which the natural horizon presented by the sea is made the standard of reference.

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[1] In practice, a single telescope usually serves for both observations.

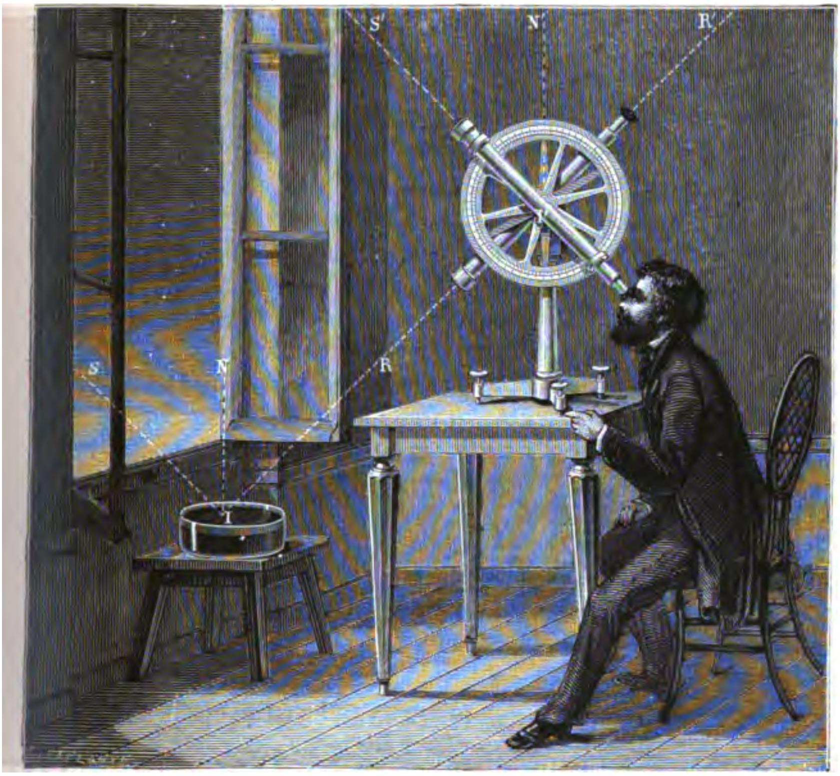

Fig. 625.—Artificial Horizon.

697. Irregular Reflection.—The reflection which we have thus far been discussing is called *regular reflection*. It is more marked as the reflecting surface is more highly polished, and (except in the case of metals) as the incidence is more oblique. But there is another kind of reflection, in virtue of which bodies, when illuminated, send out light in all directions, and thus become visible. This is called *irregular reflection* or *diffusion*. Regular reflection does not render the reflecting body visible, but exhibits images of surrounding objects. A perfectly reflecting mirror would be itself unseen, and actual mirrors are only visible in virtue of the small quantity of diffused light which they usually emit. The transformation of incident into diffused light is usually selective; so that, though the incident beam may be white, the diffused light is usually coloured. The power which a body possesses of making such selection constitutes its colour.

The word *reflection* is often used by itself to denote what we have here called *regular reflection,* and we shall generally so employ it.

698. Mirrors.—The mirrors of the ancients were of metal, usually of the compound now known as speculum-metal. Looking-*glasses* date from the twelfth century. They are plates of glass, coated at the back with an amalgam of quicksilver and tin, which forms the reflecting surface. This arrangement has the great advantage of excluding the air, and thus preventing oxidation. It is attended, however, with the disadvantage that the surface of the glass and the surface of the amalgam form two mirrors; and the superposition of the two sets of images produces a confusion which would be intolerable in delicate optical arrangements. The mirrors, or *specula* as they are called, of reflecting telescopes are usually made of *speculum-metal,* which is a bronze composed of about 32 parts of copper to 15 of tin. Lead, antimony, and arsenic are sometimes added. Of late years specula of glass coated in *front* with real silver have been extensively used; they are known as *silvered specula.* A coating of platinum has also been tried, but not with much success. The mirrors employed in optics are usually either *plane* or *spherical.*

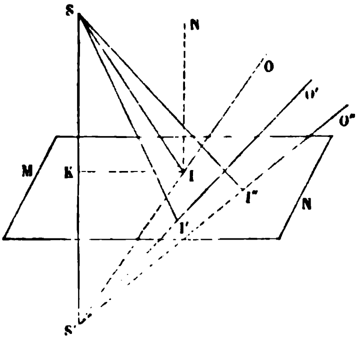

Fig. 626.—Plane Mirror.

699. Plane Mirrors.—By a plane mirror we mean any plane reflecting surface. Its effect, as is well-known, is to produce, behind the mirror, images exactly similar, both in form and size, to the real objects in front of it. This phenomenon is easily explained by the laws of reflection.

Let M N (Fig. 626) be a plane mirror, and S a luminous point. Rays S I, S I′, S I″ proceeding from this point give rise to reflected rays I O, I′ O′, I″ O″; and each of these, if produced backwards, will meet the normal S K in a point S′, which is at the same distance behind the mirror that S is in front of it.[1] The reflected rays have therefore the same directions as if they had come from S′, and the eye receives the same impression as if S′ were a luminous point.

Fig. 627 represents a pencil of rays emitted by the highest point of a candle-flame, and reflected from a plane mirror to the eye of an observer. The reflected rays are divergent (like the incident rays), and if produced backwards would meet in a point, which is the position of the image of the top of the flame.

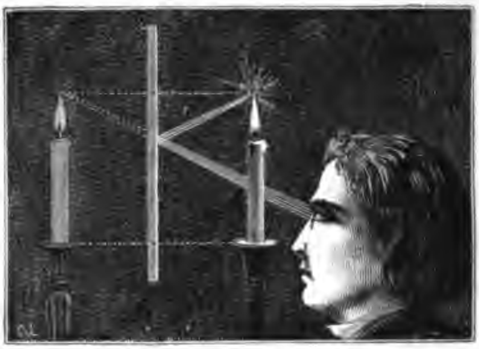

Fig. 627.—Image of Candle.

As an object is made up of points, these principles show that the image of an object formed by a plane mirror must be equal to the object, and symmetrically situated with respect to the plane of the mirror. For example, if A B (Fig. 628) is an object in front of the mirror, an eye placed at O will see the image of the point A at A′, the image of B at B′, and so on for all the other points of the object. The position of the image A′B′ depends only on the positions of the object and of the mirror, and remains stationary as the eye is moved about. It is possible, however, to find positions from which the eye will not see the image at all, the conditions of visibility being the same as if the image were a real object, and the mirror were an opening through which it could be seen.

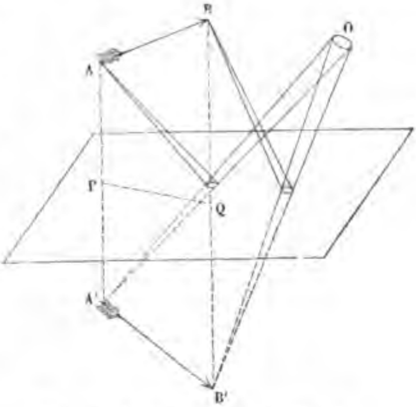

Fig. 628.—Incident and Reflected Pencils.

The images formed by a plane mirror are *erect.* They are not however exact duplicates of the objects from which they are formed, but differ from them precisely in the same way as the left foot or hand differs from the right. The image of a printed page is like the appearance of the page as seen through the paper from the back, or like the type from which the page was printed.

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[1] This is evident from the comparison of the two triangles S K I, S′ K I, bearing in mind that the angle N I S is equal to the alternate angle I S K, and N I O to K S′ I.

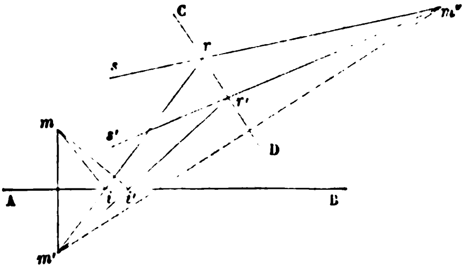

Fig. 629.—Reflection from two Mirrors.

700. Images of Images.—When rays from a luminous point *m* have been reflected from a mirror A B (Fig. 629), their subsequent course is the same as if they had come from the image *m′* at the back of the mirror. Hence, if they fall upon a second mirror C D, an image *m″* of the first image will be formed at the back of the second mirror. If, after this, they undergo a third reflection, an image of *m″* will be formed, and so on indefinitely. The figure shows the actual paths of two rays *m i r s*, *m i′ r′ s′*. They diverge first from *m*, then from *m′*, and lastly from *m″*. This is the principle of the multiple images formed by two or more mirrors, as in the following experiments.

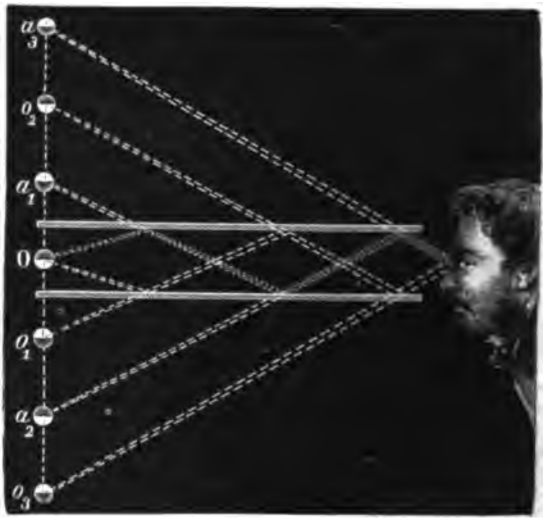

Fig. 630.—Parallel Mirrors.

701. Parallel Mirrors.—Let an object O be placed between two parallel mirrors which face each other, as in Fig. 630. The first reflections will form images $a_1\, o_1$. The second reflections will form images $a_2\, o_2$ of the first images; and the third reflections will form images $a_3\, o_3$ of the second images. The figure represents an eye receiving *the rays which form the third images*, and shows the paths which *these rays* have taken in their whole course from the object O to the eye. The rays by which the same eye sees the other images are omitted, to avoid confusing the figure. A long row of images can thus be seen at once, becoming more and more dim as they recede in the distance, inasmuch as each reflection involves a loss of light.

If the mirrors are truly parallel, all the images will be ranged in one straight line, which will be normal to the mirrors. If the mirrors are inclined at any angle, the images will be ranged on the circumference of a circle, whose centre is on the line in which the reflecting surfaces would intersect if produced. This principle is sometimes employed as a means of adjusting mirrors to exact parallelism.

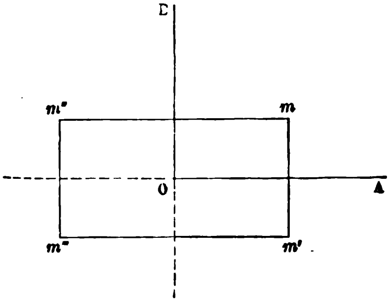

Fig. 631.—Mirrors at Right Angles.

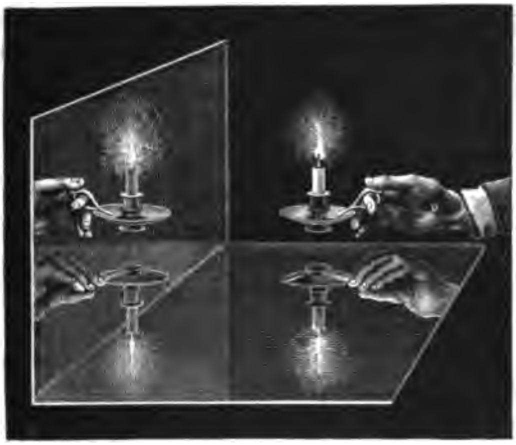

Fig. 632.—Mirrors at Right Angles.

702. Mirrors at Right Angles.—Let two mirrors O A, O B (Fig. 631), be set at right angles to each other, facing inwards, and let m be a luminous point placed between them. Images m' m'' will be formed by first reflections, and two coincident images will be formed at m''' by second reflections. No third reflection will occur, for the point m''', being behind the planes of both the mirrors, cannot be reflected in either of them. Counting the two coincident images as one, and also counting the object as one, there will be in all four images, placed at the four corners of a rectangle. Fig. 632 will give an idea of the appearance actually presented when one of the mirrors is vertical and the other horizontal. When both the mirrors are vertical, an observer sees his own image constantly bisected by their common section, in a way which appears at first sight very paradoxical.

703. Mirrors Inclined at 60 Degrees.—A symmetrical distribution of images may be obtained by placing a pair of mirrors at any angle which is an aliquot part of 360°. If, for example, they be inclined at 60° to each other, the number of images, counting the object itself as one, will be six. Their position is illustrated by Fig. 633. The object is placed in the sector A C B. The images formed by first reflections are situated in the two neighbouring sectors B C A′, A C B; the images formed by second reflections are in the sectors B′ C A″, A′ C B′, and these yield, by third reflections, two coincident images in the sector B″ C A″, which is vertically opposite to the sector A C B in which the object lies, and is therefore behind the planes of both mirrors, so that no further reflection can occur.

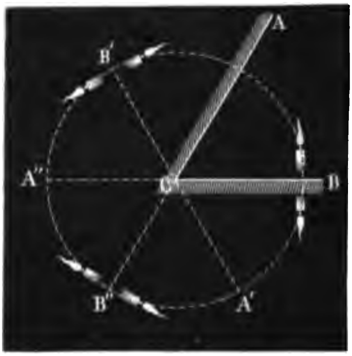

Fig. 633.—Images in Kaleidoscope.

704. Kaleidoscope.—The symmetrical distribution of images, obtained by two mirrors inclined at an angle which is an aliquot part of four right angles, is the principle of the *kaleidoscope*, an optical toy invented by Sir David Brewster. It consists of a tube containing two glass plates, extending along its whole length, and inclined at an angle of 60°. One end of the tube is closed by a metal plate, with the exception of a hole in the centre, through which the observer looks in; at the other end there are two plates, one of ground and the other of clear glass (the latter being next the eye), with a number of little pieces of coloured glass lying loosely between them. These coloured objects, together with their images in the mirrors, form symmetrical patterns of great beauty, which can be varied by turning or shaking the tube, so as to cause the pieces of glass to change their positions.

A third reflecting plate is sometimes employed, the cross-section of the three forming an equilateral triangle. As each pair of plates produces a kaleidoscopic pattern, the arrangement is nearly equivalent to a combination of three kaleidoscopes.

The kaleidoscope is capable of rendering important aid to designers.

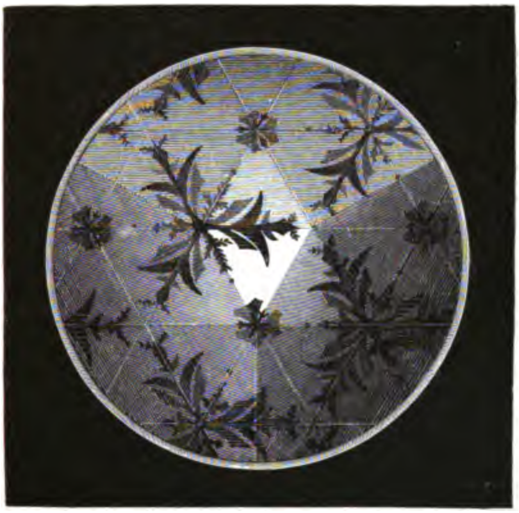

Fig 634.—Kaleidoscopic Pattern.

Fig. 634 represents a pattern produced by the equilateral arrangement of three reflectors just described.

705. Pepper's Ghost.—Many ingenious illusions have been contrived, depending on the laws of reflection from plane surfaces. We shall mention two of the most modern.

In the *magic cabinet*, there are two vertical mirrors hinged at the two back corners of the cabinet, and meeting each other at a right angle, so as to make angles of 45° with the sides, and also with the back. A spectator seeing the images of the two sides, mistakes them for the back, which they precisely resemble; and performers may be concealed behind the mirrors when the cabinet appears empty. If one of the persons thus concealed raises his head above the mirrors, it will appear to be suspended in mid-air without a body.

The striking spectral illusion known as *Pepper's Ghost* is produced by reflection from a large sheet of unsilvered glass, which is so arranged that the actors on the stage are seen through it, while other actors, placed in strong illumination, and out of the direct view of the spectators, are seen by reflection in it, and appear as ghosts on the stage.

705 A. Deviation produced by Rotation of Mirror.—Let A B (Fig. 634 A) represent a mirror perpendicular to the plane of the paper, and capable of being rotated about an axis through C, also perpendicular to the paper; and let I C represent an incident ray. When the mirror is in the position A B, perpendicular to I C, the ray will be reflected directly back upon its course; but when the mirror is turned through the acute angle A C A′, the reflected ray will take the direction C R, making with the normal C N an angle N C R, equal to the angle of incidence N C I. The deviation I C R of the reflected ray, produced by rotating the mirror, is therefore double of the angle I C N or A C A′, through which the mirror has been turned; and if, starting from the position A′ B′, we turn the mirror through a further angle θ, the reflected ray C R will be turned through a further angle 2θ. It thus appears, that, *when a plane mirror is rotated in the plane of incidence, the direction of the reflected ray is changed by double the angle through which the mirror is turned.* Conversely, if we assign a constant direction C I to the reflected ray, the direction of the incident ray R C must vary by double the angle through which the mirror is turned.

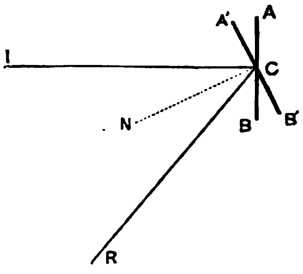

Fig. 634 A.—Effect of rotating a Mirror.

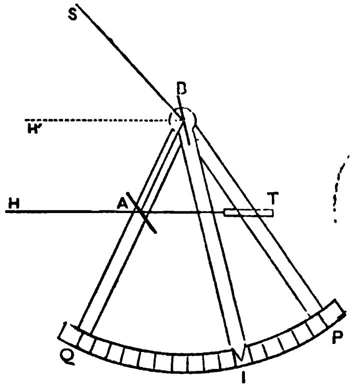

Fig. 634 B.—Sextant.

705 B. Hadley's Sextant.—The above principle is illustrated in the nautical instrument called the *sextant* or *quadrant*, which was invented by Newton, and reinvented by Hadley. It serves for measuring the angle between any two distant objects as seen from the station occupied by the observer. Its essential parts are represented in Fig. 634 B.

It has two plane mirrors A, B, one of which, A, is fixed to the frame of the instrument, and is only partially silvered, so that a distant object in the direction A H can be seen through the unsilvered part. The other mirror B is mounted on a movable arm B I, which carries an index I, traversing a graduated arc P Q. When the two mirrors are parallel, the index is at P, the zero of the graduations, and a ray H′ B incident on B parallel to H A, will be reflected first along B A, and then along A T, the continuation of H A. The observer looking through the telescope T thus sees, by two reflections, the same objects which he also sees directly through the unsilvered part of the mirror. Now let the index be advanced through an angle θ; then, by the principles of last section, the incident ray S B makes with H′ B, or H A, an angle 2θ. The angle between S B and H A would therefore be given by reading off the angle through which the index has been advanced, and doubling; but in practice the arc P Q is always graduated on the principle of marking half degrees as whole ones, so that the reading at I is the required angle 2θ. In using the instrument, the two objects which are to be observed are brought into apparent coincidence, one of them being seen directly, and the other by successive reflection from the two mirrors. This coincidence is not disturbed by the motion of the ship; but unpractised observers often find a difficulty in keeping both objects in the field of view. Dark glasses, not shown in the figure, are provided for protecting the eye in observations of the sun, and a vernier and reading microscope are provided instead of the pointer I.

706. Spherical Mirrors.—By a spherical mirror is meant a mirror whose reflecting surface is a portion (usually a very small portion) of the surface of a sphere. It is concave or convex according as the inside or outside of the spherical surface yields the reflection. The centre of the sphere (C, Fig. 635) is called the *centre of curvature* of

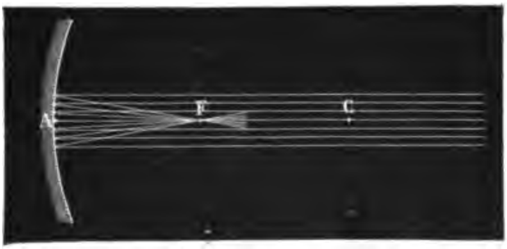

Fig. 635.—Principal Focus.

the mirror. If the mirror has a circular boundary, as is usually the case, the central point A of the reflecting surface may conveniently be called the *pole* of the mirror. *Centre of the mirror* is an ambiguous phrase, being employed sometimes to denote the pole, and sometimes the centre of curvature. The line A C is called the *principal axis* of the mirror, and any other straight line through C which meets the mirror is called a *secondary axis*.

When the incident rays are parallel to the principal axis, the reflected rays converge to a point F, which is called the *principal focus*. This law is rigorously true for parabolic mirrors (generated by the revolution of a parabola about its principal axis). For spherical mirrors it is only approximately true, but the approximation is very close if the mirror is only a very small portion of an entire sphere. In grinding and polishing the specula of large reflecting telescopes, the attempt is made to give them, as nearly as possible, the parabolic form. Parabolic mirrors are also frequently employed to reflect, in a definite direction, the rays of a lamp placed at the focus.

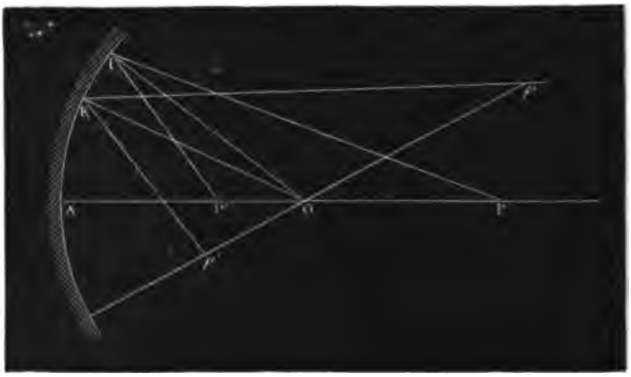

Fig. 636.—Theory of Conjugate Foci.

Rays reflected from the circumferential portion of a spherical mirror are always too convergent to concur exactly with those reflected from the central portion. This deviation from exact concurrence is called *spherical aberration*.

707. Conjugate Foci.—Let P (Fig. 636) be a luminous point situated on the principal axis of a spherical mirror, and let P I be one of the rays which it sends to the mirror. Draw the normal O I, which is simply a radius of the sphere. Then O I P is the angle of incidence, and the angle of reflection O I P′ must be equal to it; hence O I bisects an angle of the triangle P I P′, and therefore we have

$$\frac{\mathrm{IP}}{\mathrm{IP'}} = \frac{\mathrm{OP}}{\mathrm{OP'}}.$$

Let p, p' denote A P, A P′ respectively, and let r denote the radius of the sphere. Then, if the angular aperture of the mirror is small, I P is sensibly equal to p, and I P′ to p'. Substituting these approximate values, the preceding equation becomes

$$\frac{p}{p'} = \frac{p-r}{r-p'}; \text{ whence } pr + p'r = 2pp';$$

or, dividing by $pp'r$,

$$\frac{1}{p} + \frac{1}{p'} = \frac{2}{r}. \qquad (a)$$

This formula determines the position of the point P′, in which the reflected ray cuts the principal axis, and shows that it is, to the accuracy of our approximation, independent of the position of the point I; that is to say, all the rays which P sends to the mirror are reflected to the same point P′. We have assumed P to be on the principal axis. If we had taken it on a secondary axis, as at p (Fig. 636), we should have found, by the same process of reasoning, that the reflected rays would all meet in a point p' on that secondary axis. The distinction between primary and secondary axes, in the case of a spherical mirror, is in fact merely a matter of convenience, not representing any essential difference of property. Hence we can lay down the following general proposition as true within limits of error corresponding to the approximate equalities which we have above assumed as exact:—

Rays proceeding from any given point in front of a concave spherical mirror, are reflected so as to meet in another point; and the line joining the two points passes through the centre of the sphere.

It is evident that rays proceeding from the second point to the mirror would be reflected to the first. The relation between them is therefore mutual, and they are hence called *conjugate foci.* By a *focus* in general is meant a point in which a number of rays meet (or would meet if produced); and the rays which thus meet, taken collectively, are called a *pencil.* Fig. 637 represents two pencils of rays whose foci S *s* are conjugate, so that, if either of them be regarded as an incident pencil, the other will be the corresponding reflected pencil.

We can now explain the formation of images by concave mirrors. Each point of the object sends a pencil of rays to the mirror, which converge, after reflection, to the conjugate focus. If the eye of the observer be placed beyond this point of concourse, and in the path of the rays, they will present to him the same appearance as if they had come from this point as origin. The image is thus composed of points which are the conjugate foci of the several points of the object.

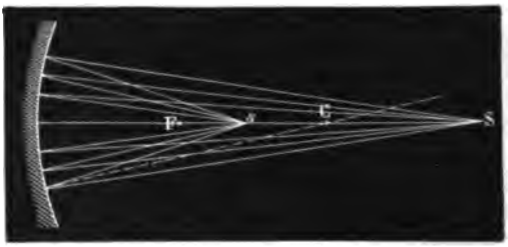

Fig. 637.—Conjugate Foci.

708. Principal Focus.—If, in formula (*a*) of last section, we make p increase continually, the term $\frac{1}{p}$ will continually decrease, and will vanish as p becomes infinite. This is the case of rays parallel to the principal axis, for parallel rays may be regarded as coming from a point at infinite distance. The formula then becomes

$$\frac{1}{p'} = \frac{2}{r}; \text{ whence } p' = \frac{r}{2};$$

that is to say, *the principal focal distance is half the radius of curvature.* This distance is often called the *focal length* of the mirror. If we denote it by f, the general formula becomes

$$\frac{1}{p} + \frac{1}{p'} = \frac{1}{f}. \qquad (b)$$

709. Discussion of the Formula.—By the aid of this formula we can easily trace the corresponding movements of conjugate foci.

If p is positive and very large, p' is a very little greater than f: that is to say, the conjugate focus is a very little beyond the principal focus.

As p diminishes, p' increases, until they become equal, in which case each of them is equal to r or $2f$; that is to say, the conjugate foci move towards each other till they coincide at the centre of curvature. This last result is obvious in itself; for rays from the centre of curvature are normal to the mirror, and are therefore reflected directly back.

As p continues to diminish, the two foci, as it were, change places; the luminous point advancing from the centre of curvature to the principal focus, while the conjugate focus moves away from the centre of curvature to infinity.

As the luminous point continues to approach the mirror, $\frac{1}{p}$ is greater than $\frac{1}{f}$, and hence $\frac{1}{p'}$, and therefore also p', must be negative. The physical interpretation of this result is that the conjugate focus is *behind* the mirror, as at *s* (Fig. 638), and that the reflected

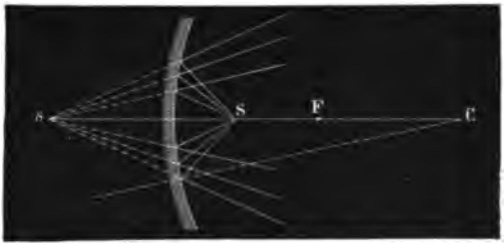

Fig. 638.—Virtual Focus.

rays diverge as if they had come from this point. Such a focus is called *virtual*, while a focus in which rays actually meet is called *real*. As the luminous point moves up from F to the mirror, the conjugate focus moves up from an infinite distance at the back, and meets it at the surface of the mirror.

If S is a real luminous point sending rays to the mirror, it must of necessity lie in front of the mirror, and p therefore cannot be negative; but when we are considering images of images this restriction no longer holds. If an incident beam, for example, converges towards a point *s* at the back of the mirror, it will be reflected to a point S in front. In this case p is negative, and p' positive. The conjugate foci S *s* have in fact changed places.

It appears from the above investigation that there are two principal cases, as regards the positions of conjugate foci of a concave mirror.

1. One focus between F and C; and the other beyond C.
2. One focus between F and the mirror; and the other behind the mirror.

In the former case, the foci move to meet each other at C; in the atter, they move to meet each other at the surface of the mirror.

710. Formation of Images.—We are now in a position to discuss the formation of images by concave mirrors. Let A B (Fig. 639) be an object placed in front of a concave mirror, at a distance greater than its radius of curvature. All the rays which diverge from A will be reflected to the conjugate focus *a*. Hence this point can be found by the following construction. Draw through A the ray A A′ parallel to the principal axis, and draw its path after reflection, which must of necessity pass through the principal focus. The intersection of this reflected ray with the secondary axis through A will be the point required. A similar construction will give the conjugate focus corresponding to any other point of the object; *b*, for example,[1] is the focus conjugate to B. Points of the object lying between A and B will have their conjugate foci between *a* and *b*. An eye placed behind the object A B will accordingly receive the same impression from the reflected rays as if the image *a b* were a real object.

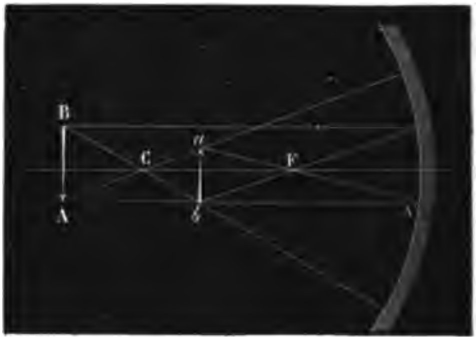

Fig. 639.—Formation of Image.

711. Size of Image.—As regards the comparative sizes of object and image, it is obvious, from similar triangles, that their linear dimensions are directly as their distances from C the centre of curvature.

Again, since C F and A A′ are parallel, we have

$$\frac{a\mathrm{F}}{\mathrm{FA'}} = \frac{a\mathrm{C}}{\mathrm{CA}} = \frac{ab}{\mathrm{AB}},$$

or

$$\frac{\text{length of image}}{\text{length of object}} = \frac{\text{distance of image from principal focus}}{\text{focal length}}; \qquad (c)$$

and by a similar construction we can prove that

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[1] It is only by accident that *b* happens to lie on A A′ in the figure.

$$\frac{\text{length of object}}{\text{length of image}} = \frac{\text{distance of object from principal focus}}{\text{focal length}}. \qquad (d)$$

This last formula affords the readiest means of calculating the size of the image when the size and position of the object are given. Both the formulæ (*c*) and (*d*) are perfectly general, both for concave and convex mirrors. They show that the object and image will be equal when they coincide at the centre of curvature, and that as they move away from this point, in opposite directions, that which moves away from the mirror continually gains in size upon the other.

Since the lines joining corresponding points of object and image cross at the point C, which lies between them when the image is real, a real image formed by a concave mirror is always inverted.

712. Experiment of the Phantom Bouquet.—Let a box open on one side be placed in front of a concave mirror, at a distance about equal to its radius of curvature, and let an inverted bouquet be suspended within it, the open side of the box being next the mirror. By giving a proper inclination to the mirror, an image of the bouquet will be obtained in mid-air, just above the top of the box. As the bouquet

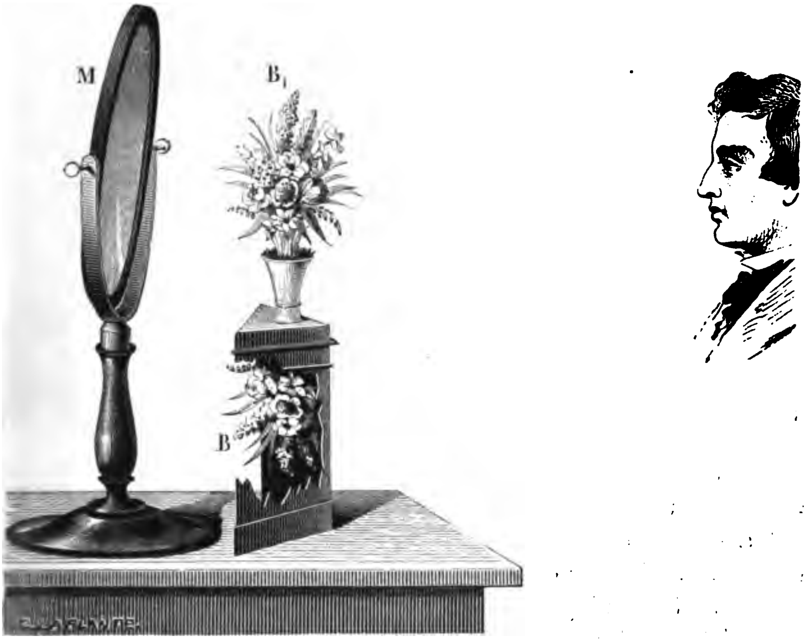

Fig. 640.—Experiment of Phantom Bouquet.

is inverted, its image is erect, and a real vase may be placed in such a position that the phantom bouquet shall appear to be standing in it. The spectator must be full in front of the mirror, and at a sufficient distance for all parts of the image to lie between his eyes and the mirror. When the colours of the bouquet are bright, the image is generally bright enough to render the illusion very complete.

713. Images on a Screen.—Such experiments as that just described can only be seen by a few persons at once, since they require the spectator to be in a line with the image and the mirror. When an image is projected on a screen, it can be seen by a whole audience at once, if the room be darkened and the image be large and bright. Let a lighted candle, for example, be placed in front of a concave mirror, at a distance exceeding the focal length, and let a screen be placed at the conjugate focus; an inverted image of the candle will be depicted on the screen. Fig. 641 represents the case in which the candle is at a distance less than the radius of curvature, and the image is accordingly magnified.

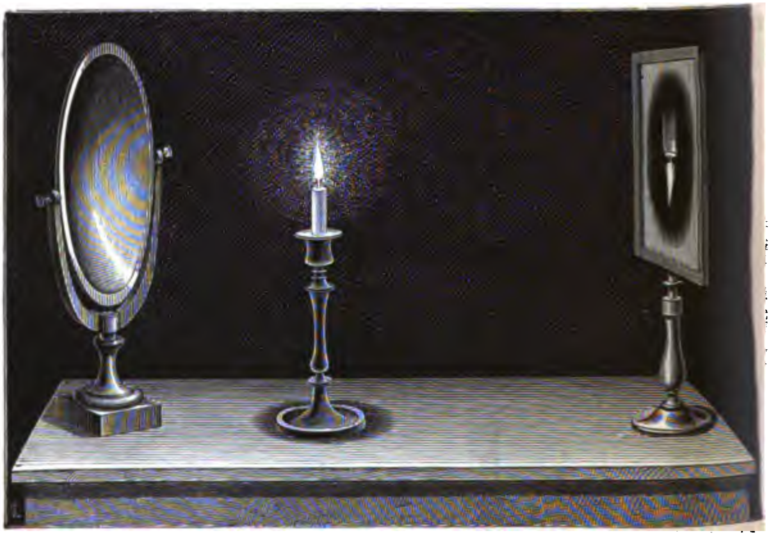

Fig. 641.—Image on Screen.

By this mode of operating, the formula for conjugate focal distances can be experimentally verified with considerable rigour, care being taken, in each experiment, to place the screen in the position which gives the most sharply defined image.

714. Difference between Image on Screen, and Image as seen in Mid-air. Caustics.—For the sake of simplicity we have made some statements regarding visible images which are not quite accurate; and we must now indicate the necessary corrections.

Images thrown on a screen have a determinate position, and are really the loci of the conjugate foci of the points of the object; but this is not rigorously true of images seen directly. They change their position to some extent, according to the position of the observer.

The actual state of things is explained by Fig. 641 A. The plane of the figure[1] is a principal plane (that is, a plane containing the principal axis) of a concave hemispherical mirror, and the incident rays are parallel to the principal axis. All the rays reflected in the plane of the figure touch a certain curve called a *caustic curve*, which has a cusp at F, the principal focus; and the direction in which the image is seen by an eye situated in the plane of the figure is determined by drawing from the eye a tangent to this caustic. If the eye be at E, on the principal axis, the point of contact will be F; but when the rays are received obliquely, as at E′, it will be at a point *a* not lying in the direction of F. For an eye thus situated, *a* is called the *primary focus*, and the point where the tangent at *a* cuts the principal axis is called the *secondary focus*. When the eye is moved in the plane of the diagram, the apparent position of the

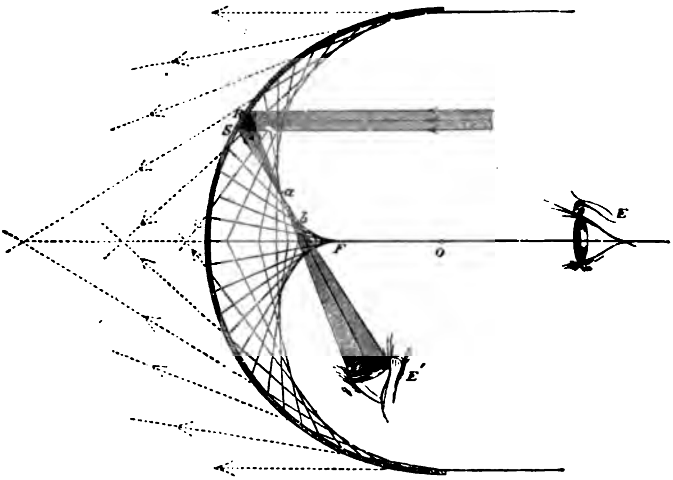

Fig. 641 A.—Position of Image in Oblique Reflection.

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[1] Figs. 641 A and 657 A are borrowed, by permission, from Mr. Osmund Airy's *Geometrical Optics*.

image (as determined by its remaining in coincidence with a cross of threads or other mark) is the primary focus; and when the eye is moved perpendicular to the plane of the diagram, the apparent position of the image is the secondary focus.[1] If we suppose the diagram to rotate about the principal axis, it will still remain true in all positions, and the surface generated by this revolution of the caustic curve is the *caustic surface.* Its form and position vary with the position of the point from which the incident rays proceed; and it has a cusp at the focus conjugate to this point.

There is always more or less blurring, in the case of images seen obliquely (except in plane mirrors), by reason of the fact that the point of contact with the caustic surface is not the same for rays entering different parts of the pupil of the eye.

A caustic curve can be exhibited experimentally by allowing the rays of the sun or of a lamp to fall on the concave surface of a strip of polished metal bent into the form of a circular arc, as in Fig. 642, the reflected light being received on a sheet of white paper on which the strip rests. The same effect may often be observed on the surface of a cup of tea, the reflector in this case being the inside of the tea-cup.

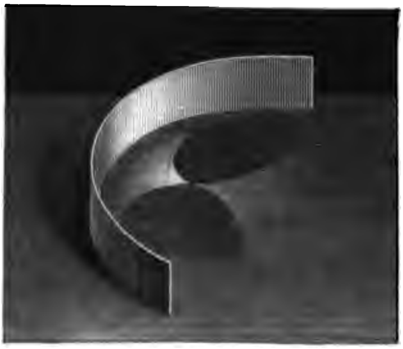

Fig. 642.—Caustic by Reflection.

The image of a luminous point received upon a screen is formed by all the rays which touch the corresponding caustic surface. The brightest and most distinct image will be formed at the cusp, which is, in fact, the conjugate focus; but there will be a border of fainter light surrounding it. This source of indistinctness in images is an example of *spherical aberration* (§ 707).

714 A. Image on a Screen by Oblique Reflection.—If we attempt to throw upon a screen the image of a luminous point by means of a concave mirror very oblique to the incident rays, we shall find that no image can be obtained at all resembling a point; but that there are two positions of the screen in which the image becomes a line

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[1] Since every ray incident parallel to the principal axis, is reflected through the principal axis. If the incident rays diverged from a point on the principal axis, they would still be reflected through the principal axis.

In the annexed figure (Fig. 641 B), which represents on a larger scale a portion of Fig. 641 A, *a c*, *b d* are rays from the highest and lowest points of the portion R S of the hemispherical mirror, which portion we suppose to be small in both its dimensions in comparison with the radius of curvature; and we may suppose the rest of the hemisphere to be removed, so that R S will represent a small concave mirror receiving a pencil very obliquely.

Then, if a screen be held perpendicular to the plane of the diagram, at *m*, where the section of the pencil by the plane of the diagram is narrowest, a blurred line of light will be formed upon it, the length of the line being perpendicular to the plane of the diagram. This is called the *primary focal line.*

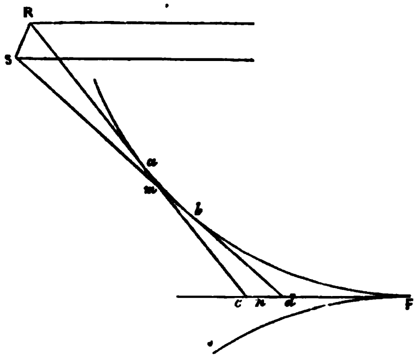

Fig. 641 B.—Formation of Focal Lines.

The *secondary focal line* is *c d*, which, if produced, passes through the centre of curvature of the mirror, and also through the point from which the incident light proceeds. This line is very sharply formed upon a screen held so as to coincide with *c d* and to be perpendicular to the plane of the diagram. Its edges are much better defined than those of the primary line; and its position in space is also more definite. If the mirror is used as a burning-glass to collect the sun's rays, ignition will be more easily obtained at one of these lines than in any intermediate position.[1]

Focal lines can also be seen directly. In this case a small element of the mirror sends all its reflected rays to the eye, the rays from opposite sides of the element crossing each other at the focal lines, before they reach the eye. It is possible, in certain positions of the eye, to see either focal line at pleasure, by altering the focal adjustment of the eye; or the two may be seen with imperfect definition crossing each other at right angles. The experiment is easily made by employing a gas flame, turned very low, as the source of light. One line is in the plane of incidence, and the other is normal to this plane.

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[1] The "elongated figure of 8" which is often mentioned in connection with the secondary focal line, is obtained by turning the screen about *n* the middle point of *c d*, so as to blur both ends of the image by bad focussing. It will be observed, from an inspection of the diagram, that *c d* is very oblique to the reflected rays.

If we neglect the blurring of the primary line, we may describe the part of the pencil lying between the two lines as a tetrahedron, of which the two lines are opposite edges.

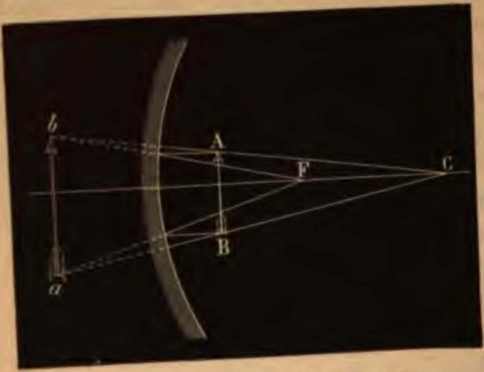

Fig. 643.—Formation of Virtual Image.

715. Virtual Image in Concave Mirror.—Let an object be placed, as in Fig. 643, in front of a concave mirror, at a distance less than that of the principal focus. The rays incident on the mirror from any point of it, as A, will be reflected as a divergent pencil, the focus from which they diverge being a point *b* at the back of the mirror. To find this point, we may trace the course of a ray through A parallel to the principal axis. Such a ray will be reflected to the principal focus F, and by producing this reflected ray backwards till it meets the secondary axis CA, the point *b*, which is the conjugate focus of A, is determined. We can find in the same way the position of *a*, the conjugate focus of B, and it is obvious that the image of AB will be erect and magnified.

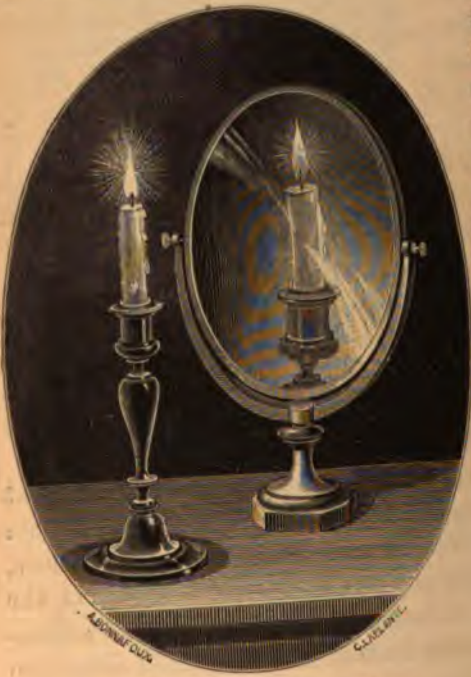

Fig. 644.—Virtual Image in Concave Mirror.

716. Remarks on Virtual Images.—A virtual image cannot be projected on a screen; for the rays which produce it do not actually pass through its place, but only seem to do so. A screen placed at *a b* would obviously receive none of the reflected light whatever.

case of a convex or a concave mirror, are directly proportional to their distances from the centre of curvature. The image is inverted or erect according as this centre does or does not lie between the object and its image. In the case of a convex mirror the centre never lies between them (if the object be real), and therefore the image is always erect.

Convex mirrors are very seldom employed in optical instruments.

The silvered globes which are frequently used as ornaments, are examples of convex mirrors, and present to the observer at one view an image of nearly the whole surrounding landscape. As the part of the mirror in which he sees this image is nearly an entire hemisphere, the deformation of the image is very notable, straight lines being reflected as curves.

718. Anamorphosis.—Much greater deformations are produced by cylindric mirrors. A cylindric mirror, when the axis of the cylinder is vertical, behaves like a plane mirror as regards the angular magnitude under which the height of the image is seen, and like a spherical mirror as regards the breadth of the image. If it be a convex cylinder, it causes bodies to appear unduly contracted horizontally in proportion to their heights. Distorted pictures are sometimes drawn upon paper, according to such a system that when they are seen

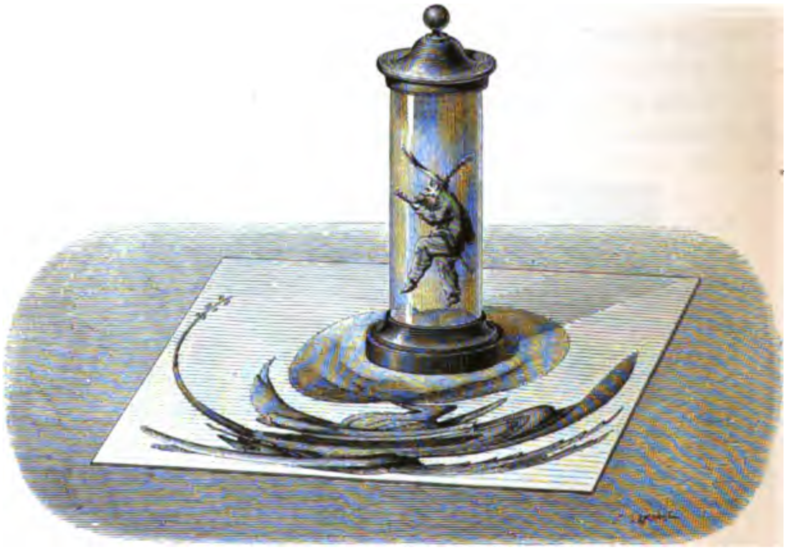

Fig. 646.—Anamorphosis.

reflected in a cylindric mirror properly placed, as in Fig. 646, the distortion is corrected, and while the picture appears a mass of confusion, the image is instantly recognized. This restoration of true proportion in a picture is called *anamorphosis*.

719. Medical Applications.—Concave mirrors are frequently used to concentrate light upon an object for the purpose of rendering it more distinctly visible.

The *ophthalmoscope* is a small concave mirror, with a small hole in its centre, through which the observer looks from behind, while he directs a beam of reflected light from a lamp into the pupil of the patient's eye. In this way (with the help sometimes of a lens) the retina can be rendered visible, and can be minutely examined.

The *laryngoscope* consists of two mirrors. One is a small plane mirror, with a handle attached, at an angle of about 45° to its plane. This small mirror is held at the back of the patient's mouth, so that the observer, looking into it, is able by reflection to see down the patient's throat, the necessary illumination being supplied by a concave mirror, strapped to the observer's forehead, by means of which the light from a lamp is reflected upon the plane mirror, which again reflects it down the throat.

CHAPTER LIX.

REFRACTION.

720. Refraction.—When a ray of light passes from one transparent medium to another, it undergoes a change of direction at the surface of separation, so that its course in the second medium makes an angle with its course in the first. This changing of direction is called *refraction.*

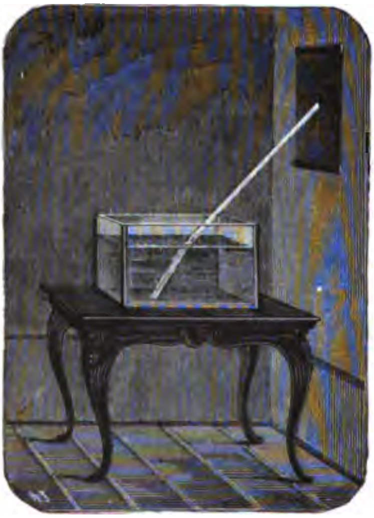

Fig. 647.—Refraction.

The phenomenon can be exhibited by admitting a beam of the sun's rays into a dark room, and receiving it on the surface of water contained in a rectangular glass vessel. The path of the beam will be easily traced by its illumination of the small solid particles which lie in its course.

The following experiment is a well-known illustration of refraction:—A coin *m n* (Fig. 648) is laid at the bottom of a vessel with opaque sides, and a spectator places himself so that the coin is just hidden from him by the side of the vessel; that is to say, so that the line *m* A in the figure passes just above his eye. Let water now be poured into the vessel, care being taken not to displace the coin. The bottom of the vessel will appear to rise, and the coin will come into sight. Hence a pencil of rays from *m* must have entered the spectator's eye. The pencil in fact undergoes a sudden bend at the surface of the water, and thus reaches the eye by a crooked course, in which the obstacle A is evaded. If the part of the pencil in air be produced backwards, its rays will approximately meet in a point *m′*, which is therefore the image of *m*. Its position is not correctly indicated in the figure, being placed too much to the left (§ 727 A).

The broken appearance presented by a stick (Fig. 649) when partly immersed in water in an oblique position, is similarly explained, the part beneath the water being lifted up by refraction.

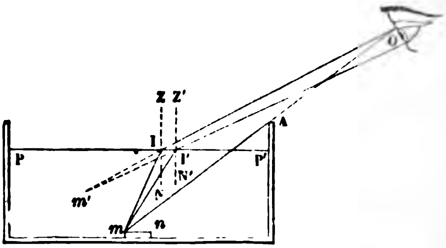

Fig. 648.—Experiment of Coin in Basin.

721. Refractive Powers of Different Media.—In the experiments of the coin and stick, the rays, in leaving the water, are bent away from the normals Z I N, Z′ I′ N′ at the points of emergence; in the experiment first described (Fig. 647), on the other hand, the rays, in passing from air into water, are bent nearer to the normal. In every case the path which the rays pursue in going is the same as they would pursue in returning; and of the two media concerned, that in which the ray makes the smaller angle with the normal is said to have greater refractive power than the other, or to be more highly refracting.

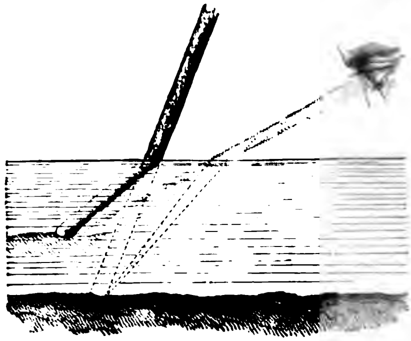

Fig. 649.—Appearance of Stick in Water.

Liquids have greater refractive power than gases, and as a general rule (subject to some exceptions in the comparison of dissimilar substances) the denser of two substances has the greater refracting power. Hence it has become customary, in enunciating some of the laws of optics, to speak of the *denser* medium and the *rarer* medium, when the more correct designations would be *more refractive* and *less refractive*.

722. Laws of Refraction.—The quantitative law of refraction was not discovered till quite modern times. It was first stated by Snell, a Dutch philosopher, and was made more generally known by Descartes, who has often been called its discoverer.

Let R I (Fig. 650) be a ray incident at I on the surface of separation of two media, and let I S be the course of the ray after refraction. Then the angles which R I and I S make with the normal are called the *angle of incidence* and the *angle of refraction* respectively; and the first law of refraction is that these angles lie in the same plane, or *the plane of refraction is the same as the plane of incidence.*

The law which connects the magnitudes of these two angles, and which was discovered by Snell, can only be stated either by reference to a geometrical construction, or by employing the language of trigonometry. Describe a circle about the point of incidence I as centre, and drop perpendiculars, from the points where it cuts the rays, on the normal. The law is that these perpendiculars R′ P′, S P, will have a constant ratio; or *the sines of the angles of incidence and refraction are in a constant ratio.* It is often referred to as the *law of sines.*

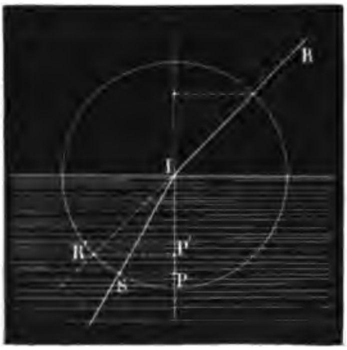

Fig. 650.—Law of Refraction.

The angle by which a ray is turned out of its original course in undergoing refraction is called its *deviation.* It is zero if the incident ray is normal, and always increases with the angle of incidence.

723. Verification of the Law of Sines.—These laws can be verified by means of the apparatus represented in Fig. 651, which is very similar to that employed by Descartes. It has a vertical divided circle, to the front of which is attached a cylindrical vessel, half-filled with water or some other transparent liquid. The surface of the liquid must pass exactly through the centre of the circle. I is a movable mirror for directing a reflected beam of solar light on the centre O. The beam must be directed centrally through a short tube attached to the mirror, and to facilitate this adjustment the tube is furnished with a diaphragm with a hole in its centre. The arm O *a* is movable about the centre of the circle, and carries a vernier for measuring the angle of incidence. The ray undergoes refraction at O; and the angle of refraction is measured by means of a second arm O R, which is to be moved into such a position that the diaphragm of its tube receives the beam centrally. No refraction occurs at emergence, since the emergent beam is normal to the surfaces of the liquid and glass; the position of the arm accordingly indicates the direction of the refracted ray. The angles of incidence and refraction can be read off at the verniers carried by the two arms; and the ratio of their sines will be found constant. The sines can also be directly measured by employing sliding-scales as indicated in the figure, the readings being taken at the extremity of each arm.

It would be easy to make a beam of light enter at the lower side of the apparatus, in a radial direction; and it would be found that the ratio of the sines was precisely the same as when the light entered from above. This is merely an instance of the general law, that the course of a returning ray is the same as that of a direct ray.

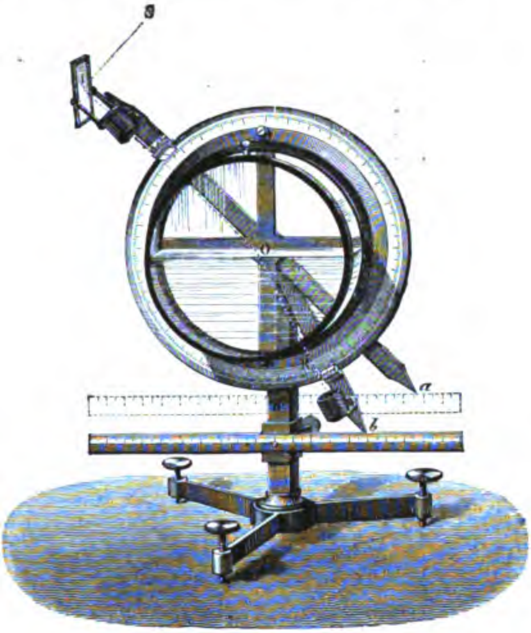

Fig. 651.—Apparatus for Verifying the Law.

724. Indices of Refraction.—The ratio of the sine of the angle of incidence to the sine of the angle of refraction, when a ray passes from one medium into another, is called the *relative index of refraction* from the former medium to the latter. When a ray passes from vacuum into any medium this ratio is always greater than unity, and is called the *absolute index of refraction*, or simply the *index of refraction*, for the medium in question. The relative index of refraction from any medium A into another B is always equal to the absolute index of B divided by the absolute index of A. The absolute index of air is so small that it may usually be neglected in comparison with those of solids and liquids; but strictly speaking, the relative index for a ray passing from air into a given substance must be multiplied by the absolute index for air, in order to obtain the absolute index of refraction for the substance.

The following table gives the indices of refraction of several substances:—

INDICES OF REFRACTION.[1]

Diamond,	2·44 to 2·755	Alcohol,	1·372
Sapphire,	1·794	Aqueous humour of eye,	1·337
Flint-glass,	1·576 to 1·642	Vitreous humour,	1·339
Crown-glass,	1·531 to 1·563	Crystalline lens, outer coat,	1·337
Rock-salt,	1·545	„ „ under coat,	1·379
Canada balsam,	1·540	„ „ central portion,	1·400
Bisulphide of carbon,	1·678	Sea water,	1·343
Linseed-oil (sp. gr. ·932)	1·482	Pure water,	1·336
Oil of turpentine (sp. gr. ·885),	1·478	Air at 0° C. and 760 mm	1·000,294

725. Critical Angle.—We see, from the law of sines, that when the incident ray is in the less refractive of the two media, to every possible angle of incidence there is a corresponding angle of refraction. This, however, is not the case when the incident ray is in the more refractive of the two media. Let S O, S′ O, S″ O (Fig. 652), be incident rays in the less refractive medium, and O R, O R′, O R″ the corresponding refracted rays. There will be a particular direction of refraction O L corresponding to the angle of incidence of 90°. Conversely, incident rays R O, R′ O, R″ O, in the more refractive

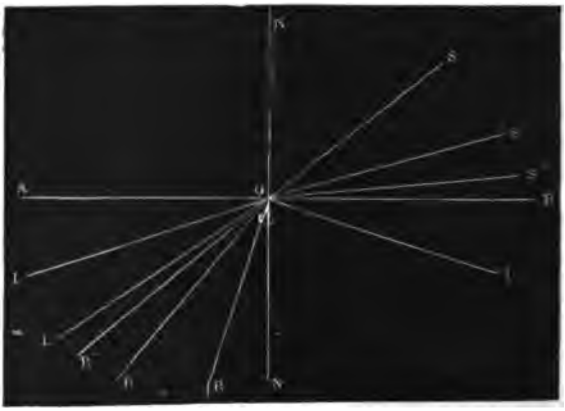

Fig. 652.—Critical Angle.

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[1] The index of refraction is always greater for violet than for red (see Chap. lxii.) The numbers in this table are to be understood as mean values.

medium, will emerge in the directions O S, O S′, O S″, and the direction of emergence for the incident ray L O will be O B, which is coincident with the bounding surface.

The angle L O N is called the critical angle, and is easily computed when the relative index of refraction is given. For let μ denote this index (the incident ray being supposed to be in the less refractive medium), then we are to have

$$\frac{\sin 90^\circ}{\sin x} = \mu, \text{ whence } \sin x = \frac{1}{\mu};$$

that is, *the sine of the critical angle is the reciprocal of the index of refraction.*

When the media are air and water, this angle is about 48° 30′. For air and different kinds of glass its value ranges from 38° to 41°.

If a ray, as I O, is incident in the more refractive medium, at an angle greater than the critical angle, the law of sines becomes nugatory, and experiment shows that such a ray undergoes internal reflection in the direction O I′, the angle of reflection being equal to the angle of incidence. Reflection occurring in these circumstances is nearly perfect, and has received the name of *total reflection. Total reflection occurs when rays are incident in the more refractive medium at an angle greater than the critical angle.*

The phenomenon of total reflection may be observed in several familiar instances. For example, if a glass of water, with a spoon in it (Fig. 653), is held above the level of the eye, the under side of the surface of the water is seen to shine like a brilliant mirror, and the lower part of the spoon is seen reflected in it. Beautiful effects of the same kind may be observed in aquariums.

726. Mirage.—An appearance as of water is frequently seen in sandy deserts, where the soil is highly heated by the sun. The observer sees in the distance the reflection of the sky and of terrestrial objects, as in the surface of a calm lake (Fig. 654). This phenomenon, called *mirage,* was first explained by Monge, who observed it in Bonaparte's Egyptian expedition.

The air near the ground becomes so highly heated that the density within a certain distance of the ground increases upwards. A ray, as M a (Fig. 655), proceeding obliquely downwards, will then be rendered, by refraction, more and more nearly horizontal as it advances, until its direction is such as to bring about total reflection; and the reflected ray is then, by successive refractions, gradually elevated till it meets the eye of the observer, who thus sees an inverted image at M′.

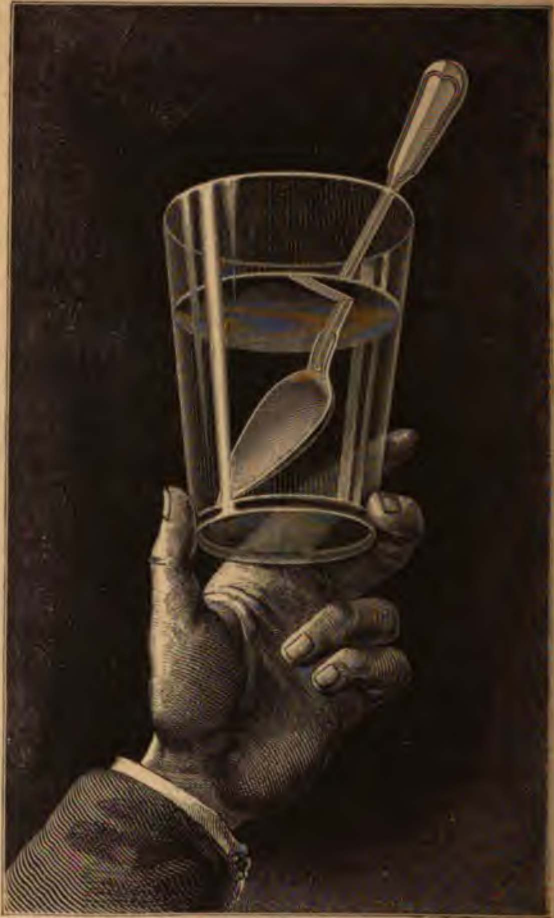

Fig. 653.—Total Reflection.

A kind of inverted mirage is often seen across masses of calm water, and is called *looming*, images of distant objects, such as ships or hills, being seen in an inverted position immediately over the objects themselves. The explanation just given of mirage will apply to this phenomenon also, if we suppose the lower strata of air to be colder and more dense than those above them.[1]

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[1] These explanations can scarcely be regarded as satisfactory. The general subject of atmospheric refraction will be treated in Chapter lxiii.

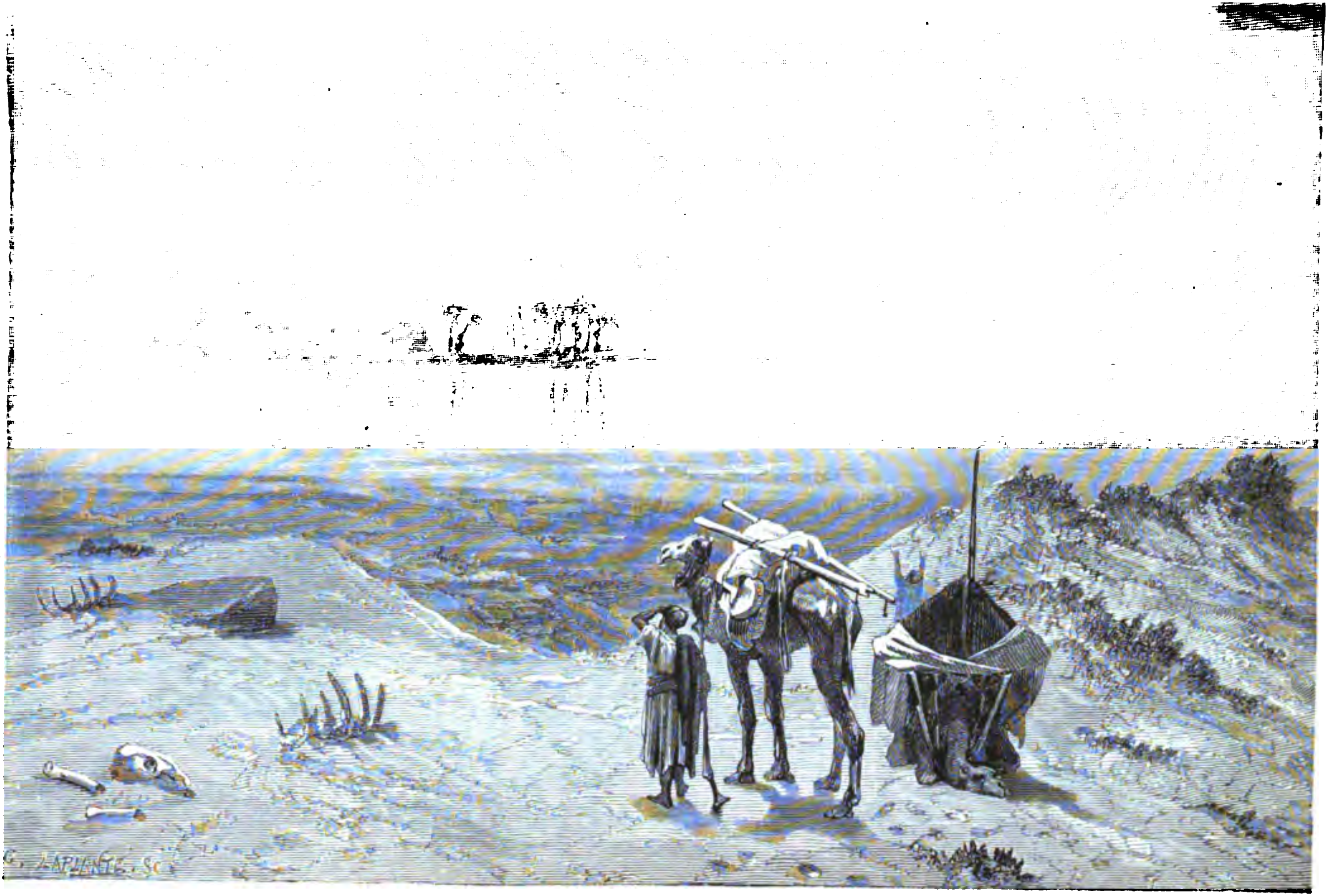
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Fig. 654.—Mirage.

Mirage may sometimes be artificially produced in the following way. A sheet-iron box is employed, with its ends cut away. Its bottom is heated externally by coals, and the observer is stationed so as to look through endwise. In general, nothing particular is observed except a fluttering due to currents of air of different densities. Sometimes, however, from causes not easily controlled, a state of calm supervenes, and one of the layers of air performs the part of a mirror, so that the observer sees the reflections of objects on the further side of the box.

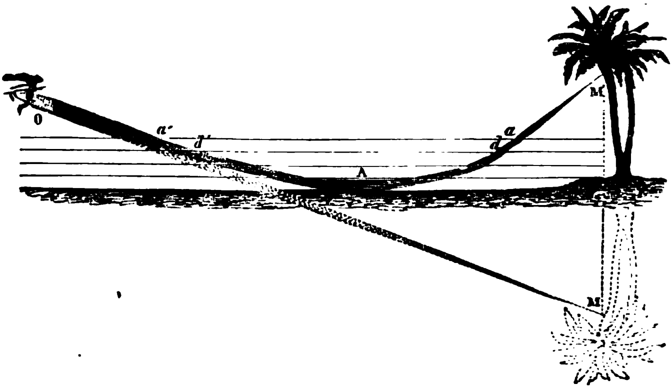

Fig. 655.—Theory of Mirage.

727. Camera Lucida.—The *camera lucida* is an instrument sometimes employed to facilitate the sketching of objects from nature. It acts by total reflection, and may have various forms, of which that proposed by Wollaston, and represented in Fig. 656, 657, is one of the commonest. The essential part is a totally-reflecting prism

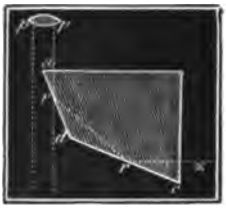

Fig. 656.—Section of Prism.

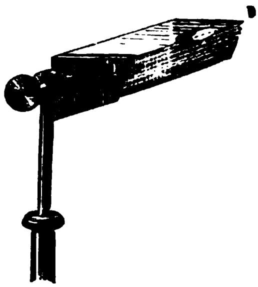

Fig. 657.—Camera Lucida.

with four angles, one of which is 90°, the opposite one 135°, and the other two each 67° 30′. One of the two faces which contain the right angle is turned towards the objects to be sketched. Rays incident normally on this face, as *x r*, make an angle greatly exceeding the critical angle with the face *c d*, and are totally reflected from it to the next face *d a*, whence they are again[1] totally reflected to the fourth face, from which they emerge normally. An eye placed so as to receive the emergent rays, will see a virtual image in a direction at right angles to that in which the object lies. In practice, the eye is held over the corner *a* of the prism, in such a position that one-half of the pupil receives these reflected rays, while the other half receives light in a parallel direction outside the prism. The observer thus sees the reflected image projected on a real background, which consists of a sheet of paper for sketching. He is thus enabled to pass a pencil over the outlines of the image, pencil image and paper being simultaneously visible. It is very desirable that the image should lie in the plane of the paper, not only because the pencil point and the image will then be seen with the same focussing of the eye, but also because parallax is thus obviated, so that when the observer shifts his eye the pencil point is not displaced on the image. As the paper, for convenience of drawing, must be at a distance of about a foot, a concave lens, with a focal length of something less than a foot is placed close in front of the prism, in drawing distant objects. By raising or lowering the prism in its stand (Fig. 657), the image of the object to be sketched may be made to coincide with the plane of the paper.

The prism is mounted in such a way that it can be rotated either about a horizontal or a vertical axis; and its top is usually covered with a movable plate of blackened metal, having a semicircular notch at one edge, for the observer to look through.

727 A. Images by Refraction at a Plane Surface.—When a point Q (Fig. 657 A), in the interior of a solid or liquid bounded by a plane surface, sends rays into air, the emergent rays, if produced backwards, will all touch a certain caustic surface, which is a surface of revolution, having for axis the normal from Q to the plane surface. The caustic has a cusp, situated on this normal, at a distance from the surface equal to the distance of Q divided by the index of refraction, and this is the position of the image as seen by an eye situated

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[1] The use of having *two* reflections is to obtain an erect image. An image obtained by one reflection would be upside down.

on the normal. For example
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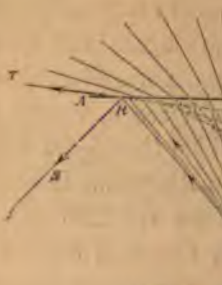

Fig. 657 A.

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728. Parallel Plate.—Rays
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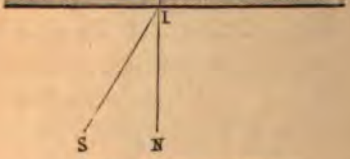

Fig. 658.—Parallel Plate.

and hence the angle of emerg
angle of incidence S I N. The
to the incident ray S I, but is n

If the luminous body is at a distance which may be regarded as infinite,—if it is a star, for example,—all the images should coincide, and form only a single image, occupying a position which does not vary with the position of the observer, provided that the plate is perfectly homogeneous, and its faces perfectly plane and parallel. A severe test is thus furnished of the fulfilment of these conditions.

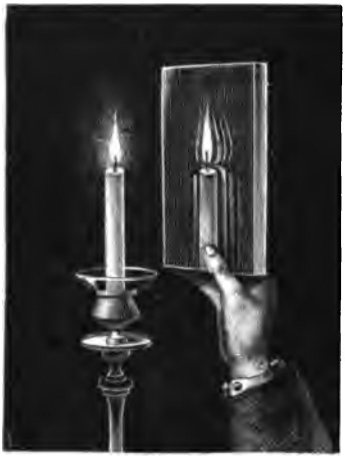

Fig. 661.—Images of Candle in Looking-glass.

Plates are sometimes tested, for parallelism and uniformity, by supporting them in a horizontal position on three points, viewing the image of a star in them with a telescope furnished with cross wires, and observing whether the image is displaced on the wires when the plate is shifted into a different position, still resting on the same three points.

730. Refraction through a Prism.—For optical purposes, any portion of a transparent body lying between two plane faces which are

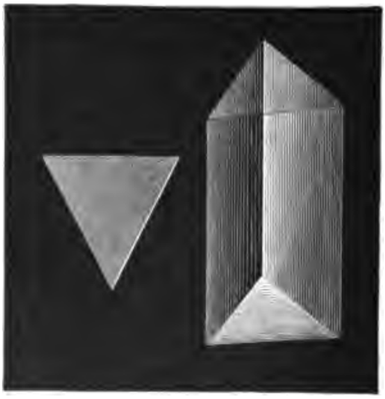

Fig. 662.—Equilateral Prism.

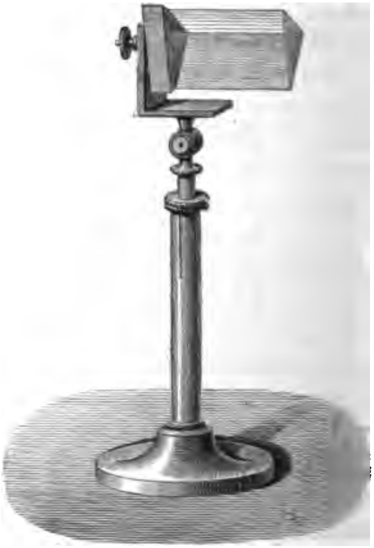

Fig. 663.—Prism mounted on Stand.

not parallel may be regarded as a prism.[1] The line in which these faces meet, or would meet if produced, is called the edge of the prism, and a section made by a plane perpendicular to them both is called a *principal section.* The prisms chiefly employed are really prisms in the geometrical sense of the word. Their principal sections are usually triangular, and are very frequently equilateral, as in Fig. 662. The stand usually employed for prisms when mounted separately is represented in Fig. 663. It contains several joints. The uppermost is for rotating the prism about its own axis. The second is for turning the prism so that its edges shall make any required angle with the vertical. The third gives motion about a vertical axis, and also furnishes the means of raising and lowering the prism through a range of several inches.

Let S I (Fig. 664) be an incident ray in the plane of a principal section of the prism. If the external medium be air, or any other substance of less refractive power than the prism, the ray in entering the prism will be bent nearer to the normal, taking such a course as I E, and in leaving the prism will be bent away from the normal, taking the course E B. The effect of these two refractions is, therefore, to turn the ray away from the edge (or refracting angle) of the prism. In practice, the prism is usually so placed that I E, the path of the ray through the prism, makes equal angles with the two faces at which refraction occurs (§ 731). If the prism is turned very far from this position, the course of the ray may be altogether different from that represented in the figure; it may, for example, enter at one face, be internally reflected at another, and come out at the third; but we at present exclude such cases from consideration.

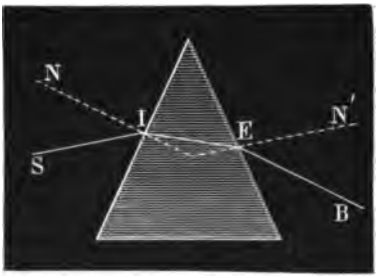

Fig. 664.—Refraction through Prism.

The direction of deviation is easily shown experimentally, by admitting a narrow beam of sunlight into a dark room, and introducing a prism in its course. It will be found that the refracted beam, in the circumstances represented in Fig. 664, is turned aside some 40° or 50° from its original course.[2]

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[1] This amounts to saying that the word *prism* in optics means *wedge.*

[2] The phenomena here described are complicated in practice by the unequal refrangibi-

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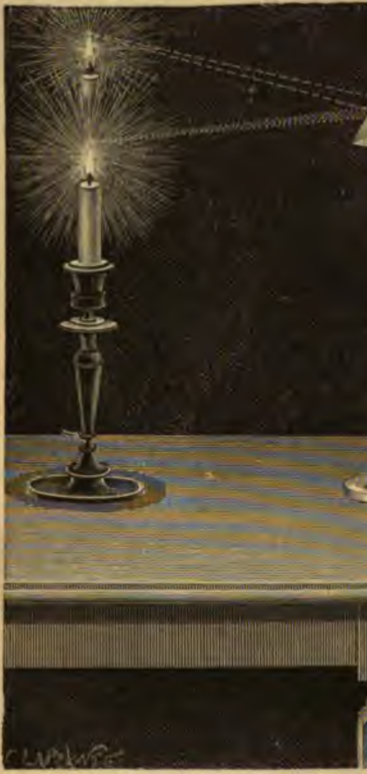

Fig. 665.—V

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731. Formulæ for Refraction t
—Let S I (Fig. 666) be an inc
section A B C of a prism. Let
r the angle of refraction M I I'.
tion by μ, we have $sin\ i = \mu\ s$

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lity of rays of different colours (Chap. lx
ploying homogeneous light, of which a spi
affords a nearly perfect example.

This last result is of great practical importance, as it enables us to calculate the index of refraction μ from measurements of the refracting angle A of the prism, and of the deviation D which occurs when the ray passes symmetrically.

When a beam of sunlight in a dark room is transmitted through a prism, it will be found, on rotating the prism about its axis, that there is a certain mean position which gives smaller deviation of the transmitted light than positions on either side of it; and that, when the prism is in this position, a small rotation of it has no sensible effect on the amount of deviation. The position determined experimentally by these conditions, and known as the *position of minimum deviation*, is the position in which the ray passes symmetrically

731 A. Construction for Deviation.—The following geometrical construction furnishes a very simple method of representing the variation of deviation with the angle of incidence:—

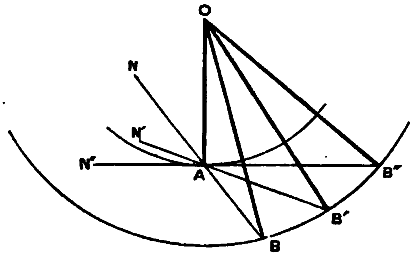

Fig. 666 A.—General Construction for Deviation.

1. When the refraction is at a single surface, describe two circular arcs about a common centre O (Fig. 666 A), the ratio of their radii being the index of refraction. Then, if the incidence is from rare to dense, draw a radius O A of the smaller circle to represent the direction of the incident ray, and let N A B be the direction of the normal to the surface at the point of incidence, so that O A N is the angle of incidence. Join O B. Then O B N is the angle of refraction, since $\frac{\sin OAN}{\sin OBN} = \frac{OB}{OA} =$ index of refraction; hence O B is parallel to the refracted ray. If the incidence is from dense to rare, we must draw O B to represent the incident ray, make O B N equal to the angle of incidence, and join O A. In either case the angle A O B is the deviation, and it evidently increases with the angle of incidence O A N, attaining its greatest value when this angle (O A N″ in the figure) is a right angle, in which case the angle of refraction O B″ N″ is the critical angle.

2. To find the deviation in refraction through a prism, describe two concentric circular arcs as before (Fig. 666 B), the ratio of their radii being the index of refraction. Draw the radius O A of the

smaller circle to represent the incident ray, N B to represent the normal at the first surface, B N′ the normal at the second surface. Then O B represents the direction of the ray in the prism, O A′ the direction of the emergent ray, and A O A′ is accordingly the total deviation.

In fact we have

O A N = angle of incidence at first surface.
O B N = „ refraction „
O B N′ = „ incidence at second surface.
O A′ N′ = „ refraction „
A O B = deviation at first surface.
B O A′ = „ second „
A B A′ = angle between normals = angle of prism.

Again, the deviation A O A′, being the angle at the centre of a circle, is measured by the arc A A′, which subtends it. To obtain the minimum deviation, we must so arrange matters that the angle A B A′ being given (= angle of prism), the arc A A′ shall be a minimum. Let A B A′, *a* B *a*′ (Fig. 666 C), be two consecutive positions, B A′ and B *a*′ being greater than B A and B *a*. Then, since the small angles A B *a*, A′ B *a*′ are equal, it is obvious, for a double reason, that the small arc A′ *a*′ is greater than A *a*, and hence the whole arc *a a*′ is greater than A A′. The deviation is therefore increased by altering the position in such a way as to make B A and B A′ depart further from equality, and is a minimum when they are equal.

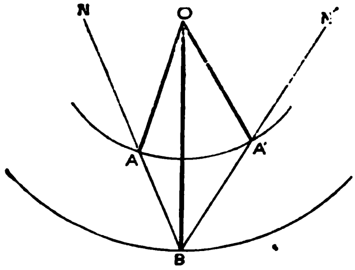

Fig. 666 B.—Application to Prism.

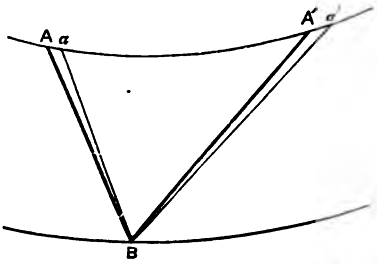

Fig. 666 C.—Proof of Minimum Deviation.

731 B. Conjugate Foci for Minimum Deviation.—When the angle of incidence is nearly that corresponding to minimum deviation, a small change in this angle has no sensible effect on the amount of deviation.

Hence a small pencil of rays sent in this direction from a luminous point, and incident near the refracting edge, will emerge with their divergence sensibly unaltered, so that if produced backwards they would meet in a virtual focus at the same distance (but of course not in the same direction) as the point from which they came.

In like manner, if a small pencil of rays converging towards a point, are turned aside by interposing the edge of a prism in the position of minimum deviation, they will on emergence converge to another point at the same distance. We may therefore assert that, neglecting the thickness of a prism, *conjugate foci are at the same distance from it, and on the same side, when the deviation is a minimum.*

732. Double Refraction.—Thus far we have been treating of what is called *single refraction.* We have assumed that to each given incident ray there corresponds only one refracted ray. This is true when the refraction is into a liquid, or into well-annealed glass, or into a crystal belonging to the cubic system.

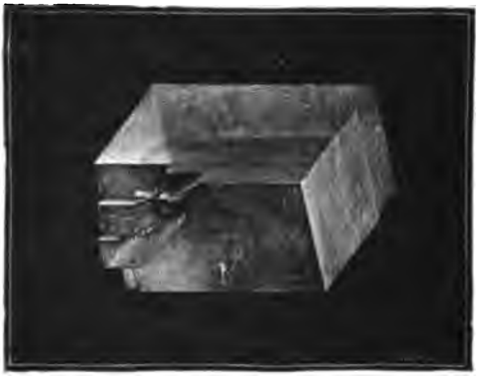

Fig. 667.—Iceland-spar.

On the other hand, when an incident ray is refracted into a crystal of any other than the cubic system, or into glass which is unequally stretched or compressed in different directions; for example, into unannealed glass, it gives rise in general to two refracted rays which take different paths; and this phenomenon is called *double refraction.* Attention was first called to it in 1670 by Bartholin, who observed it in the case of Iceland-spar, and its laws for this substance were accurately determined by Huyghens.

733. Phenomena of Double Refraction in Iceland-spar.—Iceland-spar or calc-spar is a form of crystallized carbonate of lime, and is found in large quantity in the country from which it derives its name. It is usually found in rhombohedral form, as represented in Figs. 667, 668.

To observe the phenomenon of double refraction, a piece of the spar may be laid on a page of a printed book. All the letters seen through it will appear double, as in Fig. 668; and the depth of their blackness is considerably less than that of the originals, except where the two images overlap.

In order to state the laws of the phenomena with precision, it is necessary to attend to the crystalline form of Iceland-spar.

At the corner which is represented as next us in Fig. 667 three equal obtuse angles meet; and this is also the case at the opposite corner which is out of sight. If a line be drawn through one of these corners, making equal angles with the three edges which meet there, it or any line parallel to it is called the *axis* of the crystal; the axis being properly speaking not a definite *line* but a definite *direction*.

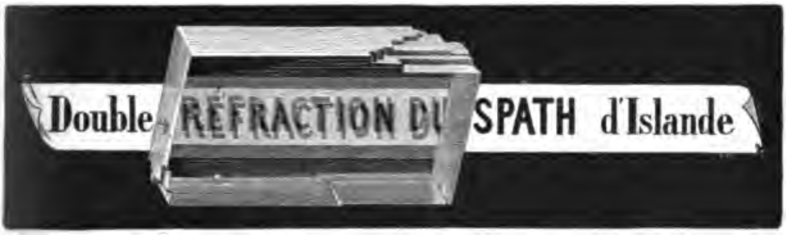

Fig. 668.—Double Refraction of Iceland-spar.

The angles of the crystal are the same in all specimens; but the lengths of the three edges (which may be called the oblique length, breadth, and thickness) may have any ratios whatever. If the crystal is of such proportions that these three edges are equal, as in the first part of Fig. 669, the axis is the direction of one of its diagonals, which is represented in the figure.

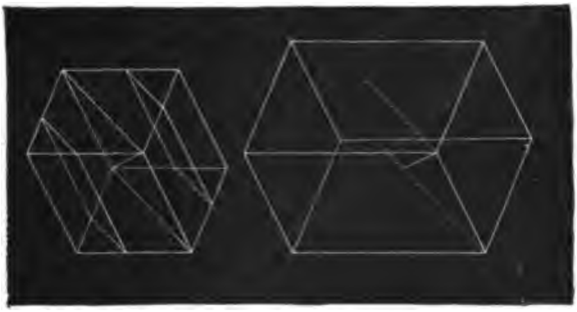

Fig. 669.—Axis of the Crystal.

Any plane containing (or parallel to) the axis is called a *principal plane* of the crystal.

If the crystal is laid over a dot on a sheet of paper, and is made to rotate while remaining always in contact with the paper, it will be observed that, of the two images of the dot, one remains unmoved, and the other revolves round it. The former is called the *ordinary*, and the latter the *extraordinary* image. It will also be observed that the former appears nearer than the latter, being more lifted up by refraction.

The rays which form the ordinary image follow the ordinary law of sines (§ 722). They are called the ordinary rays. Those which form the extraordinary image (called the extraordinary rays) do not follow the law of sines, except when the plane of incidence is perpendicular to the axis of the crystal, and in this case their index of refraction (called the extraordinary index) is different from that of the ordinary rays. The ordinary index is 1·66, and the extraordinary 1·52.

When the plane of incidence is parallel to the axis, the extraordinary ray lies in this plane, but the ratio of the sines of the angles of incidence and refraction is variable.

When the plane of incidence is oblique to the axis, the extraordinary ray generally lies in a different plane.

We shall recur to the subject of double refraction in the concluding chapter of this volume.

CHAPTER LX.

LENSES.

735. Forms of Lenses.—A lens is usually a piece of glass bounded by two surfaces which are portions of spheres. There are two principal classes of lenses.

1. *Converging* lenses or *convex* lenses, which have one or other of the three forms represented in Fig. 670. The first of these is called double convex, the second plano-convex, and the third concavo-convex. This last is also called a converging meniscus. All three are thicker in the middle than at the edges. They are called converging, because rays are always more convergent or less divergent after passing through them than before.

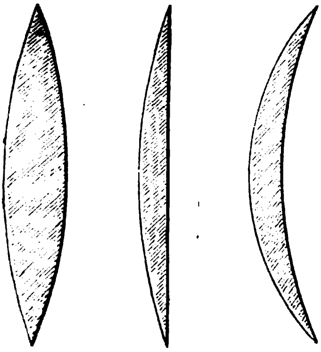

Fig. 670.—Converging Lenses.

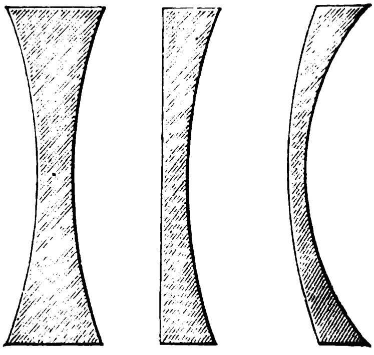

Fig. 671.—Diverging Lenses.

2. *Diverging* lenses or *concave* lenses (Fig. 671) produce the opposite effect, and are characterized by being thinner in the middle than at the edges. Of the three forms represented, the first is double concave, the second plano-concave, and the third convexo-concave also called a diverging meniscus).

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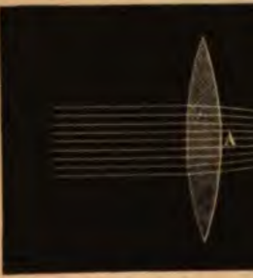

Fig. 672.—Principal

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In elementary optics it is usual to neglect the thickness of the lens. The incident and emergent rays SI, ER may then be regarded as lying in one straight line which passes through C, and we may lay down the proposition that *rays which pass through the centre of a lens undergo no deviation.* Any straight line through the centre of a lens is called a *secondary axis.*

The approximate convergence of the refracted rays to a point, when the incident rays are parallel, is true for all directions of incidence; and the point to which the emergent rays approximately converge (*f*, Fig. 675) is always situated on the secondary axis (*a c f*) parallel to the incident rays. The focal distance is sensibly the same as for rays parallel to the principal axis, unless the obliquity is considerable.

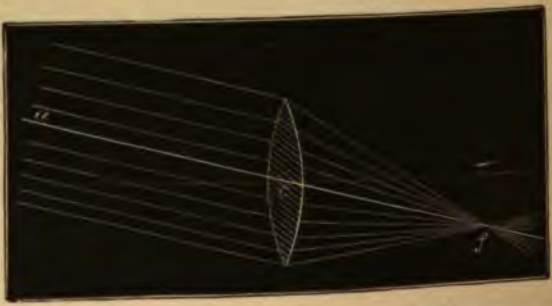

Fig. 675.—Principal Focus on Secondary Axis.

738. Conjugate Foci.—When a luminous point S sends rays to a lens (Fig. 676), the emergent rays converge (approximately) to one

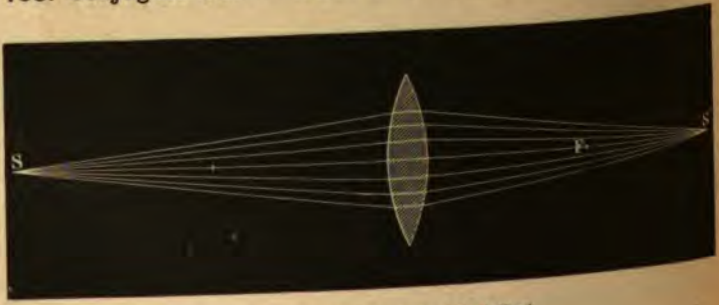

Fig. 676.—Conjugate Foci, both Real.

The distances of C from the two faces are respectively the difference between R and OC, and the difference between R′ and O′C, and we have

$$\frac{R}{R'} = \frac{OC}{O'C} = \frac{R - OC}{R' - O'C}$$

point S′; whence it follows that rays sent from S′ to the lens would converge (approximately) to S. Two points thus related are called *conjugate foci* of the lens, and the line joining them always passes through the centre of the lens; in other words, they must either be both on the principal axis, or both on the same secondary axis.

The fact that rays which come from one point go to one point is the foundation of the theory of images, as we have already explained in connection with mirrors (§ 707).

The diameters of object and image are directly as their distances from the centre of the lens, and the image will be erect or inverted according as the object and image lie on the same side or on opposite sides of this centre (§ 711). There is also, in the case of lenses, the same difference between an image seen in mid-air and an image thrown on a screen which we have pointed out in § 714.

It is to be remarked that the distinction between principal and secondary axes has much more significance in the case of lenses than of mirrors; and images produced by a lens are more distinct in the neighbourhood of the principal axis than at a distance from it.

739. Formulæ relating to Lenses.—The deviation produced in a ray by transmission through a lens will not be altered by substituting for the lens a prism bounded by planes which touch the lens at the points of incidence and emergence; and in the actual use of lenses, the direction of the rays with respect to the supposed prism is such as to give a deviation not differing much from the minimum. The expression for the minimum deviation (§ 731) is $2i-2r$ or $2i-\mathrm{A}$; and when the angle of the prism is small, as it is in the case of ordinary lenses, we may assume $\frac{i}{r}=\frac{\sin i}{\sin r}=\mu$; so that $2i$ becomes $2\mu r$ or $\mu\mathrm{A}$, and the expression for the deviation becomes

$$(\mu-1)\,\mathrm{A}, \qquad (1)$$

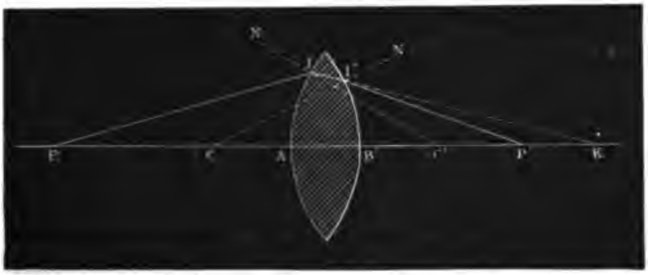

Fig. 677.—Diagram showing Path of Ray, and Normals.

A being the angle between the tangent planes (or between the normals) at the points of entrance and emergence.

Let x_1 and x_2 denote the distances of these points respectively from the principal axis, and r_1, r_2 the radii of curvature of the faces on which they lie. Then $\frac{x_1}{r_1}$, $\frac{x_2}{r_2}$ are the sines of the angles which the normals make with the axis, and the angle A is the sum or difference of these two angles, according to the shape of the lens. In the case of a double convex lens it is their sum, and if we identify the sines of these small angles with the angles themselves, we have

$$A = \frac{x_1}{r_1} + \frac{x_2}{r_2}. \qquad (2)$$

But if p_1, p_2 denote the distances from the faces of the lens to the points where the incident and emergent rays cut the principal axis, $\frac{x_1}{p_1}$, $\frac{x_2}{p_2}$ are the sines of the angles which these rays make with the axis, and the deviation is the sum or difference of these two angles, according as the conjugate foci are on opposite sides or on the same side of the lens. In the former case, identifying the angles with their sines, the deviation is $\frac{x_1}{p_1} + \frac{x_2}{p_2}$, and this, by formula (1), is to be equal to $(\mu - 1)$ A, that is, to $(\mu - 1)\left(\frac{x_1}{r_1} + \frac{x_2}{r_2}\right)$.

If the thickness of the lens is negligible in comparison with p_1, p_2, we may regard x_1 and x_2 as equal, and the equation

$$\frac{x_1}{p_1} + \frac{x_2}{p_2} = (\mu - 1)\left(\frac{x_1}{r_1} + \frac{x_2}{r_2}\right) \qquad (3)$$

will reduce to

$$\frac{1}{p_1} + \frac{1}{p_2} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right). \qquad (4)$$

If p_1 is infinite, the incident rays are parallel, and p_2 is the principal focal length, which we shall denote by f. We have therefore

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \qquad (5)$$

and

$$\frac{1}{p_1} + \frac{1}{p_2} = \frac{1}{f}. \qquad (6)$$

740. Conjugate Foci on Secondary Axis.—Let M be a luminous point on the secondary axis M O M′, O being the centre of the lens, and let M′ be the point in which an emergent ray corresponding to the incident ray M I cuts this axis. Let x denote x_1 or x_2, the distances of the points of incidence and emergence from the principal axis, and θ the obliquity of the secondary axis; then $x \cos \theta$ is the length of the perpendicular from I upon M M′, and $\frac{x \cos \theta}{\mathrm{MI}}$, $\frac{x \cos \theta}{\mathrm{M'I}}$, are the sines of the angles O M I, O M′ I respectively. But the deviation is the sum of these angles; hence, proceeding as in last section, we have

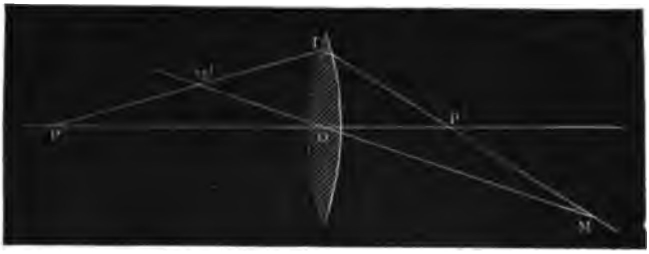

Fig. 678.—Conjugate Foci on Secondary Axis.

$$\frac{x \cos \theta}{\mathrm{MI}} + \frac{x \cos \theta}{\mathrm{M'I}} = (\mu - 1)\left(\frac{x}{r_1} + \frac{x}{r_2}\right) = \frac{x}{f}; \qquad (7)$$

and when θ is small, its cosine is sensibly equal to unity;[1] in which case the equation reduces to

$$\frac{1}{\mathrm{MI}} + \frac{1}{\mathrm{M'I}} = \frac{1}{f}. \qquad (8)$$

The fact that x does not appear in equations (6) and (8) shows that, for every position of a luminous point, there is a conjugate focus lying on the same axis as the luminous point itself, at least in so far as the approximate assumptions which we have made are allowable.

741. Discussion of the Formula for Convex Lenses.—It thus appears that the formula

$$\frac{1}{p} + \frac{1}{p'} = \frac{1}{f} \qquad (9)$$

---

[1] The quantity $\frac{1}{\mathrm{MI}} + \frac{1}{\mathrm{M'I}}$ is in fact rather greater than $\frac{1}{f}$; for in the first place, we have taken $\cos \theta$ as equal to unity, and in the second place we have assumed that the actual deviation is equal to the minimum deviation $\frac{x}{f}$. Correcting these inexact assumptions, we see that $\frac{1}{\mathrm{MI}} + \frac{1}{\mathrm{M'I}}$ is really equal to a quantity rather greater than $\frac{1}{f}$, multiplied by $\sec \theta$, which is also rather greater than unity. The effective focal length is therefore rather less for oblique than for direct pencils.

applies both to direct and oblique pencils, f denoting the principal focal length of the lens (supposed convex), and p, p' the distances of a pair of conjugate foci from the lens, on opposite sides of it. This formula being identical with equation (*b*) of § 708, leads to results similar to those already deduced in the case of concave mirrors.

As one focus advances from infinite distance to a principal focus, its conjugate moves away from the other principal focus to infinite distance on the other side. The more distant focus is always moving more rapidly than the nearer, and the least distance between them is accordingly attained when they are equidistant from the lens; in which case the distance of each of them from the lens is $2f$, and their distance from each other $4f$.

If either of the distances, as p, is less than f, the formula shows that the other distance p' is negative. The meaning is that the two foci are on the same side of the lens, and in this case one of them (the more distant of the two) must be virtual. For example, in Fig. 679, if S, S′ are a pair of conjugate foci, one of them S being between the principal focus F and the lens, rays sent to the lens by a luminous point at S, will, after emergence, diverge as if from S′; and rays coming from the other side of the lens, if they converge to S′ before incidence, will in reality be made to meet in S. As S moves towards the lens, S′ moves in the same direction more rapidly; and they become coincident at the surface of the lens. The formula in fact shows that if $\frac{1}{p}$ is very great in comparison with $\frac{1}{f}$, and positive, $\frac{1}{p'}$ must be very great and negative; that is to say, if p is a very small positive quantity, p' is a very small negative quantity.

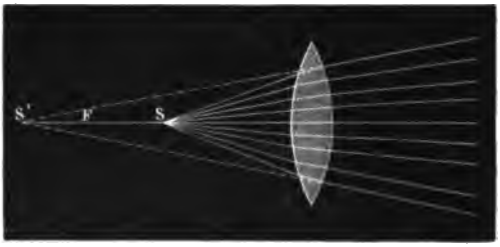

Fig. 679.—Conjugate Foci, one Real, one Virtual.

742. Formation of Real Images.—Let A B (Fig. 680) be an object in front of a lens, at a distance exceeding the principal focal length. It will have a real image on the other side of the lens. To determine the position of the image by construction, draw through any point A of the object a line parallel to the principal axis, meeting the lens in A′. The ray represented by this line will, after refraction, pass through the principal focus F; and its intersection with the secondary axis A O determines the position of *a*, the focus conjugate to A. We can in like manner determine the position of *b*, the focus conjugate to B, another point of the object; and the joining line *a b* will then be the image of the line A B. It is evident that if *a b* were the object, A B would be the image.

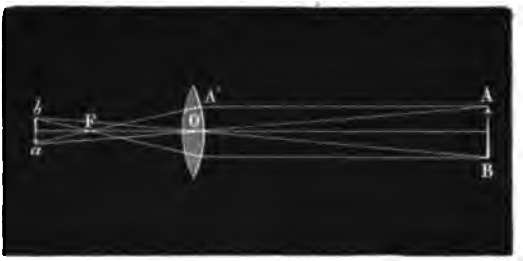

Fig. 680.—Real and Diminished Image.

Figs. 680, 681 represent the cases in which the distance of the object is respectively greater and less than twice the focal length of the lens.

In each case it is evident that $\frac{\mathrm{AB}}{ab}=\frac{\mathrm{OA}}{\mathrm{O}a}=\frac{p}{p'}$, or the *linear dimensions of object and image are directly as their distances from the centre of the lens.*

Again, since F O is parallel to a side of the triangle *a* A′ A, we have

$$\frac{\mathrm{OA}}{\mathrm{O}a}=\frac{\mathrm{FA'}}{\mathrm{F}a}=\frac{f}{p'-f}.$$

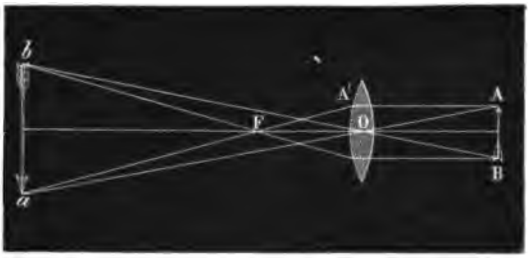

Fig. 681.—Real and Magnified Image.

And by making a similar construction with respect to the other principal focus, we can prove that

$$\frac{\mathrm{OA}}{\mathrm{O}a}=\frac{p-f}{f}.$$

We have therefore

$$\frac{\mathrm{AB}}{ab}=\frac{p-f}{f}=\frac{f}{p'-f}, \qquad (10)$$

f denoting the focal length of the lens, and *p*, *p′* the distances of A B, *a b* respectively from the lens.

743. Example.—A straight line 25^{mm} long is placed perpendicularly on the axis, at a distance of 35 centimetres from a lens of 15 centimetres' focal length; what are the position and magnitude of the image?

To determine the distance p' we have

$$\frac{1}{35}+\frac{1}{p'}=\frac{1}{15};\ \text{whence}\ p'=\frac{35\times 15}{35-15}=26\tfrac{1}{4}^{\text{cm}}$$

For the length of the image we have

$$25\,\frac{f}{p-f}=25\,\frac{15}{35-15}=18\tfrac{3}{4}^{\text{mm}}$$

743A. Image on Cross-wires.—The position of a real image seen in mid-air can be tested by means of a cross of threads, or other convenient mark, so arranged that it can be fixed at any required point. The observer must fix this cross so that it appears approximately to coincide with a selected point of the image. He must then try whether any relative displacement of the two occurs on shifting his eye to one side. If so, the cross must be pushed nearer to the lens or drawn back, according to the nature of the observed displacement which follows the general rule of parallactic displacement, that the more distant object is displaced in the same direction as the observer's eye. The cross may thus be brought into exact coincidence with the selected point of the image, so as to remain in apparent coincidence with it from all possible points of view. When this coincidence has been attained, the cross is at the focus conjugate to that which is occupied by the selected point of the object.

By employing two crosses of threads, one to serve as object, and the other to mark the position of the image, it is easy to verify the fact that when the second cross coincides with the image of the first the first cross also coincides with the image of the second.

744. Aberration of Lenses.—In the investigations of §§ 739, 740, we made several assumptions which were only approximately true. The rays which proceed from a luminous point to a lens are in fact not accurately refracted to one point, but touch a curved surface called a caustic. The cusp of this caustic is the conjugate focus, and is the point at which the greatest concentration of light occurs. It is accordingly the place where a screen must be set to obtain the brightest and most distinct image. Rays from the central parts of the lens pass very nearly through it; but rays from the circumferential portions fall short of it. This departure from exact concurrence is called *spherical aberration*. The distinctness of an image on a screen is improved by employing an annular diaphragm to cut off all except the central rays; but the brightness is of course diminished.

By holding a convex lens in a position very oblique to the incident light, a primary and secondary focal line can be exhibited on a screen, just as in the case of concave mirrors (§ 714A). The experiment, however, is rather more difficult of performance.

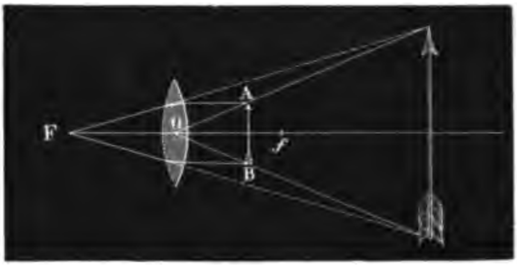

Fig. 682.—Virtual Image formed by Convex Lens.

745. Virtual Images.—Let an object AB be placed between a convex lens and its principal focus. Then the foci conjugate to the points A, B are virtual, and their positions can be found by construction from the consideration that rays through A, B, parallel to the principal axis, will be refracted to F, the principal focus on the other side. These refracted rays, if produced backward, must meet the secondary axes OA, OB in the required points. An eye placed on the other side of the lens will accordingly see a virtual image erect, magnified, and at a greater distance from the lens than the object. This is the principle of the simple microscope. The formula for the distances D, d of object and image from the lens, when both are on the same side, is

$$\frac{1}{D} - \frac{1}{d} = \frac{1}{f}, \qquad (11)$$

f denoting the principal focal length.

746. Concave Lens.—For a concave lens, if the focal length be still regarded as positive, and denoted by f, and if the distances D, d be on the same side of the lens, the formula becomes

$$\frac{1}{d} - \frac{1}{D} = \frac{1}{f}, \qquad (12)$$

which shows that d is always less than D; that is, the image is nearer to the lens than the object.

In Fig. 683, A B is the object, and *a b* the image. Rays incident from A and B parallel to the principal axis will emerge as if they came from the principal focus F. Hence the points *a, b* are determined by the intersections of the dotted lines in the figure with the secondary axes O A, O B. An eye on the other side of the lens sees the image *a b*, which is always virtual, erect and diminished.

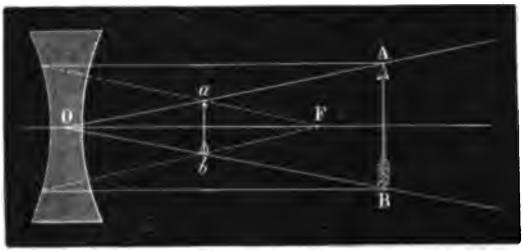

Fig. 683.—Virtual Image formed by Concave Lens.

747. Focometer.—Silbermann's focometer (Fig. 684) is an instrument for measuring the focal lengths of convex lenses, and is based on the principle (§ 741), that when the object and its image are equidistant from the lens, their distance from each other is four times the focal length. It consists of a graduated rule carrying three runners M, L, M'. The middle one L is the support for the lens which is to be examined; the other two, M, M', contain two thin plates of horn or other translucent material, ruled with lines, which are at the same distance apart in both. The sliders must be adjusted until the image of one of these plates is thrown upon the other plate, without enlargement or diminution, as tested by the coincidence of the ruled lines of the image with those of the plate on which it is cast. The distance between M and M' is then read off, and divided by 4.

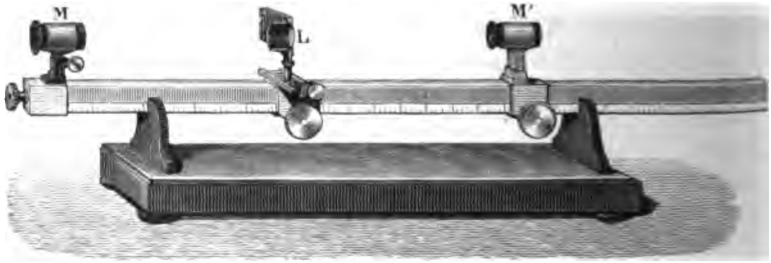

Fig. 684.—Silbermann's Focometer.

747 A. Refraction at a Single Spherical Surface.—When the bounding surface between two media is a portion of a sphere, the positions of conjugate foci relative to this surface are given by the equation

$$\frac{\mu}{p'} - \frac{1}{p} = \pm\frac{\mu-1}{r}; \qquad (13)$$

where p and p' denote the distances of object and image from the surface, *both on the same side,* r is the radius of curvature, and μ the index of refraction, which is greater or less than unity according as the incidence is from rare to dense, or from dense to rare. The upper or lower sign is to be taken according as the surface is concave or convex towards the incident light.

By making p infinite we see that the principal focus is at a distance $\frac{\mu r}{\mu-1}$ from the surface. When, as in the cornea of the human eye, the surface is convex towards the incident light, and the incidence is from rare to dense, the refracted rays are more convergent than the incident rays, but are not so convergent as if they had been

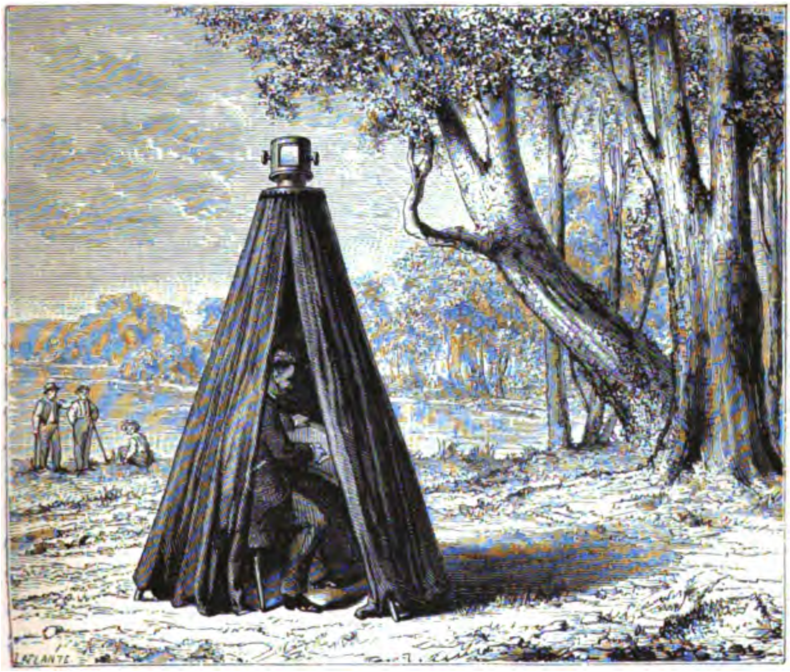

Fig. 685.—Camera Obscura for Sketching.

refracted through a plano-convex lens of the same convexity. The focal length of such a lens would, in fact, be only $\frac{r}{\mu - 1}$.

748. Camera Obscura.—The images obtained by means of a hole in the shutter of a dark room (§ 683) become sharper as the size of the hole is diminished; but this diminution involves loss of light, so that it is impossible by this method to obtain an image at once bright and sharp. This difficulty can be overcome by employing a lens. If the objects in the external landscape depicted are all at distances many times greater than the focal length of the lens, their images will all be formed at sensibly the same distance from the lens, and may be received upon a screen placed at this distance. The images thus obtained are inverted, and are of the same size as if a simple aperture were employed instead of a lens. This is the principle of the *camera obscura*, one form of which is represented in Fig. 685. It is a kind of tent surrounded by opaque curtains, and having at its top a revolving lantern, containing a lens with its axis horizontal, and a mirror placed behind it at a slope of 45°, to reflect the transmitted light downwards on to a sheet of white paper lying on the top of a table. Images of external objects are thus depicted on the paper, and their outlines can be traced with a pencil if desired. It is still better to combine lens and mirror in one, by the arrangement represented in section in Fig. 686. Rays

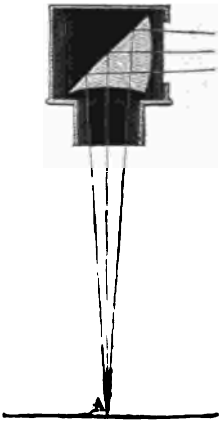

Fig. 686.—Objective of Camera.

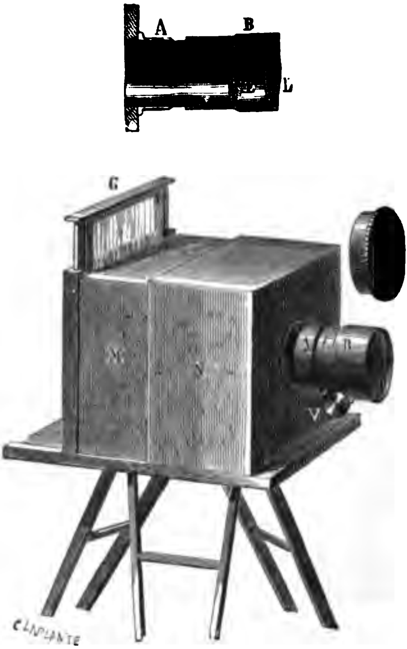

Fig. 687.—Photographic Camera.

from external objects are first refracted at a convex surface, then totally reflected at the back of the lens, which is plane, and finally emerge through the bottom of the lens, which is concave, but with a larger radius of curvature than the first surface. The two refractions produce the effect of a converging meniscus. The instrument is now only employed for purposes of amusement.

749. Photographic Camera.—The camera obscura employed by photographers (Fig. 687) is a box MN with a tube AB in front, containing an object-glass at its extremity. The object-glass is usually compound, consisting of two single lenses E, L, an arrangement which is very commonly adopted in optical instruments, and which has the advantage of giving the same effective focal length as a single lens of smaller radius of curvature, while it permits the employment of a larger aperture, and consequently gives more light. At G is a slide of ground glass, on which the image of the scene to be depicted is thrown, in setting the instrument. The focussing is performed in the first place by sliding the part M of the box in the part N, and finally by the pinion V which moves the lens. When the image has thus been rendered as sharp as possible, the sensitized plate is substituted for the ground glass.[1]

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[1] The photographic processes at present in use are very various, both optically and chemically; but are all the same in principle with the method originally employed by Talbot. This method, which was almost forgotten during the great success of Daguerre, consists in first obtaining, on a transparent plate, a picture with lights and shades reversed, called a *negative;* then placing this upon a piece of paper sensitized with chloride of silver, and exposing it to the sun's rays. The light parts of the negative allow the light to pass and blacken the paper, thus producing a positive picture. The same negative serves for producing a great number of positives.

The negative plate is usually a glass plate covered with a film of collodion (sometimes of albumen), sensitized by a salt of silver. The following is one of the numerous formulæ for this preparation. Take

Sulphuric ether, ...	800 grammes
Alcohol at 40°	200 ,,
Gun cotton, ...	5 ,,

Incorporate these ingredients thoroughly in a porcelain mortar; then add

Iodide of potassium, ...	13 grammes
Iodide of ammonium, ...	1·75 ,,
Iodide of cadmium, ...	1·75 ,,
Bromide of cadmium, ...	1·25 ,,

This mixture is poured over the plate, which is then immersed in a solution (10 per cent.

750. Use of Lenses for Purposes of Projection.—Lenses are extensively employed in the lecture-room, for rendering experiments visible to a whole audience at once, by projecting them on a screen. The arrangements vary according to the circumstances of each case, and cannot be included in a general description.

751. Solar Microscope. Magic Lantern.—In the solar microscope a convex lens of short focal length is employed to throw upon a screen a highly-magnified image of a small object placed a little beyond the principal focus. As the image is always much less bright than the object, and the more so as the magnification is greater, it is necessary that the object should be very highly illuminated. For this purpose the rays of the sun are directed upon it by means of a mirror and large lens; the latter serving to increase the solid angle of the cone of rays which fall upon the object, and thus to enable a larger portion of the magnifying lens to be utilized. The objects magnified are always transparent; and the images are formed by rays which have been transmitted through them.

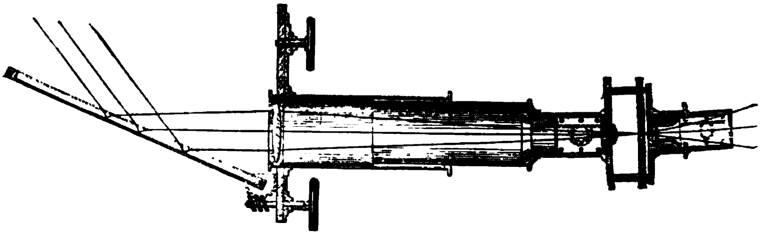

Fig. 688.—Solar Microscope.

strong) of nitrate of silver. The film of collodion is thus brought to an opal tint, and the plate, after being allowed to drain, is ready for exposure in the camera.

After being *exposed*, the picture is *developed*, by the application of a liquid for which the following is a formula;

Distilled water, ...	250 grammes.
Pyrogallic acid, ...	1 „
Crystallizable acetic acid, ...	20 „

When the picture is sufficiently developed, it is *fixed*, by the application of a solution, either of hyposulphite of soda from 25 to 30 per cent. strong, or of cyanide of potassium 3 per cent. strong, and the negative is completed.

To obtain a positive, the negative plate is laid upon a sheet of paper in a glass dish, the paper having been sensitized by immersing it first in a solution of common sea-salt 3 or 4 per cent. strong, and then in a solution of nitrate of silver 18 per cent. strong. The exposure is continued till the tone is sufficiently deep, the tint is then improved by means of a salt of gold, and the picture is fixed by hyposulphite of soda. It has then only to be washed and dried.—*D.*

The lens employed for producing the image is usually compound, consisting of a convex and a concave lens combined.

The electric light can be employed instead of the sun. The apparatus for regulating this light is usually placed within a lantern (Fig. 689), in such a position that the light is at the centre of curvature of a spherical mirror, so that the inverted image of the light coincides with the light itself. The light is concentrated on the object by a system of lenses, and, after passing through the object, traverses another system of lenses, placed at such a distance from the object as to throw a highly-magnified image of it on a screen. The whole arrangement is called the *electric* or *photo-electric microscope.*

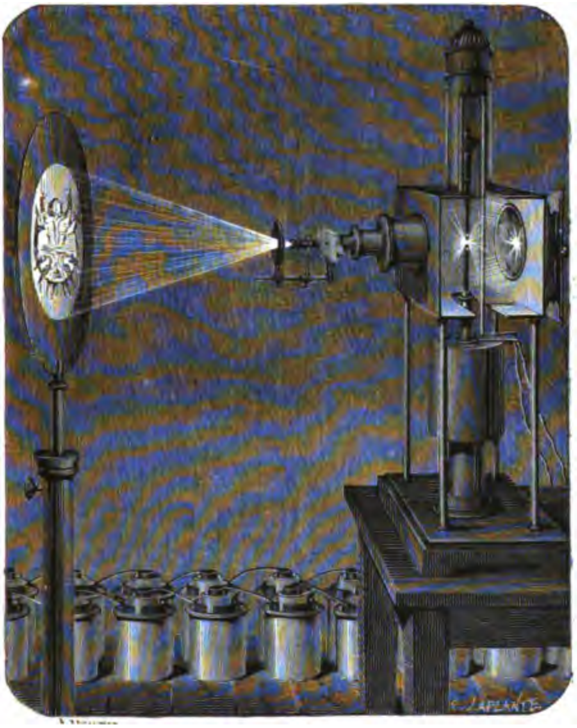

Fig. 689.—Photo-electric Microscope.

The magic lantern is a rougher instrument of the same kind, employed for projecting magnified images of transparent paintings, executed on glass slides. It has one lens for converging a beam of light on the slide, and another for throwing an image of the slide on the screen. In all these cases the image is inverted.

CHAPTER LXI.

VISION AND OPTICAL INSTRUMENTS.

752. Description of the Eye.—The human eye (Fig. 690) is a nearly spherical ball, capable of turning in any direction in its socket. Its outermost coat is thick and horny, and is opaque except in its anterior portion. Its opaque portion H is called the *sclerotica*, or in common language the white of the eye. Its transparent portion A is called the *cornea*, and has the shape of a very convex watch-glass. Behind the cornea is a diaphragm D, of annular form, called the *iris*. It is coloured and opaque, and the circular aperture C in its centre

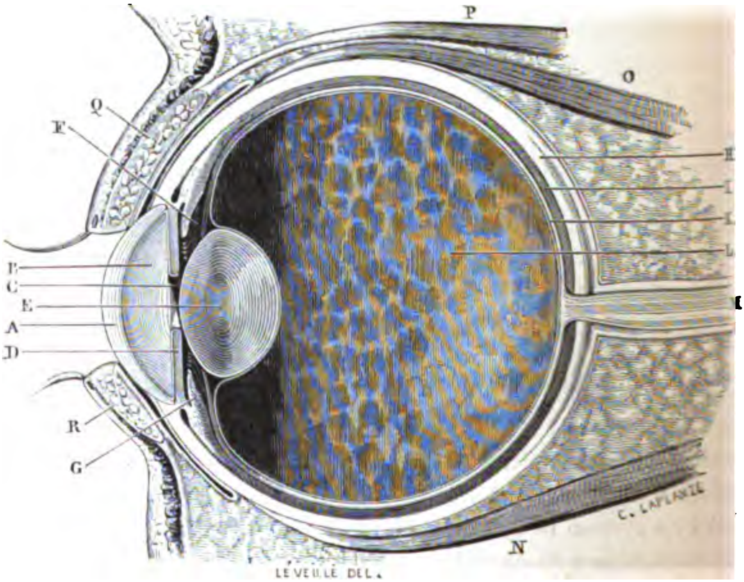

Fig. 690.—Human Eye.

is called the *pupil.* By the action of the involuntary muscles of the iris, this aperture is enlarged or contracted on exposure to darkness or light. The colour of the iris is what is referred to when we speak of the colour of a person's eyes. Behind the pupil is the *crystalline lens* E, which has greater convexity at back than in front. It is built up of layers or shells, increasing in density inwards. This latter circumstance tends to diminish spherical aberration, the effect of which, in an ordinary lens, is to make rays which pass near the outside too convergent, as compared with those which pass near the axis. The cavity B between the cornea and the crystalline is called the anterior chamber, and is filled with a watery liquid called the *aqueous humour.* The much larger cavity L, behind the crystalline, is called the posterior chamber, and is filled with a transparent jelly called the *vitreous humour,* inclosed in a very thin transparent membrane (the hyaloid membrane). The posterior chamber is inclosed, except in front, by the *choroid coat* or *uvea* I, which is saturated with an intensely black and opaque mucus, called the *pigmentum nigrum.* The choroid is lined, except in its anterior portion, with another membrane K, called the retina, which is traversed by a ramified system of nerve filaments diverging from the optic nerve M. Light incident on the retina gives rise to the sensation of vision; and there is no other part of the eye which possesses this property.

753. The Eye as an Optical Instrument.—It is clear, from the above description, that a pencil of rays entering the eye from an external point will undergo a series of refractions, first at the anterior surface of the cornea, and afterwards in the successive layers of the crystalline lens, all tending to render them convergent (see table of indices, § 724). A real and inverted image is thus formed of any external object to which the eye is directed. If this image falls on the retina, the object is seen; and if the image thus formed on the retina is sharp and sufficiently luminous, the object is seen distinctly.

754. Adaptation to Different Distances.—As the distance of an image from a lens varies with the distance of the object, it would only be possible to see objects distinctly at one particular distance, were there not special means of adaptation in the eye. Persons whose sight is not defective can see objects in good definition at all distances exceeding a certain limit. When we wish to examine the minute details of an object to the greatest advantage, we hold it at a particular distance, which varies in different individuals, and averages about eight inches. As we move it further away, we experience rather more ease in looking at it, though the diminution of its apparent size, as measured by the visual angle, renders its minuter features less visible. On the other hand, when we bring it nearer to the eye than the distance which gives the best view, we cannot see it distinctly without more or less effort and sense of strain: and when we have brought it nearer than a certain lower limit (averaging about six inches), we find distinct vision no longer possible. In looking at very distant objects, if our vision is not defective, we have very little sense of effort. These phenomena are in accordance with the theory of lenses, which shows that when the distance of an object is a large multiple of the focal length of the lens, any further increase, even up to infinity, scarcely alters the distance of the image; but that, when the object is comparatively near, the effect of any change of its distance is considerable. There has been much discussion among physiologists as to the precise nature of the changes by which we adapt our eyes to distinct vision at different distances. Such adaptation might consist either in a change of focal length, or in a change of distance of the retina. Observations in which the eye of the patient is made to serve as a mirror, giving images by reflection at the front of the cornea, and at the front and back of the crystalline, have shown that the convexity of the front of the crystalline is materially changed as the patient adapts his eye to near or remote vision, the convexity being greatest for near vision. This increase of convexity corresponds to a shortening of focal length, and is thus consistent with theory.

755. Binocular Vision.—The difficulty which some persons have felt in reconciling the fact of an inverted image on the retina with the perception of an object in its true position, is altogether fanciful and arises from confused notions as to the nature of perception.

The question as to how it is that we see objects single with two eyes, rests upon a different footing, and is not to be altogether explained by habit and association.[1] To each point in the retina of one eye there is a *corresponding point*, similarly situated, in the other. An impression produced on one of these points is, in ordinary circumstances, undistinguishable from a similar impression produced on the other, and when both at once are similarly impressed, the effect is simply more intense than if one were impressed alone; or, to describe the same phenomena subjectively, we have only one field of view for our two eyes, and in any part of this field of view we see either one image, brighter than we should see it by one alone, or else we see two overlapping images. This latter phenomenon can be readily illustrated by holding up a finger between one's eyes and a wall, and looking at the wall. We shall see, as it were, two transparent fingers projected on the wall. One of these transparent fingers is in fact seen by the right eye, and the other by the left, but our visual sensations do not directly inform us which of them is seen by the right eye, and which by the left.

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[1] Binocular vision is a subject which has been much debated. For the account here given of it, the Editor is alone responsible.

The principal advantage of having two eyes is in the estimation of distance, and the perception of relief. In order to see a point as single by two eyes, we must make its two images fall on corresponding points of the retinæ; and this implies a greater or less convergence of the optic axes according as the object is nearer or more remote. We are thus furnished with a direct indication of the distance of the object from our eyes; and this indication is much more precise than that derived from the adjustment of their focal length.

In judging of the comparative distances of two points which lie nearly in the same direction, we are greatly aided by the parallactic displacement which occurs when we change our own position.

We can also form an estimate of the nearness of an object, from the amount of change in its apparent size, contour, and bearing, produced by shifting our position. This would seem to be the readiest means by which very young animals can distinguish near from remote objects.

756. Stereoscope.—The perception of relief is closely connected with the doubleness of vision which occurs when the images on corresponding portions of the two retinæ are not similar. In surveying an object we run our eyes rapidly over its surface, in such a way as always to attain single vision of the particular point to which our attention is for the instant directed. We at the same time receive a somewhat indistinct impression of all the points within our field of view; an impression which, when carefully analyzed, is found to involve a large amount of doubleness. These various impressions combine to give us the perception of relief; that is to say, of *form in three dimensions.*

The perception of relief in binocular vision is admirably illustrated by the *stereoscope,* an instrument which was invented by Wheatstone, and reduced to its present more convenient form by Brewster. Two figures are drawn, as in Fig. 691, being perspective representations of the same object from two neighbouring points of view, such as might be occupied by the two eyes in looking at the object. Thus if the object be a cube, the right eye will have a fuller view of the right face, and the left eye of the left face. The two pictures are placed in the right and left compartments of a box, which has a partition down the centre serving to insure that each eye shall see only the picture intended for it; and over each of the compartments a half-lens is fixed, serving, as in Fig. 693, not only to magnify the picture, but at the same time to displace it, so that the two virtual images are brought into approximate coincidence. Stereoscopic pictures are usually photographs obtained by means of a double camera, having two objectives, one beside the other, which play the part of two eyes.

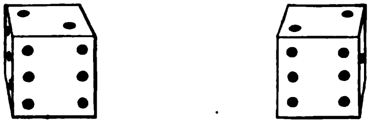

Fig. 691.—Stereoscopic Pictures.

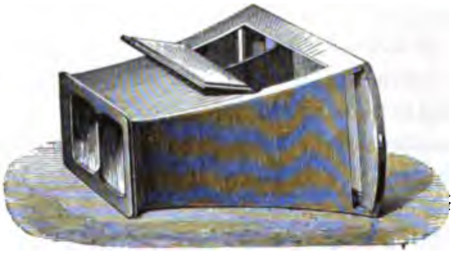

Fig. 692.—Stereoscope.

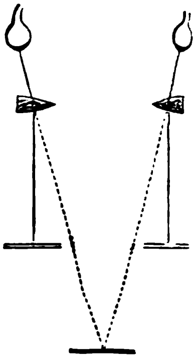

Fig. 693.—Path of Rays in Stereoscope.

When matters are properly arranged, the observer seems to see the object in relief. He finds himself able to obtain single view of any one point of the solid image which is before him; and the adjustments of the optic axes which he finds it necessary to make, in shifting his view from one point of it to another, are exactly such as would be required in looking at a solid object.

When one compartment of the stereoscope is empty, and the other contains an object, an observer, of normal vision, looking in in the ordinary way, is unable to say which eye sees the object. If two pictures are combined, consisting of two equal circles, one of them having a cross in its centre, and the other not, he is unable to decide whether he sees the cross with one eye or both.

When two entirely dissimilar pictures are placed in the two compartments, they compete for mastery, each of them in turn becoming more conspicuous than the other, in spite of any efforts which the observer may make to the contrary. A similar fluctuation will be observed on looking steadily at a real object which is partially hidden from one eye by an intervening object. This tendency to alternate preponderance renders it well nigh impossible to combine two colours by placing one under each eye in the stereoscope.

The immediate visual impression, when we look either at a real solid object, or at the apparently solid object formed by properly combining a pair of stereoscopic views, is a single picture formed of two slightly different pictures superimposed upon each other. The coincidence becomes exact at any point to which attention is directed, and to which the optic axes are accordingly made to converge, but in the greater part of the combined picture there is a want of coincidence, which can easily be detected by a collateral exercise of attention. The fluctuation above described to some extent tends to conceal this doubleness; and in looking at a real solid object, the concealment is further assisted by the blurring of parts which are out of focus.

757. Visual Angle. Magnifying Power.—The angle which a given straight line subtends at the eye is called its *visual angle*, or the *angle under which it is seen*. This angle is the measure of the length of the image of the straight line on the retina. Two discs at different distances from the eye, are said to have the same apparent size, if their diameters are seen under equal angles. This is the condition that the nearer disc, if interposed between the eye and the remoter disc, should be just large enough to conceal it from view.

The angle under which a given line is seen, evidently depends not only on its real length, and the direction in which it points, but also on its distance from the eye; and varies, in the case of small visual angles, in the inverse ratio of this distance. The *apparent length* of a straight line may be regarded as measured by the visual angle which it subtends.

By the *magnifying power* of an optical instrument, is usually meant the ratio in which it increases *apparent lengths* in this sense.

In the case of telescopes, the comparison is between an object as seen in the telescope, and the same object as seen with the naked eye at its actual distance. In the case of microscopes, the comparison is between the object as seen in the instrument, and the same object as seen by the naked eye at the least distance of distinct vision, which is usually assumed as 10 inches.

But two discs, whose diameters subtend the same angle at the eye, may be said to have the same *apparent area;* and since the areas of similar figures are as the squares of their linear dimensions, it is evident that the apparent area of an object varies as the square of the visual angle subtended by its diameter. The number expressing *magnification of apparent area* is therefore the square of the magnifying power as above defined. Frequently, in order to show that the comparison is not between apparent areas, but between apparent lengths, an instrument is said to magnify so many *diameters.* If the diameter of a sphere subtends 1° as seen by the naked eye, and 10° as seen in a telescope, the telescope is said to have a magnifying power of 10 diameters. The superficial magnification in this case is evidently 100.

The apparent length and apparent area of an object are respectively proportional to the length and area of its image on the retina.

Apparent length is measured by the plane angle, and apparent area by the solid angle, which an object subtends at the eye.

758. Spectacles.—Spectacles are of two kinds, intended to remedy two opposite defects of vision. Short-sighted persons can see objects distinctly at a smaller distance than persons whose vision is normal; but always see distant objects confused. On the other hand, persons whose vision is normal in their youth, usually become over-sighted with advancing years, so that, while they can still adjust their eyes correctly for distant vision, objects as near as 10 or 12 inches always appear blurred. Spectacles for over-sighted persons are convex, and should be of such focal length, that, when an object is held at about 10 inches distance, its virtual image is formed at the nearest distance of distinct vision for the person who is to use them. This latter distance must be ascertained by trial. Call it p inches; then, by § 745, the formula for computing the required focal length x (in inches) is

$$\frac{1}{10} - \frac{1}{p} = \frac{1}{x}.$$

For example, if 15 inches is the nearest distance at which the person can conveniently read without spectacles, the focal length required is 30 inches.

In Fig. 694, A represents the position of a small object, and A′ that of its virtual image as seen with spectacles of this kind.

Over-sight is not the only defect which the eye is liable to acquire by age; but it is the defect which ordinary spectacles are designed to remedy.

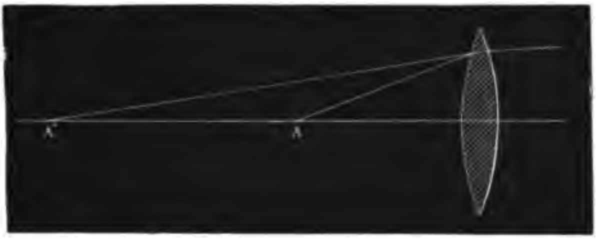

Fig. 694.—Spectacle-glass for Over-sighted Eye.

Spectacles for short-sighted persons are concave, and the focal length which they ought to have, if designed for reading, may be computed by the formula

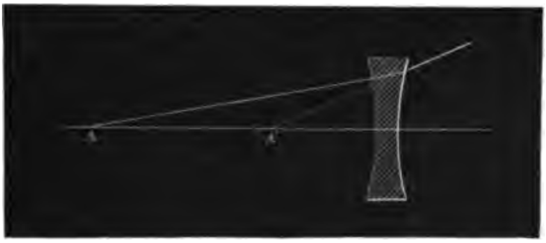

Fig. 695.—Spectacle-glass for Short-sighted Eye.

$$\frac{1}{p} - \frac{1}{10} = \frac{1}{x},$$

p denoting the nearest distance at which the person can read, and x the focal length, both in inches. If his *greatest* distance of distinct vision exceeds the focal length, he will be able, by means of the spectacles, to obtain distinct vision of objects at all distances, from 10 inches upwards.

759. Simple Magnifier.—A *magnifying-glass* is a convex lens, of shorter focal length than the human eye, and is placed at a distance somewhat less than its focal length from the object to be viewed. In Fig. 696, *a b* is the object, and A B the virtual image which is seen by the eye K. The construction which we have employed for drawing the image is one which we have several times used before. Through the point *a*, the line *a* M is drawn parallel to the principal axis. F M is then drawn from the principal focus F; O *a* is drawn from the optical centre O; and these two lines are produced till they meet in A.

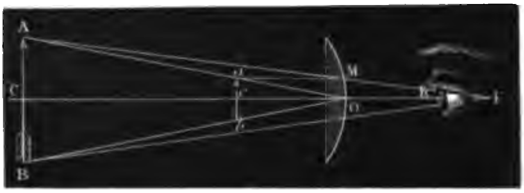

Fig. 696.—Magnifying-glass.

Distance of lens from object. In order that the image may be properly seen, its distance from the eye must fall between the limits of distinct vision; and in order that it may be seen under the largest possible visual angle, the eye must be close to the lens, and the object must be as near as is compatible with distinct vision. This and other interesting properties are established by the following investigation:—

Let θ denote the visual angle under which the observer sees the image of the portion *a c* of the object. Also let x denote the distance *c* O of the object from the lens, and y the distance O K of the lens from the eye. Then we have

$$\tan\theta = \frac{\mathrm{AC}}{\mathrm{CK}} = \frac{\mathrm{AC}'}{\mathrm{CO}+y};$$

But, by formulæ (10) and (11) of last chapter, we have

$$\mathrm{AC} = ac\,\frac{f}{f-x}, \quad \mathrm{CO} = x.\frac{f}{f-x}.$$

Substituting these values for A C and C O, and reducing, we have

$$\tan\theta = ac.\frac{f}{(x+y)f-xy}. \qquad \text{(A)}$$

This equation shows that, for a given lens and a given object, the visual angle varies inversely as the quantity $(x+y)f-xy$.

The following practical consequences are easily drawn:—

(1) If the distance $x+y$ of the eye from the object is given, the visual angle increases as the two distances x, y approach equality, and is not altered by interchanging them.

(2) If one of the two distances x, y be given, and be less than f, the other must be made as small as possible, if we wish to obtain the largest possible visual angle.

To obtain the absolute maximum of visual angle, we must select, from the various positions which make C K equal to the nearest distance of distinct vision, that which gives the largest value of A C, since the quotient of A C by C K is the tangent of the visual angle. Now A C increases as the image moves further from the lens, and hence the absolute maximum is obtained by making its distance from the lens equal to the nearest distance of distinct vision, and making the eye come up close to the lens. In this case the distance p of the object from the lens is given by the equation $\frac{1}{p}-\frac{1}{\mathrm{D}}=\frac{1}{f}$, where D denotes the nearest distance of distinct vision, and $\tan\theta$ is $\frac{ac}{p}$ or $ac\left(\frac{1}{f}+\frac{1}{\mathrm{D}}\right)$. But the greatest angle under which the body could be seen by the naked eye is the angle whose tangent is $\frac{ac}{\mathrm{D}}$; hence the visual angle (or its tangent) is increased by the lens in the ratio $1+\frac{\mathrm{D}}{f}$, which is called the *magnifying power*. If the object were in the principal focus, and the eye close to the lens, the magnifying power would be $\frac{\mathrm{D}}{f}$. In either case, the thickness of the lens being neglected, the visual angle is the angle which the object subtends at the centre of the lens, and therefore varies inversely as the distance of this centre from the object. For lenses of small focal length, the reciprocal of the focal length may be regarded as proportional to the magnifying power.

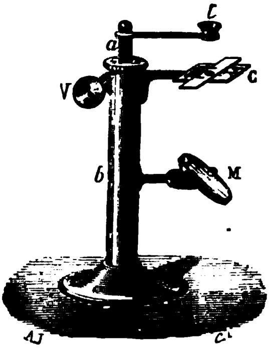

Fig. 697.—Simple Microscope.

Simple Microscope.—By a *simple microscope* is usually understood a lens mounted in a manner convenient for the examination of small objects. Fig. 697 represents an instrument of this kind. The lens l is mounted in brass, and carried at the end of an arm. It is raised and lowered by turning the milled head V, which acts on the rack *a*. C is the platform on which the object is laid, and M is a concave mirror, which can be employed for increasing the illumination of the object.

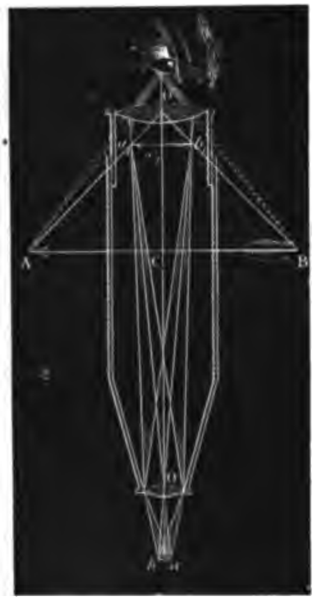

Fig. 698.—Compound Microscope.

760. Compound Microscope.—In the compound microscope, there is one lens which forms a real and greatly enlarged image of the object; and this image is itself magnified by viewing it through another lens.

In Fig. 698, $a\,b$ is the object, O is the first lens, called the *objective*, and is placed at a distance only slightly exceeding its focal length from the object; an inverted image $a_1\,b_1$ is thus formed, at a much greater distance on the other side of the lens, and proportionally larger. O′ is the second lens, called the *ocular* or *eye-piece*, which is placed at a distance a little less than its focal length from the first image $a_1\,b_1$, and thus forms an enlarged virtual image of it A B, at a convenient distance for distinct vision.

If we suppose the final image A B to be at the least distance of distinct vision from the eye placed at O′ (this being the arrangement which gives the largest visual angle), the magnifying power will be simply the ratio of the length of this image to that of the object $a\,b$, and will be the product of the two factors $\frac{\mathrm{A\,B}}{a_1\,b_1}$ and $\frac{a_1\,b_1}{a\,b}$. The former is the magnification produced by the eye-piece, and is, as we have just shown (§ 759), $1 + \frac{\mathrm{D}}{f}$. The other factor $\frac{a_1\,b_1}{a\,b}$ is the magnification produced by the objective, and is equal to the ratio of the distances $\frac{\mathrm{O}\,a_1}{\mathrm{O}\,a}$. If the objective is taken out, and replaced by another of different focal length, the readjustment will consist in altering the distance O a, leaving the distance O a_1 unchanged. The total magnification therefore varies inversely as O a, that is, nearly in the inverse ratio of the focal length of the objective. Compound microscopes are usually provided with several objectives, of various focal lengths, from which the observer makes a selection according to the magnifying power which he requires for the object to be examined. The powers most used range from 50 to 350 diameters.

The magnifying power of a microscope can be determined by direct observation, in the following way. A plane reflector pierced with a hole in its centre, is placed directly over the eye-piece (Fig. 699), at an inclination of 45°, and another plane reflector, or still better, a totally reflecting prism, as in the figure, is placed parallel to it at the distance of an inch or two, so that the eye, looking down upon the first mirror, sees, by means of two successive reflections, the image of a divided scale placed at a distance of 8 or 10 inches below the second reflector. In taking an observation, a micrometer scale engraved on glass, its divisions being at a known distance apart (say $\frac{1}{100}$ of a millimetre), is placed in the microscope as the object to be magnified; and the observer holds his eye in such a position that, by means of different parts of his pupil, he sees at once the magnified image of the micrometer scale in the microscope, and the reflected and unmagnified image of the other scale. The two images will be superimposed in the same field of view; and it is easy to observe how many divisions of the one coincide with a given number of divisions of the other. Let the divisions on the large scale be millimetres, and those on the micrometer scale hundredths of a millimetre. Then the magnifying power is 100, if one of the magnified covers one of the unmagnified divisions; and is $\frac{100\,N}{n}$, if n of the former cover N of the latter. This is on the assumption that the large scale is placed at the nearest distance of convenient vision. In stating the magnifying power of a microscope, this distance is usually reckoned as 10 inches.

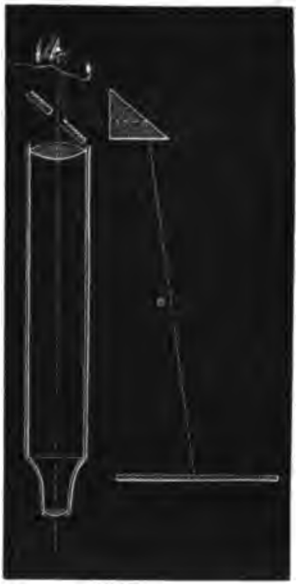

Fig. 699.
Measurement of Magnifying Power.

A short-sighted person sees an image in a microscope (whether simple or compound) under a larger visual angle than a person of normal sight; but the inequality is not so great as in the case of objects seen by the naked eye. In fact, if f be the focal length of the eye-piece in a compound microscope, or of the microscope itself if simple, and D the nearest distance of distinct vision for the observer, the visual angle under which the image is seen in the microscope is proportional to $\frac{1}{f}+\frac{1}{\mathrm{D}}$, the greatest visual angle for the naked eye being represented by $\frac{1}{\mathrm{D}}$. Both these angles increase as D diminishes, but the latter increases in a greater ratio than the former. When f is as small as $\frac{1}{10}$ of an inch, the visual angle in the microscope is sensibly the same for short as for normal sight.

Before reading off the divisions in the observation above described, care should be taken to focus the microscope in such a way, that the image of the micrometer scale is at the same distance from the eye as the image of the large scale with which it is compared. When this is done, a slight motion of the eye does not displace one image with respect to the other.

Instead of a single eye-lens, it is usual to employ two lenses separated by an interval, that which is next the eye being called the *eye-glass*, and the other the *field-glass*. This combination is equivalent to the Huyghenian or negative eye-piece employed in telescopes (§ 800).

761. Astronomical Telescope.—The astronomical refracting telescope consists essentially (like the compound microscope) of two lenses, one of which forms a real and inverted image of the object, which is looked at through the other.

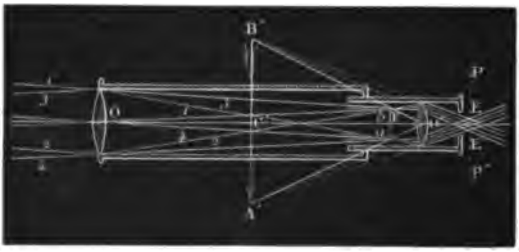

Fig. 700.—Astronomical Telescope.

In Fig. 700, O is the object-glass, which is sometimes a foot or more in diameter, and is always of much greater focal length than the eye-piece O′. The inverted image of a distant object is formed at the principal focus F. This image is represented at *a b*. The parallel rays marked 1, 2 come from the upper extremity of the object, and meet at *a*; and the parallel rays 3, 4, from the other extremity, meet at *b*. A′ B′ is the virtual image of *a b* formed by the eye-piece. Its distance from the eye can be changed by pulling out or pushing in the eye-tube; and may in practice have any value intermediate between the least distance of distinct vision and infinity, the visual angle under which it is seen being but slightly affected by this adjustment. The rays from the highest point of the object emerge from the eye-piece as a pencil diverging from A′; and the rays from the lowest point of the object form a pencil diverging from B′.

Magnification.—The angle under which the object would be seen by the naked eye is $a\,O\,b$; for the rays $a\,O$, $b\,O$, if produced, would pass through its extremities. The angle under which it is seen in the telescope, if the eye be close to the eye-lens, is $A'\,O'\,B'$ or $a\,O'\,b$.

The magnification is therefore $\frac{a\,O'\,b}{a\,O\,b}$, which is approximately the same as the ratio of the distances of the image $a\,b$ from the two lenses $\frac{O\,F}{O'\,F}$. If the eye-tube is so adjusted as to throw the image $A'\,B'$ to infinite distance, F will be the principal focus of both lenses, and the magnification is the ratio of the focal length of the object-glass to that of the eye-piece.

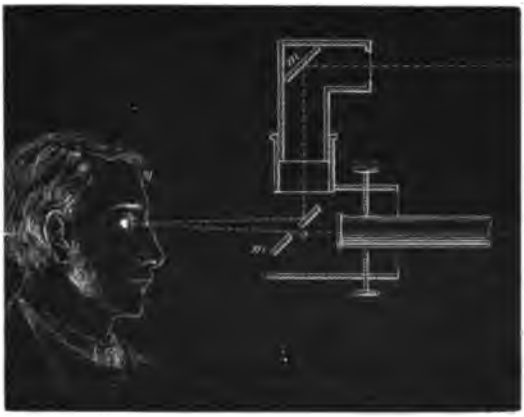

Fig. 701.—Measurement of Magnifying Power.

If the eye-tube be pushed in as far as is compatible with distinct vision (the eye being close to the lens), the magnification is greater than this in the ratio $\frac{D+f}{D}$, D denoting the nearest distance of distinct vision, and f the focal length of the eye-piece.

The magnification can be directly observed by looking with one eye through the telescope at a brick wall, while the other eye is kept open. The image will thus be superimposed on the actual wall, and we have only to observe how many courses of the latter coincide with a single course of the magnified image.

If the telescope is large, its tube may prevent the second eye from seeing the wall, and it may be necessary to employ a reflecting arrangement, as in Fig. 701, analogous to that described in connection with the microscope.

Telescopes without stands seldom magnify more than about 10 diameters. Powers of from 20 to 60 are common in telescopes with stands, intended for terrestrial purposes. The powers chiefly employed in astronomical observation are from 100 to 500.

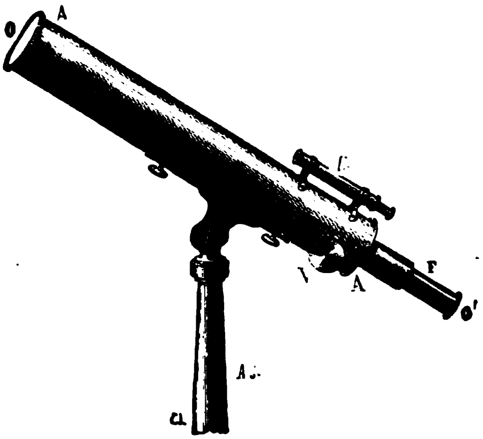

Fig. 702.—Astronomical Telescope.

Mechanical Arrangements.—The achromatic object-glass O is set in a mounting which is screwed into one end of a strong brass tube A A (Fig. 702). In the other end slides a smaller tube F containing the eye-piece O′; and by turning the milled head V in one direction or the other, the eye-piece is moved forwards or backwards.

Finder.—The small telescope *l*, which is attached to the principal telescope, is called a *finder*. This appendage is indispensable when the principal telescope has a high magnifying power; for a high magnifying power involves a small field of view, and consequent difficulty in directing the telescope so as to include a selected object within its range. The finder is a telescope of large field; and as it is set parallel to the principal telescope, objects will be visible in the latter if they are seen in the centre of the field of view of the former.

762. Best Position for the Eye.—The eye-piece forms a real and inverted image of the object-glass[1] at E E′ (Fig. 700), through which all rays transmitted by the telescope must of necessity pass. If the telescope be directed to a bright sky, and a piece of white paper held behind the eye-piece to serve as a screen, a circular spot of light will be formed upon it, which will become sharply defined (and at the same time attain its smallest size) when the screen is held in the correct position. This image (which we shall call the *bright spot*) may be regarded as marking the proper place for the pupil of the observer's eye. Every ray which traverses the centre of the object-glass traverses the centre of this spot; every ray which traverses the upper edge of the object-glass traverses the lower edge of the spot; and any selected point of the spot receives all the rays which have been transmitted by one particular point of the object-glass. An eye with its pupil anywhere within the limits of the bright spot, will therefore see the whole field of view of the telescope. If the spot and pupil are of exactly the same size, they must be made to coincide with one another, as the necessary condition of seeing the whole field of view with the brightest possible illumination. Usually in practice the spot is much smaller than the pupil, so that these advantages can be obtained without any nicety of adjustment; but to obtain the most distinct vision, the centre of the pupil should coincide as closely as possible with the centre of the spot. To facilitate this adjustment, a brass diaphragm, with a hole in its centre, is screwed into the eye-end of the telescope, the proper place for the eye being close to this hole.

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[1] Or it may be called an image of *the aperture which the object-glass fills*. It remains sensibly unchanged on removing the object-glass so as to leave the end of the telescope open.

One method of determining the magnifying power of a telescope consists in measuring the diameter of the bright spot, and comparing it with the effective aperture of the object-glass. In fact, let F and f denote the focal lengths of object-glass and eye-piece, and a the distance of the spot from the centre of the eye-piece; then $\mathrm{F}+f$ is approximately the distance of the object-glass from the same centre, and, by the formula for conjugate focal distances, we have $\frac{1}{\mathrm{F}+f}+\frac{1}{a}=\frac{1}{f}$. Multiplying both sides of this equation by $\mathrm{F}+f$, and then subtracting unity, we have $\frac{\mathrm{F}+f}{a}=\frac{\mathrm{F}}{f}$. But the ratio of the diameter of the object-glass to that of its image is $\frac{\mathrm{F}+f}{a}$; and $\frac{\mathrm{F}}{f}$ is the usual formula for the magnifying power. Hence, *the linear magnifying power of a telescope is the ratio of the diameter of the object-glass to that of the bright spot.*

763. Terrestrial Telescope.—The astronomical telescope just described gives inverted images. This is no drawback in astronomical observation, but would be inconvenient in viewing terrestrial objects. In order to re-invert the image, and thus make it erect, two additional lenses O″ O‴ (Fig. 703) are introduced between the real image $a\,b$ and the eye-lens O′. If the first of these O″ is at the distance of its principal focal length from $a\,b$, the pencils which fall upon the second will be parallel, and an erect image $a'\,b'$ will thus be formed in the principal focus of O‴. This image is viewed through the eye-lens O′, and the virtual image A′ B′ which is perceived by the eye will therefore be erect. The two lenses O″, O‴, are usually made precisely alike, in which case the two images *a b*, *a′ b′* will be equal. In the better class of terrestrial telescopes, a different arrangement is adopted, requiring one more lens; but whatever system be employed, the reinversion of the image always involves some loss both of light and of distinctness.

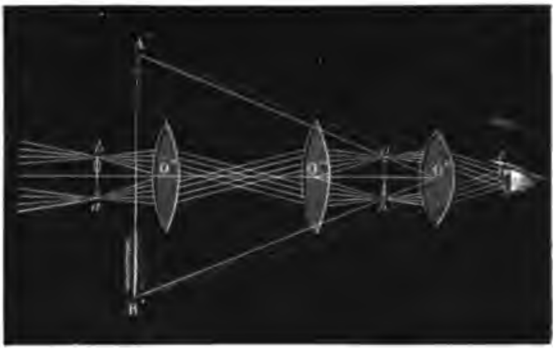

Fig. 703.—Terrestrial Eye-piece.

764. Galilean Telescope.—Besides the disadvantages just mentioned, the erecting eye-piece involves a considerable addition to the length of the instrument. The telescope invented by Galileo, and the earliest of all telescopes, gives erect images with only two lenses, and with shorter length than even the astronomical telescope. O (Fig. 704) is the object-glass, which is convex as in the astronomical telescope, and would form a real and inverted image *a b* at its principal focus; but the eye-glass O′, which is a concave lens, is interposed at a distance equal to or slightly exceeding its own focal length from the place of this image, and forms an erect virtual image A′ B′, which the observer sees.

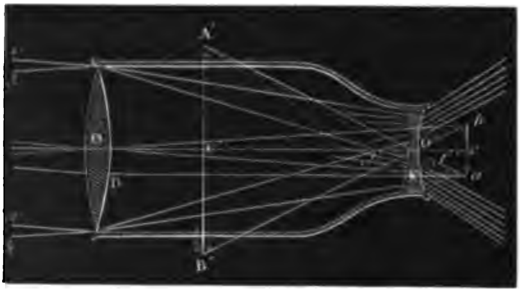

Fig. 704.—Galilean Telescope.

Neglecting the distance of his eye from the lens, the angle under which he sees the image is A′ O′ B′, which is equal to *a* O′ *b*, whereas the visual angle to the naked eye would be aOb. The magnification is therefore $\frac{aO'b}{aOb}$, which is approximately equal to $\frac{Oc}{O'c}$, c being the principal focus of the object-glass. If the instrument is focussed in such a way that the image A′ B′ is thrown to infinite distance, c is also the principal focus of the eye-lens, and the magnification is simply the ratio of the focal lengths of the two lenses. This is the same rule which we deduced for the astronomical telescope; but the Galilean telescope, if of the same power, is shorter by twice the focal length of the eye-lens, since the distance between the two lenses is the difference instead of the sum of their focal lengths.

This telescope has the disadvantage of not admitting of the employment of cross-wires; for these, in order to serve their purpose, must coincide with the real image; and no such image exists in this telescope.

There is another peculiarity in the absence of the *bright spot* above described, the image of the object-glass formed by the eye-glass being virtual. In other telescopes, if half the object-glass be covered, half the bright spot will be obliterated; but the remaining half suffices for giving the whole field of view, though with diminished brightness. In the Galilean telescope, on the contrary, if half the object-glass be covered, half the field of view will be cut off, and the remaining half will be unaffected.

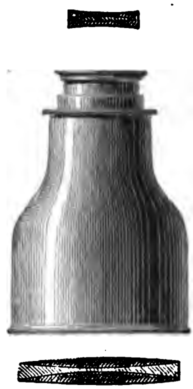

Fig. 705.—Opera-glass.

The *opera-glass*, single or binocular, is a Galilean telescope, or a pair of Galilean telescopes. In the best instruments, both object-glass and eye-glass are achromatic combinations of three pieces, as shown in section in the figure (Fig. 705); the middle piece in each case being flint, and the other two crown (§ 794).

765. Reflecting Telescopes.—In reflecting telescopes, the place of an object-glass is supplied by a concave mirror called a *speculum*, usually composed of solid metal. The real and inverted image which it forms of distant objects is, in the Herschelian telescope, viewed directly through an eye-piece, the back of the observer being towards the object, and his face towards the speculum. This construction is only applicable to very large specula; as in instruments of ordinary size the interposition of the observer's head would occasion too serious a loss of light.

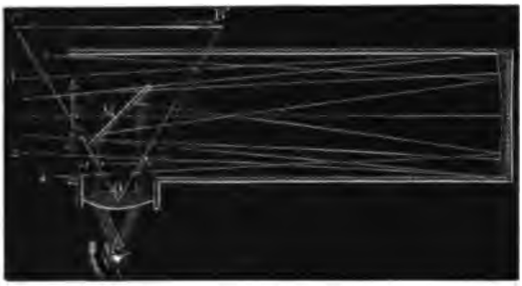

Fig. 706.—Newtonian Telescope.

An arrangement more frequently adopted is that devised by Sir Isaac Newton, and employed by him in the first reflecting telescope ever constructed. It is represented in Fig. 706. The speculum is at the bottom of a tube whose open end is directed towards the distant object to be examined. The rays 1 and 2 from one extremity of the object are reflected towards *a*, and the rays 3, 4 from the other-extremity are reflected towards *b*. A real inverted image *ab* would thus be formed at the principal focus of the concave speculum: but a small plane mirror M is interposed obliquely, and causes the real image to be formed at *a' b'* in a symmetrical position with respect to the mirror M. The eye-lens O′ transforms this into the enlarged and virtual image A′ B′.

Magnifying Power.—The approximate formula for the magnifying power is the same as in the case of the refracting telescopes already described. In fact the first image *a b* subtends, at the optical centre O (not shown in the figure) of the large speculum, an angle aOb equal to the visual angle for the naked eye; and the second image *a' b'* (which is equal to the former) subtends, at the centre of the eye-piece, an angle *a'* O′ *b'* equal to the angle under which the image is seen in the telescope. The magnifying power is therefore $\frac{a'O'b'}{aOb}$, or, what is the same thing, is the ratio of the distance of *a b* from O to the distance of *a' b'* from O′, or the ratio of the focal length of the speculum to that of the eye-piece.

In the Gregorian telescope, which was invented before that of Newton, but not manufactured till a later date, there are two concave specula. The large one, which receives the direct rays from the object, forms a real and inverted image. The smaller speculum, which is suspended in the centre of the tube, with its back to the object, receives the rays reflected from the first speculum, and forms a second real image, which is the enlarged and inverted image of the first, and is therefore erect as compared with the object. This real and erect image is then magnified by means of an eye-piece, as in the instruments previously described, the eye-piece being contained in a tube which slides in a hole pierced in the middle of the large speculum.

As this arrangement gives an erect image, and enables the observer to look directly towards the object, it is specially convenient for terrestrial observation. It is the construction almost universally adopted in reflecting telescopes of small size.

The Cassegranian telescope resembles the Gregorian, except that the second speculum is convex, and the image which the observer sees is inverted.

766. Silvered Specula.—Achromatic refracting telescopes give much better results, both as regards light and definition, than reflectors of the same size or weight; but it has been found practicable to make specula of much larger size than object-glasses. The aperture of Lord Rosse's largest telescope is 6 feet, whereas the aperture of the largest achromatic telescopes yet constructed is less than two feet, and increase of size involves increased thickness of glass, and consequent absorption of light.

The massiveness which is found necessary in the speculum in order to prevent flexure, is a serious inconvenience, as is also the necessity for frequent repolishing—an operation of great delicacy, as the slightest change in the form of the surface impairs the definition of the images. Both these defects have been to a certain extent remedied by the introduction of glass specula, covered in front with a thin coating of silver. Glass is much more easily worked than speculum-metal (which is remarkable for its brittleness in casting). and has only one-third of its specific gravity. Silver is also much superior to speculum-metal in reflecting power; and as often as it becomes tarnished it can be removed and renewed, without liability to change of form in the reflecting surface.[1]

767. Measure of Brightness.—The brightness of a surface is most naturally measured by the amount of light per unit area of its image on the retina: and therefore varies *directly as the amount of light which the surface sends to the pupil, and inversely as the apparent area of the surface.*

To avoid complications arising from the varying condition of the observer, we shall leave dilatation and contraction of the pupil out of account.

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[1] The merits of silvered specula are fully set forth in a brochure published by Mr. Browning of the Minories, entitled *A Plea for Reflectors.*

When a body is looked at through a small pinhole in a card held close to the eye, it appears much darker than when viewed in the ordinary way; and in like manner images formed by optical instruments often furnish beams of light too narrow to fill the pupil. In all such cases it becomes necessary to distinguish between *effective brightness* and *intrinsic brightness*, the former being less than the latter in the same ratio in which the cross section of the beam which enters the pupil is less than the area of the pupil. We may correctly speak of the *intrinsic brightness* of a surface *for a particular point* of the pupil; and the effective brightness will in every case be the average value of the intrinsic brightness taken over the whole pupil.

In the case of natural bodies viewed in the ordinary way, the distinction may be neglected, since they usually send light in sensibly equal amounts to all parts of the pupil.

To obtain a numerical measure of intrinsic brightness, let us denote by e the area of the pupil, by s a small area on a surface directly facing towards the eye (or the foreshortened projection of a small area inclined to the line of vision), and by r the distance between e and s. Then the quantity of light q which s sends to e per unit time, varies jointly as the area e, the apparent area or real solid angle $\frac{s}{r^2}$, and the intrinsic brightness I. We may therefore write $q = \mathrm{I}\,e\,\frac{s}{r^2} = \mathrm{I}\,s\,\frac{e}{r^2}$; and if we put ω for the solid angle $\frac{e}{r^2}$ which the pupil subtends at s, we have $q = \mathrm{I}\,s\,\omega$. *The intrinsic brightness of a small area s is therefore measured by* $\frac{q}{s\,\omega}$, *where q denotes the quantity of light which s emits per unit time, in directions limited by the small solid angle of divergence* ω.

768. Applications.—One of the most obvious consequences is that *surfaces appear equally bright at all distances* in the same direction, provided that no light is stopped by the air or other intervening medium; for q and ω both vary inversely as the square of the distance. The area of the image formed on the retina in fact varies directly as the amount of light by which it is formed.

Images formed by Lenses.—Let A B (Fig. 707) be an object, and $a\,b$ its real image formed by the lens C D, whose centre is O. Let

S denote a small area at A, and Q the quantity of light which it sends to the lens; also let s denote the corresponding area of the image, and q the quantity of light which traverses it. Then q would be identical with Q if no light were stopped by the lens; the areas S, s, are directly as the squares of the conjugate focal distances O A, O a; and the solid angles of divergence Ω and ω, for Q and q, being the solid angles subtended by the lens at A and a (for the plane angle $c\,a\,d$ in the figure is equal to the vertical angle C a D), are inversely as the squares of the conjugate focal distances. We have accordingly $\frac{\Omega}{\omega}=\frac{s}{S}$ and $S\Omega=s\omega$. The intrinsic brightness $\frac{q}{s\omega}$ of the image is therefore equal to the intrinsic brightness $\frac{Q}{S\Omega}$ of the object, except in so far as light is stopped by the lens. Precisely similar reasoning applies to virtual images formed by lenses.[1]

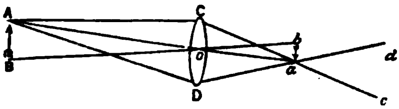

Fig. 707.—Brightness of Image.

In the case of images formed by mirrors, Ω and ω are the solid angles subtended by the mirror at the conjugate foci, and are inversely as the squares of the distances from the mirror; while S and s are directly as the squares of the distances from the centre of curvature; but these four distances are proportional (§ 407), so that the same reasoning is still applicable. If the mirror only reflects half the incident light, the image will have only half the intrinsic brightness of the object.

If the pupil is filled with light from the image, the effective brightness will be the same as the intrinsic brightness thus computed. If this condition is not fulfilled, the former will be less than the latter. When the image is greatly magnified as compared with the object, the angle of divergence is greatly diminished in comparison with the angle which the lens or mirror subtends at the object, and often becomes so small that only a small part of the pupil is utilized. This is the explanation of the great falling off of light which is observed in the use of high magnifying powers, both in microscopes and telescopes.

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[1] From the fundamental formula for refraction at a spherical surface, $\mu=\frac{OQ}{OF}\cdot\frac{AF}{AQ}$ (*Parkinson's Optics*, Art. 27), where Q and F are conjugate foci, O Q, O F their distances from the centre of curvature, and A Q, A F their distances from the refracting surface, we have at once, by squaring both sides, $\mu^2=\frac{S}{s}\,\frac{\Omega}{\omega}$. The product $s\,\omega$ is accordingly diminished in the ratio of μ^2 to 1 at the first surface of a lens, and increased again to its original value at the second surface. The conclusions deduced in the text will therefore remain true when the thickness of the lens is taken into account.

769. Brightness of Image in a Telescope.—It has been already pointed out (§ 762) that in most forms of telescope (the Galilean being an exception), there is a certain position, a little behind the eye-piece, at which a well-defined bright spot is formed upon a screen held there while the telescope is directed to any distant source of light. It has also been pointed out that this spot is the image, formed by the eye-piece, of the opening which is filled by the object-glass, and that the magnifying power of the instrument is the ratio of the size of the object-glass to the size of this bright spot.

Let s denote the diameter of the bright spot, o the diameter of the object-glass, e the diameter of the pupil of the eye; then $\frac{o}{s}$ is the linear magnifying power.

We shall first consider the case in which the spot exactly covers the pupil of the observer's eye, so that $s = e$. Then the whole light which traverses the telescope from a distant object enters the eye; and if we neglect the light stopped in the telescope, this is the whole light sent by the object to the object-glass, and is $\left(\frac{o}{e}\right)^2$ times that which would be received by the naked eye. The magnification of apparent area is $\left(\frac{o}{s}\right)^2$, which, from the equality of s and e, is the same as the increase of total light. The brightness is therefore the same as to the naked eye.

Next, let s be greater than e, and let the pupil occupy the central part of the spot. Then, since the spot is the image of the object-glass, we may divide the object-glass into two parts—a central part whose image coincides with the pupil, and a circumferential part whose image surrounds the pupil. All rays from the object which traverse the central part, traverse its image, and therefore enter the pupil; whereas rays traversing the circumferential part of the object-glass, traverse the circumferential part of the image, and so are wasted. The area of the central part (whether of the object-glass or of its image) is to the whole area as $e^2 : s^2$; and the light which the object sends to the central portion, instead of being $\left(\frac{o}{e}\right)^2$ times that which would be received by the naked eye, is only $\left(\frac{o}{s}\right)^2$ times. But $\left(\frac{o}{s}\right)^2$ is the magnification of apparent area. Hence the brightness is the same as to the naked eye. In these two cases, effective and intrinsic brightness are the same.

Lastly (and this is by far the most common case in practice), let s be less than e. Then no light is wasted, but the pupil is not filled. The light received is $\left(\frac{o}{e}\right)^2$ times that which the naked eye would receive; and the magnification of apparent area is $\left(\frac{o}{s}\right)^2$. The effective brightness of the image, is to the brightness of the object to the naked eye, as $(\frac{o}{e})^2 : (\frac{o}{s})^2$; that is, as $s^2 : e^2$; that is, as the area of the bright spot to the whole area of the pupil.

To correct for the light stopped by reflection and imperfect transparency, we have simply to multiply the result in each case by a proper fraction, expressing the ratio of the transmitted to the incident light. This ratio, for the central parts of the field of view, is about 0·85 in the best achromatic telescopes. In such telescopes, therefore, the brightness of the image cannot exceed 0·85 of the brightness of the object to the naked eye. It will have this precise value, when the magnifying power is equal to or less than $\frac{o}{e}$; and from this point upwards will vary inversely as the square of the linear magnification.

The same formulæ apply to reflecting telescopes, o denoting now the diameter of the large speculum which serves as objective; but the constant factor is usually considerably less than 0·85.

It may be accepted as a general principle in optics, that while it is possible, by bad focussing or instrumental imperfections, to obtain a confused image whose brightness shall be intermediate between the brightest and the darkest parts of the object, *it is impossible, by any optical arrangement whatever, to obtain an image whose brightest part shall surpass the brightest part of the object.*

770. Brightness of Stars.—There is one important case in which the foregoing rules regarding the brightness of images become nugatory. The fixed stars are bodies which subtend at the earth angles smaller than the *minimum visibile*, but which, on account of their excessive brightness, *appear* to have a sensible angular diameter. This is an instance of *irradiation*, a phenomenon manifested by all bodies of excessive brightness, and consisting in an extension of their apparent beyond their actual boundary. What is called, in popular language, a bright star, is a star which sends a large total amount of light to the eye.

Denoting by a the ratio of the transmitted to the whole incident light, a ratio which, as we have seen, is about 0·85 in the most favourable cases, and calling the light which a star sends to the naked eye unity, the light perceived in its image will be $a\left(\frac{o}{s}\right)^2$, or a × square of linear magnification, if the bright spot is as large as the pupil. When the eye-piece is changed, increase of power diminishes the size of the spot, and increases the light received by the eye, until the spot is reduced to the size of the pupil. After this, any further magnification has no effect on the quantity of light received, its constant value being $a\left(\frac{o}{e}\right)^2$.

The value of this last expression, or rather the value of $a\,o^2$, is the measure of what is called the *space-penetrating power* of a telescope; that is to say, the power of rendering very faint stars visible; and it is in this respect that telescopes of very large aperture, notably the great reflector of Lord Rosse, are able to display their great superiority over instruments of moderate dimensions.

We have seen that the total light in the visible image of a star remains unaltered, by increase of power in the eye-piece above a certain limit. But the visibility of faint stars in a telescope is promoted by darkening the back-ground of sky on which they are seen. Now the brightness of this back-ground varies directly as s^2, or inversely as the square of the linear magnification (s being supposed less than e). Hence it is advantageous, in examining very faint stars, to employ eye-pieces of sufficient power to render the bright spot much smaller than the pupil of the eye.

771. Images on a Screen.—Thus far we have been speaking of the brightness of images as viewed directly. Images cast upon a screen are, as a matter of fact, much less brilliant. Their brightness depends greatly on the nature of the screen, and can never exceed the brightness which the surface of the screen would exhibit if held very near the source of light. When a condensing lens is used to collect the rays of a lamp, an eye placed at the conjugate focus sees the whole lens full of light of uniform brightness, which, neglecting reflection and absorption, can be shown to be the same as that of the flame itself.[1] The illumination of a screen placed in the focus, is therefore jointly proportional to the solid angle which the lens subtends at the focus, and to the brightness of the flame; and is the same as if the screen were directly illuminated by the flame, at a distance at which the flame itself would subtend the same solid angle.

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[1] This is strictly true of *intrinsic* brightness, which is all that our reasoning requires. It is true for *effective* brightness, if the image is large enough to cover the pupil, and if the lens is at a proper distance for distinct vision.

772. Cross-wires of Telescopes.—We have described in § 743A a mode of marking the place of a real image by means of a cross of threads. When telescopes are employed to assist in the measurement of angles, a contrivance of this kind is almost always introduced. A cross of silkworm threads, in instruments of low power, or of spider threads in instruments of higher power, is stretched across a metallic frame just in front of the eye-piece. The observer must first adjust the eye-piece for distinct vision of this cross, and must then (in the case of theodolites and other surveying instruments) adjust the distance of the object-glass until the object which is to be observed is also seen distinctly in the telescope. The image of the object will then be very nearly in the plane of the cross. If it is not exactly in the plane, parallactic displacement will be observed when the eye is shifted, and this must be cured by slightly altering the distance of the object-glass. When the adjustment has been completed, the cross always marks one definite point of the object, however the eye be shifted. This coincidence will not be disturbed by pushing in or pulling out the eye-piece; for the frame which carries the cross is attached to the body of the telescope, and the coincidence of the cross with a point of the image is real, so that it could be observed by the naked eye, if the eye-piece were removed. The adjustment of the eye-piece merely serves to give distinct vision, and this will be obtained simultaneously for both the cross and the object.

773. Line of Collimation.—The employment of *cross-wires* (as these crossing threads are called) enormously increases our power of making accurate observations of direction, and constitutes one of the greatest advantages of modern over ancient instruments.

The line which is regarded as the line of sight, or as the direction in which the telescope is pointed, is called the *line of collimation.* If we neglect the curvature of rays due to atmospheric refraction, we may define it as the *line joining the cross to the object whose image falls on it.* More rigorously, the line of collimation is the *line joining the cross to the optical centre of the object-glass.* When it is desired to adjust the line of collimation,—for example, to make it truly perpendicular to the horizontal axis on which the telescope is mounted, the adjustment is performed by shifting the frame which carries the wires, slow-motion screws being provided for this purpose.

Telescopes for astronomical observation are often furnished with a number of parallel wires, crossed by one or two in the transverse direction; and the line of collimation is then defined by reference to an imaginary cross, which is the centre of mean position of all the actual crosses.

774. Micrometers.—Astronomical micrometers are of various kinds, some of them serving for measuring the angular distance between two points in the same field of view, and others for measuring their apparent direction from one another. They generally consist of spider threads placed in the principal focus of the object-glass, so as to be in the same plane as the images of celestial objects, one or more of the threads being movable by means of slow-motion screws, furnished with graduated circles, on which parts of a turn can be read off.

One of the commonest kinds consists of two parallel threads, which can thus be moved to any distance apart, and can also be turned round in their own plane.

CHAPTER LXII.

DISPERSION. STUDY OF SPECTRA.

775. Newtonian Experiment.—In the chapter on refraction, we have postponed the discussion of one important phenomenon by which it is usually accompanied, and which we must now proceed to explain. The following experiment, which is due to Sir Isaac Newton, will furnish a fitting introduction to the subject.

On an extensive background of black, let three bright strips be laid in line, as in the left-hand part of Fig. 708, and looked at through a prism with its refracting edge parallel to the strips. We shall suppose the edge to be upward, so that the image is raised above the object. The images, as represented in the right-hand part of Fig. 708, will have the same horizontal dimensions as the strips, but will be greatly extended in the vertical direction; and each image, instead of having the uniform colour of the strips from which it is derived, will be tinted with a gradual succession of colours from top to bottom. Such images are called *spectra.*

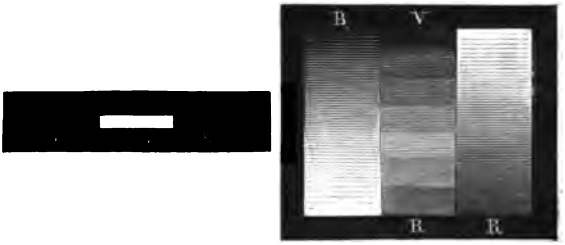

Fig. 708.—Spectra of White and Coloured Strips.

If one of the strips (the middle one in the figure) be white, its spectrum will contain the following series of colours, beginning at the top: *violet, blue, green, yellow, orange, red.*

If one of the strips be blue (the left-hand one in the figure), its image will present bright colours at the upper end; and these will be identical with the colours adjacent to them in the spectrum of white. The colours which form the lower part of the spectrum of white will either be very dim and dark in the spectrum of blue, or will be wanting altogether, being replaced by black.

If the other strip be red, its image will contain bright colours at the lower or red end, and those which belong to the upper end of the spectrum of white will be dim or absent. Every colour that occurs in the spectrum of blue or of red will also be found, and in the same horizontal line, in the spectrum of white.

If we employ other colours instead of blue or red, we shall obtain analogous results; every colour will be found to give a spectrum which is identical with part of the spectrum of white, both as regards colour and position, but not generally as regards brightness.

We may occasionally meet with a body whose spectrum consists only of one colour. The petals of some kinds of convolvulus give a spectrum consisting only of blue, and the petals of nasturtium give only red.

776. Composite Nature of Ordinary Colours.—This experiment shows that the colours presented by the great majority of natural bodies are composite. When a colour is looked at with the naked eye, the sensation experienced is the joint effect of the various elementary colours which compose it. The prism serves to resolve the colour into its components, and exhibit them separately. The experiment also shows that a mixture of all the elementary colours in proper proportions produces white.

777. Solar Spectrum.—The coloured strips in the foregoing experiment may be illuminated either by daylight or by any of the ordinary sources of artificial light. The former is the best, as gas-light and candle-light are very deficient in blue and violet rays.

Colour, regarded as a property of a coloured (opaque) body, is the power of selecting certain rays and reflecting them either exclusively or in larger proportion than others. The spectrum presented by a body viewed by reflected light, as ordinary bodies are, can thus only consist of the rays, or a selection of the rays, by which the body is illuminated.

A beam of solar light can be directly resolved into its constituents by the following experiment, which is also due to Newton, and was the first demonstration of the composite character of solar light.

Let a beam of sun-light be admitted through a small opening into a dark room. If allowed to fall normally on a white screen, it produces (§ 683) a round white spot, which is an image of the sun. Now let a prism be placed in its path edge-downwards, as in Fig. 709; the beam will thus be deflected upwards, and at the same time resolved into its component colours. The image depicted on the screen will be a many-coloured band, resembling the spectrum of white described in § 775. It will be of uniform width, and rounded off at the ends, being in fact built up of a number of overlapping discs, one for each kind of elementary ray. It is called the *solar spectrum.*

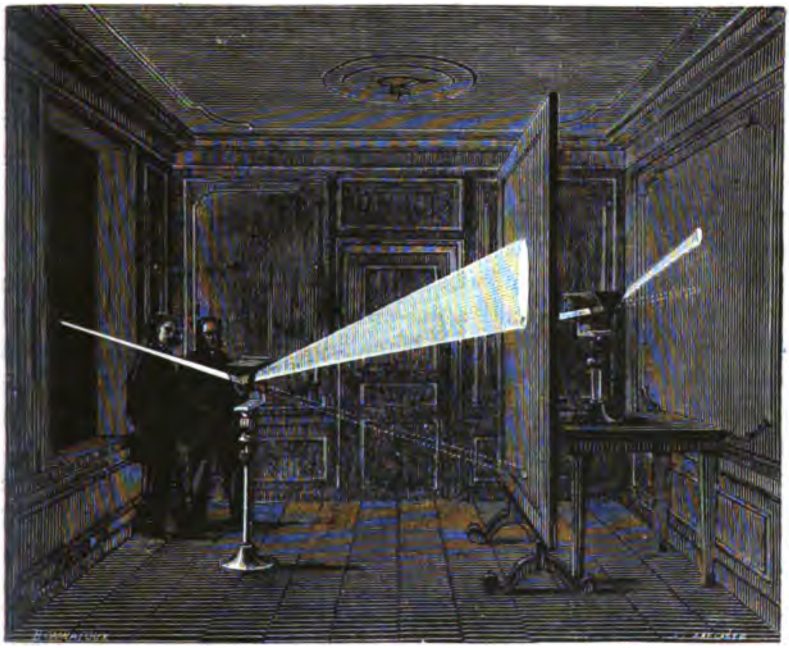

Fig. 709.—Solar Spectrum by Newton's Method.

The rays which have undergone the greatest deviation are the violet. They occupy the upper end of the spectrum in the figure. Those which have undergone the least deviation are the red. Of all visible rays, the violet are the most, and the red the least refrangible; and the analysis of light into its components by means of the prism is due to difference of refrangibility. If a small opening is made in the screen, so as to allow rays of only one colour to pass, it will be found, on transmitting these through a second prism behind the screen, as in Fig. 709, that no further analysis can be effected, and the whole of the image formed by receiving this transmitted light on a second screen will be of this one colour.

778. Mode of obtaining a Pure Spectrum.—The spectra obtained by the methods above described are built up of a number of overlapping images of different colours. To prevent this overlapping, and obtain each elementary colour pure from all admixture with the rest, we must in the first place employ as the object for yielding the images a very narrow line; and in the second place we must take care that the images which we obtain of this line are not blurred, but have the greatest possible sharpness. A spectrum possessing these characteristics is called pure.

The simplest mode of obtaining a pure spectrum consists in looking through a prism at a fine slit in the shutter of a dark room. The edges of the prism must be parallel to the slit, and its distance from the slit should be five feet or upwards. The observer, placing his eye close to the prism, will see a spectrum; and he should rotate the prism on its axis until he has brought this spectrum to its smallest angular distance from the real slit, of which it is the image.

Let E (Fig. 710) be the position of the eye, S that of the slit. Then the extreme red and violet images of the slit will be seen at R, V, at distances from the prism sensibly equal to the real distance of S (§ 731 B); and the other images, which compose the remainder of the spectrum, will occupy positions between R and V The spectrum, in this mode of operating, is virtual.

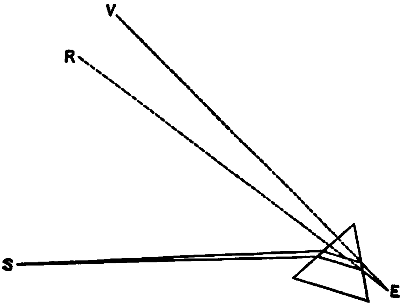

Fig. 710.—Arrangement for seeing a pure Spectrum.

To obtain a real spectrum in a state of purity, a convex lens must be employed. Let the lens L (Fig. 711) be first placed in such a position as to throw a sharp image of the slit S upon a screen at I. Next let a prism P be introduced between the lens and screen, and rotated on its axis till the position of minimum deviation is obtained, as shown by the movements of the impure spectrum which travels about the walls of the room. Then if the screen be moved into the position RV, its distance from the prism being the same as before, a pure spectrum will be depicted upon it. A similar result can be obtained by placing the prism between the lens and the slit, but the adjustments are rather more troublesome. Direct sun-light, or sun-light reflected from a mirror placed outside the shutter, is necessary for this experiment, as sky-light is not sufficiently powerful. It is usual, in experiments of this kind, to employ a movable mirror called a *heliostat*, by means of which the light can be reflected in any required direction. Sometimes the movements of the mirror are obtained by hand; sometimes by an ingenious clock-work arrangement, which causes the reflected beam to keep its direction unchanged notwithstanding the progress of the sun through the heavens.

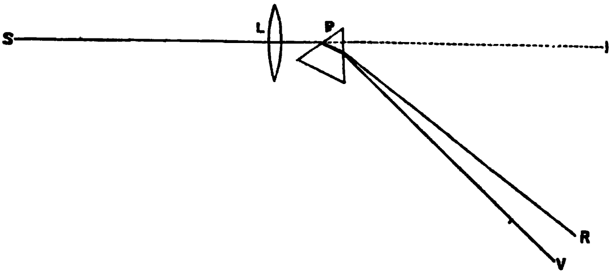

Fig. 711.—Arrangement for Pure Spectrum on Screen.

The advantage of placing the prism in the position of minimum deviation is two-fold. First, the adjustments are facilitated by the equality of conjugate focal distances, which subsists in this case and in this only. Secondly and chiefly, this is the only position in which the images are not blurred. In any other position it can be shown[1] that a small cone of homogeneous incident rays is no longer a cone (that is, its rays do not accurately pass through one point) after transmission through the prism.

The method of observation just described was employed by Wollaston, in the earliest observations of a pure spectrum ever obtained. Fraunhofer, a few years later, independently devised the same method, and carried it to much greater perfection. Instead of looking at the virtual image with the naked eye, he viewed it through a telescope, which greatly magnified it, and revealed several features never before detected. The prism and telescope were at a distance of 24 feet from the slit.

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[1] Helmholtz, *Physiological Optics*. Second Part, § 19.

779. Dark Lines in the Solar Spectrum.—When a pure spectrum of solar light is examined by any of these methods, it is seen to be traversed by numerous dark lines, constituting, if we may so say, dark images of the slit. Each of these is an indication that a particular kind of elementary ray is wanting[1] in solar light. Every elementary ray that is present gives its own image of the slit in its own peculiar colour; and these images are arranged in strict contiguity, so as to form a continuous band of light passing by perfectly gradual transitions through the whole range of simple colour, except at the narrow intervals occupied by the dark lines. Fig. 1, Plate III., is a rough representation of the appearance thus presented. If the slit is illuminated by a gas flame, or by any ordinary lamp, instead of by solar light, no such lines are seen, but a perfectly continuous spectrum is obtained. The dark lines are therefore not characteristic of light in general, but only of solar light.

Wollaston saw and described some of the more conspicuous of them. Fraunhofer counted about 600, and marked the places of 354 upon a map of the spectrum, distinguishing some of the more conspicuous by the names of letters of the alphabet, as indicated in fig. 1. These lines are constantly referred to as reference marks for the accurate specification of different portions of the spectrum. They always occur in precisely the same places as regards colour, but do not retain exactly the same relative distances one from another, when prisms of different materials are employed, different parts of the spectrum being unequally expanded by different refracting substances.[2] The inequality, however, is not so great as to introduce any difficulty in the identification of the lines.

The dark lines in the solar spectrum are often called Fraunhofer's lines. Fraunhofer himself called them the "fixed lines."

780. Invisible Rays of the Spectrum.—The brightness of the solar spectrum, however obtained, is by no means equal throughout, but is greatest between the dark lines D and E; that is to say, in the yellow and the neighbouring colours orange and light green; and falls off gradually on both sides.

The heating effect upon a small thermometer or thermopile increases in going from the violet to the red, and still continues to increase for a certain distance beyond the visible spectrum at the red end. Prisms and lenses of rock-salt should be employed for this

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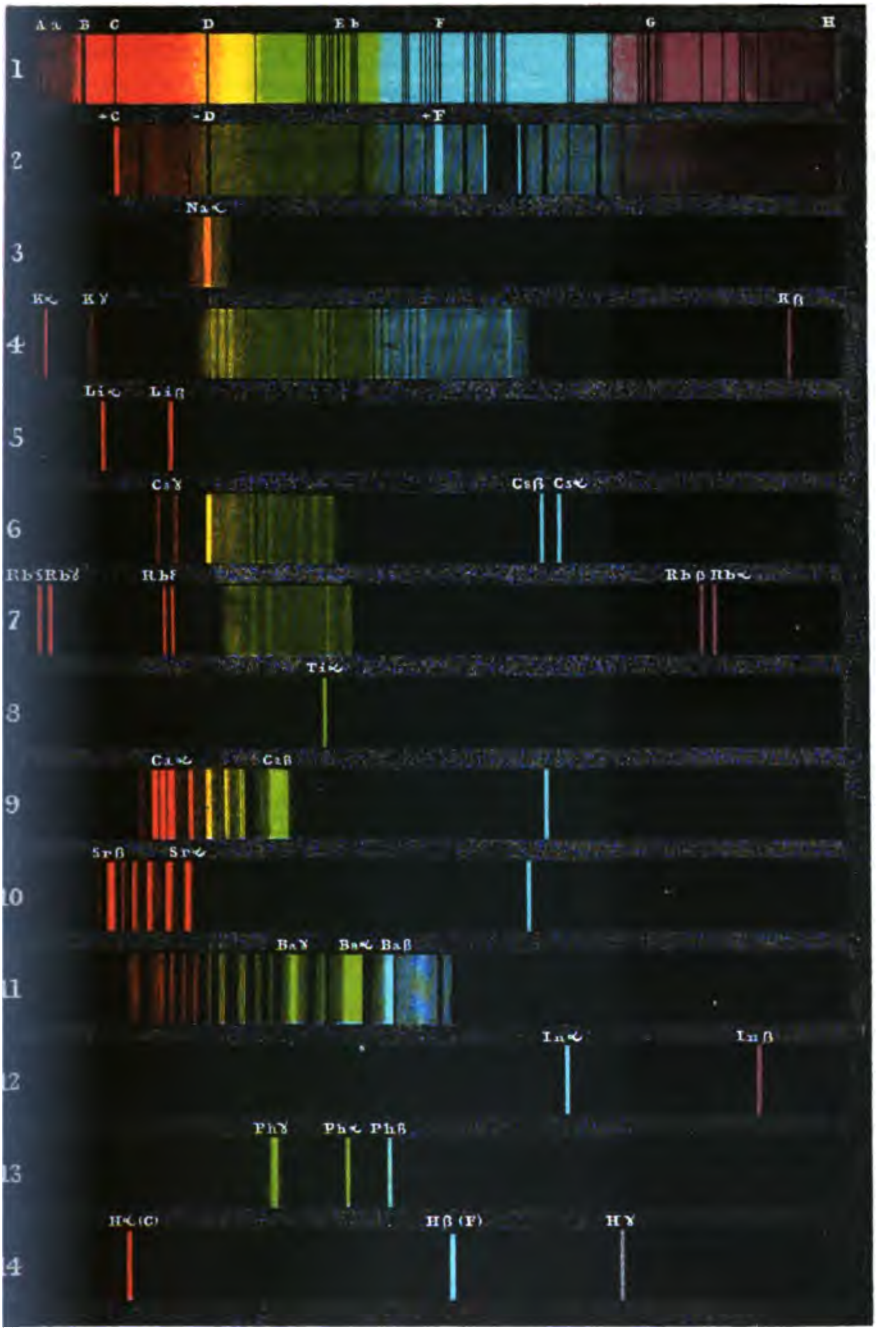
[1] Probably not absolutely wanting, but so feeble as to appear black by contrast.

[2] This property is called the *irrationality of dispersion.*

SPECTRA OF VARIOUS SOURCES OF LIGHT.

PLATE III.



1. *The Sun.* 2. *The Sun's edge.* 3. *Sodium.* 4. *Potassium.* 5. *Lithium.* 6. *Caesium.* 7. *Rubidium.* 8 *Thallium.*
9. *Calcium* 10. *Strontium.* 11 *Barium.* 12. *Indium.* 13. *Phosphorus.* 14. *Hydrogen.*

investigation, as glass largely absorbs the invisible rays which lie beyond the red.

When the spectrum is thrown upon the sensitized paper employed in photography, the action is very feeble in the red, strong in the blue and violet, and is sensible to a great distance beyond the violet end. When proper precautions are taken to insure a very pure spectrum, the photograph reveals the existence of dark lines, like those of Fraunhofer, in the invisible ultra-violet portion of the spectrum. The strongest of these have been named L, M, N, O, P.

781. Phosphorescence and Fluorescence.—There are some substances which, after being exposed in the sun, are found for a long time to appear self-luminous when viewed in the dark, and this without any signs of combustion or sensible elevation of temperature. Such substances are called *phosphorescent.* Sulphuret of calcium and sulphuret of barium have long been noted for this property, and have hence been called respectively *Canton's phosphorus*, and *Bologna*

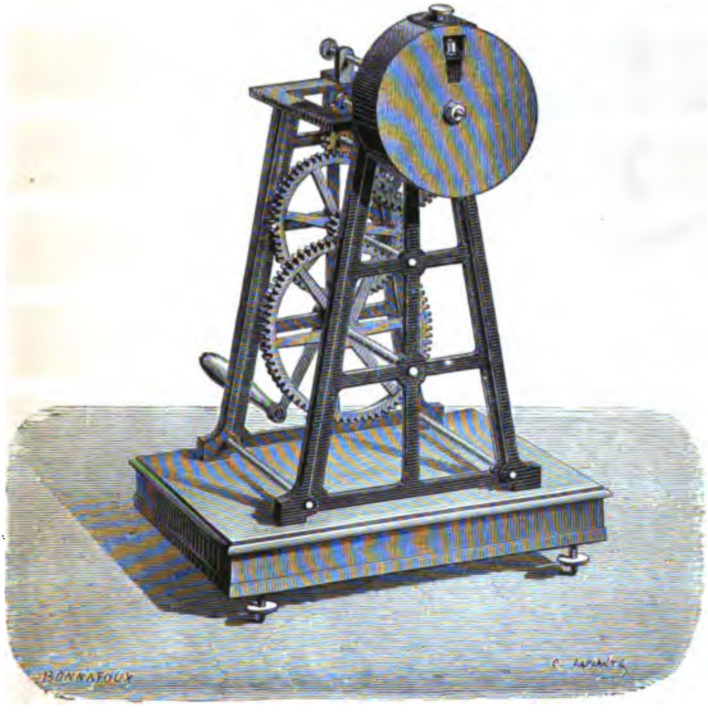


Fig. 712.—Becquerel's Phosphoroscope.

phosphorus. The phenomenon is chiefly due to the action of the violet and ultra-violet portion of the sun's rays.

More recent investigations have shown that the same property exists in a much lower degree in an immense number of bodies, their phosphorescence continuing, in most cases, only for a fraction of a second after their withdrawal from the sun's rays. E. Becquerel has contrived an instrument, called the *phosphoroscope,* which is extremely appropriate for the observation of this phenomenon. It is represented in Fig. 712. Its most characteristic feature is a pair of rigidly connected discs (Fig. 713), each pierced with four openings, those of the one being not opposite but midway between those of the other. This pair of discs can be set in very rapid rotation by means of a series of wheels and pinions. The body to be examined is attached to a fixed stand between the two discs, so that it is alternately exposed on opposite sides as the discs rotate. One side is turned towards the sun, and the other towards the observer, who accordingly only sees the body when it is not exposed to the sun's rays. The cylindrical case within which the discs revolve, is fitted into a hole in the shutter of a dark room, and is pierced with an opening on each side exactly opposite the position in which the body is fixed. The body, if not phosphorescent, will never be seen by the observer, as it is always in darkness except when it is hidden by the intervening disc. If its phosphorescence lasts as long as an eighth part of the time of one rotation, it will become visible in the darkness.

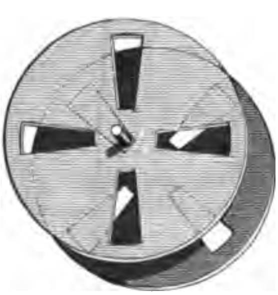


Fig. 713.—Discs of Phosphoroscope.

Nearly all bodies, when thus examined, show traces of phosphorescence, lasting, however, in some cases, only for a ten-thousandth of a second.

The phenomenon of *fluorescence,* which is illustrated in Plate II. accompanying § 618, appears to be essentially identical with phosphorescence. The former name is applied to the phenomenon, if it is observed while the body is actually exposed to the source of light, the latter to the effect of the same kind, but usually less intense, which is observed after the light from the source is cut off. Both forms of the phenomenon occur in a strongly-marked degree in the same bodies. Canary-glass, which is coloured with oxide of uranium, is a very convenient material for the exhibition of fluorescence. A thick piece of it, held in the violet or ultra-violet portion of the solar spectrum, is filled to the depth of from ⅛ to ¼ of an inch with a faint nebulous light. A solution of sulphate of quinine is also frequently employed for exhibiting the same effect, the luminosity in this case being bluish. If the solar spectrum be thrown upon a screen freshly washed with sulphate of quinine, the ultra-violet portion will become visible by fluorescence; and if the spectrum be very pure, the presence of dark lines in this portion will be detected.

The light of the electric lamp is particularly rich in ultra-violet rays, this portion of its spectrum being much longer than in the case of solar light, and about twice as long as the spectrum of luminous rays. Prisms and lenses of quartz should be employed for this purpose, as this material is specially transparent to the highly-refrangible rays. Flint-glass prisms, however, if of good quality, answer well in operating on solar light. The luminosity produced by fluorescence has sensibly the same tint in all parts of the spectrum in which it occurs, and depends upon the fluorescent substance employed. Prismatic analysis is not necessary to the exhibition of fluorescence. The phenomenon is very conspicuous when the electric discharge of a Holtz's machine or a Ruhmkorff's coil is passed near fluorescent substances, and it is faintly visible when these substances are examined in bright sunshine. The light emitted by a fluorescent substance is found by analysis not to be homogeneous, but to consist of rays having a wide range of refrangibility.

The ultra-violet rays, though usually styled invisible, are not altogether deserving of this title. By keeping all the rest of the spectrum out of sight, and carefully excluding all extraneous light, the eye is enabled to perceive these highly refrangible rays. Their colour is described as lavender-gray or bluish white, and has been attributed, with much appearance of probability, to fluorescence of the retina. The ultra-red rays, on the other hand, are never seen; but this may be owing to the fact, which has been established by experiment, that they are largely, if not entirely, absorbed before they can reach the retina.

782. Recomposition of White Light.—The composite nature of white light can be established by actual synthesis. This can be done in several ways.

1. If a second prism, precisely similar to the first, but with its refracting edge turned the contrary way, is interposed in the path of

the coloured beam, very near its place of emergence from the first prism, the deviation produced by the second prism will be equal and opposite to that produced by the first, the two prisms will produce the effect of a parallel plate, and the image on the screen will be a white spot, nearly in the same position as if the prisms were removed.

2. Let a convex lens (Fig. 714) be interposed in the path of the coloured beam, in such a manner that it receives all the rays, and that the screen and the prism are at conjugate focal distances. The image thus obtained on the screen will be white, at least in its central portions.

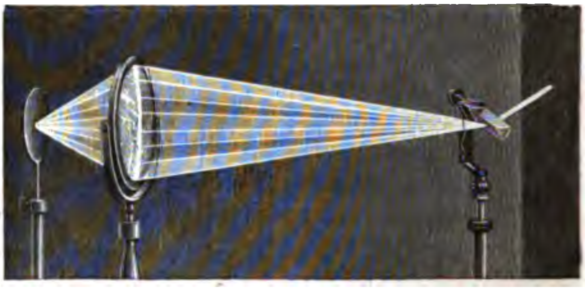


Fig. 714.—Recomposition by Lens.

3. Let a number of plane mirrors be placed so as to receive the successive coloured rays, and to reflect them all to one point of a screen, as in Fig. 715. The bright spot thus formed will be white, or approximately white.

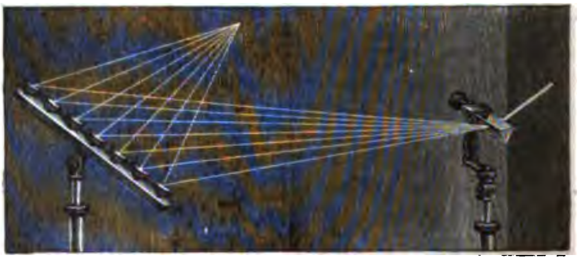


Fig. 715.—Recomposition by Mirrors.

More complete information respecting the mixture of colours will be given in the next chapter.

783. Spectroscope.—When we have obtained a pure spectrum by any of the methods above indicated, we have in fact effected an analysis of the light with which the slit is illuminated. In recent years, many forms of apparatus have been constructed for this purpose, under the name of *spectroscopes.*

A spectroscope usually contains, besides a slit, a prism, and a telescope (as in Fraunhofer's method of observation), a convex lens called a *collimator*, which is fixed between the prism and the slit, at the distance of its principal focal length from the latter. The effect of this arrangement is, that rays from any point of the slit emerge parallel, as if they came from a much larger slit (the virtual image of the real slit) at a much greater distance. The prism (set at minimum deviation) forms a virtual image of this image at the same distance, but in a different direction, on the principle of Fig. 711.

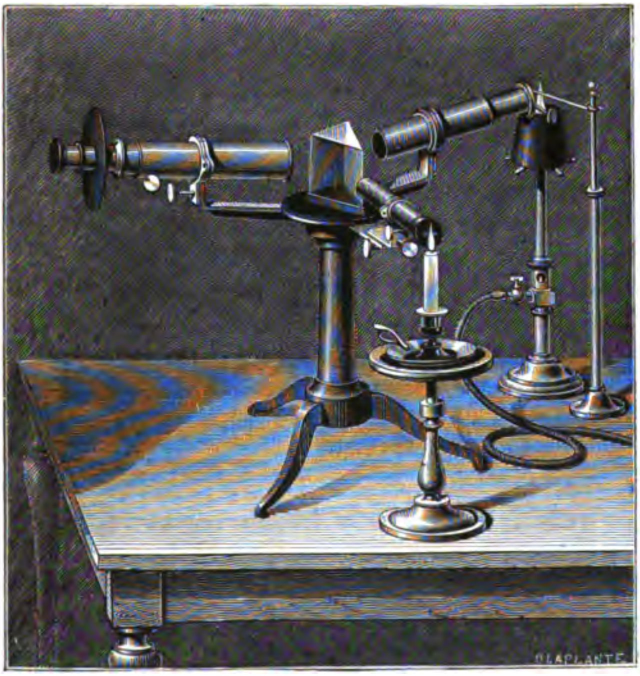


Fig. 716.—Spectroscope.

To this second virtual image the telescope is directed, being focussed as if for a very distant object.

Fig. 716 represents a spectroscope thus constructed. The tube of the collimator is the further tube in the figure, the lens being at the end of the tube next the prism, while at the far end, close to the lamp flame, there is a slit (not visible in the figure) consisting of an opening between two parallel knife-edges, one of which can be moved to or from the other by turning a screw. The knife-edges must be very true, both as regards straightness and parallelism, as it is often necessary to make the slit exceedingly narrow. The tube on the left hand is the telescope, furnished with a broad guard to screen the eye from extraneous light. The near tube, with a candle opposite its end, is for purposes of measurement. It contains, at the end next the candle, a scale of equal parts, engraved or photographed on glass. At the other end of the tube is a collimating lens, at the distance of its own focal length from the scale; and the collimator is set so that its axis and the axis of the telescope make equal angles with the near face of the prism. The observer thus sees in the telescope, by reflection from the surface of the prism, a magnified image of the scale, serving as a standard of reference for assigning the positions of the lines in any spectrum which may be under examination. This arrangement affords great facilities for rapid observation.

Another plan is, for the arm which carries the telescope to be movable round a graduated circle, the telescope being furnished with cross-wires, which the observer must bring into coincidence with any line whose position he desires to measure.

Arrangements are frequently made for seeing the spectra of two different sources of light in the same field of view, one half of the length of the slit being illuminated by the direct rays of one of the sources, while a reflector, placed opposite the other half of the slit, supplies it with reflected light derived from the other source. This method should always be employed when there is a question as to the exact coincidence of lines in the two spectra. The reflector is usually an equilateral prism. The light enters normally at one of its faces, is totally reflected at another, and emerges normally at the third, as in the annexed sketch (Fig. 717, where the dotted line represents the path of a ray.

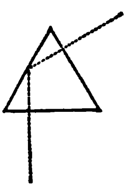


Fig. 717. Reflecting Prism.

A one-prism spectroscope is amply sufficient for the ordinary purposes of chemistry. For some astronomical applications a much greater dispersion is required. This is attained by making the light pass through a number of prisms in succession, each being set in the proper position for giving minimum deviation to the rays which have passed through its predecessor. Fig. 718 represents the ground plan of such a battery of prisms, and shows the gradually increasing width of a pencil as it passes round the series of nine prisms on its way from the collimator to the telescope. The prisms are usually connected by a special arrangement, which enables the observer, by a single movement, to bring all the prisms at once into the proper position for giving minimum deviation to the particular ray under examination, a position which differs considerably for rays of different refrangibilities.

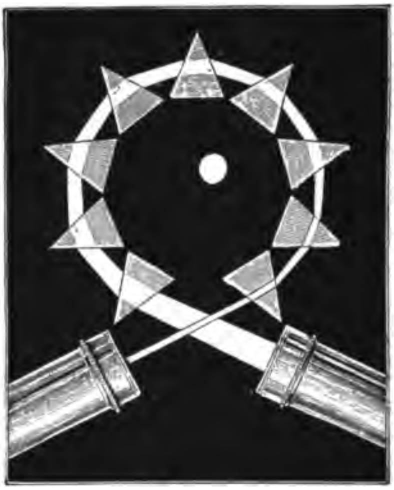


Fig. 718.—Train of Prisms.

784. Use of Collimator.—The introduction of a collimating lens, to be used in conjunction with a prism and observing telescope, is due to Professor Swan.[1] Fraunhofer employed no collimator; but his prism was at a distance of 24 feet from the slit, whereas a distance of less than 1 foot suffices when a collimator is used.

It is obvious that homogeneous light, coming from a point at the distance of a foot, and falling upon the whole of one face of a prism —say an inch in width, cannot all have the incidence proper for minimum deviation. Those rays which very nearly fulfil this condition, will concur in forming a tolerably sharp image, in the position which we have already indicated. The emergent rays taken as a whole, do not diverge from any one point, but are tangents to a virtual caustic (§ 714). An eye receiving any portion of these rays, will see an image in the direction of a tangent from the eye to the caustic; and this image will be the more blurred as the deviation is further from the minimum. When the naked eye is employed, and the prism is so adjusted that the centre of the pupil receives rays of minimum deviation, a distance of five or six feet between the prism and slit is sufficient to give a sharp image; but if we employ an observing telescope whose object-glass is five times larger in diameter than the pupil of the eye, we must increase the distance between the prism and slit five-fold to obtain equally good definition. A collimating lens, if achromatic and of good quality, gives the advantage of good definition without inconvenient length.

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[1] *Trans. Roy. Soc. Edinburgh*, 1847 and 1856.

When exact measures of deviation are required, it confers the further advantage of altogether dispensing with a very troublesome correction for parallax.

785. Different Kinds of Spectra.—The examination of a great variety of sources of light has shown that spectra may be divided into the following classes:—

1. The solar spectrum is characterized, as already observed, by a definite system of dark lines interrupting an otherwise continuous succession of colours. The same system of dark lines is found in the spectra of the moon and planets, this being merely a consequence of the fact that they shine by the reflected light of the sun. The spectra of the fixed stars also contain systems of dark lines, which are different for different stars.

2. The spectra of incandescent solids and liquids are completely continuous, containing light of all refrangibilities from the extreme red to a higher limit depending on the temperature.

3. Flames not containing solid particles in suspension, but merely emitting the light of incandescent gases, give a discontinuous spectrum, consisting of a finite number of bright lines. The continuity of the spectrum of a gas or candle flame, arises from the fact that nearly all the light of the flame is emitted by incandescent particles of solid carbon,—particles which we can easily collect in the form of soot. When a gas-flame is fed with an excessive quantity of air, as in Bunsen's burner, the separation of the solid particles of carbon from the hydrogen with which they were combined, no longer takes place; the combustion is purely gaseous, and the spectrum of the flame is found to consist of bright lines. When the electric light is produced between metallic terminals, its spectrum contains bright lines due to the incandescent vapour of these metals, together with other bright lines due to the incandescence of the oxygen and nitrogen of the air. When it is taken between charcoal terminals, its spectrum is continuous; but if metallic particles be present, the bright lines due to their vapours can be seen as well.

The spectrum of the electric discharge in a Geissler's tube consists of bright lines characteristic of the gas contained in the tube.

786. Spectrum Analysis.—As the spectrum exhibited by a compound substance when subjected to the action of heat, is frequently found to be identical with the spectrum of one of its constituents, or to consist of the spectra of its constituents superimposed,[1] the spectroscope affords an exceedingly ready method of performing qualitative analysis.

If a salt of a metal which is easily volatilized is introduced into a Bunsen lamp-flame, by means of a loop of platinum wire, the bright lines which form the spectrum of the metal will at once be seen in a spectroscope directed to the flame; and the spectrum of the Bunsen flame itself is too faint to introduce any confusion. For those metals which require a higher temperature to volatilize them, electric discharge is usually employed. Geissler's tubes are commonly used for gases.

Plate III. contains representations of the spectra of several of the more easily volatilized metals, as well as of phosphorus and hydrogen; and the solar spectrum is given at the top for comparison. The bright lines of some of these substances are precisely coincident with some of the dark lines in the solar spectrum.

The fact that certain substances when incandescent give definite bright lines, has been known for many years, from the researches of Brewster, Herschel, Talbot, and others; but it was for a long time thought that the same line might be produced by different substances, more especially as the bright yellow line of sodium was often seen in flames in which that metal was not supposed to be present. Professor Swan, having ascertained that the presence of the 2,500,000th part of a grain of sodium in a flame was sufficient to produce it, considered himself justified in asserting, in 1856, that this line was always to be taken as an indication of the presence of sodium in larger or smaller quantity.

But the greatest advance in spectral analysis was made by Bunsen and Kirchhoff, who, by means of a four-prism spectroscope, obtained accurate observations of the positions of the bright lines in the spectra of a great number of substances, as well as of the dark lines in the solar spectrum, and called attention to the identity of several of the latter with several of the former. Since the publication of their researches, the spectroscope has come into general use among chemists, and has already led to the discovery of four new metals, cesium, rubidium, thallium, and indium.

787. Reversal of Bright Lines. Analysis of the Sun's Atmosphere.

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[1] These appear to be merely examples of the dissociation of the elements of a chemical compound at high temperatures.

—It may seem surprising that, while incandescent solids and liquids are found to give continuous spectra, containing rays of all refrangibilities, the solar spectrum is interrupted by dark lines indicating the absence or relative feebleness of certain elementary rays. It seems natural to suppose that the deficient rays have been removed by selective absorption, and this conjecture was thrown out long since. But where and how is this absorption produced? These questions have now received an answer which appears completely satisfactory.

According to the theory of exchanges, which has been explained in connection with the radiation of heat (§ 312c, 326), every substance which emits certain kinds of rays to the exclusion of others, absorbs the same kind which it emits; and when its temperature is the same in the two cases compared, its emissive and absorbing power are precisely equal for any one elementary ray.

When an incandescent vapour, emitting only rays of certain definite refrangibilities, and therefore having a spectrum of bright lines, is interposed between the observer and a very bright source of light, giving a continuous spectrum, the vapour allows no rays of its own peculiar kinds to pass; so that the light which actually comes to the observer consists of transmitted rays in which these particular kinds are wanting, together with the rays emitted by the vapour itself, these latter being of precisely the same kind as those which it has refused to transmit. It depends on the relative brightness of the two sources whether these particular rays shall be on the whole in excess or defect as compared with the rest. If the two sources are at all comparable in brightness, these rays will be greatly in excess, inasmuch as they constitute the whole light of the one, and only a minute fraction of the light of the other; but the light of the electric lamp, or of the lime-light, is usually found sufficiently powerful to produce the contrary effect; so that if, for example, a spirit-lamp with salted wick is interposed between the slit of a spectroscope and the electric light, the bright yellow line due to the sodium appears black by contrast with the much brighter back-ground which belongs to the continuous spectrum of the charcoal points. By employing only some 10 or 15 cells, a light may be obtained, the yellow portion of which, as seen in a one-prism spectroscope, is sensibly equal in brightness to the yellow line of the sodium flame, so that this line can no longer be separately detected, and the appearance is the same whether the sodium flame be interposed or removed.

The dark lines in the solar spectrum would therefore be accounted for by supposing that the principal portion of the sun's light comes from an inner stratum which gives a continuous spectrum, and that a layer external to this contains vapours which absorb particular rays, and thus produce the dark lines. The stratum which gives the continuous spectrum might be solid, liquid, or even gaseous, for the experiments of Frankland and Lockyer have shown that, as the pressure of a gas is increased, its bright lines broaden out into bands, and that the bands at length become so wide as to join each other and form a continuous spectrum[1]

Hydrogen, potassium, sodium, calcium, barium, magnesium, zinc, iron, chromium, cobalt, nickel, copper, and manganese have all been proved to exist in the sun by the accurate identity of position of their bright lines with certain dark lines in the sun's spectrum.

The strong line D, which in a good instrument is seen to consist of two lines near together, is due to sodium; and the lines C and F are due to hydrogen. No less than 450 of the solar dark lines have been identified with bright lines of iron.

788. Telespectroscope. Solar Sierra.—For astronomical investigations, the spectroscope is usually fitted to a telescope, and takes the place of the eye-piece, the slit being placed in the principal focus of the object-glass, so that the image is thrown upon it, and the light which enters it is the light which forms one strip (so to speak) of the image, and which therefore comes from one strip of the object. A telescope thus equipped is called a telespectroscope. Extremely interesting results have been obtained by thus subjecting to examination a strip of the sun's edge, the strip being sometimes tangential to the sun's disk, and sometimes radial. When the former arrangement is adopted, the appearance presented is that depicted in fig. 2, Plate III., consisting of a few bright lines scattered through a background of the ordinary solar spectrum. The bright lines are due to an outer layer called the *sierra* or *chromosphere*, which is thus proved to be vaporous. The ordinary solar spectrum which accompanies it, is due to that part of the sun from which most of our light is derived. This part is called the *photosphere*, and if not solid or liquid, it must consist of vapour so highly compressed that its properties approximate to those of a liquid.

When the slit is placed radially, in such a position that only a small portion of its length receives light from the body of the sun, the spectra of the photosphere and chromosphere are seen in immediate contiguity, and the bright lines in the latter (notably those of hydrogen, No. 14, Plate III.) are observed to form continuations of some of the dark lines of the former.

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[1] The gradual transition from a spectrum of bright lines to a continuous spectrum may be held to be an illustration of the continuous transition which can be effected from the condition of ordinary gas to that of ordinary liquid (§ 246 A).

The chromosphere is so much less bright than the photosphere, that, until a few years since, its existence was never revealed except during total eclipses of the sun, when projecting portions of it (from which it derives its name of *sierra*) were seen extending beyond the dark body of the moon. The spectrum of these projecting portions, which have been variously called "prominences," "red flames," and "rose-coloured protuberances," was first observed during the "Indian eclipse" of 1868, and was found to consist of bright lines, including those of hydrogen. From their excessive brightness, M. Janssen, who was one of the observers, expressed confidence that he should be able to see them in full sunshine; and the same idea had been already conceived and published by Mr. Lockyer. The expectation was shortly afterwards realized by both these observers, and the chromosphere has ever since been an object of daily observation. The visibility of the chromosphere lines in full sunshine, depends upon the principle that, while a continuous spectrum is extended, and therefore made fainter, by increased dispersion, a bright line in a spectrum is not sensibly broadened, and therefore loses very little of its intrinsic brightness (§ 791). Very high dispersion, attainable only by the use of a long train of prisms, is necessary for this purpose.

Still more recently, by opening the slit to about the average width of the prominence-region, as measured on the image of the sun which is thrown on the slit, it has been found possible to see the whole of an average-sized prominence at one view. This will be understood by remembering that a bright line as seen in a spectrum is a monochromatic image of the illuminated portion of the slit, or when a telespectroscope is used, as in the present case, it is a monochromatic image of one strip of the image formed by the object-glass, namely, that strip which coincides with the slit. If this strip then contains a prominence in which the elementary rays C and F (No. 2, Plate III.) are much stronger than in the rest of the strip, a red image of the prominence will be seen in the part of the spectrum corresponding to the line C, and a blue image in the place corresponding to the line F. This method of observation requires greater dispersion than is necessary for the mere detection of the chromosphere lines; the dispersion required for enabling a bright-line spectrum to predominate over a continuous spectrum being always nearly proportional to the width of the slit (§ 791).

Of the nebulæ, it is well known that some have been resolved by powerful telescopes into clusters of stars, while others have as yet proved irresolvable. Huggins has found that the former class of nebulæ give spectra of the same general character as the sun and the fixed stars, but that some of the latter class give spectra of bright lines, indicating that their constitution is gaseous.

789. Displacement of Lines consequent on Celestial Motions.—According to the undulatory theory of light, which is now universally accepted, the fundamental difference between the different rays which compose the complete spectrum, is a difference of wave-frequency, and, as connected with this, a difference of wave-length in any given medium, the rays of greatest wave-frequency or shortest wave-length being the most refrangible.

Doppler first called attention to the change of refrangibility which must be expected to ensue from the mutual approach or recess of the observer and the source of light, the expectation being grounded on reasoning which we have explained in connection with acoustics (§ 653 A).

Doppler adduced this principle to explain the colours of the fixed stars, a purpose to which it is quite inadequate; but it has rendered very important service in connection with spectroscopic research. Displacement of a line towards the more refrangible end of the spectrum, indicates approach, displacement in the opposite direction indicates recess, and the velocity of approach or recess admits of being calculated from the observed displacement.

When the slit of the spectroscope crosses a spot on the sun's disc, the dark lines lose their straightness in this part, and are bent, sometimes to one side, sometimes to the other. These appearances clearly indicate uprush and downrush of gases in the sun's atmosphere in the region occupied by the spot.

Huggins has observed a displacement of the F line towards the red end, in the spectrum of Sirius, as compared with the spectrum of the sun or of hydrogen. The displacement is so small as only to admit of measurement by very powerful instrumental appliances; but, small as it is, calculation shows that it indicates a motion of recess at the rate of about 30 miles per second.[1]

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[1] The observed displacement corresponded to recess at the rate of 41·4 miles per second;

790. Spectra of Artificial Lights.—The spectra of the artificial lights in ordinary use (including gas, oil-lamps, and candles) differ from the solar spectrum in the relative brightness of the different colours, as well as in the entire absence of dark lines. They are comparatively strong in red and green, but weak in blue; hence all colours which contain much blue in their composition appear to disadvantage by gas-light.

It is possible to find artificial lights whose spectra are of a completely different character. The salts of strontium, for example, give red light, composed of the ingredients represented in spectrum No. 10, Plate III., and those of sodium yellow light (No. 3, Plate III.) If a room is illuminated by a sodium flame (for example, by a spirit-lamp with salt sprinkled on the wick), all objects in the room will appear of a uniform colour (that of the flame itself), differing only in brightness, those which contain no yellow in their spectrum as seen by day-light being changed to black. The human countenance and hands assume a ghastly hue, and the lips are no longer red.

A similar phenomenon is observed when a coloured body is held in different parts of the solar spectrum in a dark room, so as to be illuminated by different kinds of monochromatic light. The object either appears of the same colour as the light which falls upon it, or else it refuses to reflect this light and appears black. Hence a screen for exhibiting the spectrum should be white.

791. Brightness and Purity.—The laws which determine the brightness of images generally, and which have been expounded at some length in the preceding chapter, may be applied to the spectroscope. We shall, in the first instance, neglect the loss of light by reflection and imperfect transmission.

Let Δ denote the *prismatic dispersion*, as measured by the angular separation of two specified monochromatic images when the naked eye is applied to the last prism, the observing telescope being removed. Then, putting m for the linear magnifying power of the

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but 12·0 of this must be deducted for the motion of the earth in its orbit at the season of the year when the observation was made. The remainder, 29·4, is therefore the rate at which the distance between the sun and Sirius is increasing.

In a more recent paper, read while this volume was going through the press, Dr. Huggins gives the results of observations with more powerful instrumental appliances. The recess of Sirius is found to be only 20 miles per second. Arcturus is approaching at the rate of 50 miles per second. Community of motion has been established in certain sets of stars; and the belief previously held by astronomers, as to the direction in which the solar system is moving with respect to the stars as a whole, is fully confirmed.

telescope, $m\Delta$ is the angular separation observed when the eye is applied to the telescope. We shall call $m\Delta$ the *total dispersion*.

Let θ denote the angle which the breadth of the slit subtends at the centre of the collimating lens, and which is measured by $\frac{\text{breadth of slit}}{\text{focal length of lens}}$. Then θ is also the apparent breadth of any absolutely monochromatic image of the slit, formed by rays of minimum deviation,as seen by an eye applied either to the first prism, the last prism, or any one of the train of prisms. The change produced in a pencil of monochromatic rays by transmission through a prism at minimum deviation, is in fact simply a change of direction, without any change of mutual inclination; and thus neither brightness nor apparent size is at all affected. In ordinary cases, the bright lines of a spectrum may be regarded as monochromatic, and their apparent breadth, as seen without the telescope, is sensibly equal to θ. Strictly speaking, the effect of prismatic dispersion in actual cases, is to increase the apparent breadth by a small quantity, which, if all the prisms are alike, is proportional to the number of prisms; but the increase is usually too small to be sensible.

Let I denote the intrinsic brightness of the source as regards any one of its (approximately) monochromatic constituents; in other words, the brightness which the source would have if deprived of all its light except that which goes to form a particular bright line. Then, still neglecting the light stopped by the instrument, the brightness of this line as seen without the aid of the telescope will be I; and as seen in the telescope it will either be equal to or less than this, according to the magnifying power of the telescope and the effective aperture of the object-glass (§ 769). If the breadth of the slit be halved, the breadth of the bright line will be halved, and its brightness will be unchanged. These conclusions remain true so long as the bright line can be regarded as practically monochromatic.

The brightness of any part of a *continuous* spectrum follows a very different law. It varies directly as the width of the slit, and inversely as the prismatic dispersion. Its value without the observing telescope, or its maximum value with a telescope, is $\frac{\theta}{\Delta}i$, where i is a coefficient depending only on the source.

The *purity* of any part of a continuous spectrum is properly measured by the ratio of the *distance between two specified monochromatic images* to the *breadth of either*, the distance in question being measured from the centre of one to the centre of the other.

This ratio is unaffected by the employment of an observing telescope, and is $\frac{\Delta}{\theta}$.

The ratio of the brightness of a bright line to that of the adjacent portion of a continuous spectrum forming its back-ground, is $\frac{\Delta I}{\theta i}$, assuming the line to be so nearly monochromatic that the increase of its breadth produced by the dispersion of the prisms is an insignificant fraction of its whole breadth. As we widen the slit, and so increase θ, we must increase Δ in the same ratio, if we wish to preserve the same ratio of brightness. As $\frac{\Delta}{\theta}$ is increased indefinitely, the predominance of the bright lines does not increase indefinitely, but tends to a definite limit, namely, to the predominance which they would have in a perfectly pure spectrum of the given source.

The loss of light by reflection and imperfect transmission, increases with the number of surfaces of glass which are to be traversed; so that, with a long train of prisms and an observing telescope, the actual brightness will always be much less than the theoretical brightness as above computed.

The actual purity is always less than the theoretical purity, being greatly dependent on freedom from optical imperfections; and these can be much more completely avoided in lenses than in prisms. It is said that a single good prism, with a first-class collimator and telescope, (as originally employed by Swan,) gives a spectrum much more free from blurring than the modern multiprism spectroscopes, when the total dispersion $m\Delta$ is the same in both the cases compared.

792. Chromatic Aberration.—The unequal refrangibility of the different elementary rays is a source of grave inconvenience in connection with lenses. The focal length of a lens depends upon its index of refraction, which of course increases with refrangibility, the focal length being shortest for the most refrangible rays. Thus a lens of uniform material will not form a single white image of a white object, but a series of images, of all the colours of the spectrum, arranged at different distances, the violet images being nearest, and the red most remote. If we place a screen anywhere in the series of images, it can only be in the right position for one colour. Every other colour will give a blurred image, and the superposition of them all produces the image actually formed on the screen. If the object be a uniform white spot on a black ground, its image on the screen will consist of white in its central parts, gradually merging into a coloured fringe at its edge. Sharpness of outline is thus rendered impossible, and nothing better can be done than to place the screen at the focal distance corresponding to the brightest part of the spectrum. Similar indistinctness will attach to images observed in mid-air, whether directly or by means of another lens. This source of confusion is called *chromatic aberration.*

793. Possibility of Achromatism.—In order to ascertain whether it was possible to remedy this evil by combining lenses of two different materials, Newton made some trials with a compound prism composed of glass and water (the latter containing a little sugar of lead), and he found that it was not possible, by any arrangement of these two substances, to produce deviation of the transmitted light without separation into its component colours. Unfortunately he did not extend his trials to other substances, but concluded at once that an *achromatic* prism (and hence also an achromatic lens) was an impossibility; and this conclusion was for a long time accepted as indisputable. Mr. Hall, a gentleman of Worcestershire, was the first to show that it was erroneous, and is said to have constructed some achromatic telescopes; but the important fact thus discovered did not become generally known till it was rediscovered by Dollond, an eminent London optician, in whose hands the manufacture of achromatic instruments attained great perfection.

794. Conditions of Achromatism.—The conditions necessary for achromatism are easily explained. The angular separation between the brightest red and the brightest violet ray transmitted through a prism is called the *dispersion* of the prism, and is evidently the difference of the deviations of these rays. These deviations, for the position of minimum deviation of a prism of small refracting angle A, are $(\mu'-1)$ A and $(\mu''-1)$ A, μ' and μ'' denoting the indices of refraction for the two rays considered—§ 739, equation (1)—and their difference is $(\mu''-\mu')$ A. This difference is always small in comparison with either of the deviations whose difference it is, and its ratio to either of them, or more accurately its ratio to the value of $(\mu-1)$ A for the brightest part of the spectrum, is called the *dispersive power* of the substance. As the common factor A may be omitted, the formula for the dispersive power is evidently $\frac{\mu''-\mu'}{\mu-1}$.

If this ratio were the same for all substances, as Newton supposed, achromatism would be impossible; but in fact its value varies greatly, and is greater for flint than for crown glass. If two prisms of these substances, of small refracting angles, be combined into one, with their edges turned opposite ways, they will achromatize one another if $(\mu'' - \mu')A$, or the product of deviation by dispersive power, is the same for both. As the deviations can be made to have any ratio we please by altering the angles of the prisms, the condition is evidently possible.

The deviation which a simple ray undergoes in traversing a lens, at a distance x from the axis, is $\frac{x}{f}$, f denoting the focal length of the lens (§ 739), and the separation of the red and violet constituents of a compound ray is the product of this deviation by the dispersive power of the material. If a convex and concave lens are combined, fitting closely together, the deviations which they produce in a ray traversing both, are in opposite directions, and so also are the dispersions. If we may regard x as having the same value for both (a supposition which amounts to neglecting the thicknesses of the lenses in comparison with their focal lengths) the condition of no resultant dispersion is that

$$\text{dispersive power} \times \frac{1}{f}$$

has the same value for both lenses. *Their focal lengths must therefore be directly as the dispersive powers of their materials.* These latter are about ·033 for crown and ·052 for flint glass. A converging achromatic lens usually consists of a double convex lens of crown fitted to a diverging meniscus of flint. In every achromatic combination of two pieces, the direction of resultant *deviation* is that due to the piece of smaller dispersive power.

The definition above given of dispersive power is rather loose. To make it accurate, we must specify, by reference to the "fixed lines," the precise positions in the spectrum of the two rays whose separation we consider.

Since the distances between the fixed lines have different proportions for crown and flint glass, achromatism of the whole spectrum is impossible. With two pieces it is possible to unite any two selected rays, with three pieces any three selected rays, and so on. It is considered a sign of good achromatism when no colours can be brought into view by bad focussing except purple and green.

795. Achromatic Eye-pieces.—The eye-pieces of microscopes and astronomical telescopes, usually consist of two lenses of the same kind of glass, so arranged as to counteract, to some extent, the spherical and chromatic aberrations of the object-glass. The *positive* eye-piece, invented by Ramsden, is suited for observation with cross-wires or micrometers; the *negative* eye-piece, invented by Huyghens, is not adapted for purposes of measurement, but is preferred when distinct vision is the sole requisite. These eye-pieces are commonly called achromatic, but their achromatism is in a manner spurious. It consists not in bringing the red and violet images into true coincidence, but merely in causing one to cover the other as seen from the position occupied by the observer's eye.

In the best opera-glasses (§ 764), the eye-piece, as well as the object-glass, is composed of lenses of flint and crown so combined as to be achromatic in the more proper sense of the word.

796. Rainbow.—The unequal refrangibility of the different elementary rays furnishes a complete explanation of the ordinary phenomena of rainbows. The explanation was first given by Newton, who confirmed it by actual measurement.

It is well known that rainbows are seen when the sun is shining on drops of water. Sometimes one bow is seen, sometimes two, each of them presenting colours resembling those of the solar spectrum. When there is only one bow, the red arch is above and the violet below. When there is a second bow, it is at some distance outside of this, has the colours in reverse order, and is usually less bright.

Rainbows are often observed in the spray of cascades and fountains, when the sun is shining.

In every case, a line joining the observer to the sun is the axis of the bow or bows; that is to say, all parts of the length of the bow are at the same angular distance from the sun.

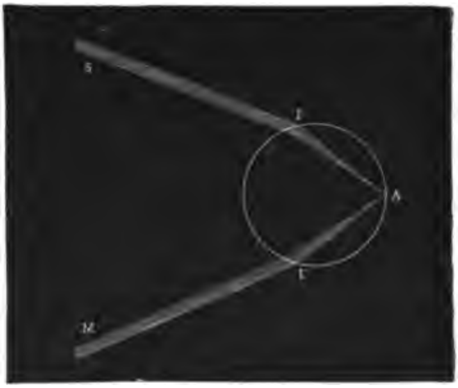

Fig. 719.—Production of Primary Bow.

The formation of the primary bow is illustrated by Fig. 719. A ray of solar light, falling on a spherical drop of water, in the direction S I, is refracted at I, then reflected internally from the back of the drop, and again refracted into the air in the direction I′M. If we take different points of incidence, we shall obtain different directions of emergence, so that the whole light which emerges from the drop after undergoing, as in the figure, two refractions and one reflection, forms a widely-divergent pencil. Some portions of this pencil, however, contain very little light. This is especially the case with those rays which, having been incident nearly normally, are returned almost directly back, and also with those which were almost tangential at incidence. The greatest condensation, as regards any particular species of elementary ray, occurs at that part of the emergent pencil which makes the smallest obtuse angle or the greatest acute angle with the direction of incidence. As the obtuse angle is the measure of the deviation, the direction of greatest condensation is the direction of *minimum deviation*. It is by means of rays which have undergone this minimum deviation, that the observer sees the corresponding colour in the bow; and the deviation which they have undergone is evidently equal to the angular distance of this part of the bow from the sun.

The minimum deviation will be greatest for those rays which are most refrangible. If the figure, for example, be supposed to represent the circumstances of minimum deviation for violet, we shall obtain smaller deviation in the case of red, even by giving the angle I A I′ the same value which it has in the case of minimum deviation for violet, and still more when we give it the value which corresponds to the minimum deviation of red. The most refrangible colours are accordingly seen furthest from the sun. The effect of the rays which undergo other than minimum deviation, is to produce a border of white light on the side remote from the sun; that is to say, on the inner edge of the bow.[1]

The condensation which accompanies minimum deviation, is merely a particular case of the general mathematical law that magnitudes remain nearly constant in the neighbourhood of a maximum or minimum value. A small parallel pencil S I incident at and around the precise point which corresponds to minimum deviation, will thus have deviations which may be regarded as equal, and will accordingly remain sensibly parallel at emergence. A parallel pencil incident on any other part of the drop, will be divergent at emergence.

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[1] When the drops are very uniform in size, a series of faint *supernumerary bows*, alternately purple and green, is sometimes seen beneath the primary bow. These bows are produced by the mutual interference of rays which have undergone other than minimum deviation, and the interference arises in the following way. Any two parallel directions of emergence, for rays of a given refrangibilty, correspond in general to two different points of incidence on any given drop, one of the two incident rays being more nearly normal and the other more nearly tangential to the drop than the ray of minimum deviation. These two rays have pursued dissimilar paths in the drop, and are in different phases when they reach the observer's eye. The difference of phase may amount to one, two, three, or more, exact wave-lengths, and thus one, two, three, or more supernumerary bows may be formed. The distances between the supernumerary bows will be greater as the drops of water are smaller. This explanation is due to Dr. Thomas Young.

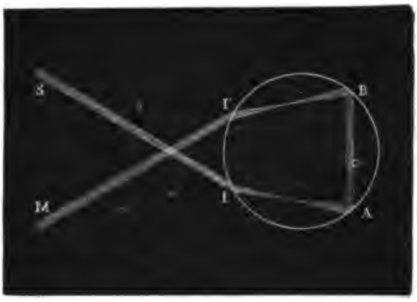

Fig. 720.—Production of Secondary Bow.

The indices of refraction for red and violet rays from air into water are respectively $\frac{108}{81}$ and $\frac{109}{81}$, and calculation shows that the distances from the centre of the sun to the parts of the bow in which these colours are strongest should be the supplements of 42° 2′ and 40° 17′ respectively. These results agree with observation. The angles 42° 2′ and 40° 17′ are the distance from the *antisolar point*, which is always the centre of the bow.

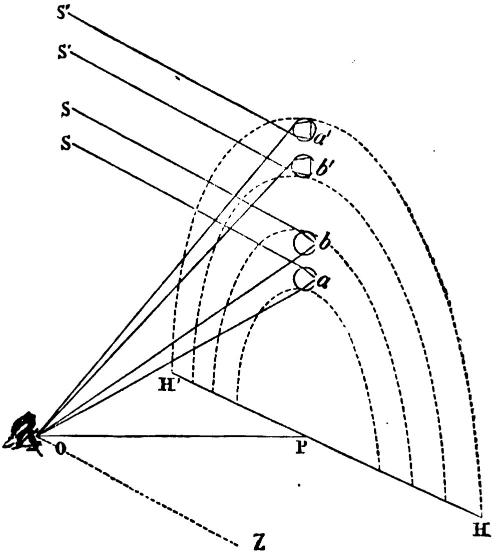

Fig. 721.—Relative Positions.

The rays which form the secondary bow have undergone two internal reflections, as represented in Fig. 720, and here again a special concentration occurs in the direction of minimum deviation. This deviation is greater than 180°, and is greatest for the most refrangible rays. The distance of the arc thus formed from the sun's centre, is 360° *minus* the deviation, and is accordingly least for the most refrangible rays. Thus the violet arc is nearest the sun, and the red furthest from it, in the secondary bow.

Some idea of the relative situations of the eye, the sun, and the drops of water in which the two bows are formed, may be obtained from an inspection of Fig. 721.

CHAPTER LXIII.

COLOUR.

797. Colour as a Property of Opaque Bodies.—A body which reflects (by irregular reflection) all the rays of the spectrum in equal proportion, will appear of the same colour as the light which falls upon it; that is to say, in ordinary cases, white or gray. But the majority of bodies reflect some rays in larger proportion than others, and are therefore coloured, their colour being that which arises from the mixture of the rays which they reflect. A body reflecting no light would be perfectly black. Practically, white, gray, and black differ only in brightness. A piece of white paper in shadow appears gray, and in stronger shadow black.

798. Colour of Transparent Bodies.—A transparent body, seen by transmitted light, is coloured, if it is more transparent to some rays than to others, its colour being that which results from mixing the transmitted rays. No new ingredient is added by transmission, but certain ingredients are more or less completely stopped out.

Some transparent substances appear of very different colours according to their thickness. A solution of chloride of chromium, for example, appears green when a thin layer of it is examined, while a greater thickness of it presents the appearance of reddish brown. In such cases, different kinds of rays successively disappear by selective absorption, and the transmitted light, being always the sum of the rays which remain unabsorbed, is accordingly of different composition according to the thickness.

When two pieces of coloured glass are placed one behind the other, the light which passes through both has undergone a double process of selective absorption, and therefore consists mainly of those rays which are abundantly transmitted by both glasses; or to speak broadly, the colour which we see in looking through the combination is not the sum of the colours of the two glasses, but their common part. Accordingly, if we combine a piece of glass coloured red with oxide of copper, and transmitting light which consists almost entirely of red rays, with a blue or violet glass of about the same depth of tint, and transmitting hardly any red, the combination will be almost black. The light transmitted through two glasses of different colour, and of the same depth of tint, is always less than would be transmitted by a double thickness of either; and the colour of the transmitted light is in most cases a colour which occupies in the spectrum an intermediate place between the two given colours. Thus, if the two glasses are yellow and blue, the transmitted light will, in most cases, be green, since most natural yellows and blues when analyzed by a prism show a large quantity of green in their composition. Similar effects are obtained by mixing coloured liquids.

799. Colours of Mixed Powders.—"In a coloured powder, each particle is to be regarded as a small transparent body which colours light by selective absorption. It is true that powdered pigments when taken in bulk are extremely opaque. Nevertheless, whenever we have the opportunity of seeing these substances in compact and homogeneous pieces before they have been reduced to powder, we find them transparent, at least when in thin slices. Cinnabar, chromate of lead, verdigris, and cobalt glass are examples in point.

"When light falls on a powder thus composed of transparent particles, a small part is reflected at the upper surface; the rest penetrates, and undergoes partial reflection at some of the surfaces of separation between the particles. A single plate of uncoloured glass reflects $\frac{1}{25}$ of normally incident light; two plates $\frac{1}{13}$, and a large number nearly the whole. In the powder of such glass, we must accordingly conclude that only about $\frac{1}{25}$ of normally incident light is reflected from the first surface, and that all the rest of the light which gives the powder its whiteness comes from deeper layers. It must be the same with the light reflected from blue glass; and in coloured powders generally only a very small part of the light which they reflect comes from the first surface; it nearly all comes from beneath. The light reflected from the first surface is white, except when the reflection is metallic. That which comes from below is coloured, and so much the more deeply the further it has penetrated. This is the reason why coarse powder of a given material is more deeply coloured than fine, for the quantity of light returned at each successive reflection depends only on the number of reflections and not on the thickness of the particles. If these are large, the light must penetrate so much the deeper in order to undergo a given number of reflections, and will therefore be the more deeply coloured.

"The reflection at the surfaces of the particles is weakened if we interpose between them, in the place of air, a fluid whose index of refraction more nearly approaches their own. Thus powders and pigments are usually rendered darker by wetting them with water, and still more with the more highly refracting liquid, oil.

"If the colours of powders depended only on light reflected from their first surfaces, the light reflected from a mixed powder would be the sum of the lights reflected from the surfaces of both. But most of the light, in fact, comes from deeper layers, and having had to traverse particles of both powders, must consist of those rays which are able to traverse both. The resultant colour therefore, as in the case of superposed glass plates, depends not on addition but rather on subtraction. Hence it is that a mixture of two pigments is usually much more sombre than the pigments themselves, if these are very unlike in the average refrangibility of the light which they reflect. Vermilion and ultramarine, for example, give a black-gray (showing scarcely a trace of purple, which would be the colour obtained by a true mixture of lights), each of these pigments being in fact nearly opaque to the light of the other."[1]

800. Mixtures of Colours.—By the colour resulting from the mixture of two lights, we mean the colour which is seen when they both fall on the same part of the retina. Propositions regarding mixtures of colours are merely subjective. The only objective differences of colour are differences of refrangibility, or if traced to their source, differences of wave-frequency. All the colours in a pure spectrum are objectively simple, each having its own definite period of vibration by which it is distinguished from all others. But whereas, in acoustics, the quality of a sound as it affects the ear varies with every change in its composition, in colour, on the other hand, very different compositions may produce precisely the same visual impression. Every colour that we see in nature can be exactly imitated by an infinite variety of different combinations of elementary rays.

To take, for example, the case of white. Ordinary white light consists of all the colours of the spectrum combined; but any one of the elementary colours, from the extreme red to a certain point in yellowish green, can be combined with another elementary colour on the other side of green in such proportion as to yield a perfect imitation of ordinary white. The prism would instantly reveal the differences, but to the naked eye all these whites are completely undistinguishable one from another.

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[1] Translated from Helmholtz's *Physiological Optics*, § 20.

801. Methods of Mixing Colours.—The following are some of the best methods of mixing colours (that is coloured lights):—

1. By combining reflected and transmitted light; for example, by looking at one colour through a piece of glass, while another colour is seen by reflection from the near side of the glass. The lower sash of a window, when opened far enough to allow an arm to be put through, answers well for this purpose. The brighter of the two coloured objects employed should be held inside the window, and seen by reflection; the second object should then be held outside in such a position as to be seen in coincidence with the image of the first. As the quantity of reflected light increases with the angle of incidence, the two colours may be mixed in various proportions by shifting the position of the eye. This method is not however adapted to quantitative comparison, and can scarcely be employed for combining more than two colours.

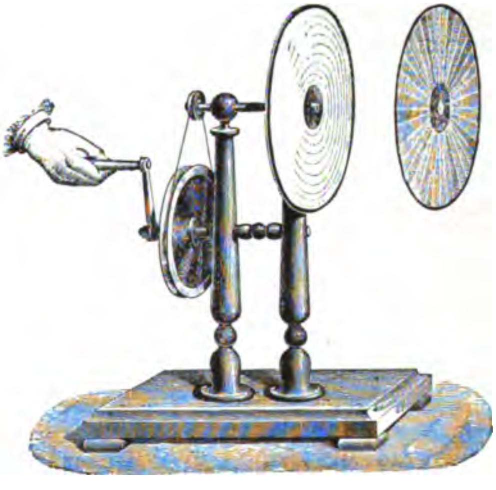

Fig. 722.—Rotating Disc.

2. By employing a rotating disc (Fig. 722) composed of differently coloured sectors. If the disc be made to revolve rapidly, the sectors will not be separately visible, but their colours will appear blended into one on account of the persistence of visual impressions. The proportions can be varied by varying the sizes of the sectors. Coloured discs of paper, each having a radial slit, are very convenient for this purpose, as any moderate number of such discs can be combined, and the sizes of the sectors exhibited can be varied at pleasure.

The mixed colour obtained by a rotating disc is to be regarded as a *mean* of the colours of the several sectors—a mean in which each of these colours is assigned a weight proportional to the size of its sector. Thus, if the 360 degrees which compose the entire disc consist of 100° of red paper, 100° of green, and 160° of blue, the intensity of the light received from the red when the disc is rotating will be only $\frac{10}{36}$ of that which would be received from the red sector when seen at rest; and the total effect on the retina is represented by $\frac{10}{36}$ of the intensity of the red, *plus* $\frac{10}{36}$ of the intensity of the green, *plus* $\frac{16}{36}$ of the intensity of the blue; or if we denote the colours of the sectors by their initial letters, the effect may be symbolized by the formula $\frac{10\,R+10\,G+16\,B}{36}$. Denoting the resultant colour by C, we have the symbolic equation

$$10\,R+10\,G+16\,B=36\,C;$$

and the resultant colour may be called the mean of 10 parts of red, 10 of green, and 16 of blue. Colour-equations, such as the above, are frequently employed, and may be combined by the same rules as ordinary equations.

3. By causing two or more spectra to overlap. We thus obtain mixtures which are the *sums* of the overlapping colours.

If, in the experiment of § 778, we employ, instead of a single straight slit, a pair of slits meeting at an angle, so as to form either an X or a V, we shall obtain mixtures of all the simple colours two and two, since the coloured images of one of the slits will cross those of the other. The display of colours thus obtained upon a screen is exquisitely beautiful, and if the eye is placed at any point of the image (for example, by looking through a hole in the screen), the prism will be seen filled with the colour which falls on this point.

802. Experiments of Helmholtz and Maxwell.—Helmholtz, in an excellent series of observations of mixtures of simple colours, employed a spectroscope with a V-shaped slit, the two strokes of the V being at right angles to one another; and by rotating the V he was able to diminish the breadth and increase the intensity of one of the two spectra, while producing an inverse change in the other. To isolate any part of the compound image formed by the two overlapping spectra, he drew his eye back from the eye-piece, so as to limit his view to a small portion of the field.

But the most effective apparatus for observing mixtures of simple colours is one devised by Professor Clerk Maxwell, by means of which any two or three colours of the spectrum can be combined in any required proportions. In principle, this method is nearly equivalent to looking through the hole in the screen in the experiment above described.

Let P (Fig. 723) be a prism, in the position of minimum deviation; L a lens; E and R conjugate foci for rays of a particular refrangibility, say red; E and V conjugate foci for rays of another given refrangibility, say violet. If a slit is opened at R, an eye at E will receive only red rays, and will see the lens filled with red light. If this slit be closed, and a slit opened at V, the eye, still placed at E, will see the lens filled with violet light. If both slits be opened, it will see the lens filled with a uniform mixture of the two lights; and if a third slit be opened, between R and V, the lens will be seen filled with a mixture of three lights.

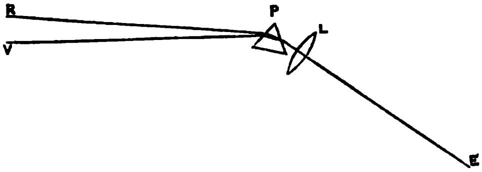

Fig. 723.—Principle of Maxwell's Colour-box.

Again, from the properties of conjugate foci, if a slit is opened at E, its spectral image will be formed at R V, the red part of it being at R, and the violet part at V.

The apparatus was inclosed in a box painted black within. There was a slit fixed in position at E, and a frame with three movable slits at R V. When it was desired to combine colours from three given parts of the spectrum, specified by reference to Fraunhofer's lines, the slit E was first turned towards the light, giving a real spectrum in the plane R V, in which Fraunhofer's lines were visible, and the three movable slits were set at the three specified parts of the spectrum. The box was then turned end for end, so that light was admitted (reflected from a large white screen placed in sunshine) at the movable slits, and the observer, looking in at the slit E, saw the resultant colour.

803. Results of Experiment.—The following are some of the principal results of experiments on the mixture of coloured lights:—

1. Lights which appear precisely alike to the naked eye yield identical results in mixtures; or employing the term *similar* to express apparent identity as judged by the naked eye, *the sums of similar lights are themselves similar*. It is by reason of this physical fact, that colour-equations yield true results when combined according to the ordinary rules of elimination.

In the strict application of this rule, the same observer must be the judge of similarity in the different cases considered. For

2. Colours may be similar as seen by one observer, and dissimilar as seen by another; and in like manner, colours may be similar as seen through one coloured glass, and dissimilar as seen through another. The reason, in both cases, is that selective absorption depends upon real composition, which may be very different for two merely similar lights. Most eyes are found to exhibit selective absorption of a certain kind of elementary blue, which is accordingly weakened before reaching the retina.

3. Every colour, except purple, is similar to a colour of the spectrum either pure or diluted, and all purples are similar to mixtures of red and blue with or without dilution. By *diluting* a colour we mean mixing it with white, gray, or black. Brown colours are obtained by diluting red, orange, or yellow of feeble intensity.

4. Between any four colours, given in intensity as well as in kind, one colour-equation subsists; expressing the fact that, when we have the power of varying their intensities at pleasure, there is one definite way of making them yield a *match*, that is to say, a pair of similar colours. Any colour (intensity included) can therefore be completely specified by three numbers, expressing its relation to three arbitrarily selected colours. This is analogous to the theorem in statics that a force acting at a given point can be specified by three numbers denoting its components in three arbitrarily selected directions.

5. Between any five colours (intensity included) a match can be made in one definite way by taking means;[1] for example, by mounting the colours on two rotating discs. If we had the power of illuminating one disc more strongly than the other in any required ratio, four colours would be theoretically sufficient; and we can, in fact, do what is nearly equivalent to this, by employing black as one of our five colours. Taking means of colours is analogous to finding centres of gravity. In following out the analogy, a colour (given in kind merely) must be represented by a material point given in position merely, and the intensity of the colour must be represented by the mass of the material point. The means of two given colours will be represented by points in the line joining two given points. The means of three given colours will be represented by points lying

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[1] Propositions 4 and 5 are not really independent, but represent different aspects of one physical (or rather physiological) law.

within the triangle formed by joining three given points, and the means of four given colours will be represented by points within a tetrahedron whose four corners are given. When we have five colours given, we have five points given, and of these generally no four will lie in one plane. Call them A, B, C, D, E. Then if E lies within the tetrahedron A B C D, we can make the centre of gravity of A, B, C, and D coincide with E, and the colour E can be matched by a mean of the other four colours. If E lies outside the tetrahedron, let the planes which contain the tetrahedron be produced indefinitely. Then if E lies in the external solid angle which is vertical to the solid angle A of the tetrahedron, the point A lies within the tetrahedron E B C D, and the colour A is the match. Lastly, let E lie in the external space which is separated from the tetrahedron by the plane B C D. Then the point where this plane is cut by the line joining A E represents the match, for it is a mean of A, E, and is also a mean of B, C, D.

With six given colours, six different matches can be made, and six colour-equations will thus be obtained, the consistency of which among themselves will be a test of the accuracy both of theory and observation, as only three of the six can be really independent. Experiments which have been conducted on this plan have given very consistent results.

804. Cone of Colour.—All combinations of colour (intensity included) can be represented geometrically by means of a cone or pyramid within which all possible colours will have their definite places. The vertex will represent total blackness, or the complete absence of light; and colours situated on the same line passing through the vertex will differ only in intensity of light. Any cross-section of the cone will contain all colours, except so far as intensity is concerned, and the colours residing on its perimeter will be the colours of the spectrum ranged in order, with purple to fill up the interval between violet and red. It appears from Maxwell's experiments, that the true form of the cross-section is approximately triangular,[1] with red, green, and blue at the three corners. When all the colours (intensity included) have been assigned their proper places in the cone, a straight line joining any two of them passes through colours which are means of these two; and if two lines are drawn from the vertex to any two colours, the parallelogram constructed on these two lines will have at its further corner the colour which is the sum of these two colours. A certain axial line of the cone will contain white or gray at all points of its length, and may be called the *line of white.*

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[1] The shape of the triangle is a mere matter of convenience, not involving any question of fact.

It is convenient to distinguish three qualities of colour which may be called *hue*, *depth*, and *brightness*. *Brightness* or *intensity* of light is represented by distance from the vertex of the cone. *Depth* depends upon angular distance from the line of white, and is the same for all points on the same line through the vertex. *Paleness* or *lightness* is the opposite of depth, and is measured by angular nearness to the line of white. *Hue* or *tint* is that which is often *par excellence* termed colour. If we suppose a plane, containing the line of white, to revolve about this line as axis, it will pass successively through different tints; and in any one position it contains only two tints, which are separated from each other by the line of white, and are complementary.

Red is complementary to	Bluish green.
Orange ,, ,,	Sky blue.
Yellow ,, ,,	Violet blue.
Greenish yellow ,,	Violet.
Green ,, ,,	Pink.

Any two colours, of complementary tint, give white when mixed in proper proportions; and any three colours can be mixed in such proportions as to yield white, unless they are all on the same side of a plane drawn through the line of white.

According to Maxwell, the orange and yellow of the spectrum can be exactly reproduced by mixtures of red and green, and the extreme colours of the spectrum (crimson and violet) can be reproduced (approximately at least) by mixtures of red and blue.

805. Three Primary Colour-sensations.—All authorities are now agreed in accepting the doctrine, first propounded by Dr. Thomas Young, that there are three elements of colour-sensation; or, in other words, three distinct physiological actions, which, by their various combinations, produce our various sensations of colour. Each is excitable by light of various wave-lengths lying within a wide range, but has a maximum of excitability for a particular wave-length, and is affected only to a slight degree by light of wave-length very different from this. The cone of colour is theoretically a triangular pyramid, having for its three edges the colours which correspond to these three wave-lengths; but it is probable that we cannot obtain one of the three elementary colour-sensations quite from admixture of the other two, and the edges of the pyramid are thus practically rounded off. One of these sensations is excited in its greatest purity by the green near Fraunhofer's line *b*, another by the extreme red, and the third by a part of the spectrum lying somewhere in deep blue or violet, its precise position being difficult to determine by reason of the feebleness of the light at this end of the spectrum.

Helmholtz ascribes these three actions to three distinct sets of nerves, having their terminations in different parts of the thickness of the retina—a supposition which aids in accounting for the approximate achromatism of the eye, for the three sets of nerve-terminations may thus be at the proper distances for receiving distinct images of red, green, and blue respectively, the focal length of a lens being shorter for blue than for red.

Light of great intensity, whatever its composition, seems to produce a considerable excitement of all three elements of colour-sensation. If a spectroscope, for example, be directed first to the clouds and then to the sun, all parts of the spectrum appear much paler in the latter case than in the former.

The popular idea that red, yellow, and blue are the three primaries, is quite wrong as regards mixtures of lights or combinations of colour-sensations. The idea has arisen from facts observed in connection with the mixture of pigments and the transmission of light through coloured glasses. We have already pointed out the true interpretation of observations of this nature, and have only now to add that in attempting to construct a theory of the colours obtained by mixtures of pigments, the law of substitution of *similars* cannot be employed. Two pigments of *similar* colour will not in general give the same result in mixtures.

806. Accidental Images.—If we look steadily at a bright stained-glass window, and then turn our eyes to a white wall, we see an image of the window with the colours changed into their complementaries. The explanation is that the nerves which have been strongly exercised in the perception of the bright colours have had their sensibility diminished, so that the balance of action which is necessary to the sensation of white no longer exists, but those elements of sensation which have not been weakened preponderate. The subjective appearances arising from this cause are called *negative accidental images.* Many well-known effects of contrast are similarly explained. White paper, when seen upon a background of any one colour, often appears tinged with the complementary colour; and stray beams of sunlight entering a room shaded with yellow holland blinds, produce blue streaks when they fall upon a white tablecloth.

In some cases, especially when the object looked at is painfully bright, there is a *positive* accidental image; that is, one of the same colour as the object; and this is frequently followed by a negative image. A positive accidental image may be regarded as an extreme instance of the persistence of impressions.

807. **Colour-blindness.**—What is called colour-blindness has been found, in every case which has been carefully investigated, to consist in the absence of the elementary sensation corresponding to red. To persons thus affected the solar spectrum appears to consist of two decidedly distinct colours with white or gray at their place of junction, which is a little way on the less refrangible side of the line F. One of these two colours is doubtless nearly identical with the normal sensation of blue. It attains its maximum about midway between F and G, and extends beyond G as far as the normally visible spectrum. The other colour extends a considerable distance into what to normal eyes is the red portion of the spectrum, attaining its maximum about midway between D and E, and becoming deeper and more faint till it vanishes at about the place where to normal eyes crimson begins. The scarlet of the spectrum is thus visible to the colour-blind, not as scarlet but as a deep dark colour, probably a kind of dark green, orange and yellow as brighter shades of the same colour, while bluish-green appears nearly white.

It is obvious from this account that what is called "colour-blindness" should rather be called *dichroic vision*, normal vision being distinctively designated as *trichroic*. To the dichroic eye any colour can be matched by a mixture of yellow and blue, and a match can be made between any three (instead of four) given colours. Objects which have the same colour to the trichroic eye have also the same colour to the dichroic eye.

808. **Colour and Musical Pitch.**—As it is completely established that the difference between the colours of the spectrum is a difference of vibration-frequency, there is an obvious analogy between colour and musical pitch; but in almost all details the relations between colours are strikingly different from the relations between sounds.

The compass of visible colour, including the lavender rays which lie beyond the violet, and are perhaps visible not in themselves, but by the fluorescence which they produce on the retina, is, according to Helmholtz, about an octave and a fourth; but if we exclude the lavender, it is almost exactly an octave. Attempts have been made to compare the successive colours of the spectrum with the notes of the gamut; but much forcing is necessary to bring out any trace of identity, and the gradual transitions which characterize the spectrum, and constitute a feature of its beauty, are in marked contrast to the transitions *per saltum* which are required in music.

CHAPTER LXIV.

WAVE THEORY OF LIGHT.

809. Principle of Huygens.[1]—The propagation of waves, whether of sound or light, is a propagation of energy. Each small portion of the medium experiences successive changes of state, involving changes in the forces which it exerts upon neighbouring portions. These changes of force produce changes of state in these neighbouring portions, or in such of them as lie on the forward side of the wave, and thus a disturbance existing at any one part is propagated onwards.

Let us denote by the name *wave-front* a continuous surface drawn through particles which have the same phase; then each wave-front advances with the velocity of light, and each of its points may be regarded as a secondary centre from which disturbances are continually propagated. This mode of regarding the propagation of light is due to Huygens, who derived from it the following principle, which lies at the root of all practical applications of the undulatory theory: *The disturbance at any point of a wave-front is the resultant* (given by the parallelogram of motions) *of the separate disturbances which the different portions of the same wave-front in any one of its earlier positions, would have occasioned if acting singly.* This principle involves the physical fact that rays of light are not affected by crossing one another; and its truth, which has been experimentally tested by a variety of consequences, must be taken as an indication that the amplitudes of luminiferous vibrations are infinitesimal in comparison with the wave-lengths. A similar law applies to the resultant of small disturbances generally, and is called by writers on dynamics the law of "superposition of small motions." It is analogous to the arithmetical principle that, when a and b are very small fractions, the product of $1+a$ and $1+b$ may be identified with

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[1] For the spelling of this name see remarks by Lalande, *Mémoires de l'Académie*, 1773.

$1+a+b$, the term $a\,b$, which represents the mutual influence of two small changes, being negligible in comparison with the sum $a+b$ of the small changes themselves.

810. Explanation of Rectilinear Propagation.—In a medium in which light travels with the same velocity in all parts and in all directions, the waves propagated from any point will be concentric spheres, having this point for centre, and the lines of propagation, in other words the rays of light, will be the radii of these spheres. It can in fact be shown that the only part of one of these waves which needs to be considered, in computing the resultant disturbance of an external point, is the part which lies directly between this external point and the centre of the sphere. The remainder of the wave-front can be divided into small parts, each of which, by the mutual interference of its own subdivisions, gives a resultant effect of zero at the given point. We express these properties by saying that *in a homogeneous and isotropic medium the wave-surface is a sphere, and the rays are normal to the wave-fronts.* This class of media includes gases, liquids, crystals of the cubic system, and well-annealed glass.

If a medium be homogeneous but not isotropic, disturbances emanating from a point in it will be propagated in waves which will retain their form unchanged as they expand in receding from their source, but this form will not generally be spherical. The rays of light in such a medium will be straight, proceeding directly from the centre of disturbance, and any one ray will cut all the wave-fronts at the same angle; but this angle will generally be different for different rays. In this case, as in the last, the disturbance produced at any point may be computed by merely taking into account that small portion of a wave-front which lies directly between the given point and the source,—in other words, which lies on or very near to the ray which traverses the given point.

A disturbance in such a medium usually gives rise to two sets of waves, having two distinct forms, and these remarks apply to each set separately.

The tendency of the different parts of a wave-front to propagate disturbances in other directions besides the single one to which such propagation is usually confined, is manifested in certain phenomena which are included under the general name of *diffraction*.

The only wave-fronts with which it is necessary to concern ourselves are those which belong to waves emanating from a single point,—that is to say, either from a surface really very small, or from a surface which, by reason of its distance, subtends a very small solid angle at the parts of space considered.

811. Application to Refraction.—When waves are propagated from one medium into another, the principle of Huygens leads to the following construction:—

Let A E (Fig. 724) represent a portion of the surface of separation between two media, and A B a portion of a wave-front in the first medium; both portions being small enough to be regarded as plane.

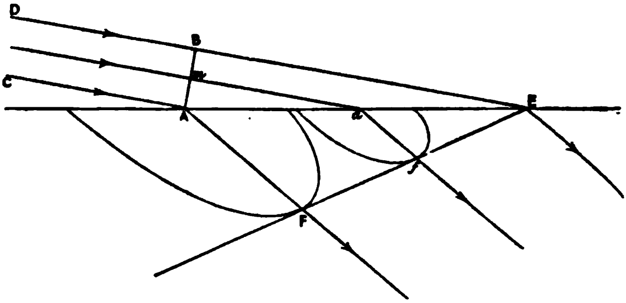

Fig. 724.—Huygens' Construction for Wave-front.

Then straight lines C A, D B E, normal to the wave-front, represent rays incident at A and E. From A as centre, describe a wave-surface, of such dimensions that light emanating from A would reach this surface in the same time in which light in air travels the distance B E, and draw a tangent plane (perpendicular to the plane of incidence) through E to this surface. Let F be the point of contact (which is not necessarily in the plane of incidence). Then the tangent plane E F is a wave-front in the second medium, and A F is a ray in the second medium; for it can be shown that disturbances propagated from all points in the wave-front A B will just have reached E F when the disturbance propagated from B has reached E. For example, a ray proceeding from *m*, the middle point of the line A B, will exhaust half the time in travelling to the middle point *a* of A E, and the remaining half in travelling through *a f*, equal and parallel to half of A F.

When the wave-surfaces in both media are spherical, the planes of incidence and refraction A B E, A F E coincide, the angle B A E (Fig. 725) between the first wave-front and the surface of separation is the same as the angle between the normals to these surfaces, that is to say, is the angle of incidence; and the angle A E F between the surface of separation and the second wave-front is the angle of refraction. The sine of the former is $\frac{BE}{EA}$, and the sine of the latter is $\frac{AF}{EA}$. The ratio $\frac{\sin i}{\sin r}$ is therefore $\frac{BE}{AF}$. But B E and A F are the distances travelled in the same time in the two media. Hence the sines of the angles of incidence and refraction are directly as the velocities of propagation of the incident and refracted light. The *relative index* of refraction from one medium into another is therefore the *ratio of the velocity of light in the first medium to its velocity in the second;* and *the absolute index of refraction of any medium is inversely as the velocity of light in that medium.*

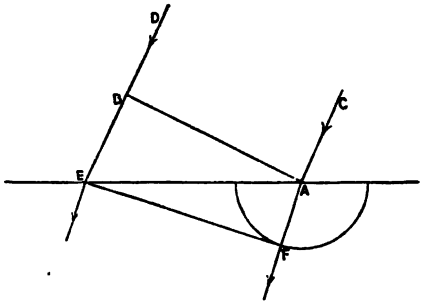

Fig. 725.—Wave-front in Ordinary Refraction.

812. Application to Reflection.—The explanation of reflection is precisely similar. Let C A, D E (Fig. 726) be parallel rays incident at A and E; A B the wave-front. As the successive points of the wave-front arrive at the reflecting surface, hemispherical waves diverge from the points of incidence; and by the time that B reaches E, the wave from A will have diverged in all directions to a distance equal to B E. If then we describe in the plane of incidence a semicircle, with centre A and radius equal to B E, the tangent E F to this semicircle will be the wave-front of the reflected light, and A F will be the reflected ray corresponding to the incident ray C A. From the equality of the right-angled triangles A B E, E F A, it is evident that the angles of incidence and reflection are equal.

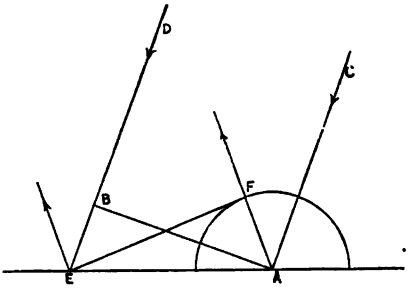

Fig. 726.—Wave-front in Reflection.

813. Newtonian Explanation of Refraction.—In the Newtonian theory, the change of direction which a ray experiences at the bounding surface of two media, is attributed to the preponderance of the attraction of the denser medium upon the particles of light. As the resultant force of this attraction is normal to the surface, the tangential component of velocity remains unchanged, and the normal component is increased or diminished according as the incidence is from rare to dense or from dense to rare. Let μ denote the relative index of refraction from rare to dense. Let v, v' be the velocities of light in the rarer and denser medium respectively, and i, i' the angles which the rays in the two media make with the normal. Then the tangential components of velocity in the two media are $v \sin i$, $v' \sin i'$ respectively, and these by the Newtonian theory are equal; whence $\frac{v'}{v}=\frac{\sin i}{\sin i'}=\mu$; whereas according to the undulatory theory $\frac{v'}{v}=\frac{1}{\mu}$. In the Newtonian theory, the velocity of light in any medium is directly as the absolute index of refraction of the medium; whereas, in the undulatory theory, the reverse rule holds.

The main design of Foucault's experiment with the rotating mirror (§ 687), in its original form, was to put these opposite conclusions to the test of direct experiment. For this purpose it was not necessary to determine the velocity of the rotating mirror, since it affected both the observed displacements alike. The two images were seen in the same field of view, and were easily distinguished by the greenness of the water-image. In every trial the water-image was more displaced than the air-image, indicating longer time and slower velocity; and the measurements taken were in complete accordance with the undulatory theory, while the Newtonian theory was conclusively disproved.

814. Principle of Least Time.—The path by which light travels from one point to another is in the generality of cases that which occupies least time. For example, in ordinary cases of reflection (except from very concave[1] surfaces), if we select any two points, one on the incident and the other on the reflected ray, the sum of their distances from the point of incidence is less than the sum of their distances from any neighbouring point on the reflecting surface. In this case, since only one medium is concerned, distance is proportional to time. When a ray in air is refracted into water, if we select any two points, one on the incident and the other on the refracted ray, and call their distances from any point of the refracting surface s, s' respectively, and the velocities of propagation in the two media v, v', then the sum of $\frac{s}{v}$ and $\frac{s'}{v'}$ is generally less when s and s' are measured to the point of incidence than when they are measured to any neighbouring point on the surface. $\frac{s}{v}$ is evidently the time of going from the first point to the refracting surface, and $\frac{s'}{v'}$ the time from the refracting surface to the second point.

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[1] Suppose an ellipse described, having the two selected points for foci, and passing through the point of incidence. If the curvature of the reflecting surface in the plane of incidence is greater than the curvature of this ellipse, the length of the path is a maximum, if less, a minimum. This follows at once from the constancy of the sum of the focal distances in an ellipse.

The proposition as above enunciated admits of certain exceptions, the time being sometimes a maximum instead of a minimum. The really essential condition (which is fulfilled in both these opposite cases) is that all points on a small area surrounding the point of incidence give sensibly *the same time.* The component waves sent from all parts of this small area will be in the same phase, and will propagate a ray of light by their combined action.

When the two points considered are conjugate foci, and there is no aberration, this condition must be fulfilled by all the rays which pass through both; and the *time of travelling from one focus to the other is the same for all the rays.* Spherical waves diverging from one focus will, after incidence, become spherical waves converging to or diverging from its conjugate focus. An effect of this kind can be beautifully exhibited to the eye by means of an elliptic dish containing mercury. If agitation is produced at one focus of the ellipse by dipping a small rod into the liquid at this point, circular waves will be seen to converge towards the other focus. A circular dish exhibits a similar result somewhat imperfectly; waves diverging from a point near the centre will be seen to converge to a point symmetrically situated on the other side of the centre.

When the second point lies on a caustic surface formed by the reflection or refraction of rays emanating from the first point, all points on an area of sensible magnitude in the neighbourhood of the point of incidence would give sensibly the same time of travelling as the actual point of incidence, so that the light which traverses a point on a caustic may be regarded as coming from an area of sensible magnitude instead of (as in the case of points not on the caustic) an excessively small area. An eye placed at a point on a caustic will see this portion of the surface filled with light.

As the velocity of light is inversely proportional to the index of refraction μ, the time of travelling a distance s with constant velocity may be represented by μs, and if a ray of light passes from one point to another by a crooked path, made up of straight lines $s_1, s_2, s_3, \ldots$ lying in media whose absolute indices are $\mu_1, \mu_2, \mu_3, \ldots$, the expression $\mu_1 s_1 + \mu_2 s_2 + \mu_3 s_3 + \ldots$ represents the time of passage. This expression, which may be called *the sum of such terms as* μs, must therefore fulfil the above condition; that is to say, the points of incidence on the surfaces of separation must be so situated that this sum either remains absolutely constant when small changes are supposed to be made in the positions of these points, or else retains that approximate constancy which is characteristic of maxima and minima. Conversely, all lines from a luminous point which fulfil this condition, will be paths of actual rays.

815. Terrestrial Refraction.[1]—The atmosphere may be regarded as homogeneous when we confine our attention to small portions of it, and hence it is sensibly true, in ordinary experiments where no great distances are concerned, that rays of light in air are straight, just as it is true in the same limited sense that the surface of a liquid at rest is a horizontal plane. The surface of an ocean is not plane, but approximately spherical, its curvature being quite sensible in ordinary nautical observations, where the distance concerned is merely that of the visible sea-horizon; and a correction for curvature is in like manner required in observing levels on land. If the observer is standing on a perfectly level plain, and observing a distant object at precisely the same height as his eye above the plain, it will appear to be below his eye, for a horizontal *plane* through his eye will pass above it, since a perfectly level *plain* is not *plane*, but shares in the general curvature of the earth. It is easily proved that the apparent depression due to this cause is half the angle between the verticals at the positions of the observer and of the object observed. But experience has shown that this apparent depression is to a considerable extent modified by an opposite disturbing cause, called *terrestrial refraction.* When the atmosphere is in its normal condition, a ray of light from the object to the observer is not straight, but is slightly concave downwards.

This curvature of a nearly horizontal ray is not due to the curvature of the earth and of the layers of equal density in the earth's atmosphere, as is often erroneously supposed, but would still exist and with no sensible change in its amount, if the earth's surface were plane, and the directions of gravity everywhere parallel. It is due to the fact that light travels faster in the rarer air above than in the denser air below, so that time is saved by deviating slightly to the upper side of a straight course. The actual amount of curvature (as determined by surveying) is from $\frac{1}{2}$ to $\frac{1}{10}$ of the curvature of the earth; that is to say, the radius of curvature of the ray is from 2 to 10 times the earth's radius.

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[1] For the leading idea which is developed in §§ 815-817, the Editor is indebted to suggestions from Professor James Thomson.

816. Calculation of Curvature of Ray.—In order to calculate the radius of curvature from physical data, it is better to approach the subject from a somewhat different point of view.

The wave-fronts of a ray in air are perpendicular to the ray; and if the ray is nearly horizontal, its wave-fronts will be nearly vertical. If two of these wave-fronts are produced downwards until they meet, the distance of their intersection from the ray will be the radius of curvature. Let us consider two points on the same wave-front, one of them a foot above the other; then the upper one being in rarer air will be advancing faster than the lower one, and it is easily shown that the difference of their velocities is to the velocity of either as 1 foot is to the radius of curvature.

Put ρ for the radius of curvature in feet, v and $v+\delta v$ for the two velocities, μ and $\mu-\delta\mu$ for the indices of refraction of the air at the two points. Then we have

$$\frac{1}{\rho}=\frac{\delta v}{v}=\frac{\delta\mu}{\mu}=\delta\mu, \text{ nearly.} \qquad (1)$$

Now it has been ascertained, by direct experiment, that the value of $\mu-1$ for air, within ordinary limits of density, is sensibly proportional to the density (even when the temperature varies), and is $\cdot 0002943$ or $\frac{1}{3400}$ at the density corresponding to the pressure 760^{mm} (at Paris) and temperature 0°C. The difference of density at the two points considered, supposing them both to be at the same temperature, will be to the density of either as 1 foot is to the "height of the homogeneous atmosphere" in feet, which call H (§ 111 A). Then $\frac{\delta\mu}{\mu-1}$ will be $\frac{1}{\mathrm{H}}$, and the value of $\frac{1}{\rho}$ in (1) may be written

$$\frac{1}{\rho}=\frac{\delta\mu}{\mu-1}(\mu-1)=\frac{1}{\mathrm{H}}(\mu-1)=\frac{1}{\mathrm{H}}\;\frac{1}{3400}. \qquad (2)$$

Hence ρ is 3400 times the height of the homogeneous atmosphere. But this height is about 5 miles, or $\frac{1}{800}$ of the earth's radius. The value of ρ is therefore about $4\frac{1}{4}$ radii of the earth. This is on the assumptions that the barometer is at 760$^{\text{mm}}$, the thermometer at 0°C., and that there is no change of temperature in ascending. If we depart from these assumptions, we have the following consequences:—

I. If the barometer is at any other height, the factor $\frac{1}{H}$ remains unaltered, and the other factor $\mu-1$ varies directly as the pressure.

II. If the temperature is $t°$ Centigrade, H is changed in the direct ratio of $1+a\,t$, a denoting the coefficient of expansion. The first factor $\frac{1}{H}$ is therefore changed in the inverse ratio of $1+a\,t$. The second factor is changed in the same ratio. The curvature of the ray therefore varies inversely as $(1+a\,t)^2$.

III. Suppose the temperature decreases upwards at the rate of $\frac{1}{n}$ of a degree Centigrade per foot. The expansion due to $\frac{1}{n}$ of a degree Centigrade is $\frac{1}{273\,n}$. The first factor $\frac{\delta\mu}{\mu-1}$, or $\frac{\text{difference of density}}{\text{density}}$, will therefore become $\frac{1}{H}-\frac{1}{273\,n}$, which, if we put $n=540$ (corresponding to 1° Fahr. in 300 feet), and reckon H as 26,000, is approximately $\frac{1}{26000}-\frac{1}{147000}$ or $\frac{1}{H}\left(1-\frac{1}{6}\right)$. The second factor of the expression for $\frac{1}{\rho}$ is unaffected. It appears, then, that decrease of temperature upwards at the rate of 1°C. in 540 feet, or 1° F. in 300 feet (which is the generally-received average), makes the curvature of the ray five-sixths of what it would be if the temperature were uniform.[1]

Combining this correction with correction II., it appears that, with a mean temperature of 10° C. or 50° F., and barometer at 760$^{\text{mm}}$, the curvature of a nearly horizontal ray (taking the earth's curvature as unity) is

$$\frac{1}{4\frac{1}{4}}\times\left(\frac{273}{283}\right)^2\times\frac{5}{6}=\frac{1}{5\cdot5}\text{ nearly.}$$

This is in perfect agreement with observation, the received average (obtained as an empirical deduction from observation) being $\frac{1}{5}$ or $\frac{1}{6}$.

817. Curvature of Inclined Rays.—Thus far we have been treating of nearly horizontal rays. To adapt our formula for $\frac{1}{\rho}$ ((2) § 816) to the case of an oblique ray, we have merely to multiply it by $\cos\theta$.

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[1] If the temperature decreases upwards at the rate of 1°C. in n feet, or 1°F. in n' feet the first factor of the expression for $\frac{1}{\rho}$ (which would be $\frac{1}{H}$ at uniform temperature) becomes approximately $\frac{1}{H}\left(1-\frac{96}{n}\right)$ or $\frac{1}{H}\left(1-\frac{53}{n'}\right)$.

θ denoting the inclination of the ray to the horizontal, or the inclination of the wave-front to the vertical. For, if we still compare two points a foot apart, on the same wave-front, and in the same vertical plane with each other and with the ray, their difference of height will be the product of 1 foot by cos θ, and $\frac{\delta v}{v}$ will therefore be less than before in the ratio cos θ.

Hence it can be shown that the earth's curvature, so far from being the cause of terrestrial refraction, rather tends in ordinary circumstances to diminish it, by increasing the average obliquity of a ray joining two points at the same level.

The general formula for the curvature of a ray (lying in a vertical plane) at any point in its length, may be written

$$\begin{aligned}\frac{1}{\rho} &= \frac{1}{H}\left(1 - \frac{96}{n}\right)(\mu - 1)\cos\theta \\ &= \frac{1}{H}\left(1 - \frac{53}{n'}\right)(\mu - 1)\cos\theta,\end{aligned} \qquad (3)$$

n denoting the number of feet of ascent which give a decrease of 1° C., and n' the number of feet which give a decrease of 1° F. The unit of length for H and may be anything we please.

818. Astronomical Refraction.—Astronomical refraction, in virtue of which stars appear nearer the zenith than they really are, can be reduced to these principles; but it is simpler, in the case of stars not more than 70° or 80° from the zenith, to regard the earth and the layers of equal density in the atmosphere as plane, and to assume that the final result is the same as if the rays from the star were refracted at once out of vacuum into the horizontal stratum of air in which the observer's eye is situated. If z be the apparent and z' the true zenith distance of the star, we shall thus have $\sin z' = \mu \sin z$, whence it may be shown that the value of $z' - z$, in terms of $\frac{\text{arc}}{\text{radius}}$, is approximately $(\mu - 1)\tan z$.

819. Mirage.—Of the three circumstances which govern the curvature of a ray in air at any point of its course, viz. *pressure, temperature,* and *rate of change of temperature,* the third is in general the most important. If the temperature decreases upwards at the rate of $\frac{1}{53}$ of 1° F. per foot of height, the correction for refraction will vanish altogether, and rays will be straight. A more rapid decrease than this will render them concave upwards.

Fig. 727, which is intended to explain the *mirage* of the desert,[1] is constructed on the hypothesis that a stratum of air, extending from the ground to a little above the observer's eye E, has so rapid a decrease of temperature upwards, that the curvature of rays which traverse it is four times greater than the opposite curvature of rays in the air above it. On tracing the course of the different rays represented in the figure as entering the eye from the object AB, it is obvious that there will be two images; one of them A′B′ erect and lifted up, the other A″B″ inverted and depressed. It is scarcely necessary to remark that vertical distances are greatly exaggerated in the figure as compared with horizontal distances.

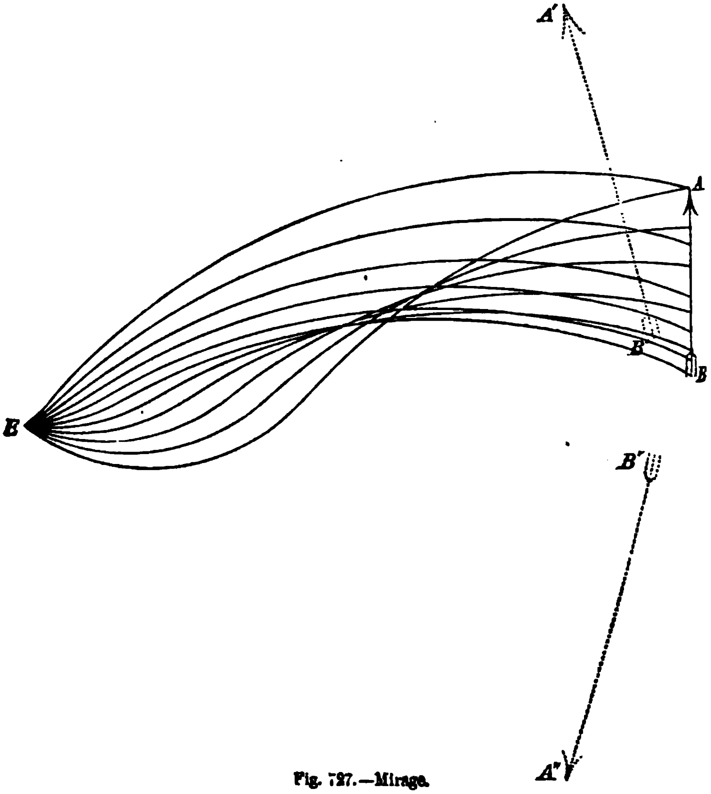

Fig. 727.—Mirage.

In general, if there be a plane stratum of air of maximum density, with regular diminutions of density on both sides, rays which cross it at a very small angle will be bent round to it again, and may thus cross it backwards and forwards any number of times. Generally speaking, an image formed after an odd number of such crossings will be inverted, and after an even number erect. Images of high orders (that is, formed by rays which have made numerous crossings) will be crowded together in the neighbourhood of the true direction of the object, and will merely produce confusion.

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[1] See § 726 for a description of the phenomena.

In every case of inverted images, the rays which form them must have crossed *one another* an odd number of times, and images formed by rays which have crossed one another an even number of times, or not at all, will be erect. Reversal of curvature in rays is not *necessary* for inversion, nor for the formation of multiple images; and a phenomenon bearing a strong resemblance to *total reflection* may be produced by a thin stratum of air in which the refractive index diminishes very rapidly in one direction only. Suppose the stratum to be horizontal, and to diminish in refractive index upwards. Then rays entering it nearly horizontally from below will be bent downwards in traversing it; and if their original inclination to the vertical exceeds a certain limiting angle (analogous to the critical angle in total reflection), they will not be able to get through it, but will emerge again at its lower side. Two nearly parallel rays entering it in the same vertical plane will cross one another within it, and emerge in the same directions as if they had been reflected from a horizontal mirror in or above it. An observer receiving such rays after emergence will see images elevated and inverted; and he may at the same time see the real objects directly by rays which have not reached the stratum. A stratum *below* the observer's eye, and rapidly diminishing in refractive index *downwards*, may in like manner produce inverted images below the real objects. Masses of air of different refractive indices, arranged in irregular forms, may produce double or multiple images; but these will generally be so blurred and distorted as to be barely recognizable.

In dealing with air, it is not important to distinguish between difference of density and difference of refractive index; for $\mu - 1$ is sensibly proportional to the density even in comparing dry air with moist, or hot with cold.

An extremely beautiful imitation of mirage may be obtained by arranging, in a vessel with plate-glass sides, three clear liquids, one above another, such that the middle liquid, while intermediate in density, has the highest index of refraction. Very distinct triple images (the middle one inverted) may thus be obtained of all the objects in a landscape.[1]

820. Diffraction Fringes.—When a beam of direct sunlight is admitted into a dark room through a narrow slit, a screen placed at any distance to receive it will show a line of white light, bordered with coloured fringes which become wider as the slit is narrowed. They also increase in width as the screen is removed further off. If they are viewed through a piece of red glass which allows only red rays to pass, they will appear as a succession of bands alternately bright and dark.

To explain their origin, we shall suppose the sun's rays (which may be reflected from an external mirror) to be perpendicular to the plane of the slit,[2] so that the wave-fronts are parallel to this plane, and we shall, in the first instance, confine our attention to light of a particular wave-length; for example, that of the light transmitted by the red glass. Then, if the slit be uniform through its whole length, the positions of the bright and dark bands will be governed by the following laws:—

1. The darkest parts will be at points whose distances from the two edges of the slit differ by an exact number of wave-lengths. If the difference be one wave-length, the light which arrives at any instant from different parts of the width of the slit is in all possible phases, and the disturbance produced by the nearer half of the slit cancels that produced by the remoter half. If the difference be n wave-lengths, we can divide the slit into n parts, such that the effect due to each part is thus *nil*.[3]

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[1] The experiment succeeds well with saturated and filtered solution of alum for the lowest liquid, pure water for the highest, and clear whisky, with white sugar dissolved in it, for the intermediate liquid. The solution of alum should be introduced first, then the water, and lastly the sugared whisky, its specific gravity (which increases with the quantity of sugar) having been carefully adjusted by trial before introducing it. It should be introduced (by means of a pipette) in sufficient quantity to form a layer about a quarter of an inch thick. This combination of liquids, in a vessel 6 inches square, has been found to retain its power of giving triple images for several days.

[2] That is, to the plane of the two knife-edges by which the slit is bounded. This condition can only be strictly fulfilled for a single point on the sun's disk. Every point on the sun's surface sends out its own waves as an independent source; and waves from one point cannot interfere with waves from another. In the experiment as described in the text the fringes due to different parts of the sun's surface are all produced at once on the screen, and overlap each other.

[3] The following explanation will serve to establish the legitimacy of the reasoning here employed:—

Each element of the length of the slit tends to produce a system of circular rings (the screen being supposed parallel to the plane of the slit). If the width of the slit is uniform,

2. The brightest parts will be at points whose distances from the two edges of the slit differ by an exact number of wave-lengths *plus* a half. Let the difference be $n + \frac{1}{2}$; then we can divide the slit into n inefficient parts and one efficient part, this latter having only half the width of one of the others.

Each colour of light has its own alternate bands of brightness and darkness, the distance from band to band being greatest for red and least for violet. The superposition of all the bands constitutes the coloured fringes which are seen.

This experiment furnishes the simplest answer to the objection formerly raised to the undulatory theory, that light is not able, like sound, to pass round an obstacle, but can only travel in straight lines. In this experiment light does pass round an obstacle, and turns more and more away from a straight line as the slit is narrowed.

When the slit is not exceedingly narrow, the light sent in oblique directions is quite insensible in comparison with the direct light, and no fringes are visible. "We have reason to think that when *sound* passes through a very large aperture, or when it is reflected from a large surface (which amounts nearly to the same thing), it is hardly sensible except in front of the opening, or in the direction of reflection."[1]

There are several other modes of producing diffraction fringes, which our limits do not permit us to notice. We proceed to describe the mode of obtaining a *pure spectrum* by diffraction.

821. Diffraction by a Grating.—If a piece of glass is ruled with parallel equidistant scratches (by means of a dividing engine and diamond point) at the rate of some hundreds or thousands to the inch, we shall find, on looking through it at a slit or other bright line (the glass being held so that the scratches are parallel to the slit), that a number of spectra are presented to view, ranged at nearly equal distances, on both sides of the slit. If the experiment is made under favourable circumstances, the spectra will be so pure as to show a number of Fraunhofer's lines.

Instead of viewing the spectra with the naked eye, we may with advantage employ a telescope, focussed on the plane of the slit; or we may project the spectra on a screen, by first placing a convex lens so as to form an image of the slit (which must be very strongly

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these systems will be precisely alike, and will have for their resultant a system of straight bands, parallel to the slit and touching the rings. These are the bands described in the text. Hence, to determine the illumination of any point of the screen, it is only necessary to attend, as in the text, to the nearest points of the two edges of the slit.

[1] Airy, *Undulatory Theory*. Art. 28

illuminated) on the screen, and then interposing the ruled glass in the path of the beam.

A piece of glass thus ruled is called a *grating*.[1] A grating for diffraction experiments consists essentially of a number of parallel strips alternately transparent and opaque.

The distance between the "fixed lines" of the spectra, and the distance from one spectrum to the next, are found to depend on the distance of the strips measured from centre to centre, in other words, on the number of scratches to the inch, but not at all on the relative breadths of the transparent and opaque strips. This latter circumstance only affects the brightness of the spectra.

Diffraction spectra are of great practical importance—

1. As furnishing a uniform standard of reference in the comparison of spectra.

2. As affording the most accurate method of determining the wave-lengths of the different elementary rays of light.

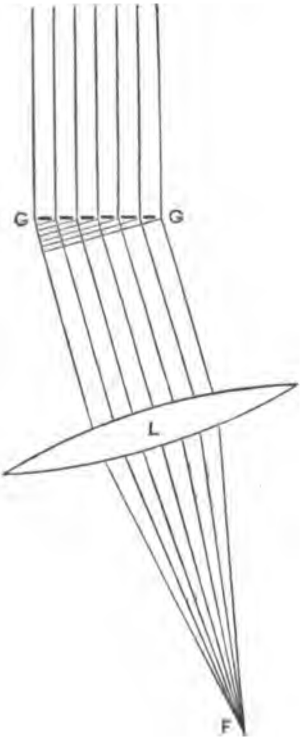

Fig. 728.
Principle of Diffraction Spectrum.

822. Principle of Diffraction Spectra.—Let GG (Fig. 728) be a grating, receiving light from an infinitely[2] distant point lying in a direction perpendicular to the plane of the grating, so that the wave-fronts of the incident light are parallel to this plane. Let a convex lens L be placed on the other side of the grating, and let its axis make an acute angle θ with the rays incident on the grating. Then the light collected at its principal focus F consists of all the light incident upon the lens parallel to its axis. Let s denote the distance between the rulings, measured from centre to centre, so that if, for example, there are 1000 lines to the inch, s will be $\frac{1}{1000}$ of an inch; and suppose first that $s \sin \theta$ is exactly equal to the wave-length λ of one of the elementary kinds of light. Then, of all the light which falls upon the lens parallel to its axis, the left-hand portion in the figure is most retarded (having travelled farthest), and the right-hand portion least, the retardation, in comparing each transparent interval with the next, being constant, and equal to $s \sin \theta$, as is evident from an inspection of the figure. Now, for the particular kind of light for which $\lambda = s \sin \theta$, this retardation is exactly a wave-length, and all the transparent intervals send light of the same phase to the focus F; so that, if there are 1000 such intervals, the resultant amplitude of vibration of F is 1000 times the amplitude due to one interval alone. For light of any other wave-length this coincidence of phase will not exist. For example, if the difference between λ and $s \sin \theta$ is $\frac{1}{1000}\lambda$, the difference of phase between the lights received from the 1st and 2d intervals will be $\frac{1}{1000}\lambda$, between the 1st and 3d $\frac{2}{1000}\lambda$, between the 1st and 501st $\frac{500}{1000}\lambda$, or just half a wave-length, and so on. The 1st and 501st are thus in complete discordance, as are also the 2d and 502d, &c. Light of every wave-length except one is thus almost completely destroyed by interference, and the light collected at F consists almost entirely of the particular kind defined by the condition

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[1] The Hon. J. W. Strutt has recently shown that very effective gratings may be made by copying engraved glass gratings photographically. The method appears likely to be of great practical service, engraved glass gratings of sufficient size being extremely expensive and difficult to procure.

[2] It is not *necessary* that the source should be infinitely distant (or the incident rays parallel); but this is the simplest case, and the most usual case in practice.

$$\lambda = s \sin \theta. \qquad (1)$$

The purity of the diffraction spectrum is thus explained.

If a screen be held at F, with its plane perpendicular to the principal axis, any point on this screen a little to one side of F will receive light of another definite wave-length, corresponding to another direction of incidence on the lens, and a pure spectrum will thus be depicted on the screen.

823. Practical Application.—In the arrangement actually employed for accurate observation, the lens L L is the object-glass of a telescope with a cross of spider-lines at its principal focus F. The telescope is first pointed directly towards the source of light, and is then turned to one side through a measured angle θ. Any fixed line of the spectrum can thus be brought into apparent coincidence with the cross of spider-lines, and its wave-length can be computed by the formula (1).

The spectrum to which formula (1) relates is called the *spectrum of the first order.*

There is also a spectrum of the second order, corresponding to values of θ nearly twice as great, and for which the equation is

$$2\lambda = \sin\theta. \tag{2}$$

For the spectrum of the third order, the equation is

$$3\lambda = \sin\theta; \tag{3}$$

and so on, the explanation of their formation being almost precisely the same as that above given. There are two spectra of each order, one to the right, and the other at the same distance to the left of the direction of the source. In Ångstrom's observations,[1] which are the best yet taken, all the spectra, up to the sixth inclusive, were observed, and numerous independent determinations of wave-length were thus obtained for several hundred of the dark lines of the solar spectrum.

The source of light was the infinitely distant image of an illuminated slit, the slit being placed at the principal focus of a collimator, and illuminated by a beam of the sun's rays reflected from a mirror.

The purity of a diffraction spectrum increases with the number of lines on the grating which come into play, provided that they are exactly equidistant; and may therefore be increased either by increasing the size of the grating, or by ruling its lines closer together. The gratings employed by Ångstrom were about $\frac{3}{4}$ of an inch square, the closest ruled having about 4500 lines, and the widest 1500.

As regards brightness, diffraction spectra are far inferior to those obtained by prisms. To give a maximum of light, the opaque intervals should be perfectly opaque, and the transparent intervals perfectly transparent; but even under the most favourable conditions, the whole light of any one of the spectra cannot exceed about $\frac{1}{10}$ of the light which would be received by directing the telescope to the slit. The greatest attainable intrinsic brightness in any part of a diffraction spectrum is thus not more than $\frac{1}{10}$ of the intrinsic brightness in the same part of a prismatic spectrum, obtained with the same slit, collimator, and observing telescope, and with the same angular separation of fixed lines. The brightness of the spectra partly depends upon the ratio of the breadths of the transparent and opaque intervals. In the case of the spectra of the first order, the best ratio is that of equality, and equal departures from equality in opposite directions give identical results; for example, if the breadth of the transparent intervals is to the breadth of the opaque either as 1 : 5 or as 5 : 1, it can be shown that the quantity of light in the first spectrum is just a quarter of what it would be with the breadths equal.

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[1] Ångstrom, *Recherches sur la Spectre Solaire.* Upsal, 1868.

When a diffraction spectrum is seen with the naked eye, the cornea and crystalline of the eye take the place of the lens L L, and form a real image on the retina at F.

824. Standard Spectrum.—The simplicity of the law connecting wave-length with position, in the spectra obtained by diffraction, offers a remarkable contrast to the "irrationality" of the dispersion produced by prisms. Diffraction spectra may thus be fairly regarded as natural standards of comparison; and, in particular, the limiting form (if we may so call it) to which the diffraction spectra tend, as sin θ becomes small enough to be identified with θ, so that deviation becomes simply proportional to wave-length, is generally and deservedly accepted by spectroscopists as the *absolute standard of reference.* This limiting form is often briefly designated as "the diffraction spectrum;" it differs in fact to a scarcely appreciable extent from the first, or even the second and third spectra furnished in ordinary cases by a grating.

The diffraction spectrum differs notably from prismatic spectra in the much greater relative extension of the red end. Owing to this circumstance, the brightest part of the diffraction spectrum of solar light is nearly in its centre.

The first three columns of numbers in the subjoined table indicate the approximate distances between the fixed lines B, D, E, F, G in certain prismatic spectra, and in the standard diffraction spectrum, the distance from B to G being in each case taken as 1000:—

	Flint-glass. Angle of 60°.	Bisulphide of Carbon. Angle of 60°.	Diffraction, or Difference of Wave-length.	Difference of Wave-frequency.
B to D, . .	220	194	381	278
D to E, . .	214	206	243	232
E to F, . .	192	190	160	184
F to G, . .	374	410	216	306
	1000	1000	1000	1000

In the standard diffraction spectrum, deviation is simply proportional to wave-length, and therefore the distance between two colours represents the difference of their wave-lengths. It has been suggested that a more convenient reference-spectrum would be constructed by assigning to each colour a deviation proportional to its wave-frequency (or to the reciprocal of its wave-length), so that the distance between two colours will represent the difference between their wave-frequencies. The result of thus disposing the fixed lines is shown in the last column of the above table. It differs from prismatic spectra in the same direction, but to a much less extent than the diffraction spectrum.

It has been suggested by Mr. Stoney as extremely probable, that the bright lines of spectra are in many cases harmonics of some one fundamental vibration. Three of the four bright lines of hydrogen have wave-frequencies exactly proportional to the numbers 20, 27, and 32; and in the spectrum of chloro-chromic acid all the lines whose positions have been observed (31 in number) have wave-frequencies which are multiples of one common fundamental.

825. Wave-lengths.—Wave-lengths of light are commonly stated in terms of a unit of which 10^{10} make a metre,—hence called the *tenth-metre*. The following are the wave-lengths of some of the principal "fixed lines" as determined by Ångstrom:[1]—

WAVE-LENGTHS IN TENTH-METRES.

A . . .	7604	E . . .	5269
B . . .	6867	F . . .	4861
C . . .	6562	G . . .	4307
D_2 . . .	5895	H_1 . . .	3968
D_1 . . .	5889	H_2 . . .	3933

The velocity of light as determined by Foucault is 298 million metres per second, or 298×10^{16} tenth-metres per second. The number of waves per second for any colour is therefore 298×10^{16} divided by its wave-length as above expressed. Hence we find approximately:—

For A	392	millions	of millions	per second.
„ D	506	„	„	„
„ H	754	„	„	„

826. Colours of Thin Films. Newton's Rings.—If two pieces of glass, with their surfaces clean, are brought into close contact, coloured fringes are seen surrounding the point where the contact is closest. They are best seen when light is obliquely reflected to the eye from the surfaces of the glass, and fringes of the complementary colours may be seen by transmitted light. A drop of oil placed on the surface of clean water spreads out into a thin film, which exhibits similar fringes of colour; and in general, a very thin film of any transparent substance, separating media whose indices of refraction are different from its own, exhibits colour, especially when viewed by obliquely reflected light. In the first experiment above-mentioned, the thin film is an air-film separating the pieces of glass. In soap-bubbles or films of soapy water stretched on rings, a similar effect is produced by a small thickness of water separating two portions of air.

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[1] The wave-lengths of the spectral lines of all elementary substances will be found in Dr. W. M. Watts' *Index of Spectra.*

The colours, in all these cases, when seen by reflected light, are produced by the mutual interference of the light reflected from the two surfaces of the thin film. An incident ray undergoes, as explained in § 729, a series of reflections and refractions; and we may thus distinguish, for light of any given refrangibility, several systems of waves, all of which originally came from the same source. These systems give by their interference a series of alternately bright and dark fringes; and when ordinary white light is employed, the fringes are broadest for the colours of greatest wave-length. Their superposition thus produces the observed colours. The colours seen by transmitted light may be similarly explained.

The first careful observations of these coloured fringes were made by Newton, and they are generally known as *Newton's rings.*

CHAPTER LXV.

POLARIZATION AND DOUBLE REFRACTION.

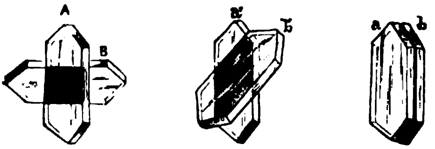

Fig. 729.—Tourmaline Plates.

827. Polarization.—When a piece of the semi-transparent mineral called tourmaline is cut into slices by sections parallel to its axis, it is found that two of these slices, if laid one upon the other in a particular relative position, as A, B (Fig. 729), form an opaque combination. Let one of them, in fact, be turned round upon the other through various angles (Fig. 729). It will be found that the combination is most transparent in two positions differing by 180°, one of them $a\,b$ being the natural position which they originally occupied in the crystal; and that it is most opaque in the two positions at right angles to these. It is not necessary that the slices should be cut from the same crystal. Any two plates of tourmaline with their faces parallel to the axes of the crystals from which they were cut, will exhibit the same phenomenon. The experiment shows that light which has passed through one such plate is in a peculiar and so to speak unsymmetrical condition. It is said to be *plane-polarized.* According to the undulatory theory, a ray of common light contains vibrations in all planes passing through the ray, and a ray of plane-polarized light contains vibrations in one plane only. Polarized light cannot be distinguished from common light by the naked eye; and for all experiments in polarization two pieces of apparatus must be employed—one to produce polarization, and the other to show it. The former is called the *polarizer*, the latter the *analyzer;* and every apparatus that serves for one of these purposes will also serve for the other. In the experiment above described, the plate next the eye is the analyzer. The usual process in examining light with a view to test whether it is polarized, consists in looking at it through an analyzer, and observing whether any change of brightness occurs as the analyzer is rotated. When the light of the blue sky is thus examined, a difference of brightness can always be detected according to the position of the analyzer, especially at the distance of about 90° from the sun. In all such cases there are two positions, differing by 180°, which give a minimum of light, and the two positions intermediate between these give a maximum of light.

The extent of the changes thus observed is a measure of the completeness of the polarization of the light.

828. Polarization by Reflection.—Transmission through tourmaline is only one of several ways in which light can be polarized. When a beam of light is reflected from a polished surface of glass, wood, ivory, leather, or any other non-metallic substance, at an angle of from 50° to 60° with the normal, it is more or less polarized, and in like manner a reflector composed of any of these substances may be employed as an analyzer. In so using it, it should be rotated about an axis parallel to the incident rays which are to be tested, and the observation consists in noting whether this rotation produces changes in the amount of reflected light.

Malus' Polariscope (Fig. 730) consists of two reflectors A, B, one serving as polarizer and the other as analyzer, each consisting of a pile of glass plates. Each of these reflectors can be turned about a horizontal axis; and the upper one (which is the analyzer) can also be turned about a vertical axis, the amount of rotation being measured on the horizontal circle C C. To obtain the most powerful effects, each of the reflectors should be set at an angle of about 33° to the vertical, and a strong beam of common light should be allowed to fall upon the lower pile in such a direction as to be reflected vertically upwards. It will thus fall upon the centre of the upper pile, and the angles of incidence and reflection on both the piles will be about 57°. The observer looking into the upper pile, in such a

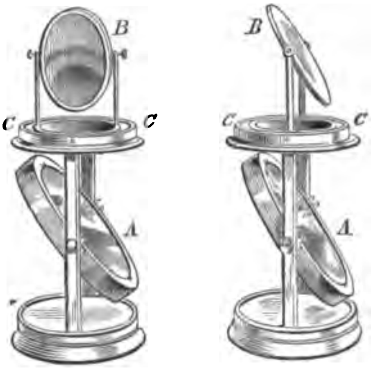

Fig. 730.—Malus' Polariscope.

direction as to receive the reflected beam, will find that, as the upper pile is rotated about a vertical axis, there are two positions (differing by 180°) in which he sees a black spot in the centre of the field of view, these being the positions in which the upper pile refuses to reflect the light reflected to it from the lower pile. They are 90° on either side of the position in which the two piles are parallel; this latter, and the position differing from it by 180°, being those which give a maximum of reflected light.

For every reflecting substance there is a particular angle of incidence which gives a maximum of polarization in the reflected light. It is called the *polarizing angle* for the substance, and its tangent is always equal to the index of refraction of the substance; or what amounts to the same thing, it is that particular angle of incidence which is the complement of the angle of refraction, so that the refracted and reflected rays are at right angles.[1] This important law was discovered experimentally by Sir David Brewster.

The reflected ray under these circumstances is in a state of almost complete polarization; and the advantage of employing a *pile* of plates consists merely in the greater intensity of the reflected light thus furnished. The transmitted light is also polarized; it diminishes in intensity, but becomes more completely polarized, as the number of plates is increased. The reflected and the transmitted light are in fact mutually complementary, being the two parts into which common light has been decomposed; and their polarizations are accordingly opposite, so that, if both the transmitted and reflected beams are examined by a tourmaline, the maxima of obscuration will be obtained by placing the axis of the tourmaline in the one case parallel and in the other perpendicular to the plane of incidence.

It is to be noted that what is lost in reflection is gained in transmission, and that polarization never favours reflection at the expense of transmission.

829. Plane of Polarization.—That particular plane in which a ray of polarized light, incident at the polarizing angle, is most copiously reflected, is called the *plane of polarization* of the ray. When the polarization is produced by reflection, the plane of reflection is the

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[1] Adopting the indices of refraction given in the table § 724, we find the following values for the polarizing angle for the undermentioned substances:—

Diamond, . . .	67° 43′ to 70° 3′	Pure Water, ...	53° 11′
Flint-glass, . .	57° 36′ to 58° 40′	Air, ...	45°
Crown-glass, . .	56° 51′ to 57° 23′		

plane of polarization. According to Fresnel's theory, which is that generally received, the vibrations of light polarized in any plane are perpendicular to that plane (§ 841). The vibrations of a ray reflected at the polarizing angle are accordingly to be regarded as perpendicular to the plane of incidence and reflection, and therefore as parallel to the reflecting surface.

830. Polarization by Double Refraction.—We have described in §732 some of the principal phenomena of double refraction in uniaxal crystals. We have now to mention the important fact that the two rays furnished by double refraction are polarized, the polarization in this case being more complete than in any of the cases thus far discussed. On looking at the two images through a plate of tourmaline, or any other analyzer, it will be found that they undergo great variations of brightness as the analyzer is rotated, one of them becoming fainter whenever the other becomes brighter, and the maximum brightness of either being simultaneous with the absolute extinction of the other. If a second piece of Iceland-spar be used as the analyzer, four images will be seen, of which one pair become dimmer as the other pair become brighter, and either of these pairs can be extinguished by giving the analyzer a proper position.

831. Theory of Double Refraction.—The existence of double refraction admits of a very natural explanation on the undulatory theory. In uniaxal crystals it is assumed that the elasticity of the luminiferous æther is the same for all vibrations executed in directions perpendicular to the axis; and that, for vibrations in other directions, the elasticity varies solely according to the inclination of the direction of vibration to the axis. There are two classes of doubly-refracting uniaxal crystals, called respectively *positive* and *negative*. In the former the elasticity for vibrations perpendicular to the axis is a maximum; in the latter it is a minimum. Iceland-spar belongs to the latter class; and as small elasticity implies slow propagation, a ray propagated by vibrations perpendicular to the axis will, in this crystal, travel with minimum velocity; while the most rapid propagation will be attained by rays whose vibrations are parallel to the axis.

Consider any plane oblique to the axis. Through any point in this plane we can draw one line perpendicular to the axis; and the line at right angles to this will have smaller inclination to the axis than any other line in the plane. These two lines are the directions of least and greatest resistance to vibration; the former is the direction of vibration for an ordinary, and the latter for an extraordinary ray. The velocity of propagation is the same for the ordinary rays in all directions in the crystal, so that the wave-surface for these is spherical; but the velocity of propagation for the extraordinary rays differs according to their inclination to the axis, and their wave-surface is a spheroid whose polar diameter is equal to the diameter of the aforesaid sphere. The sphere and spheroid touch one another at the extremities of this diameter (which is parallel to the axis of the crystal), and the ordinary and extraordinary rays in this particular direction coincide, so that the double refraction becomes single. The course of the two rays produced in the crystal by a given ray incident on a plane face, may be determined by Huygens' construction, which has been described in § 811. The ordinary index is the ratio of the velocity in air to the velocity of the ordinary ray. The extraordinary index (so called) is the ratio of the velocity in air to the velocity of the slowest extraordinary rays in the case of positive crystals, or to the velocity of the swiftest extraordinary rays in the case of negative crystals. In both cases the extraordinary index is that value of $\frac{\text{sine of incidence}}{\text{sine of refraction}}$ which differs most from the ordinary index. The extraordinary index is applicable to refraction at a plane surface parallel to the axis, when the plane of incidence is perpendicular to the axis.

Tourmaline, like Iceland-spar, is a negative uniaxal crystal; and its use as a polarizer depends on the property which it possesses of absorbing the ordinary much more rapidly than the extraordinary ray, so that a thickness which is tolerably transparent to the latter is almost completely opaque to the former.

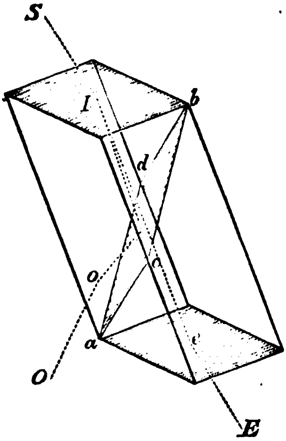

Fig. 731.—Nicol's Prism.

832. Nicol's Prism.—One of the most convenient and effective contrivances for polarizing light, or analyzing it when polarized, is that known, from the name of its inventor, as Nicol's prism. It is made by slitting a rhomb of Iceland-spar along a diagonal plane *acbd* (Fig. 731), and cementing the two pieces together in their natural position by Canada balsam, a substance whose refractive index is intermediate between the ordinary and extraordinary indices of the crystal.[1] A ray of common light SI undergoes double refraction on entering the prism. Of the two rays thus formed, the ordinary ray is totally reflected on meeting the first surface of the balsam, and passes out at one side of the crystal, as *o*O; while the extraordinary ray is transmitted through the balsam as through a parallel plate, and finally emerges at the end of the prism, in the direction *e*E, parallel to the original direction SI. This apparatus has nearly all the convenience of a tourmaline plate, with the advantages of much greater transparency and of complete polarization.

In Foucault's prism, which is extensively used instead of Nicol's, the Canada balsam is omitted, and there is nothing but air between the two pieces. This change has the advantage of shortening the prism (because the critical angle of total reflection depends on the relative index of refraction of the two media), but gives a smaller field of view, and rather more loss of light by reflection.

833. **Colours produced by Elliptic Polarization.**—Very beautiful colours may be produced by the peculiar action of polarized light. For example, if a piece of selenite (crystallized gypsum) about the thickness of paper, is introduced between the polarizer and analyzer of any polarizing arrangement, and turned about into different directions, it will in some positions appear brightly coloured, the colour being most decided when the analyzer is in either of the two critical positions which give respectively the greatest light and the greatest darkness. The colour is changed to its complementary by rotating the analyzer through a right angle; but rotation of the piece of selenite, when the analyzer is in either of the critical positions, merely alters the depth of the colour without changing its tint, and in certain critical positions of the selenite there is a complete absence of colour. Thicker plates of selenite restore the light when extinguished by the analyzer, but do not show colour.

834. **Explanation.**—The following is the explanation of these appearances. Let the analyzer be turned into such a position as to produce complete extinction of the plane-polarized light which comes to it from the polarizer; and let the plane of polarization and the plane perpendicular thereto (and parallel to the polarized rays) be

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[1] *a* and *b* are the corners at which three equal obtuse angles meet (§ 733). The ends of the rhomb which are shaded in the figure are rhombuses. Their diagonals drawn through *a* and *b* respectively will lie in one plane, which will contain the axis of the crystal, and will cut the plane of section *a c b d* at right angles. The length of the rhomb is about three and a half times its breadth.

called the two *planes of reference.* Let the slice of selenite be laid so that the polarized rays pass through it normally. Then there are two directions, at right angles to each other, which are the directions of greatest and least elasticity in the plane of the slice. Unless the slice is laid so that these directions coincide with the two planes of reference, the plane-polarized light which is incident upon it will be broken up into two rays, one of which will traverse it more rapidly than the other. Referring to the diagram of Lissajous' figures (Fig. 604), let the sides of the rectangle be the directions of greatest and least elasticity, and let the diagonal line in the first figure be the direction of the vibrations of an incident ray,—this diagonal accordingly lies in one of the two planes of reference. In traversing the slice, the component vibrations in the directions of greatest and least elasticity will be propagated with unequal velocities; and if the incident ray be homogeneous, the emergent light will be elliptically polarized; that is to say, its vibrations, instead of being rectilinear, will be elliptic, precisely on the principle[1] of Blackburn's pendulum (§ 677 A). The shape of the ellipse depends, as in the case of Lissajous' figures, on the amount of retardation of one of the two component vibrations as compared with the other, and this is directly proportional to the thickness of the slice. The analyzer resolves these elliptic vibrations into two rectilinear components parallel and perpendicular to the original direction of vibration, and suppresses one of these components, so that only the other remains. Thus if the ellipse in the annexed figure (Fig. 732) represent the vibrations of the light as it emerges from the selenite, and CD, EF be tangents parallel to the original direction of vibration, the perpendicular distance between these tangents, AB, is the component vibration which is not suppressed when the analyzer is so turned that all the light would be suppressed if the selenite were removed. By rotating the analyzer, we shall obtain vibrations of various amplitudes, corresponding to the distances between parallel tangents drawn in various directions.

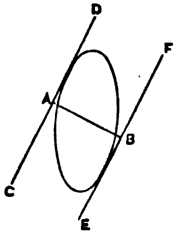

Fig. 732.—Colours of Selenite Plates.

For a certain thickness of selenite the ellipse will become a circle, and we have thus what is called *circularly-polarized* light, which is characterized by the property that rotation of the analyzer produces no change of intensity. Circularly-polarized light is not however identical with ordinary light; for the interposition of an additional thickness of selenite converts it into elliptically (or in a particular case into plane) polarized light (§ 840).

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[1] The principle is that, whereas displacement of a particle parallel to either of the sides of the rectangle calls out a restoring force directly opposite to the displacement, displacement in any other direction calls out a restoring force inclined to the direction of displacement, being in fact the resultant of the two restoring forces which its two components parallel to the sides of the rectangle would call out.

The above explanation applies to homogeneous light. When the incident light is of various refrangibilities, the retardation of one component upon the other is greatest for the rays of shortest wavelength. The ellipses are accordingly different for the different elementary colours, and the analyzer in any given position will produce unequal suppression of different colours. But since the component which is suppressed in any one position of the analyzer, is the component which is not suppressed when the analyzer, is turned through a right angle, the light yielded in the former case *plus* the light yielded in the latter must be equal to the whole light which was incident on the selenite.[1] Hence the colours exhibited in these two positions must be complementary.

It is necessary for the exhibition of colour in these experiments that the plate of selenite should be very thin, otherwise the retardation of one component vibration as compared with the other will be greater by several complete periods for violet than for red, so that the ellipses will be identical for several different colours, and the total non-suppressed light will be sensibly white in all positions of the analyzer.

Two thick plates may however be so combined as to produce the effect of one thin plate. For example, two selenite plates, of nearly equal thickness, may be laid one upon the other, so that the direction of greatest elasticity in the one shall be parallel to that of least elasticity in the other. The resultant effect in this case will be that due to the difference of their thicknesses. Two plates so laid are said to be *crossed.*

835. Colours of Plates perpendicular to Axis.—A different class of appearances are presented when a plate, cut from a uniaxal crystal by sections perpendicular to the axis, is inserted between the polarizer and the analyzer. Instead of a broad sheet of uniform colour, we have now a system of coloured rings, interrupted when the analyzer is in one of the two critical positions, by a black or white cross, as at A, B (Fig. 733).

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[1] We here neglect the light absorbed and scattered; but the loss of this does not sensibly affect the *colour* of the whole. It is to be borne in mind that the intensity of light is measured by the *square* of the amplitude, and is therefore the simple sum of the intensities of its two components when the resolution is rectangular.

836. Explanation.—The following is the explanation of these appearances. Suppose, for simplicity, that the analyzer is a plate of tourmaline held close to the eye. Then the light which comes to the eye from the nearest point of the plate under examination (the foot of a perpendicular dropped upon it from the eye), has traversed the plate normally, and therefore parallel to its optic axis. It has therefore not been resolved into an ordinary and an extraordinary ray, but has emerged from the plate in the same condition in which it entered, and is therefore black, gray, or white according to the position of the analyzer, just as it would be if the plate were removed. But the light which comes obliquely to the eye from any other part of the plate, has traversed the plate obliquely, and has undergone double refraction. Let E (Fig. 734) be the position of the eye, E O a perpendicular on the plate, P a point on the circumference of a circle described about O as centre. Then, since E O is parallel to the axis of the plate, the direction of vibration for the ordinary ray at P is perpendicular to the plane E O P, and is tangential to the circle. The direction of vibration for the extraordinary ray lies in the plane E O P, is nearly perpendicular to E O (or to the axis), if the angle O E P is small, and deviates more from perpendicularity to the axis as the angle O E P increases. Both for this reason, and also on account of the greater thickness traversed, the retardation of one ray upon the other is greater as P is taken further from O; and from the symmetry of the circumstances, it

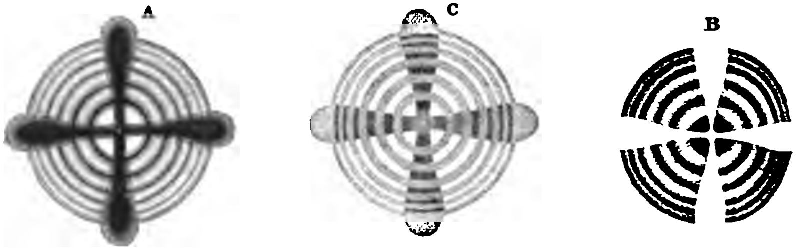

Fig. 733.—Rings and Cross.

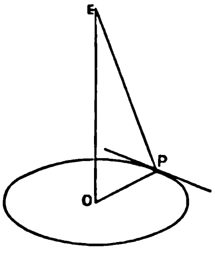

Fig. 734.
Theory of Rings and Cross.

must be the same at the same distance from O all round. In consequence of this retardation, the light which emerges at P in the direction P E is elliptically polarized; and by the agency of the analyzer it is accordingly resolved into two components, one of which is suppressed. With homogeneous light, rings alternately dark and bright would thus be formed at distances from O corresponding to retardations of 0, ½, 1, 1½, 2, 2½, . . . complete periods; and it can be shown that the radii of these rings would be proportional to the numbers 0, $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$: . . . The rings are larger for light of long than of short wave-length; and the coloured rings actually exhibited when white light is employed, are produced by the superposition of all the systems of monochromatic rings. The monochromatic rings for red light are easily seen by looking at the actual rings through a piece of red glass.

Let O, P, Fig. 735, be the same points which were denoted by these letters in Fig. 734, and let A B be the direction of vibration of the light incident on the crystal at P. Draw A C, D B parallel to O P, and complete the rectangle A C B D. Then the length and breadth of this rectangle are approximately the directions of vibration of the two components, one of which loses upon the other in traversing the crystal. The vibration of the emergent ray is represented by an ellipse inscribed in the rectangle A C B D (§ 676, note 2); and when the loss is half a period, this ellipse shrinks into a straight line, namely, the diagonal C D. Through C and D draw lines parallel to A B; then the distance between these parallels represents the double amplitude of the vibration which is transmitted when there has been a retardation of half a period, and is greater than the distance between the tangents in the same direction to any of the inscribed ellipses. A retardation of another half period will again reduce the inscribed ellipse to the straight line A B, as at first. The position D C corresponds to the brightest and A B to the darkest part of any one of the series of rings for a given wave-length of light, the analyzer being in the position for suppressing all the light if the crystal were removed. When the analyzer is turned into the position at right angles to this, A B corresponds to the brightest, and D C to the darkest parts of the rings. It is to

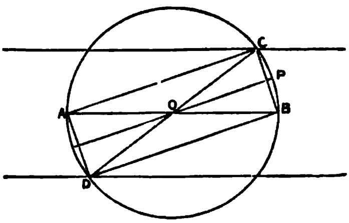

Fig. 735.—Theory of Rings and Cross.

be remembered that amount of retardation depends upon distance from the centre of the rings, and is the same all round. The two diagonals of our rectangle therefore correspond to different sizes of rings.

If the analyzer is in such a position with respect to the point P considered, that the suppressed vibration is parallel to one of the sides of the rectangle (in other words, if O P, or a line perpendicular to O P, is the direction of suppression) the retardation of one component upon the other has no influence, inasmuch as one of the two components is completely suppressed and the other is completely transmitted. There are, accordingly, in all positions of the analyzer, a pair of diameters, coinciding with the directions of suppression and non-suppression, which are alike along their whole length and free from colour.

Again if P is situated at B or at 90° from B, the corner C of the rectangle coincides with B or with A, and the rectangle, with all its inscribed ellipses, shrinks into the straight line A B. The two diameters coincident with and perpendicular to A B are therefore alike along their whole length and uncoloured.

The two colourless crosses which we have thus accounted for, one of them turning with the analyzer and the other remaining fixed with the polarizer, are easily observed when the analyzer is not near the critical positions. In the critical positions, the two crosses come into coincidence; and these are also the positions of maximum blackness or maximum whiteness for the two crosses considered separately. Hence the conspicuous character of the cross in either of these positions, as represented at A, B, Fig. 733. As the analyzer is turned away from these positions, the cross at first turns after it with half its angular velocity, but soon breaks up into rings, somewhat in the manner represented at C, which corresponds to a position not differing much from A.

837. Biaxal Crystals.—Crystals may be divided optically into three classes:—

1. Those in which there is no distinction of different directions, as regards optical properties. Such crystals are said to be optically *isotropic*.

2. Those in which the optical properties are the same for all directions equally inclined to one particular direction called the optic axis, but vary according to this inclination. Such crystals are called *uniaxal*.

3. All remaining crystals (excluding compound and irregular formations) belong to the class called *biaxal.* In any homogeneous elastic solid, there are three cardinal directions called *axes of elasticity,* possessing the same distinctive properties which belong to the two principal planes of vibration in Blackburn's pendulum (§ 677 A); that is to say, if any small portion of the solid be distorted by forcibly displacing one of its particles in one of these cardinal directions, the forces of elasticity thus evoked tend to urge the particle *directly* back; whereas displacement in any other direction calls out forces whose resultant is generally oblique to the direction of displacement, so that when the particle is released it does not fly back through the position of equilibrium, but passes on one side of it, just as the bob of Blackburn's pendulum generally passes beside and not through the lowest point which it can reach.

In biaxal crystals, the resistances to displacement in the three cardinal directions are all unequal; and this is true not only for the crystalline substance itself, but also for the luminiferous æther which pervades it, and is influenced by it.[1] The construction given by Fresnel for the wave-surface in any crystal is as follows:—First take an ellipsoid, having its axes parallel to the three cardinal directions, and of lengths depending on the particular crystalline substance considered. Then let any plane sections (which will of course be ellipses) be made through the centre of this ellipsoid, let normals to them be drawn through the centre, and on each normal let points be taken at distances from the centre equal to the greatest and least radii of the corresponding section. The locus of these points is the complete wave-surface, which consists of two sheets cutting one another at four points. These four points of intersection are situated upon the normals to the two *circular sections* of the ellipsoid, and the two *optic axes,* from which *biaxal* crystals derive their name, are closely related to these two circular sections. The optic axes are the directions of *single wave-velocity,* and the normals to the two circular sections are the directions of *single ray-velocity.* The direction of advance of a wave is always regarded as normal to the front of the wave, whereas the direction of a ray (defined by the condition of traversing two apertures placed in its path) always passes through the centre of the wave-surface, and is not in general normal to the front. Both these pairs of directions of single velocity are in the plane which contains the greatest and least axes of the ellipsoid.

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[1] The cardinal directions are however believed not to be the same for the æther as for the material of the crystal.

When two axes of the ellipsoid are equal, it becomes a spheroid, and the crystal is uniaxal. When all three axes are equal, it becomes a sphere, and the crystal is isotropic.

Experiment has shown that biaxal crystals expand with heat unequally in three cardinal directions, so that in fact a spherical piece of such a crystal is changed into an ellipsoid[1] when its temperature is raised or lowered. A spherical piece of a uniaxal crystal in the same circumstances changes into a spheroid; and a spherical piece of an isotropic crystal remains a sphere.

It is generally possible to determine to which of the three classes a crystal belongs, from a mere inspection of its shape as it occurs in nature. Isotropic crystals are sometimes said to be symmetrical about a point, uniaxal crystals about a line, biaxal crystals about neither. The following statement is rather more precise:—

If there is one and only one line about which if the crystal be rotated through 90° or else through 120° the crystalline form remains in its original position, the crystal is uniaxal, having that line for the axis. If there is more than one such line, the crystal is isotropic, while, if there is no such line, it is biaxal. Even in the last case, if there exist a plane of crystalline symmetry, such that one half of the crystal is the reflected image of the other half with respect to this plane, it is also a plane of optical symmetry, and one of the three cardinal directions for the æther is perpendicular to it.[2]

Glass, when in a strained condition, ceases to be isotropic, and if inserted between a polarizer and an analyzer, exhibits coloured streaks or spots, which afford an indication of the distribution of strain through its substance. The experiment is shown sometimes with unannealed glass, which is in a condition of permanent strain, sometimes with a piece of ordinary glass which can be subjected to force at pleasure by turning a screw. Any very small portion of a piece of strained glass has the optical properties of a crystal, but different portions have different properties, and hence the glass as a whole does not behave like one crystal.

The production of colour by interposition between a polarizer and an analyzer, is by far the most delicate test of double refraction. Many organic bodies (for example, grains of starch) are thus found to be doubly refracting; and microscopists often avail themselves of this means of detecting diversities of structure in the objects which they examine.

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[1] This fact furnishes the best possible definition of an ellipsoid for persons unacquainted with solid geometry.

[2] The optic axes either lie in the plane of symmetry, or lie in a perpendicular plane and are equally inclined to the plane of symmetry.

For the precise statement here given, the Editor is indebted to Professor Stokes.

838. Rotation of Plane of Polarization.—When a plate of quartz (rock-crystal), even of considerable thickness, cut perpendicular to the axis, is interposed between the polarizer and analyzer, colour is exhibited, the tints changing as the analyzer is rotated; and similar effects of colour are produced by employing, instead of quartz, a solution of sugar, inclosed in a tube with plane glass ends.

If homogeneous light is employed, it is found that if the analyzer is first adjusted to produce extinction of the polarized light, and the quartz or saccharine solution is then introduced, there is a partial restoration of light. On rotating the analyzer through a certain angle, there is again complete extinction of the light; and on comparing different plates of quartz, it will be found that the angle through which the analyzer must be rotated is proportional to the thickness of the plate. In the case of solutions of sugar, the angle is proportional jointly to the length of the tube and the strength of the solution.

The action thus exerted by quartz or sugar is called *rotation of the plane of polarization*, a name which precisely expresses the observed phenomena. In the case of ordinary quartz, and solutions of sugar-candy, it is necessary to rotate the analyzer in the direction of watch-hands as seen by the observer, and the rotation of the plane of polarization is said to be *right-handed*. In the case of what is called *left-handed* quartz, and of solutions of non-crystallizable sugar, the rotation of the plane of polarization is in the opposite direction, and the observer must rotate the analyzer against watch-hands.

The amount of rotation is different for the different elementary colours, and has been found to be inversely as the square of the wave-length. Hence the production of colour.

839. Magneto-optic Rotation.—Faraday made the remarkable discovery that the plane of polarization can be rotated in certain circumstances by the action of magnetism. Let a long rectangular piece of "heavy-glass" (silico-borate of lead) be placed longitudinally between the poles of the powerful electro-magnet represented in Fig. 432 (Part III.), which is for this purpose made hollow in its axis, so that an observer can see through it from end to end. Let a Nicol's prism be fitted into one end of the magnet, to serve as polarizer, and another into the other end to serve as analyzer, and let one of them be turned till the light is extinguished. Then, as long as no current is passed round the electro-magnet, the interposition of the heavy-glass will produce no effect; but the passing of a current, while the heavy-glass is in its place between the poles, produces rotation of the plane of polarization in the same direction as that in which the current circulates. The amount of rotation is directly as the strength of current, and directly as the length of heavy-glass traversed by the light. Flint-glass gives about half the effect of heavy-glass, and all transparent solids and liquids exhibit an effect of the same kind in a more or less marked degree.

A steel magnet, if extremely powerful, may be used instead of an electro-magnet; and in all cases, to give the strongest effect, the lines of magnetic force should coincide with the direction of the transmitted ray.

Faraday regarded these phenomena as proving the direct action of magnetism upon light; but it is now more commonly believed that the direct effect of the magnetism is to put the particles of the transparent body in a peculiar state of strain, to which the observed optical effect is due.

In every case tried by Faraday, the direction of the rotation was the same as the direction in which the current circulated; but certain substances[1] have since been found which give rotation against the current. The law for the relative amounts of rotation of different colours is approximately the same as in the case of quartz.

The direction of rotation is with watch-hands as seen from one end of the arrangement, and against watch-hands as seen from the other; so that the same piece of glass, in the same circumstances, behaves like right-handed quartz to light entering it at one end, and like left-handed quartz to light entering it at the other.

The rotatory power of quartz and sugar appears to depend upon a certain unsymmetrical arrangement of their molecules, an arrangement somewhat analogous to the thread of a screw; right-handed and left-handed screws representing the two opposite rotatory powers. It is worthy of note that the two kinds of quartz crystallize in different forms, each of which is unsymmetrical, one being like the image of the other as seen in a looking-glass. Pasteur has conducted extremely interesting researches into the relations existing between substances which, while in other respects identical or nearly identical, differ as regards their power of producing rotation. For the results we must refer to treatises on chemistry.

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[1] One such substance is a solution of $Fe^2 Cl^3$ (old notation) in methylic (not methylated) alcohol.

840. Circular Polarization. Fresnel's Rhomb.—We have explained in § 834 the process by which elliptic polarization is brought about, when plane-polarized light is transmitted through a thin plate of selenite. To obtain circular polarization (which is merely a case of elliptic), the plate must be of such thickness as to retard one component more than the other by a *quarter of a wave-length*, and must be laid so that the directions of the two component vibrations make angles of 45° with the plane of polarization. Plates specially prepared for this purpose are in general use, and are called *quarter-wave plates*. They are usually of mica, which differs but little in its properties from selenite. It is impossible, however, in this way to obtain complete circular polarization of ordinary white light, since different thicknesses are required for light of different wave-lengths, the thickness which is appropriate for violet being too small for red.

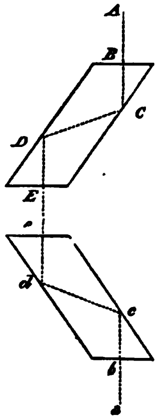

Fig. 736.
Two Fresnel's Rhombs.

Fresnel discovered that plane-polarized light is elliptically polarized by *total internal reflection* in glass, whenever the plane of polarization of the incident light is inclined to the plane of incidence. The rectilinear vibrations of the incident light are in fact resolved into two components, one of them in, and the other perpendicular to, the plane of incidence; and one of these is retarded with respect to the other in the act of reflection, by an amount depending on the angle of incidence. He determined the magnitude of this angle for which the retardation is precisely $\frac{1}{8}$ of a wave-length; and constructed a *rhomb*, or oblique parallelopiped of glass (Fig. 736), in which a ray, entering normally at one end, undergoes two successive reflections at this angle (about 55°), the plane of reflection being the same in both. The total retardation of one component on the other is thus $\frac{1}{4}$ of a wave-length; and if the rhomb is in such a position that the plane in which the two reflections take place is at an angle of 45° to the plane of polarization of the incident light, the emergent light is circularly-polarized. The effect does not vary much with the wave-length, and sensibly white circularly polarized light can accordingly be obtained by this method.

When circularly-polarized light is transmitted through a Fresnel's rhomb, or through a quarter-wave plate, it becomes plane-polarized, and we have thus a simple mode of distinguishing circularly-polarized light from common light; for the latter does not become polarized when thus treated. Two quarter-wave plates, or two Fresnel's rhombs, may be combined either so as to assist or to oppose one another. By the former arrangement, which is represented in Fig. 736, we can convert plane-polarized light into light polarized in a perpendicular plane, the final result being therefore the same as if the plane of polarization had been rotated through 90°. The several steps of the process are illustrated by the five diagrams of Fig. 737, which represent the vibrations of the five portions A C, C D, D *d*, *dc ca* of the ray which traverses the two rhombs in the preceding figure. The sides of the square are parallel to the directions of resolution; the initial direction of vibration is one diagonal of the square, and the final direction is the other diagonal; a gain or loss of half a complete vibration on the part of either component being just sufficient to effect this change.

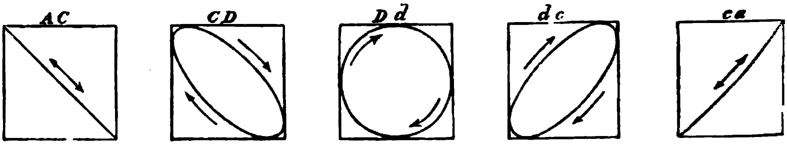

Fig. 737.—Form of Vibration in traversing the Rhombs.

841. Direction of Vibration of Plane-polarized Light.—The plane of polarization of plane-polarized light may be defined as the plane in which it is most copiously reflected. It is perpendicular to the plane in which the light refuses to be reflected (at the polarizing angle); and is identical with the original plane of reflection, if the polarization was produced by reflection. This definition is somewhat arbitrary, but has been adopted by universal consent.

When light is polarized by the double refraction of Iceland-spar, or of any other uniaxal crystal, it is found that the plane of polarization of the ordinary ray is the plane which contains the axis of the crystal. But the distinctive properties of the ordinary ray are most naturally explained by supposing that its vibrations are perpendicular to the axis. Hence we conclude that the direction of vibration in plane-polarized light is normal to the so-called plane of polarization, and therefore that, in polarization by reflection, the vibrations of the reflected light are parallel to the reflecting surface.

This is Fresnel's doctrine. MacCullagh, however, reversed this hypothesis, and maintained that the direction of vibration is *in* the plane of polarization. Both theories have been ably expounded; but Stokes contrived a crucial experiment in diffraction, which confirmed Fresnel's view;[1] and in his classical paper on "Change of Refrangibility," he has deduced the same conclusion from a consideration of the phenomena of the polarization of light by reflection from excessively fine particles of solid matter in suspension in a liquid.[2]

842. Vibrations of Ordinary Light.—Ordinary light agrees with circularly-polarized light in always yielding two beams of equal intensity when subjected to double refraction; but it differs from circularly-polarized light in not becoming plane-polarized by transmission through a Fresnel's rhomb or a quarter-wave plate. What, then, can be the form of vibration for common light? It is probably very irregular, consisting of ellipses of various sizes, positions, and forms (including circles and straight lines), rapidly succeeding one another. By this irregularity we can account for the fact that beams of light from different sources (even from different points of the same flame, or from different parts of the sun's disc), cannot, by any treatment whatever, be made to exhibit the phenomena of mutual interference; and for the additional fact that the two rectangular components into which a beam of common light is resolved by double refraction, cannot be made to interfere, even if their planes of polarization are brought into coincidence by one of the methods of rotation above described.

Certain phenomena of interference show that a few hundred consecutive vibrations of common light may be regarded as similar; but as the number of vibrations in a second is about 500 millions of millions, there is ample room for excessive diversity during the time that one impression remains upon the retina.

843. Polarization of Radiant Heat.—The fundamental identity of radiant heat and light is confirmed by thermal experiments on polarization. Such experiments were first successfully performed by Forbes in 1834, shortly after Melloni's invention of the thermo-multiplier. He first proved the polarization of heat by tourmaline;

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[1] *Cambridge Transactions.* 1850.

[2] *Philosophical Transactions,* 1852; pp. 530, 531.

next by transmission through a bundle of very thin mica plates, inclined to the transmitted rays; and afterwards by reflection from the multiplied surfaces of a pile of thin mica plates placed at the polarizing angle. He next succeeded in showing that polarized heat, even when quite obscure, is subject to the same modifications which doubly refracting crystallized bodies impress upon light, by suffering a beam of heat, after being polarized by transmission, to pass through an interposed plate of mica, serving the purpose of the plate of selenite in the experiment of § 833, the heat traversing a second mica bundle before it was received on the thermo-pile. As the interposed plate was turned round in its own plane, the amount of heat shown by the galvanometer was found to fluctuate just as the amount of light received by the eye under similar circumstances would have done. He also succeeded in producing circular polarization of heat by a Fresnel's rhomb of rock-salt. These results have since been fully confirmed by the experiments of other observers.

PROBLEMS.

[Words inclosed within square brackets [ ] have been interpolated by the Editor.]

I.—DYNAMICS AND HYDROSTATICS.

1. Two projectiles are successively discharged vertically upwards from the same point, with a velocity of 100 metres per second. What must be the interval of time between their discharges that the second may move for 8·7 sec. before meeting the first?

2. In an Atwood's machine the equal weights at the two ends of the thread are each 100 grammes. What must be the additional weight laid upon one of them that the space traversed in the first two seconds of the fall may be 4 decimetres?

3. A body is thrown horizontally from the top of a tower 100 m. high, with a velocity of 30 metres per sec. When and where will it strike the ground?

4 Two bodies are successively dropped from the same point, with an interval of $\frac{1}{2}$ of a second. When will the distance between them be 1 metre?

5. A stone is dropped into a well, and after 2 sec. is heard to strike the bottom. What is the depth?

6. Explain the well-known fact that a straight stick, loaded with lead at one end, can be more easily balanced vertically on the finger when the loaded end is upwards than when it is downwards.

7. Find the centre of gravity of a sphere 1 decimetre in radius, having in its interior a spherical excavation whose centre is at a distance of 5 centimetres from the centre of the large sphere [and whose radius is 4 centimetres].

8. Two small spheres, one of lead, weighing 100 grammes, the other of ivory, weighing 25 gr., are connected together by a thread 0·75 cm. long, and mounted on the rod of the centrifugal force apparatus [Fig. 37]. Find the position of [unstable] equilibrium.

9. A round table is supported on one central leg. At what points of its circumference must weights of 4, 5 and 6 kilog. be placed, that the resultant pressure may act at the centre?

10. A glass globe is full of air at atmospheric pressure (750 mm.) It is exhausted till the pressure is only x mm. Hydrogen is then admitted till atmospheric pressure is again established, and the mixture is then exhausted till the pressure is again reduced to x mm. Hydrogen is then a second time admitted till atmospheric pressure is established. If the weight of air in the globe at the conclusion of this operation is $\frac{1}{1000}$ of the weight of the hydrogen, what is the value of x? The temperature is supposed constant throughout the operation, and the specific gravity of hydrogen as compared with air is 0·0692.

11. A piece of iron, when plunged in a vessel full of water, makes 10 grammes run over. When placed in a vessel full of mercury, it floats, displacing 78 grammes of mercury. Required the weight, volume, and specific gravity of the iron.

12. A cylinder of steel, 22 cm. long, is to be counterpoised by a cylinder of platinum of the same diameter. What must be the length of the platinum cylinder? (sp. gr. of steel 7·5, of platinum 22·5.)

13. Two liquids are mixed. The total volume is 3 litres, with a sp. gr. of 0·9. The sp. gr. of the first liquid is 1·3, of the second 0·7. Find their volumes.

14. A curved tube has two vertical legs, one having a section of 1 sq. cm., the other of 10 sq. cm. Water is poured in, and stands at the same height in both legs. A piston, weighing 5 kilogrammes, is then allowed to descend, and press with its own weight upon the surface of the liquid in the larger leg. Find the elevation thus produced in the surface of the liquid in the smaller leg.

15. A frustum of a cone of cork, the radii of its ends being 2 and 1 decim. respectively, and its height 1 decim., floats freely in water, with its axis vertical. Find how much of its axis is immersed, the sp. gr. of cork being 0·24.

16. What volume of platinum must be attached to a litre of iron, that the system may float freely at all depths in mercury?

17. What must be the thickness of a hollow sphere of platinum with an external radius of 1 decim., that it may barely float in water?

18. A sphere of cork, 3 cm. in radius, is weighted with a sphere of gold. What must be the radius of the latter that the system may barely float in alcohol?

19. An alloy of gold and silver has density D. The density of gold is

d, that of silver d'. Find the proportions by weight of the two metals in the alloy, supposing that neither expansion nor contraction occurs in its formation.

20. Given the weight of a body in air, and in water at maximum density; deduce its weight in vacuo.

21. A vertical cylinder, 1 decim. in diameter and 3 decim. high, communicates at its lower part with a tube 1 centim. in diameter, which is bent up and continued vertically to a sufficient height, its upper end being left open. The cylinder is half full of mercury, its upper half being occupied by air at atmospheric pressure. What additional weight of air must be forced in to produce a fall of 10 cm. in the level of the mercury in the cylinder?

22. An open manometer, formed of a bent tube of iron whose two branches are parallel and vertical, and of a glass tube of larger size, contains mercury at the same level in both branches, this level being higher than the junction of the iron with the glass tube. What must be the ratio of the sections of the two tubes, that the mercury may ascend half a metre in the glass tube when a pressure of 6 atmospheres is exerted in the opposite branch?

23. A receiver A, with a capacity of 3 litres, can be put in communication either with a forcing pump P, or with the external air. The former communication is established by a valve R, and the latter by a cock R′. The receiver A is initially filled with air at 0° C. and 760 mm. The pump P is supplied from a gasometer containing carbonic acid, at the constant pressure 760 mm. and temperature 0° C., and when R is open the capacity of the pump-barrel is 2 litres.

R′ is closed. One stroke of the pump is taken, and when time has been allowed for the gases to become thoroughly mixed, R′ is opened for an instant, so that equilibrium of pressure is established between A and the external air. R′ is then closed, a second stroke is taken, and so on, the cock R′ being opened for an instant after each stroke. How many strokes must be taken, that not more than a centigramme of air may be left in the receiver? The external pressure is supposed to remain constant at 760 mm.

24. There is a glass tube a metre long, with an internal section of 1 square centimetre, the external section being 2 sq. cm., and consequently the section of the glass itself 1 sq. cm.

This tube, being supposed closed at one end by a flat stopper without thickness and without weight, is filled with mercury and inverted in a deep vessel of the same liquid; 10 cubic centimetres of air at the external pressure and temperature are then introduced, and the tube is left to itself in a vertical position. Required the volume of the air in the tube when equilibrium is attained; and the difference between the internal and external level of the mercury. Specific gravity of glass, 2·49; external pressure, 760 mm.

25. Given that the sp. gr. of the solution, containing 85 parts of water and 15 of salt, which is employed for graduating Baumé's hydrometer for acids, is 1·116, establish the formula $D = \frac{144}{144 - N}$, which gives the sp. gr. in terms of the degree read off at the surface of the liquid.

26. In the graduation of Baumé's hydrometer for spirits, a solution containing 90 parts of water and 10 of salt is employed, its sp. gr. being 1·084. Deduce the formula

$$D = \frac{129}{119 + N}.$$

27. Find, to the nearest millimetre, the edge of a regular tetrahedron of coinage gold, of the value of 1000 francs, the sp. gr. of this gold being 18 [and the value of 1 gramme of it being 3·1 francs].

28. The pressure indicated by a siphon barometer is 750 mm., and when mercury is poured into the open branch till the barometric chamber is reduced to half its former volume, the pressure indicated is 740 mm. Deduce the true pressure.

29. A cylindrical test-tube, 1 decim. long, is plunged, mouth downwards, into mercury. How deep must it be plunged that the volume of the inclosed air may be diminished by one-half?

30. In an air-pump, whose receiver has a capacity of 1 litre, it is found that three strokes reduce the pressure from 0·760 m. to 0·315 m. The experiment is repeated after a body has been introduced into the receiver, and it is found that the same number of strokes reduce the pressure from 0·760 m. to 0·200 m. Deduce the volume of the body.

31. A cylindrical test-tube, 1 decim. in height and 2 centim. in diameter, floats upright in water, its mouth being downwards, and its top being just level with the surface of the water. To what height does the liquid rise in its interior?

32. A test-tube floats upright in water, with its mouth downwards. To make its top come down to the surface of the water, it is found necessary to load it with a weight, which, added to its own weight, gives a total weight P. The experiment is repeated with the open end upwards, and the weight which is then necessary to bring its top down to the level of the water, including the weight of the tube itself, is Q. Deduce the atmospheric pressure.

33. A cylinder of wood, 1 decim. long, and with a sp. gr. 0·96, floats

upright in water. The vessel is placed in a receiver in which the air can be compressed to 40 atmospheres. Find the change produced by this pressure in the position of the cylinder.

II.—HEAT.

34. A truly conical vessel contains a certain quantity of mercury at 0° C. To what temperature must the vessel and its contents be raised that the depth of the liquid may be increased by $\frac{1}{180}$ of itself?

35. What temperature is denoted by the same number in the Centigrade as in the Fahrenheit scale? Is there more than one temperature which fulfils this condition?

36. A Graham's compensating pendulum is formed of an iron rod, whose length at 0° C. is l, carrying a cylindrical vessel of glass, which at the same temperature has an internal radius r, and height h. Find the depth x of mercury at 0° C. which is necessary for compensation, supposing that the compensation consists in keeping the centre of gravity of the mercury at a constant distance from the axis of suspension.

37. A brass tube contains mercury, with a piece of platinum immersed in it; and the level of the liquid is marked by a scratch on the inside of the tube. On applying heat, it is found that the liquid still stands at this mark. Deduce the ratio of the weight of the platinum to that of the mercury.

38. A glass tube, closed at one end and drawn out at the other, is filled with dry air, and raised to a temperature x at atmospheric pressure. It is then hermetically sealed. When it has been cooled to the temperature 100° C, it is inverted over mercury, and its pointed end is broken off beneath the surface of the liquid. The mercury rises to the height of 19 centimetres in the tube, the external pressure remaining at 76 cm. as at the commencement of the experiment. The tube is re-inverted, and weighed with the mercury which it contains. The weight of this mercury is found to be 200 grammes; when completely full it contains 300 grammes of mercury. Deduce the temperature x.

39. A glass tube, whose interior is a right circular cylinder, 2 millimetres in diameter at 0° C., contains a column of mercury, whose length at this temperature is 2 decim. What will be the length of this column of mercury when the temperature is 80° C., the co-efficient of expansion of mercury being $\frac{1}{5550}$, and the co-efficient of cubical expansion of glass $\frac{1}{38700}$?

40. A closed globe, whose external volume at 0° C. is 10 litres, is immersed in air at 15° C. and at a pressure of 0·77 m. Required (1) the

loss of weight which it experiences from the action of the air; (2) the change which this loss would undergo if the pressure became 0·768 m. and the temperature 17° C.

41. Some dry air is inclosed in a horizontal thermometric tube, by means of an index of mercury. At 0° C. and 0·760 m. the air occupies 720 divisions of the tube, the tube being divided into parts of equal capacity. At an unknown temperature and pressure, the same air occupies 960 divisions. The tube being immersed in melting ice, and the latter pressure being still maintained, the air occupies 750 divisions. Required the temperature and pressure.

42. At what temperature will the density of oxygen at the pressure 0·20 m., be the same as that of hydrogen at 0° C. at the pressure 1·60 m.?

43. What is the interior volume at 0° C. of a glass bulb which at 25° C. is exactly filled by 53 grammes of mercury?

44. A barometer at one time reads 770 mm., with a temperature 85° C., and at another time 760 mm., with a temperature 5° C. Find the ratio of the true barometric heights.

45. The specific gravity of copper at 0° C. is 8·8; its co-efficient of linear expansion is $\frac{1}{58100}$. What will be the length at 30° of a roll of copper wire weighing 15 kilogrammes, the section of the wire at 10° C. being 4 square millimetres?

46. The normal density of air being 0·000154 of that of brass, what change is produced in the apparent weight of a kilogramme of brass when the pressure and temperature of the air change from 713 mm. and −19° C. to 781 mm. and +36° C.?

47. A cylindrical tube of glass is divided into 300 equal parts. It is loaded with mercury, and sinks to the 50th division in water at 10° C. To what division will it sink in water at 50° C.? The volumes of a given mass of water at 10° and 50° are as 1·000268 and 1·01205.

48. Find the co-efficient of expansion of air, when zero Fahrenheit is the starting point, and the degree Fahrenheit the unit interval.

49. What must be the pressure of air at 15° C., that its density may be the same as that of hydrogen at 0° C. and 760 mm.? Density of hydrogen 0·0692.

50. At what temperature does a litre of dry air at 760 mm. weigh 1 gramme?

51. In a cubic metre of air at 20° C., 11·56 grammes of vapour are found. What is the relative humidity of this air?

52. Calculate the weight of 15 litres of air saturated with aqueous

vapour, at 20° C. and 750 mm. The maximum tension of vapour at 20° is 17·39 mm.

53. There is a bent tube, terminating at one end in a large bulb, and simply closed at the other. A column of mercury stands at the same height in the two branches, and thus separates two quantities of air at the same pressure. The air in the bulb is saturated with moisture ; that in the opposite branch is perfectly dry. The length of the column of dry air is known, and also its initial pressure, the temperature of the whole being 0° C. Calculate the displacement of the mercurial column when the temperature of the apparatus is raised to 100° C. The bulb is supposed to have enough water in it to keep the air constantly saturated; and is also supposed to be so large that the volume of the moist air is not sensibly affected by the displacement of the mercurial column.

54. A litre of alcohol, measured at 0° C., is contained in a brass vessel weighing 100 grammes, and [after being raised to 58° C.] is immersed in a kilogramme of water at 10° C., contained in a brass vessel weighing 200 grammes. The temperature of the water is thereby raised to 27°. What is the specific heat of alcohol? The specific gravity of alcohol is 0·8 ; the specific heat of brass is 0·1.

55. A copper vessel, weighing 1 kilogramme, contains 2 kilogr. of water. A thermometer, composed of 100 grammes of glass and 200 gr. of mercury, is completely immersed in this water. All these bodies are at the same temperature, 0° C. If 100 grammes of steam at 100° C. are passed into the vessel, and condensed in it, what will be the temperature of the whole apparatus when equilibrium has been attained, supposing that there is no loss of heat externally. The specific heat of mercury is 0·033 ; of copper, 0·095 ; of glass, 0·177.

III.—ACOUSTICS AND OPTICS.

56. The specific gravity of platinum being taken as 22, and that of iron as 7·8, what must be the ratio of the lengths of two wires, one of platinum and the other of iron, both of the same section, that they may vibrate in unison when stretched with equal forces?

57. Two strings of the same length and section are formed of materials whose specific gravities are respectively d and d'. Each of these strings is stretched with a weight equal to [1000 times] its own weight. What is the musical interval between the notes which they will yield?

58. A pipe gives a note of 100 vibrations per second at the temperature 10° C. What must be the temperature of the air that the same pipe may yield a note higher by a major fifth?

59. What is the least height that a plane mirror can have, that the whole of a given vertical object may be seen reflected in it at one view?

60. The flame of a candle is placed on the axis of a concave spherical mirror at the distance of 1·54 m., and its image is formed at the distance of 0·45 m. What is the radius of curvature of the mirror?

61. On the axis of a concave spherical mirror of 1 m. radius, an object 9 cm. high is placed at a distance of 2 m. Find the size and position of the image.

62. What is the size of the circular image of the sun which is formed at the principal focus of a mirror of 20 m. radius? The apparent diameter of the sun is 30′.

63. In front of a concave spherical mirror of 2 metres' radius is placed a concave luminous arrow, 1 decimetre long, perpendicular to the principal axis, and at the distance of 5 metres from the mirror. What are the position and size of the image? A small plane reflector is then placed at the principal focus of the spherical mirror, at an inclination of 45° to the principal axis, its polished side being next the mirror. What will be the new position of the image?

64. A pencil of parallel rays fall upon a sphere of glass of 1 metre radius. Find the principal focus of rays near the axis, the index of refraction of glass being 1·5.

65. What is the focal length of a double-convex lens of diamond, the radius of curvature of each of its faces being 4 millimetres? Index of refraction 2·487.

66. An object 8 centimetres high is placed at 1 metre distance on the axis of an equi-convex lens of ordinary crown-glass, the radius of curvature of its faces being 0·4 m. Find the size and position of the image.

67. What is the ratio of the magnifying power of a diamond lens to that of a glass lens of the same focal length [supposed small]? Index for glass, 1·5; for diamond, 2·481.

68. A Gregorian telescope is constructed in the following manner:—The rays after reflection from the objective speculum form a real image at the principal focus. Continuing their course, they meet a small concave mirror, which reflects them so that they form a second image, inverted with respect to the first, and consequently erect with respect to the object. This second image is viewed through an eye-piece, whose tube passes through a hole in the objective. Investigate a formula for the magnifying power of the telescope.

69. Two converging lenses, with a common focal length of 0·05 m., are at a distance of 0·03 m. apart, and their axes coincide. What image

will this system give of a circle 0·01 m. in diameter, placed successively at different distances on the prolongation of the common axis?

70. A convex lens of vocal length f, is cemented to a concave lens of focal length f'. What is the focal length of the system?

71. The stem of a siren carries a plane mirror, thin, polished on both sides, and parallel to the axis of the stem. The siren gives a note of 345 vibrations per second. The revolving plate has 15 holes. A fixed source of light sends to the mirror a horizontal pencil of parallel rays. What space is traversed in a minute by a point of the reflected pencil at a distance of 4 metres from the axis of the siren? This axis is supposed vertical.

72. A lamp and a taper are at a distance of 4·15 m. from each other; and it is known that their illuminating powers are as 6 to 1. At what distance from the lamp, in the straight line joining the flames, must a screen be placed that it may be equally illuminated by them both?

73. A ray of light falls perpendicularly on the surface of an equilateral prism of glass with a refracting angle of 60°. What will be the deviation produced by the prism? Index of refraction of glass 1·5.

74. What is the length of the cone of the *umbra* thrown by the earth? and what is the diameter of a cross section of it made at a distance equal to that of the moon?

The radius of the sun is 112 radii of the earth; the distance of the moon from the earth is 60 radii of the earth; and the distance of the sun from the earth is 24,000 radii of the earth. Atmospheric refraction is to be neglected.

75. A sphere of glass lying upon a horizontal plane receives the sun's rays. What must be the height of the sun above the horizon that the principal focus of the sphere may be in this horizontal plane?

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